PRACTICE PROBLEMS ON FIRST ORDER FIRST DEGREE DIFFERENTIAL EQUATIONS

TYPE- I

Solve the following equations.

1. $(2x^2 + 3y^2 - 7) x\, dx + (3x^2 + 2y^2 - 8) y\, dy = 0$

2. $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0$

3. $\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + (x + \log x - x \sin y)dy = 0$

4. $\frac{y\,dx - x\,dy}{(x+y)^2} = \frac{dx}{2\sqrt{1-x^2}}$

5. $(x^2 - x\tan^2 y + \sec^2 y)\, dy = (\tan y - 2xy - y)\, dx$

6. $[y \sin(xy) + xy^2 \cos(xy)]dx + [x \sin(xy) + x^2 y \cos(xy)]\, dy = 0$

7. $x\, dx + y\, dy = \frac{a(x\, dy - y\, dx)}{x^2 + y^2}$

8. $(2xy \cos x^2 - 2xy + 1)\, dx + (\sin x^2 - x^2)\, dy = 0$

9. $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$

10. $2\left(1 + x^2\sqrt{y}\right)y\,dx + \left(x^2\sqrt{y} + 2\right)x\, dy = 0$

11. $\left(x\sqrt{1 - x^2 y^2} - y\right)dy + \left(x + y\sqrt{1 - x^2 y^2}\right)dx = 0$

12. $\frac{y}{x^2}\cos\left(\frac{y}{x}\right)dx - \frac{1}{x}\cos\left(\frac{y}{x}\right)dy + 2x\,dx = 0$

13. $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0,\, given\ y(0) = 4$

14. $\frac{dy}{dx} = -\frac{4x^3 y^2 + y \cos(xy)}{2x^4 y + x \cos(xy)}$

15. $(2x^2 + 3y^2 - 7) x\, dx + (3x^2 + 2y^2 - 8) y\, dy = 0$

16. $(\tan y + x)dx + (x \sec^2 y - 3y)dy = 0$

17. $[1 + \log(xy)]dx + \left(1 + \frac{x}{y}\right)dy = 0$

ANSWERS

1. $x^4 + y^4 + 3x^2 y^2 - (7x^2 + 8y^2) = c$

2. $e^{xy^2} + x^4 - y^3 = c$

3. $xy + y \log x + x \cos y = c$

4. $\frac{y}{x+y} + \frac{1}{2}\sin^{-1}x = c$

5. $x \tan y - x^2 y - xy - \tan y = c$

6. $xy \sin(xy) = c$

7. $\frac{x^2}{2} + \frac{y^2}{2} + a \tan^{-1}\left(\frac{x}{y}\right) = c$

8. $y \sin x^2 - x^2 y + x = c$

9. $y \sin x + x \sin y + xy = c$

10. $2xy + \frac{2}{3}x^3 y^{3/2} = c$

11. $x^2 - y^2 + xy\sqrt{1 - x^2 y^2} + \sin^{-1}xy = c$

12. $x^2 - \sin\left(\frac{y}{x}\right) = c$

13. $x + ye^{x/y} = 4$

14. $x^4 y^2 + \sin xy = c$

15. $x^4 + 3x^2 y^2 - 7x^2 + y^4 - 8y^2 = 0$

16. $2x \tan y + x^2 - 3y^2 = c$

17. $y + x \log(xy) = c$

TYPE – II

Solve the following equations.

1. $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right) dx + \frac{1}{4}(x + xy^2)dy = 0$

2. $(x^4 e^x - 2m\, xy^2)\, dx + 2mx^2\, y\, dy = 0$

3. $(x^2 + y^2 + 1)dx - 2xy\, dy = 0$ **4.** $(y - 2x^3)dx - x\,(1 - xy)\, dy = 0$

5. $(2x \log x - xy)dy + 2y dx = 0$

6. $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$

7. $\left(xy^2 - e^{1/x^3}\right)dx - x^2 y\, dy = 0$ **8.** $(x^4 + y^4)\, dx - xy^3\, dy = 0$

9. $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$

10. $y(xy + e^x)dx - e^x dy = 0$

11. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

12. $(xy^3 + y)dx + 2(x^2 y^2 + x + y^4)dy = 0$ **13.** $(2xy^2 - y)dx + xdy = 0$

14. $y(1 + xy)\, dx + x\,(1 - xy)\, dy = 0$ **15.** $(y - xy^2)dx - (x + x^2 y)dy = 0$

16. $y(1 + xy)\, dx + x(1 + xy + x^2 y^2)\, dy = 0$ **17.** $\frac{dy}{dx} = -\frac{x^2 y^3 + 2y}{2x - 2x^3 y^2}$

18. $(xy \sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)\, x\, dy = 0$

19. $(x^4 + y^4)dx - xy^3\, dy = 0$

20. $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$

21. $y(x + y)dx - x(y - x)dy = 0$ **22.** $(x^2 + y^2)dx - (x^2 + xy)dy = 0$

23. $(x^2 - xy + y^2)dx - xydy = 0$

24. $\left[2x \sin h\left(\frac{y}{x}\right) + 3y\cos h\left(\frac{y}{x}\right)\right] dx - 3x.\cos h\left(\frac{y}{x}\right).dy = 0$

25. $\left(\frac{y}{x}secy - \tan y\right) dx - (x - secylog\, x)dy = 0$

26. If y^n is an integrating factor of $y(2x^2 y + e^x)dx - (e^x + y^2)dy = 0$ find n and solve the equation.

27. If y^n is an integrating factor of $(2xy^2 + e^x y)dx - e^x dy = 0$ then find n and solve the equation.

28. If y^n is an integrating factor of $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$. Find n and solve the equation.

29. If x^n is an integrating factor of $y\, dx - x\, dy + (1 + x^2)dx + x^2 \sin y\, dy = 0$ Find n and solve the equation.

30. If x^n is an integrating factor of $(y - 2x^3)dx - x(1 - xy)dy = 0$ then find n and solve the equation.

31. If x^n is an integrating factor of $(x^4 e^x - 2mxy^2)dx + 2mx^2 y\, dy = 0$ find n and solve the equation.

32. If $f(x)$ a function of x only is an integrating factor of $(x^4 e^x - 2mxy^2)dx + 2mx^2 ydy = 0$. Find $f(x)$ and then solve the equation.

33. If $(x + y)^k$ is an integrating factor of $(4x^2 + 2xy + 6y)\,dx + (2x^2 + 9y + 3x)\,dy = 0$. find k and solve the equation.

34. If $(x + y + 1)^n$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ find n and solve the equation.

ANSWERS

1. $x^6 + 3x^4y + x^4y^3 = c$

2. $x^2 e^x + my^2 = cx^2$

3. $x^2 - y^2 - 1 = cx$

4. $-\frac{y}{x} - x^2 + \frac{y^2}{2} = c$

5. $2y\log x - \frac{y^2}{2} = c$

6. $4x^4y + 4x^3y^2 - x^4 = c$

7. $\frac{1}{3}e^{1/x^3} - \frac{y^2}{2x^2} = c$

8. $4x^4 \log x - y^4 = cx^4$

9. $x^2 e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$

10. $\frac{x^2}{2} + \frac{e^x}{y} = c$

11. $x\left(y + \frac{2}{y^2}\right) + y^2 = c$

12. $3x^2y^4 + 6xy^2 + 2y^6 = c$

13. $x^2y - x = cy$

14. $\frac{1}{xy} - \log\left(\frac{x}{y}\right) = c$

15. $\log\frac{x}{y} = c + xy$

16. $\frac{1}{2x^2y^2} + \frac{1}{xy} - \log y = c$

17. $\frac{1}{3}\log\frac{x}{y^2} - \frac{1}{3x^2y^2} = c$

18. $x \sec xy = cy$

19. $4x^4 \log x - y^4 = cx^4$

20. $\frac{cy^3}{x^2} = e^{-x/y}$

21. $\log\sqrt{xy} - \frac{y}{2x} = c$

22. $\frac{y}{x} = \log\frac{x}{(x-y)^2} + c$

23. $\log(x - y) + \frac{y}{x} = c$

24. $x^{-2/3}.\sinh\left(\frac{y}{x}\right) = c$

25. $y\log x - x\sin y = c$

26. $n = -2, \frac{2x^3}{3} + \frac{e^x}{y} - y = c$

27. $n = -2, x^2y + e^x = cy$

28. $n = -4, x^2y^3e^y + x^2y^2 + x = cy^3$

29. $n = -2, y + 1 - x^2 + x\cos y = cx$

30. $n = -2, \frac{y}{x} + x^2 - \frac{y^2}{2} = c$

31. $n = -4, e^x + my^2x^{-2} = c$

32. $f(x) = x^{-4}, e^x + my^2x^{-2} = c$

33. $k = 1, x^4 + 2x^3y + 3x^2y + x^2y^2 + 6y^2x + 3y^3 = c$

34. $n = -4, \frac{xy}{(x+y+1)^3} = c$

TYPE – III

Solve the following differential equations.

1. $\frac{dy}{dx} + \frac{4x}{x^2+1} \cdot y = \frac{1}{(x^2+1)^3}$

2. $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right)y = 1$

3. $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$

4. $x(x - 1)\frac{dy}{dx} - (x - 2)y = x^3(2x - 1)$

5. $\frac{dy}{dx}\cosh x = 2\cosh^2 x.\sinh x - y\sin hx$

6. $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

7. $(1 + x + xy^2)dy + (y + y^3)dx = 0$

8. $(1 + y^2)dx = \left(e^{\tan^{-1}y} - x\right)dy$

9.　$(y + 1)\,dx + [x - (y + 2)e^y]\,dy = 0$

10.　$(1 + \sin y)\dfrac{dx}{dy} = [2y\cos y - x(\sec y + \tan y)]$

11.　$(1 + y^2)dx = (\tan^{-1} y - x)dy$

12.　$\dfrac{dy}{dx} + 2y\tan x = \sin x$ at $y = 0, x = \dfrac{\pi}{3}$

13.　$\dfrac{dy}{dx} + \dfrac{2x}{x^2+1}\cdot y = \dfrac{1}{(x^2+1)^2}$ given that $y = 0$ when $x = 1$

14.　$x\log x.\dfrac{dy}{dx} + y = 2\log x$

15.　$dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$

16.　$\dfrac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

17.　$\dfrac{dy}{dx} = e^{x-y}(e^x - e^y)$

18.　$\dfrac{dy}{dx} + x^3\sin^2 y + x\sin 2y = x^3$

19.　$\dfrac{dy}{dx} + (2x\tan^{-1}y - x^3)(1 + y^2) = 0$

20.　$y\dfrac{dy}{dx} + \dfrac{4x}{3} - \dfrac{y^2}{3x} = 0$

21.　$e^x(x + 1)dx + (y^2 e^{2y} - xe^x)dy = 0$

22.　$\dfrac{dx}{dy} = e^{y-x}(e^y - e^x)$

23.　$x\cos y\dfrac{dy}{dx} + \sin y = x\sin^2 y$

24.　$3y^2\dfrac{dy}{dx} + 2y^3 x = 4x^3 e^{x^2}$

25.　$\dfrac{dy}{dx} = \dfrac{e^y}{x^2} - \dfrac{1}{x}$

26.　$(xy^2 - e^{1/x^3})dx - x^2 y\,dy = 0$

27.　$\dfrac{dy}{dx} = \dfrac{y^3}{e^{2x}+y^2}$

28.　$\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1$

29.　$(x + 1)\dfrac{dy}{dx} - y = e^x(x + 1)^2$

ANSWERS

1.　$y(x^2 + 1)^2 = \tan^{-1}x + c$

2.　$y = x^2 + ce^{1/x}.x^2$

3.　$y = \sqrt{(1 - x^2)} + c(1 - x^2)$

4.　$y = x^3 + \dfrac{cx^2}{x-1}$

5.　$y.\cos hx = \dfrac{2}{3}\cosh^3 x + c$

6.　$yx\sec x = \tan x + c$

7.　$xy + \tan^{-1}y = c$

8.　$xe^{\tan^{-1}y} = \dfrac{1}{2}e^{2\tan^{-1}y} + c$

9.　$(y + 1)(x - e^y) = c$

10.　$x.(1 + \sin y) = y^2\cos y + c\cos y$

11.　$x = \tan^{-1} y - 1 + ce^{-\tan^{-1}y}$

12.　$y\sec^2 x = \sec x - 2$

13.　$y(x^2 + 1) = \tan^{-1}x - \dfrac{\pi}{4}$

14.　$y\log x = (\log x)^2 + c$

15.　$2r\sin^2\theta + \sin^4\theta = c$

16.　$\tan y = \dfrac{1}{2}(x^2 - 1) + ce^{-x^2}$

17.　$e^y = e^x - 1 + ce^{-e^x}$

18.　$\tan y.e^{x^2} = \dfrac{1}{2}e^{x^2}(x^2 - 1) + c$

19.　$\tan^{-1}y = \dfrac{(x^2-1)}{2} + ce^{-x^2}$

20.　$y^2 x^{-2/3} + 2x^{4/3} = c$

21.　$xe^{x-y} = -(y^2 - 2y + 2)e^y + c$

22.　$e^x = e^y - 1 + ce^{-e^y}$

23.　$\cosec y + x(\log x + c) = 0$

24.　$2y^3 e^{x^2} = e^{2x^2}(2x^2 - 1) + c$

25.　$2x e^{-y} = 2cx^2 + 1$

26.　$\dfrac{y^2}{x^2} = \dfrac{2}{3}e^{1/x^3} + c$

27.　$e^{-2x}y^2 + 2\log y = c$

28.　$ye^{2\sqrt{x}} = 2\sqrt{x} + c$

29.　$y = (x + 1)(e^x + c)$