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Recitation 2

1) A coin is tossed twice, Alice claims that event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses in head. Is she Right? Does it make a Difference if the coin is fair or unfair? How can we generalize Alice's Reasoning?

→ We have two events
A → first toss is head
B → Second toss is head

~~we have to compare~~

→ $p(A\cap B \mid A)$ and $p(A\cap B \mid A\cup B)$

↑ first toss is head — First toss is head

(i) $p(A\cap B\mid A) = \dfrac{p((A\cap B)\cap A)}{p(A)} = \dfrac{p(A\cap B)}{p(A)}$

(ii) $p(A\cap B\mid A\cup B) = \dfrac{p((A\cap B)\cap(A\cup B))}{p(A\cup B)} = \dfrac{P(A\cap B)}{P(A\cup B)}$

Also from Question we know $p(A\cup B) \geq p(A)$

$\therefore$ (i) $\geq$ (ii)

$\therefore$ Alice is Right Regardless of fair/unfair coin.

in case of fair coin
$\Omega = \{HH, HT, TH, TT\}$

$p(A\cap B\mid A) = \dfrac{1/4}{2/4} = \dfrac{1}{2}$ | $p(A\cap B\mid A\cup B) = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

Generalization, A, B and C be the event, $B\subset C$
$A\cap B = A\cap C$ $(A\subset B\subset C)$
then event A is at least as likely if we know that B has occurred than if we know that C has occurred.
Alice explains this $C = A\cup B$.

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2) We roll two fair 6-sided Dice. Each one of the 36 possible outcomes is assumed to be equally likely.

a) Find the prob Doubles are Rolled.

b) Given that the Roll Results in a sum of 4 or less, find the conditional prob that doubles are Rolled.

c) Find the probability that atleast one die Roll is a 6

d) Given that the two dice land on Different numbers, find the conditional prob that atleast one die Roll is a 6.

→ a) Each Possible outcome $= 1/36$

Doubles possibility $= 6 \Rightarrow$ prob $= \frac{1}{36} \times 6 = \frac{1}{6}$

b) $\Omega(\text{sum} \leq 4) = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$

$\therefore \text{Prob}(\text{Doubles} \mid \text{sum} \leq 4) = \frac{2}{6} = \frac{1}{3}$

c) There are 11 possible outcomes

outcome $= \{(6,6), (6,5), (6,4), (6,3), (6,2), (6,1), (1,6), (2,6), (3,6), (4,6), (5,6)\}$

$\therefore$ prob $= 11/36$

d) Outcomes of two same no $= \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$\therefore$ outcomes of two not same $= 36 - 6 = 30$

$p(\text{outcome not same}) = \frac{30}{36}$.

atleast one die Roll is 6 $= \{(6,1), (6,2), (6,3), (6,4), (6,5), (1,6), (2,6), (3,6), (4,6), (5,6)\}$

$= 10$

$p(\text{one die Roll is } 6 \mid \text{outcome of two not same}) = \frac{10}{30} = \frac{1}{3}$

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3) You enter a chess tournament where your probability of winning a game is 0.3 against half of the players (call them type 1), 0.4 against a quarter of the player (call them type 2) and 0.5 against the remaining quarter of players (call them type 3). You play against a randomly chosen opponent.

a) What is the prob of winning?

b) Suppose you win, what is the prob that you had an opponent of type 1?

→ a) $P(A_1) = 0.5$
$P(A_2) = 0.25$
$P(A_3) = 0.25$

Let W be winning event

$\cancel{P(W|A_1) = 0.3}$ $\quad P(W|A_1) = 0.3$
$\cancel{P(W|A_2) = 0.4}$ $\quad P(W|A_2) = 0.4$
$\cancel{P(W|A_3) = 0.5}$ $\quad P(W|A_3) = 0.5$

Total prob theorem says

$$P(W) = P(A_1) \times P(W|A_1) + P(A_2) \times P(W|A_1) + P(A_3) \times P(W|A_3)$$
$$= 0.5 \times 0.3 + 0.25 \times 0.4 + 0.25 \times 0.5$$
$$= 0.15 + 0.1 + 0.125$$
$$P(W) = 0.375 \quad -(i)$$

0.125
0.100
0.150
0.375

b) Cause-Effect Question

I know $P(W|A_1) = 0.3$

$\cancel{P(W|A_1) = \frac{P(W \cap A_1)}{P(A_1)} = P(A_1) \times P(}$

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$$p(A_1|W) = \frac{p(A_1 \cap W)}{p(W)} = \frac{p(A_1) \times p(W|A_1)}{p(W)}$$

$$= \frac{p(A_1) \times p(W|A_1)}{0.375} \quad \text{(From i)}$$

$$= \frac{0.5 \times 0.3}{0.375}$$

$$p(A_1|W) = 0.4 \rightarrow \text{Inference.}$$

4) The Monty Hall problem. This is a much Discussed puzzle, based on old American game show, You are told that a prize is equally likely to be found behind any of three closed doors in front of you. You point to one of the doors. A friend opens for you one of the Remaining two doors, After making sure that the prize is not behind it. At this point, You can stick to your initial choice, or switch to the other unopened door. You win the prize if it lies behind your final choice of door. Consider the following strategies.

→ ~~a) Stick to your initial door~~

prob of winning = 1/3

prob of loosing = 2/3.

a) You pick a door and stay with it — prob of winning = 1/3

b) Switch to other unopened door

If we picked wrong initially (2/3) then the other unopened door must have car. (switching = 2/3)

while if I stay (1/3)

∴ Switching to unopened door ~~increases the chances of~~ winning.

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