Conditioning and Bayes' Rule

(Always Expect the unexpected)

0 prob → not impossible
↳ unlikely to happen.

$p(A|B)$ = probability of A, given that B occurred.

— B is new universe.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ Assuming $P(B) \neq 0$

$$P(B|B) = \frac{P(B \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$P(A \cap B) = P(B) \times P(A|B)$$
$$= P(A) \times P(B|A)$$

If $A \cap B = \phi$

$P(A \cup B) = P(A) + P(B)$

C occurred

$p(A \cup B|C) = p(A|C) + p(B|C)$

example

Event A → Airplane is flying above

Event B → Something registers on Radar screen

Multiplication

$p(A \cap B \cap C) = p(A) \times p(B|A) \times p(C|A \cap B)$

Teacher's Signature:......

proof

$$p(A \cap B \cap C) = p((A \cap B) \cap C)$$

$$p(A \cap B \cap C) = p(A \cap B) \times p(C \mid A \cap B)$$

$$= p(A) \times p(B \mid A) \times p(C \mid A \cap B)$$

Total prob theorem

→ Divide and conquer

→ Partition of sample space into A_1, A_2, A_3

→ Have $P(B \mid A_i)$ for every i

$$\left.\begin{aligned} p(B) = {} & p(A_1) \times p(B \mid A_1) \\ & + p(A_2) \times p(B \mid A_2) \\ & + p(A_3) \times p(B \mid A_3) \end{aligned}\right\}$$

Given $p(A_1) + p(A_2) + p(A_3) = 1$

→ basically weighted averages

<u>Bayes's Rule</u>

We know $P(B \mid A_i)$ for each i.

wish to Revise $P(A_i \mid B)$

$$p(A_i \mid B) = \frac{p(A_i \cap B)}{p(B)} = \frac{p(A_i) \times p(B \mid A_i)}{p(B)}$$

$$= \frac{p(A_i) \times p(B \mid A_i)}{\sum_j P(A_j) \times P(B \mid A_j)}$$

(Cause Effect Model)

$$A \xrightarrow[P(B \mid A)]{} B$$

$$A \xleftarrow[p(A_i \mid B)]{} B \quad \text{Inference}$$