

ASSIGNMENT 6

QUESTION NUMBER 1

To Find: $\frac{\partial f}{\partial x}$ for a forward difference of second order

SOLUTION

$$f(x + \Delta x) = f(x) + \frac{\partial f}{\partial x} * (\Delta x) + \frac{\partial^2 f}{\partial x^2} * (\Delta x)^2 + \frac{\partial^3 f}{\partial x^3} * (\Delta x)^3 + \dots \text{ (Equation I)}$$

$$f(x + 2\Delta x) = f(x) + \frac{\partial f}{\partial x} * (2\Delta x) + \frac{\partial^2 f}{\partial x^2} * (2\Delta x)^2 + \frac{\partial^3 f}{\partial x^3} * (2\Delta x)^3 + \dots \text{ (Equation II)}$$

we need a second order formula

Hence we need to eliminate the $\frac{\partial^2 f}{\partial x^2}$ term

*4*Equation I – Equation II gives*

$$4*(f(x + \Delta x)) - f(x + 2\Delta x) = 3*f(x) + 2*\frac{\partial f}{\partial x}*(\Delta x) - 2*\frac{\partial^3 f}{\partial x^3}*(\Delta x)^3 + \text{other terms}$$

$$\Rightarrow \frac{4*(f(x + \Delta x)) - f(x + 2\Delta x) - 3*f(x)}{2\Delta x} = \frac{\partial f}{\partial x} - \left(\frac{\partial^3 f}{\partial x^3} * (\Delta x)^2 + \text{other terms} \right)$$

The bracketed part is a second order error

Hence, a second order formula is:

$$\frac{\partial f}{\partial x} = \frac{4*(f(x + \Delta x)) - f(x + 2\Delta x) - 3*f(x)}{2\Delta x}$$