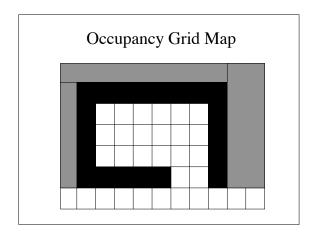
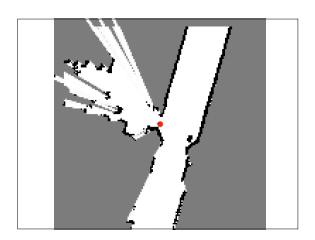
Lecture 13: Occupancy Grids

CS 344R/393R: Robotics Benjamin Kuipers





Occupancy Grid Map

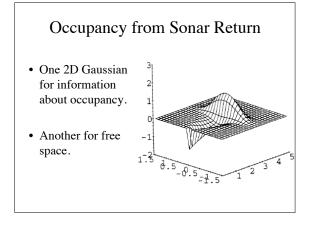
- Maps the environment as an array of cells.
 - Cell sizes range from 5 to 50 cm.
- Each cell holds a probability value
 - that the cell is occupied.
- Useful for combining different sensor scans, and even different sensor modalities.
 - Sonar, laser, IR, bump, etc.
- No assumption about type of features.
 - Static world, but with frequent updates.

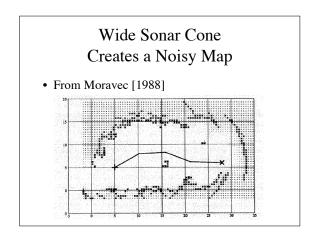
A Bit of History

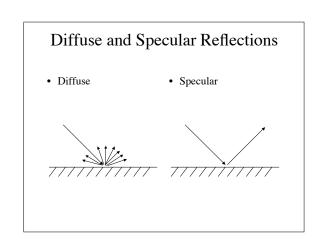
- Occupancy grids were first popularized by *Hans Moravec* and *Alberto Elfes* at CMU.
- *Kurt Konolige* at SRI made a number of valuable contributions.
 - Konolige's Erratic robot is the ancestor to the Amigobot. Konolige developed Saphira, too.
- Hugh Durrant-Whyte and John Leonard (then at Oxford) used landmarks and Kalman filters as an alternative.
- Sebastian Thrun (then CMU, now Stanford) has done very impressive metrical mapping work, which we will study.

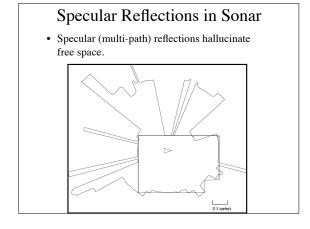
Sonar Sensors Give Evidence of Obstacles P(AIB) Incident Energy Transducer (S)

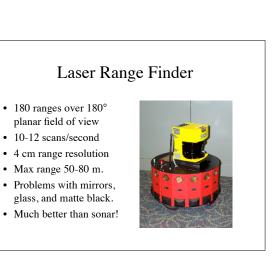
Sonar Sweeps a Wide Cone Obstacle could be anywhere on the arc at distance D. The space closer than D is likely to be free.





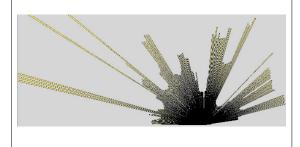


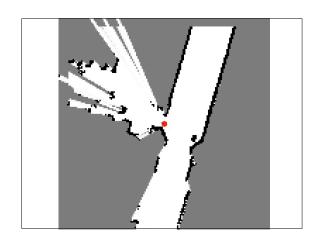




Laser Rangefinder Image

• 180 narrow beams at 1° intervals.





Occupancy Grid Cells C_{ij}

- The proposition occ(i,j) means:
 - The cell C_{ij} is occupied.
- **Probability**: p(occ(i,j)) has range [0,1].
- **Odds**: o(occ(i,j)) has range $[0,+\infty)$.

$$o(A) = \frac{p(A)}{p(\neg A)}$$

- **Log odds**: $\log o(occ(i,j))$ has range $(-\infty,+\infty)$
- Each cell C_{ii} holds the value $\log o(occ(i,j))$
 - $-C_{ij} = 0$ corresponds to p(occ(i,j)) = 0.5

Probabilistic Occupancy Grids

• We will apply Bayes Law

$$p(A \mid B) = \frac{p(B \mid A) * p(A)}{p(B)}$$

- where A is occ(i,j)
- and B is an observation r=D
- We can simplify this by using the log odds representation.

Bayes Law Using Odds

- Bayes Law: $p(A \mid B) = \frac{p(B \mid A) * p(A)}{p(B)}$
- Likewise: $p(\neg A \mid B) = \frac{p(B \mid \neg A) * p(\neg A)}{p(B)}$
- so: $o(A \mid B) = \frac{p(A \mid B)}{p(\neg A \mid B)} = \frac{p(B \mid A) * p(A)}{p(B \mid \neg A) * p(\neg A)}$ = $\lambda(B \mid A) * o(A)$
- where:

$$o(A \mid B) = \frac{p(A \mid B)}{p(\neg A \mid B)} \qquad \lambda(B \mid A) = \frac{p(B \mid A)}{p(B \mid \neg A)}$$

- Easy Update Using Bayes Law
- Bayes' Law can be written:

$$o(A \mid B) = \lambda(B \mid A) * o(A)$$

• Take log odds to make multiplication into addition.

$$\log o(A \mid B) = \log \lambda(B \mid A) + \log o(A)$$

• Easy update for cell contents.

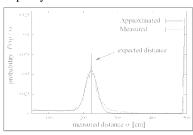
Occupancy Grid Cell Update

- Cell C_{ii} holds $\log o(occ(i,j))$.
- Evidence r=D means sensor r returns D.
- For each cell C_{ij} accumulate evidence from each sensor reading.

$$\log o(A \mid B) = \log \lambda(B \mid A) + \log o(A)$$
$$\log o(occ(i, j))$$
$$+ \log \lambda(r = D \mid occ(i, j))$$
$$= \log o(occ(i, j) \mid r = D)$$

Sensor Model $p(r=D \mid occ(i,j))$

• Probability of range-reading given known occupancy at a known distance.



Update Values for λ

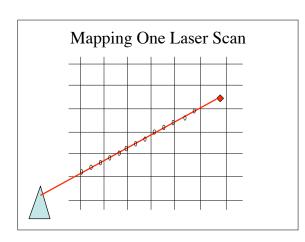
• If the laser terminates at C_{ij} at distance D

$$\lambda(r = D \mid occ(i, j)) = \frac{p(r = D \mid occ(i, j))}{p(r = D \mid \neg occ(i, j))} \approx \frac{.06}{.005} = 12$$

- so $\log_2 \lambda \approx +3.5$

• If the laser passes through C_{ii} .

$$\begin{split} \lambda(r > D \mid occ(i,j)) &= \frac{p(r > D \mid occ(i,j))}{p(r > D \mid \neg occ(i,j))} \approx \frac{.45}{.90} = .5 \\ &- \text{ so } \log_2 \lambda \approx -1.0 \end{split}$$



Future Attraction: SLAM

- To build an accurate map, we assume that robot pose (x,y,θ) is known accurately.
 - This is usually not true.
- *Localization* means using sensor input to estimate the robot pose (x,y,θ) .
- Simultaneous Localization and Mapping (SLAM) uses the existing map and current sensor input for localization.
 - Once localized, use sensors to update the map

Mapping Assignments (4 and 5)

- We will give you laser range-sensor traces.
 - Few specular reflections; no spreading cone.
 - Off-line computation; no physical control.
- For **Assignment 4**, you will have accurate pose information (x,y,θ) .
 - You build an accurate occupancy grid map.
- For **Assignment 5**, you will do simultaneous localization and mapping (SLAM).
 - You are given laser and odometry sensor values.

Implementation Hints

- Use 10×10 cm² grid cells.
 - But make cell size a parameter and try others.
- To display the grid:
 - Black means occupied
 - White means free
 - Grey means unknown
- Experiment with different shade mappings.
 - Make it both useful and attractive.

Implementation Hints

- Robot pose (x,y,θ) and laser endpoints (p,q) are high-resolution values.
 - Grid cells correspond to extended regions.
- Put cell centers at integer coordinates so *rounding* quickly gives cell coordinates.
- Increment C_{ij} for endpoint of laser beam.
- Step regularly along free part of the beam, decrementing C_{ij} for free cells.

