# REVIEW REPORT

## **Keywords**

Coriolis force, Strength of gravity, Rotating disk reactor, Asymptotic solution & expansion, Lubrication approximation, Creeping flow, Surface tension, Viscous Newtonian fluids, Fingering. Non-dimensional groups used: Strength of gravity  $\gamma = \frac{\rho g h o^2}{\mu \omega R}$  effect of surface tension  $\alpha = \frac{\sigma h o^3}{\mu \omega R^4}$ 

### Introduction

We discussed in the paper a Newtonian viscous liquid coating a vertical rotating disk in the creeping flow regime. Experiments were performed at different rotation speeds and constant volume for measuring the thickness profile at a steady state. While the maximum liquid load of the rotating disk varied with rotation rate and liquid viscosity, the value of signifying the ratio of gravity to viscous forces was the same in all the cases,  $\gamma = 0.30$ . A lubrication analysis for the time evolution of the film thickness that accounted for gravity, surface tension, and viscous forces was solved numerically to steady-state to obtain profiles of experimental and quantitative value. The lubrication equation at steady state was solved analytically in the absence of surface tension to get constant height contours circular and symmetric about the horizontal axis(y=0). However, to obtain an accurate solution, knowledge of the height variation across the contours is required, obtained by considering the surface tension. On including this effect, we derived an asymptotic solution to predict thickness.

#### Results and Discussion

The given research paper is of high interest. It deals with one of the specific cases of fluid motion, rotation under gravity which is not studied and experimented on to a more significant extent. The Paper involved a good amount of discussion on the fluid properties (including the effect of gravity, surface tension, and viscosity of liquid layer) in the illustration of the rotating disk.

The advancements that can be withdrawn from the research paper are debated/discussed briefly below:

- It highlights the difference between the instances when the disk is rotating horizontally and rotating vertically.
- Paper's steady-state assumption is correctly taken as measured when the heights at many points become constant. Hence uniformity is achieved.
- The researchers have not mistreated the physical errors, such as errors due to considering the disk to be plain. Instead, they have taken an average value at different points for high precision.
- Mentioned the reason for the displacement of the centre of contours from the axis of rotation to its left. The Paper shows the reason for the same, both numerically and analytically.
- The Paper mentions the effect of considering surface tension term in the experiment. It showed that it is necessary to capture the thickness variation across the contours.
- Graphs plotted in the Paper completely match experimental results and correctly show the increasing asymmetry with the decrease in  $\omega$ .
- The Paper's analytical derivation is correct and provides a good match with experimental data.

The major topics that were covered from Transport Phenomena in the Research paper are as follows:

- Dimensionless groups
- Rotation of fluids
- Viscosity v/s Surface tension v/s gravitational forces, a comparison
- Shear forces

## **Drawbacks and Improvements**

- While calculating the thickness, Coriolis force is ignored without mentioning the reason. By inspection of other such related papers, we found that Coriolis can be neglected if  $\eta >> \rho \omega h^2$ , which is indeed the case here (5 orders of magnitude greater).
- At any given x, it was observed that the fluid profile is symmetric about y = 0. However, the explanation for this observation is missing from the paper.
- Upon comparison with the case of a horizontal rotating disc, some differences were observed. The formation of fingers is a characteristic in the horizontal disc case, which occurs due to atmospheric forces' intervention when the profile expands. Why are fingers not forming in the case of a vertical disc could have been explained.
- They have assumed that the contact line is pinned to the circumference of the disc. In other words, the fluid is considered to wet the entire disc. There are a few problems with this:
  - a]  $(\gamma)max$  is 0.3, and the experiment is done for gamma at least an order of magnitude lower than gamma max. This means we are working with 31.6 % of the maximum volume that the disc can bear. With such a low volume of fluid in control, assuming that the fluid wets the entire surface may not be correct.
  - b] Since we are assuming at the beginning of the experiment itself that the fluid wets the entire surface, we have no scope left for the profile to expand. Thus, we cannot observe the Fingering effect that happens in the horizontal case.
  - c] To solve the non dimensionalized governing equation, the boundary conditions assumed the radial component of the flux to be zero, which means fluid can neither overflow the surface nor leave any part of the surface dry. This is a serious issue since we are putting too many constraints on the liquid.
- The reason for the increasing left-right asymmetry of the fluid profile about y=0, with decreasing angular velocity, is not mentioned. The reason for the asymmetry in the height of the fluid profile is not explained in the paper. The asymmetry is observed due to the conservation of mass (Area x velocity = constant).
- As gamma increases, the profile maxima moves away from the disc centre. The reason for the same is missing.
- The non-dimensional term  $(\gamma)max$  is termed "strength of gravity". In our opinion, the term is very misleading. Although it is the ratio of g and  $(mu\omega)$ , what varies in this experiment is essentially omega, and in rare cases, mu; g is just a constant. Thus, the gamma represents the effect of the change of omega and the fluid viscosity instead of the gravity. Hence the name should reflect that.
- The same is the case with gamma max, which translates to the maximum strength of gravity which is not the intended meaning. It just says that the gamma reaches a const value at max possible volume load for all cases.
- By  $\gamma = 0.28$ , the centre of the innermost circle reaches the periphery of the disk, and the circular characteristics touch the boundary (-1,0). Hence the ad hoc model fails here.

#### References

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