Probilistic Systems Analysis And Applied Probability

6.041

framework about uncertainity

↳ models
↳ Sample Space
↳ probability law → mainly to occur.

Axioms of probability
several exams

Sample space

exp done → something happen
↳ Coin, Dice, Card game → (Sample space)

[list of all possibility]
↳ [set]

(Coin → H, T)

→ list must be:
1) mutually exclusive. (This happens that does not happen)
2) collectively exhaustive.

(Belief)

(H
T + Raining
T + not Raining)

relations of Details (some we keep, some rejects neglects)
(Art, judgement)

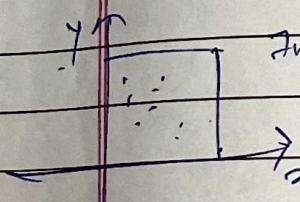
→ When Rolling Dice twice, I think as one experiment not 2 Separate experiment

2,3 is Different than 3,2

outcome → overall outcome at the end of exp. (2,3) is outcome Not 2.

Sample space → Continuous enemy

$\Omega = \{(x,y) \mid 0 \le x, y \le 1\}$



Infinite set.

Assigning Probabilities (Relative likelyhoods)

Event → Assign prob to subset instead of Entire event

$0 < prob < 1$

↑ ↑

Not going to happen

Axioms → Ground Rules for models to obey:

1. Non-negativity $P(A) \ge 0$
2. Normalization $P(\Omega) = 1$ Ω = Sample space
3. Additive If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

∩ → And. ∪ → OR

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$P(\{s_1, s_2 \dots s_n\}) = P(s_1) + P(s_2) + P(s_3) + P(s_4) \dots P(s_n)$

$1 = P(\Omega) = P(A \cup A^c)$

$= P(A) + P(A^c)$

$P(A) = 1 - P(A^c) \overset{(n)}{\leq} 1$

Union of 3 sets

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$ — Disjoint & exhaustive

$= P((A \cup B) \cup C)$

$= P(A \cup B)$

$= P(A \cup B) + P(C)$ (Given they are Disjoints)

$= P(A) + P(B) + P(C)$

$A_1 \dots A_m$ (Disjoint)

$P(A_1 \dots A_m) = P(A_1) + \dots P(A_m)$

$= P(\{s_1\}) + \dots + P(\{s_k\})$

$= P(s_1) + \dots P(s_k)$

Discrete Uniform Law (Reduced to counting)

~~If all outcomes be equally likely.~~

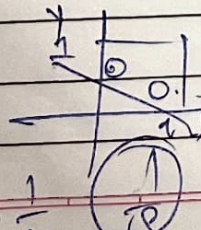
Uniform.

Then,

$P(A) = \frac{\text{no. of elements of } A}{\text{total number of sample points}}$

Continuous Uniform Law

Two "Random" numbers in $[0,1]$



Prob = Area.

Single point = 0

$P(x + y \leq 1/2)$

$P(x, y \in \{0 \leq 1, 0 \leq 1\})$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

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Example

Probability law; w/ countably

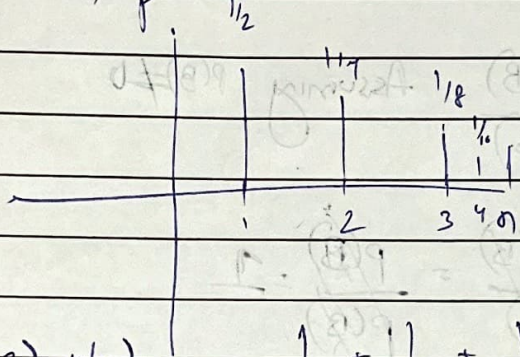
rep flipping a coin till heads

outcome has no bounds

Sample space = {1, 2 . . }

$p(n) = 2^{-n}$, $n = 1, 2$ (Prob law)

p(outcome is even)



$p(2, 4, 6 \ldots) = p(2) + p(4) \ldots = \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} \ldots = \frac{1}{3}$

Countable additivity axiom ~~(Must be Disjoint)~~

$P(A_1 \cup A_2 \ldots A_n) = P(A_1) + P(A_2) \ldots P(A_n)$

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