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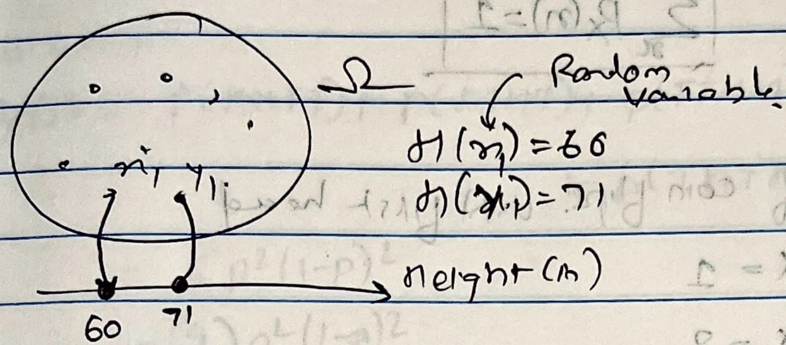
Lecture 5.

Discrete Random Variable

Random Variable



Function to calculate Height takes input a Random Var ω to generate a output.

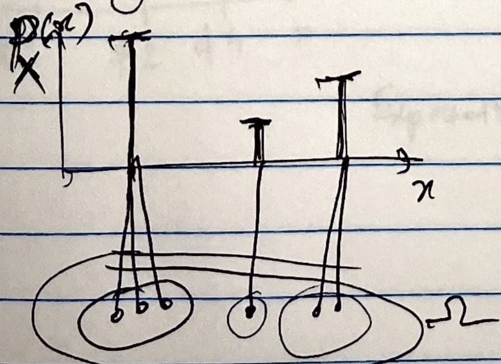
Functions of Random Vars are also Random Variables

Random Variables Can be Discrete or continuous.

Random Variable X

numerical Value x

Probability mass Function



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$$p_X(x) = P(X=x)$$
$$= P(\{\omega \in \Omega \text{ s.t. } X(\omega)=x\})$$

$\boxed{p_X(x) \geq 0}$ $\boxed{\sum_x p_X(x)=1}$

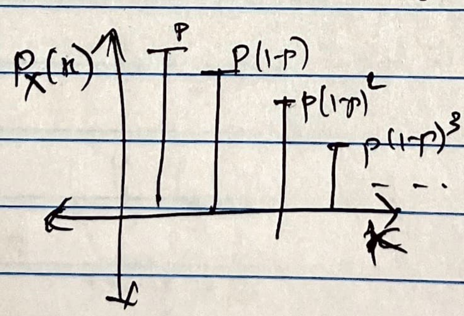
example: X = number of coin flips until first head

H = $X = 1$

TH $X = 2$

$\underbrace{TTT\cdots T}_{k-1}H$ $X = k$

$$p_X(k) = P(X=k)$$
$$= P(\underbrace{TT\cdots T}_{k-1}H)$$
$$= (1-p)^{k-1}\, p \quad \Big\} \text{ Geometric PMF.}$$



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Binomial PMF

$X \to$ number of heads in n independent coin tosses.

$p(H) = p$

Let $n = 4$

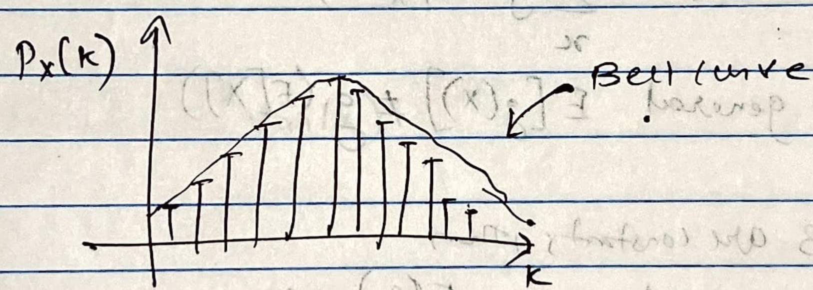
$P_X(2) = p(HHTT) + p(HTHT) + p(TTHH) + p(THHT) + p(THTH) + p(HTTH)$

$= 6p^2(1-p)^2$

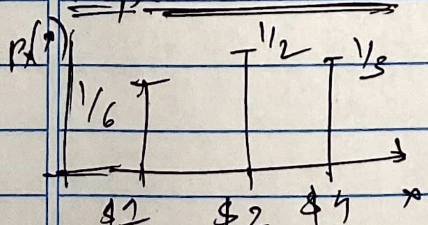
$= \binom{4}{2} p^2 (1-p)^2$

In general

$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$ $\quad k = 0, 1, \ldots, n.$



Expected Value



$\frac{1}{6} \times 1 + \frac{1}{2} \times 2 + \frac{1}{3} \times 4 = 2.5$

Expected Val $= \sum_x P_X(x) \cdot x$

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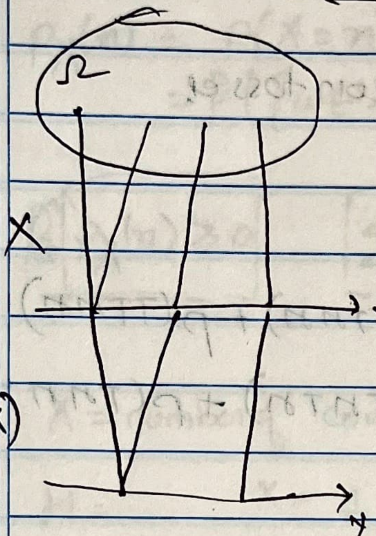
Example

Properties of Expectations



$Y = g(X)$

$E[X]$ let X be a r.v. and let $Y = g(X)$

$$E[Y] = \sum_y y\, p_Y(y)$$

$$E[Y] = \sum_x g(x)\, p_X(x)$$

In general! $E[g(X)] \neq g(E[X])$

If α, β are constants, then

$E(\alpha) = \alpha$, $E(2) = 2$

$E[\alpha X] = \alpha E[X]$

$$E[\alpha X] = \sum_x g(x)\, p_X(x)$$

$$= \sum_x \alpha x\, p_X(x)$$

$$= \alpha \sum_x x\, p_X(x)$$

$$= \alpha E[x]$$

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$$E[\alpha X+\beta] = E[\alpha X]+\beta$$
$$= \alpha E[X]+\beta$$

Variance

$$E[g(x)] = \sum_{x} g(x)\, p_x(x)$$

square $= E[X^2] = \sum_{x} x^2 p_x(x)$

variance

$$\mathrm{Var}(X) = E[(X-E[X])^2]$$
$$= \sum_{x} (x-E[X])^2 p_x(x)$$
$$= E[X^2] - (E[X])^2$$

$\mathrm{Var}(X) \geq 0$

$\mathrm{Var}(\alpha X+\beta) = \alpha^2 \mathrm{Var}(X)$