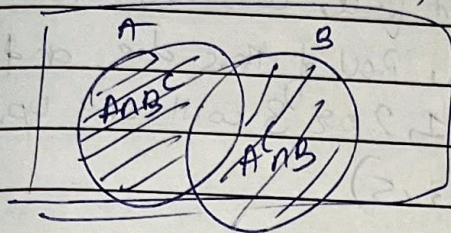


Recitation 1

Page No.:

Date: / /

1) Prove $P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$



→ prove

$$(A \cap B^c) \cup (A^c \cap B)$$

are Disjoint, so

we can Right them

$$P((A \cap B^c) \cup (A^c \cap B)) = P(A \cap B^c) + P(A^c \cap B)$$

Also from the Drawing $P(A) = P(A \cap B^c) + P(A \cap B)$ — (i)

Similar $P(B) = P(A^c \cap B) + P(A \cap B)$ — (ii)

Add i and ii

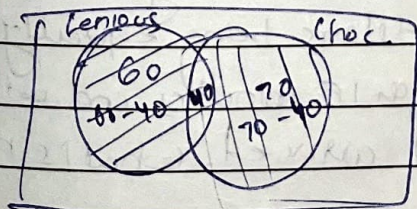
$$P(A) + P(B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B) + P(A \cap B)$$

$$P(A) + P(B) - 2P(A \cap B) = P(A \cap B^c) + P(A^c \cap B)$$

$$\therefore P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$$

2) Out of the students in the class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a random selected student is neither a genius nor a chocolate lover.

→



Assume 100 Students

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 60 + 70 - 40$$

$$= 130 - 40 = 90$$

$$\text{Universe} = 100$$

$$n(A^c \cap B^c) = 100 - 90$$

$$= 10$$

$$P(A^c \cap B^c) = \frac{10}{100}$$

$$= \frac{1}{10}$$

Teacher's Signature:.....

- 3.) A 6-sided die is loaded in a way that each even face is twice as likely as each odd face. Construct a probabilistic model for a single Roll of this die, and find the probability that a 1, 2 or 3 will come up.

$$\rightarrow P(A = 2, 4, 6) = 2P(A = 1, 3, 5)$$

~~$$\text{Let } P(A = 2, 4, 6) = x$$~~

~~$$\text{So } P(A = 1, 3, 5) = x$$~~

$$\text{Let } P(A = 1, 3, 5) = x \quad \text{total} = 3x$$

$$\therefore P(A = 2, 4, 6) = 2x \quad \text{total} = 6x$$

$$\therefore P(\{1, 2, 3\}) = P(1) + P(2) + P(3) \\ = x + 2x + x = 4x$$

But we also know

$$P(A = 1, 3, 5) + P(A = 2, 4, 6) = 1$$

$$3x + 6x = 1$$

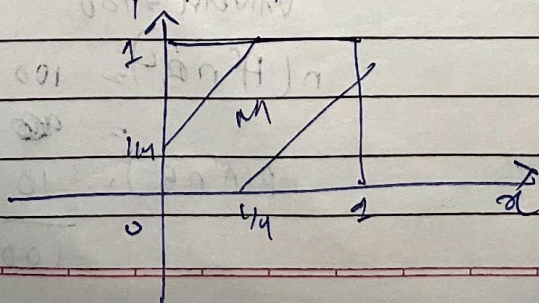
$$9x = 1$$

$$x = \frac{1}{9}$$

$$\therefore P(\{1, 2, 3\}) = 4x = 4 \times \frac{1}{9} = \frac{4}{9}$$

- 4.) Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the prob that they will meet?

\rightarrow Assume a Sample Space $\Omega = [0, 1] \times [0, 1]$ Square



Now, we also know to calculate the meeting each other.

$$|x - y| \leq 15 \text{ minutes}$$

$$|x - y| \leq \frac{1}{4} \text{ (hr)}$$

∴ we map this on a figure.

$$\therefore M = \{(x, y) \mid |x - y| \leq \frac{1}{4}, 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

Area = prob (From continuous models)

$$\therefore \text{Area} = \text{Area of square} - 2(\text{Area of triangle})$$

$$= 1 - 2 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$$

Area of $M = 1$ minus area of two triangles

$$1 - 2 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{7}{16}$$

$$P(\text{They meet}) = \frac{7}{16}$$