Lecture 3: Independence

Review

Conditional prob

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule

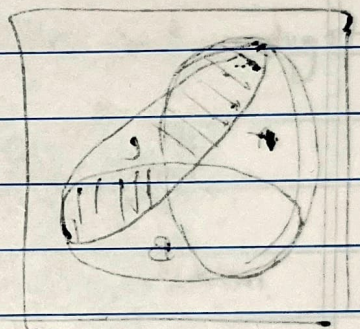
$$P(A \cap B) = P(B) \times P(A|B)$$
$$= P(A) \times P(B|A)$$

Total prob theorem

$$P(B) = P(A) \times P(B|A) + P(A^c) \times P(B|A^c)$$

Bayes Rule

$$P(A_i|B) = \frac{P(A_i) \times P(B|A_i)}{P(B)}$$



What is Independent events?

→ Think it as this way assume the prob that First toss is Heads and Then the second toss is also Heads is P, likewise if the first toss is tails, and second toss is heads also has prob P, it Really doesn't matters what my First toss was as the prob to get H on second toss is always P, thus they all called Independent.

∴ Independence of two events

$P(B|A) = P(B)$, occurrence of A provides no info about B occurrence.

Also we know $P(A \cap B) = P(A) \times P(B|A)$

$= P(A) \times P(B)$ (If Independent)

the prop is also Symmetric

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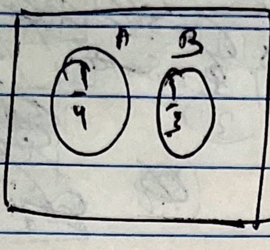
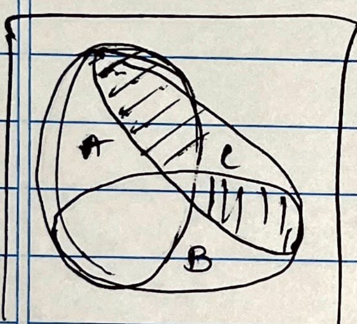


→ This is exclusive not Independent as it tells me if A occured, certainly B did not occured.

$p(A|B) = 0$ ≠ $p(A) = \frac{1}{4}$ ∴ not independent.

Conditional Independence

$p(A \cap B | C) = p(A|C) \times p(B|C)$



A and B are Independent If C occured, The intersection of C with A and C Are Disjoint ∴ They are exclusive not Independent.

The "King's" Siblings

→ The King comes from a family of two children, what's the probability that his sibling is female?

Ans)

BB 1/4, 1/3	BG 1/4, 1/3
GB 1/4, 1/3	GG 1/4

Or did not happen

Sibling is a girl $= \frac{2}{3}$