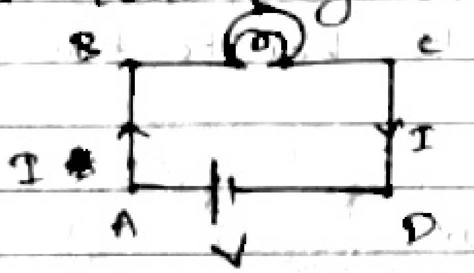
DC circuit

Souce of power is battery or d.c. power supply, conductors used to carry the current and load (e.g. lamp, heater, motor)



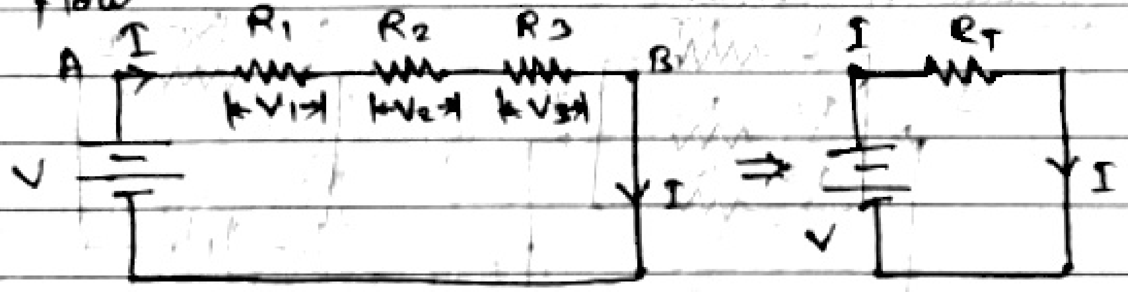
closed path followed by a direct current dc is called as dc ckt.

- Load for dc ckt is usually a resistance.
- Depending upon load / resistances connected in series, parallel or series-parallel fashion dc ckts are classified as:

1) Series ckt 2) Parallel ckt 3) Series-parallel ckt

1) Series ckt :-

Resistances are connected end to end so that there is only one path for the current to flow



$$R_T = R_1 + R_2 + R_3$$

As per Ohm's law

volg drop across resi. R_1, $V_1 = IR_1$

" " " R_2, $V_2 = IR_2$

" " R_3, $V_3 = IR_3$

For series ckt, sum of voltage drops = Applied volg

$$\therefore V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$
$$\frac{V}{I} = R_1 + R_2 + R_3$$

$\frac{V}{I}$ = Total resi.

$$R_T = R_1 + R_2 + R_3$$

For series dc ckt:

- Current flowing thr' each resistance is same
- Applied volg = sum of different volg. drops
- Every resistor of ckt has its own volg. drop.

Voltage divider rule:-

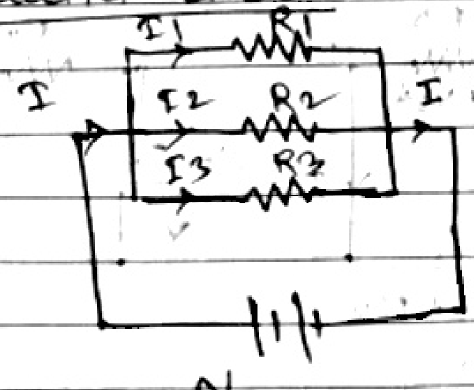
Voltage drop across the resistances can be obtained as:

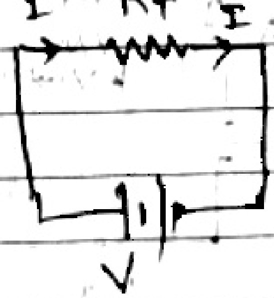
$$V_1 = IR_1 = \frac{V}{R_T} R_1 \quad \left(\because I = \frac{V}{R_T}\right)$$

$$V_2 = IR_2 = \frac{V R_2}{R_T}$$

$$V_3 = IR_3 = \frac{V R_3}{R_T}$$

Parallel circuit





ckt in which one end of each resistance is joined to a common point and the other end of each resi. is joined to anothe

common point, so that there are many paths for current flow as the number of resistances is called a parallel circuit.
Total current 'I' is divided into three parts I_1 flowing thr' R_1, I_2 flowing thr' R_2 and I_3 flowing thr' R_3
Voltage across each resistance is same.
By ohm's law
Current thr' resi. R_1, $I_1 = \frac{V}{R_1}$

" R_2, $I_2 = \frac{V}{R_2}$

" R_3, $I_3 = \frac{V}{R_3}$

For parallel ckt sum of the branch currents is equal to total current.

$$I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

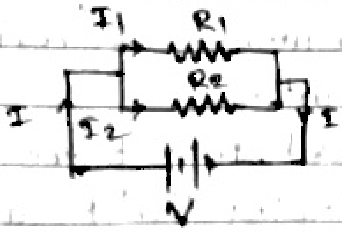
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_T} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

For parallel ckt
- voltage drop across each resi. is same
- Total current = sum of branch currents
- every resi. has it's own current

Current divider rule: #1



In a parallel ckt, two resistances R_1 and R_2 are connected in parallel across a battery of V volt. Current I is divided into two parts: I_1 flowing thr' R_1 and I_2 flowing thr' R_2.

Total equivalent resi. R_T is given as

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_1+R_2}{R_1R_2}$$

so, $$R_T = \frac{R_1R_2}{R_1+R_2}$$

$$V = IR_T = I\frac{R_1R_2}{R_1+R_2}$$

Current thr' R_1 is

$$I_1 = \frac{V}{R_1}$$

$$= \frac{I\frac{R_1R_2}{R_1+R_2}}{R_1}$$

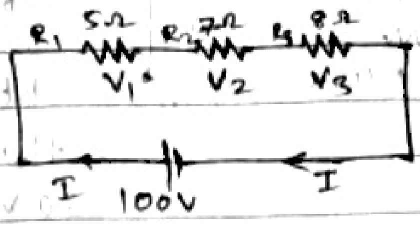
$$\therefore I_1 = I\frac{R_2}{R_1+R_2}$$

Current thr' R_2 is

$$I_2 = \frac{V}{R_2}$$

$$I_2 = \frac{R_1}{R_1+R_2}$$

1 Determine the current thr' and the voltages across three resistances of ohmic values 5Ω, 7Ω and 8Ω connected in series across 100V.



Total resi. $= R_T = 5+7+8 = 20\Omega$

$$I = \frac{V}{R_T} = \frac{100}{20} = 5A$$

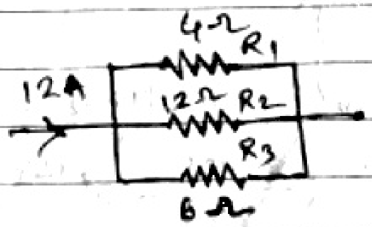
Voltage across 5Ω resi = $5I = 25V$

" 7Ω resi = $7I = 35V$

" 8Ω = $8I = 40V$

$$V = V_1 + V_2 + V_3$$
$$= 100V$$

2



3 resistances are connected in ||el as shown in the dia. If total current is 12A, how is this current divided among the various resistors?

Equivalent total Resi. $= R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{12} + \frac{1}{6} = \frac{3+1+2}{12} = \frac{6}{12} = 0.5\Omega$$

$$\therefore R_{eq} = 2\Omega$$

Voltage across ckt $= V = I R_{eq} = 12 \times 2 = 24V$.

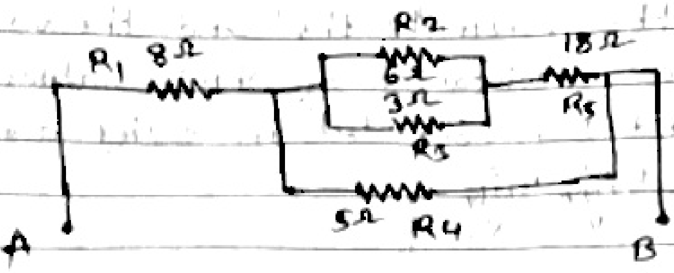
Current thr' 4Ω resi. $= \frac{V}{R_1} = \frac{24}{4} = 6A$

Current thr' 12Ω resi $= \frac{V}{R_2} = \frac{24}{12} = 2A$

Current thr' 6Ω resi $= \frac{V}{R_3} = \frac{24}{6} = 4A$

Sums on Resistance for series + parallel co[mbination]

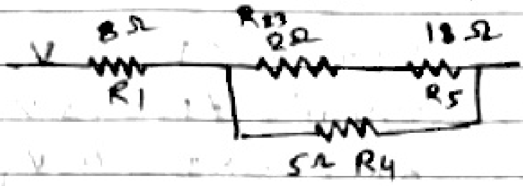
1.



Calculate effective resi of the ckt and cu[rrent] thr' 8Ω resⁿ 60 V is applie[d] betⁿ pts. A &

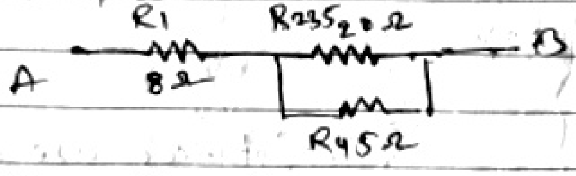
$R_2 = 6\Omega, R_3 = 3\Omega$

$R_2 \parallel R_3 = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 3}{9} = 2\Omega$



R_{23} & R_5 are in series, so get added.

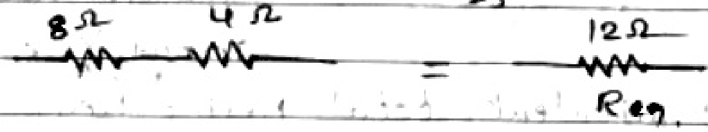
$R_{235} = 2\Omega + 18\Omega = 20\Omega$



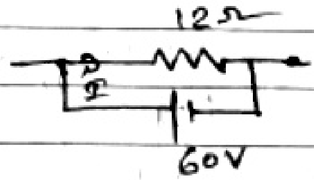
Now $R_{235} \parallel R_4$ ∴ $R_{2354} = 20\Omega \parallel 5\Omega$

$= \frac{20 \times 5}{25} = 4\Omega$

∴

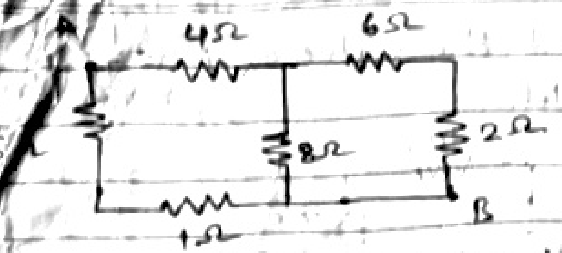


∴ $R_{eq.} = 12\Omega$

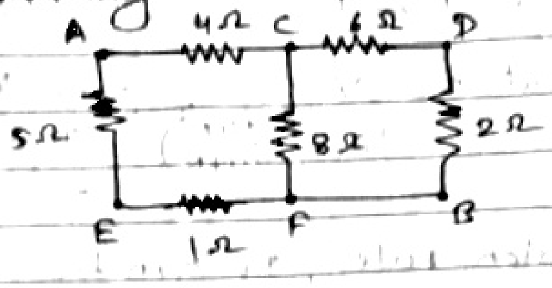


∴ $I = \frac{V}{R} = \frac{60}{12} = 5\text{Amp.}$

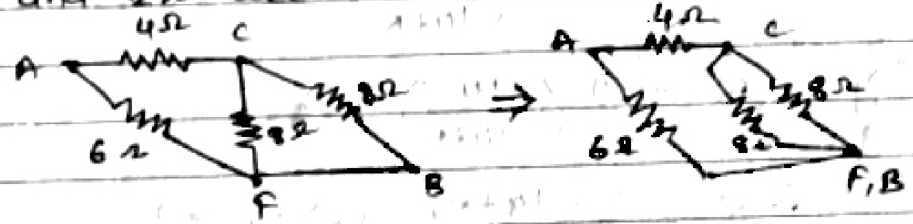
∴ Current thr' 8Ω res. = 5Amp.



Naming the nodes for the given ckt

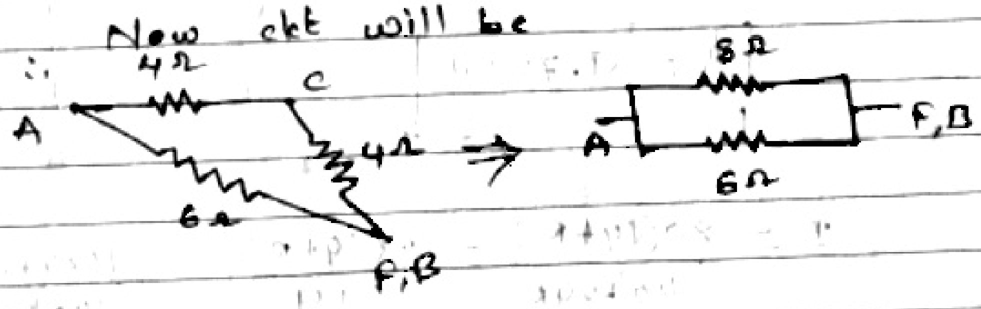


Resistors 5Ω and 1Ω are in series. Also resistors 6Ω and 2Ω are in series. So new ckt will be



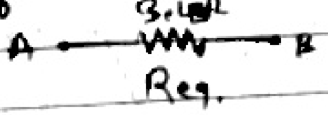
$$8\Omega \,||\, 8\Omega = \frac{8\times 8}{8+8} = \frac{64}{16} = 4\Omega$$

$\therefore$ Now ckt will be

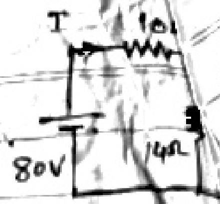


$$6\Omega \,||\, 8\Omega = \frac{8\times 6}{8+6} = \frac{48}{14} = 3.43\Omega$$

$\therefore$ Effective resistance bet pts. A & B is

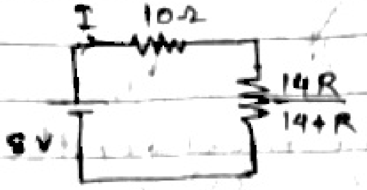


2 In the ckt shown, find the value of R.



14Ω + R in parallel.

$\therefore \frac{14R}{14+R}$



Total circuit current $I = \frac{V}{10 + \left(\frac{14R}{14+R}\right)} = \frac{80(14+R)}{140+24R}$ —— ①

By current division rule, I_1 can be calculated as

$$I_1 = I\left(\frac{14}{14+R}\right)$$

$$3 = I\left(\frac{14}{14+R}\right)$$

$$\therefore I = \frac{3(14+R)}{14} \quad \text{—— ②}$$

Solving eq's ① & ②

$$\boxed{R = 9.722\,\Omega}$$

$$I = \frac{80(14+R)}{140+24R} = \frac{3(14+R)}{14}$$

$$= 80 \times 14 = 3(140+24R)$$

$$= \frac{112}{3} = 140+24R$$

11

$$\frac{1120+80R}{140+24R} = \frac{42+3R}{14}$$

$14(1120+80R) =$

$(42+3R)(140+24R)$

$15680 + 1120R =$

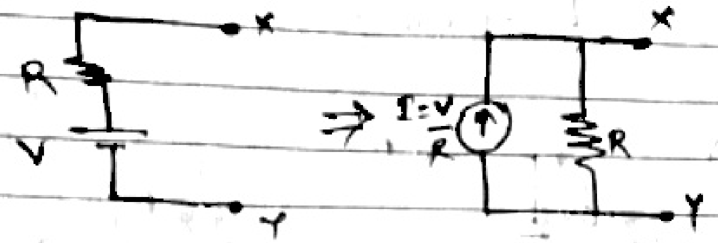
$5880 + 1008R + 420R + 72R^2$

Source Transformation

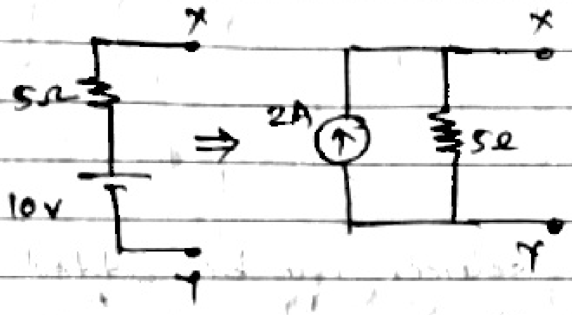
Case I

A voltage source with a series resistance can be converted into equivalent current source with parallel resi.

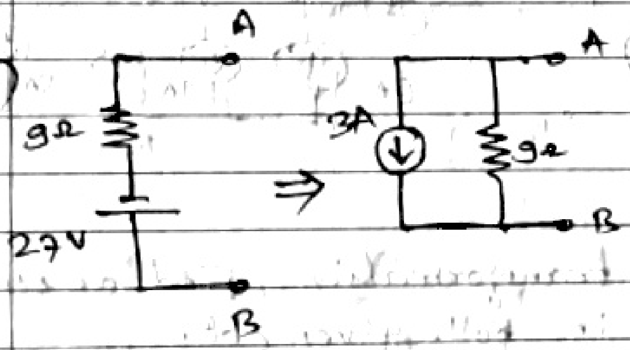
Eg



1)

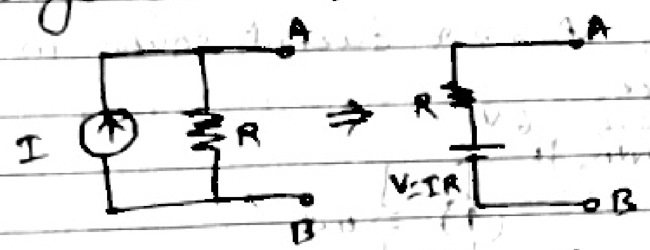


2)

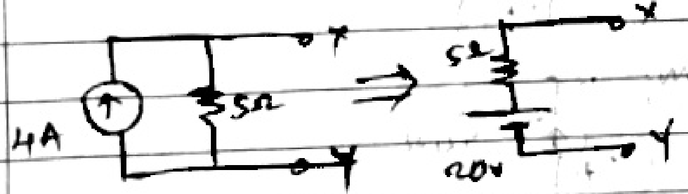


Case II

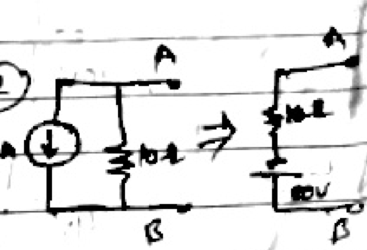
A current source with a parallel resistance can be converted into an equivalent voltage source with series resistance.



①



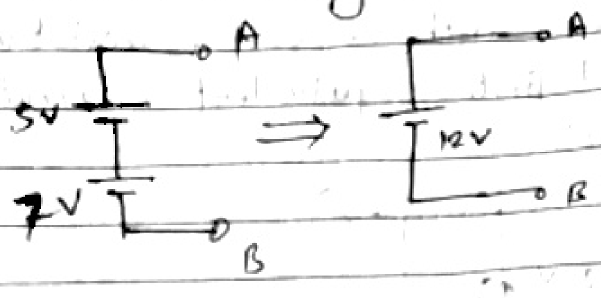
②



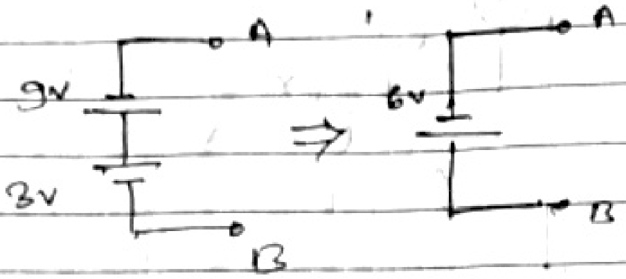
Case III

Series voltage sources can be added.

①



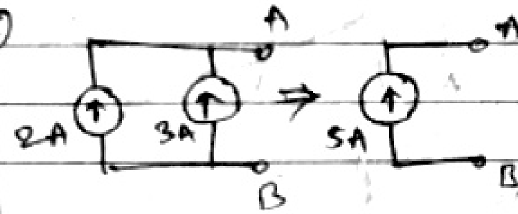
②



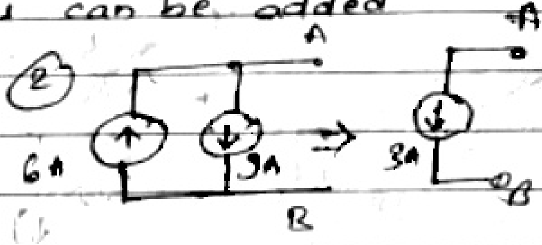
Case IV

Parallel current sources can be added

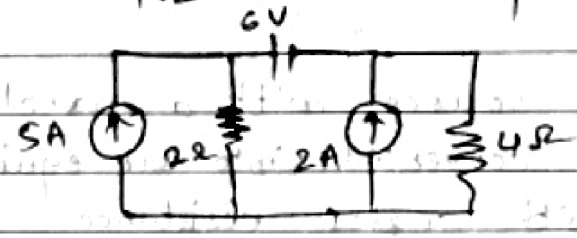
①



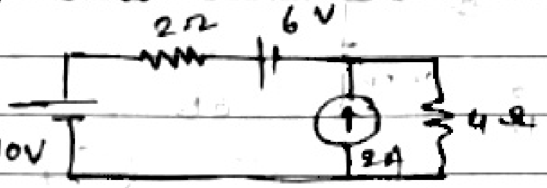
②

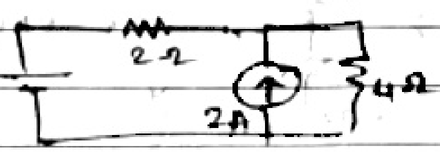


\#1 Using source transformation, find current in 4Ω res^n in foll. given ckt.

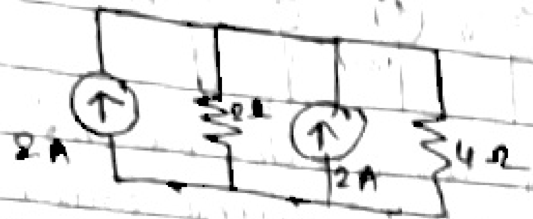


Converting 5A & 2Ω current source in equi. volg. source





Converting 4V in series with 2Ω res^n into equivalent current source, we get



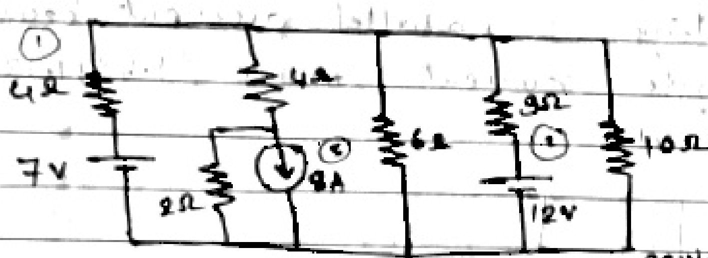
Adding two parallel current sources

By current division method current in 4Ω resi^r can be found as

$$I_2 = 4 \frac{2}{2+4}$$

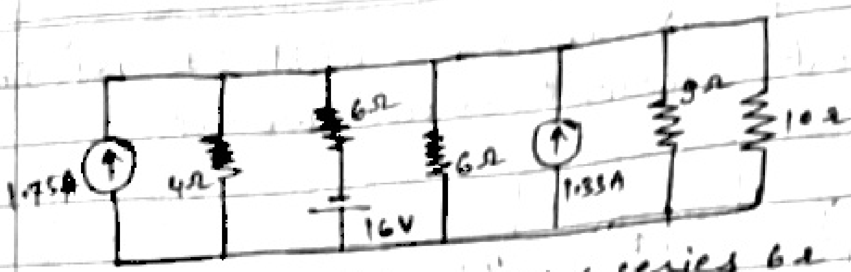
$$I_2 = 1.33\,A$$

2 By source transformation, find current in 10Ω resistor in the ckt shown:

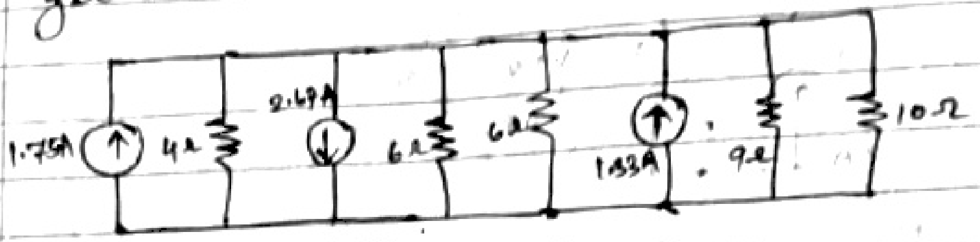


Converting 2 volg. sources into equ. current source and one current source into equ. voltage source

For volg. source 3 two series resistors can be added



Now converting 16V & series 6Ω resi volg. source into equivalent current source we get



4Ω || 6Ω || 6Ω || 9Ω

Converting parallel resistors into equivalent single resr.

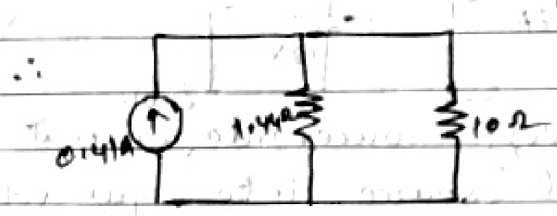
$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{6} + \frac{1}{6} + \frac{1}{9}$$

$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{9} = \frac{9+12+4}{36} = \frac{25}{36}$$

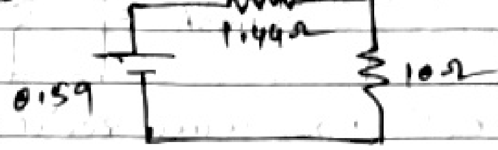
$\therefore R_{eq} = 1.44\Omega$

Converting 3 parallel current sources in single current source $(1.75 + 1.33 - 2.67)$A $= 0.41$A

$\therefore$



Now converting current source into eq. volg source

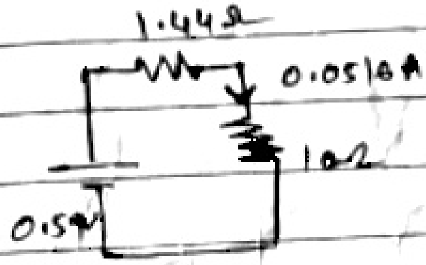


Using Ohm's law,
current in 10Ω resistor can be given as

$$I = \frac{V}{R} \qquad R = (1.44 + 10)\Omega$$

$$= \frac{0.59}{1.44+10}$$

$$I = 0.0516\ A$$



Elements of Electrical and Electronics Engineering

Module 1 DC Circuits

Classification:

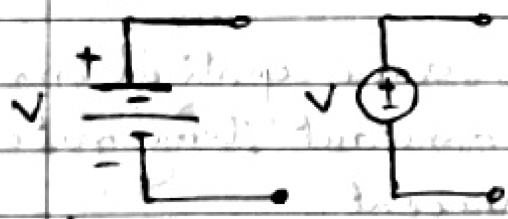
Electric Energy Sources
- Voltage sources
  - Independent Voltage sources
    - Ideal Volt. sources
    - Practical volg. s.
  - Dependent Voltage sources
- Current sources
  - Independent current sources
    - Ideal
    - Practical
  - Dependent current sources

Independent Voltage sources:

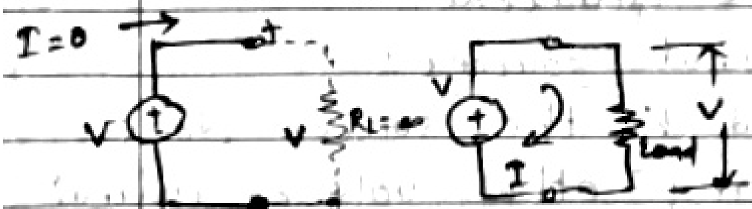
Terminal voltage of the source is not dependent on any other voltage source or current source.

• **Ideal voltage sources:-**

Symbol:



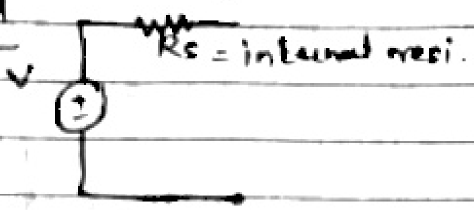
* Source resistance is zero
• Constant terminal volg. = V without Load ($R_L = \infty$) or with Load.
* Terminal volg. is independent of Load.



(1) with $R_L = \infty$ (2) with load

Practical voltage source :

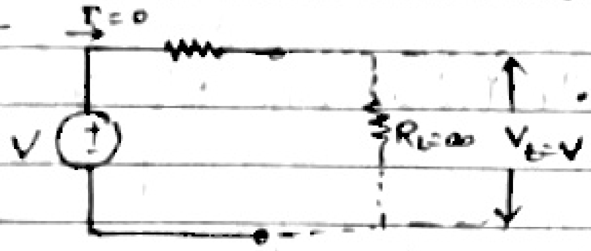
Symbol



• With internal series resistance R_s.

2 cases

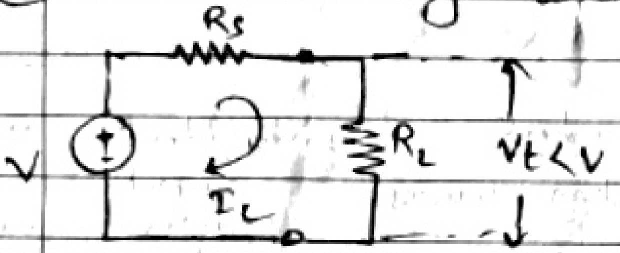
(a)



• Terminal voltage with no load or $R_L = \infty$

• $I = 0$, so no drop across R_s, so terminal volg $V_t = V$

(b) Terminal volg. with Load.



• When load is connected, finite load current I flows.

• Some volg. drop across R_s

• So terminal volg. $V_t < V$

• With increase in load current I_L, terminal volg. decreases.

as $V_t = V - IR_s$

Current Sources :- In some applications it is reqd. to have constant time rate of ēs or constant current.

Independent Current sources :-

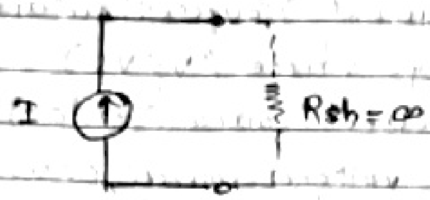
Current supplied by the source does not depend on any other volg or current source.

2 Types:-

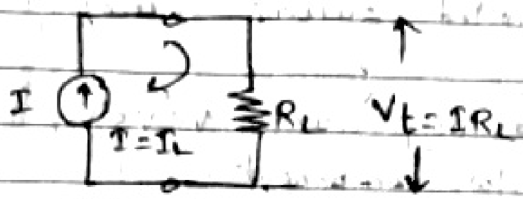
1) Ideal current source
2) Practical current source

Ideal current source

Symbol



- Internal shunt resistance $R_{sh} = \infty$

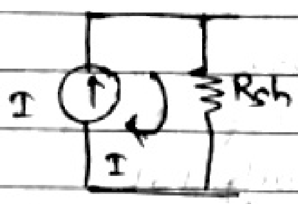


With load

- When load resi. R_L is connected across current source, const. current I flows thr' load. i.e. $I_L = I$
- Volg. across load $V_t = IR_L$.

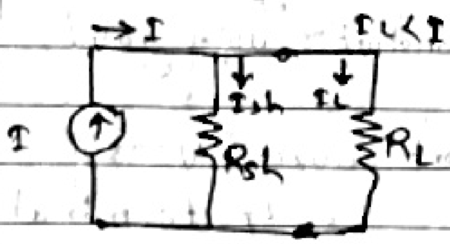
Practical current source

Symbol



R_{sh} = Internal shunt resistance

- Finite internal shunt resi.



- When load R_L is connected load current I_L will start flowing thr' it.
- But due to internal R_{sh} source current I gets divided betn R_{sh} & R_L.

∴ Current thr' R_L is less than I

∴ $I_L = I - I_{sh}$ $\qquad I_L < I$

Dependent Source

If volg. or current of a source depends upon some other volg. or current, it is called as dependent or controlled source.

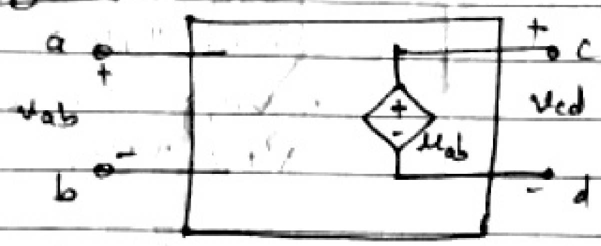
4 types: Depending on whether the control variable is volg. or current and controlled source is a volg. source or current source, there are 4 types

1) Volg.-controlled volg. source (VCVS)
2) Volg.-controlled current source (VCCS)
3) Current-controlled voltage source (CCVS)
4) Current-controlled current source (CCCS)

① Voltage-controlled volg. source (VCVS)

- 4 terminal n/w component.

Symbol



- It establishes a volg. v_{cd} betn two points c and d in the ckt that is proportional to volg. v_{ab} betn two points a and b.

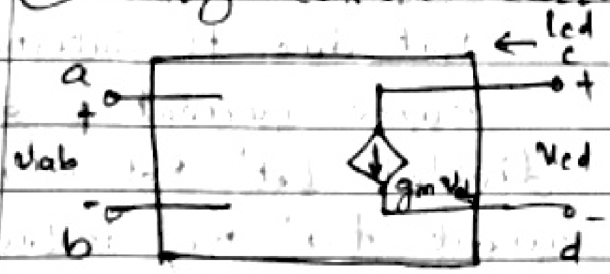
component as a volg. source.

$$v_{cd} = \mu v_{ab}$$

v_{cd} depends upon control volg. v_{ab}.

μ - dimensionless constant, volg. gain.

② Voltage-controlled current source (VCCS)



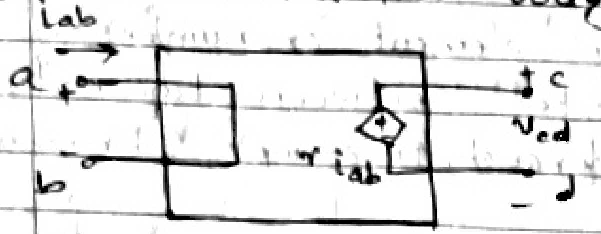
- 4 terminal n/w component that establishes a current i_{cd} in a branch of the ckt that is proportional to the voltage v_{ab} betn two points a and b.

$i_{cd} = g_m v_{ab}$ — g_m - transconductance or mutual conductance

i_{cd} depends only on control volg. and const. g_m.

unit is A/V or Simens(S)

③ Current - controlled voltage source (CCVS)



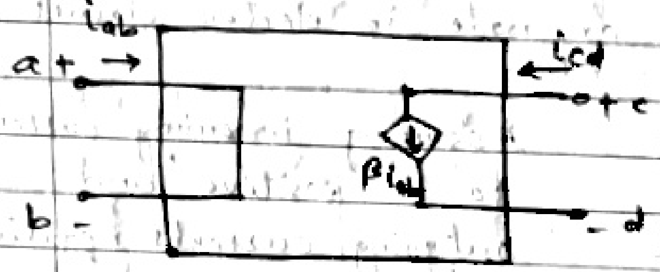
• 4 terminal n/w component that establishes a voltage v_{cd} between two points c and d in the circuit that is proportional to the current i_{ab} in some branch of the ckt.

$v_{cd} = r\,i_{ab}$

r - transresistance or mutual resistance

It is constant and has unit V/Amp or ohm.

④ Current - controlled current source (CCCS)

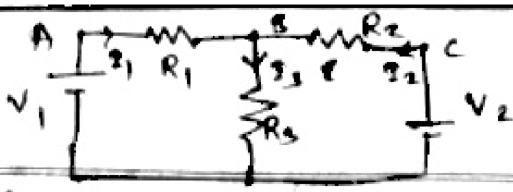


• 4 terminal n/w component that establishes a current i_{cd} in one branch of a circuit that is proportional to the current i_{ab} in some branch of the network.

$i_{cd} = \beta\, i_{ab}$

Current i_{cd} depends only on the control current i_{ab} and dimensionless const. β

β - current gain



Kirchhoff's Laws

- Useful for analysis of electric circuit.

Terms used.

1) Node :- It is a junction where two or more circuit elements are connected together. In above ckt, A, B, C, D are nodes.

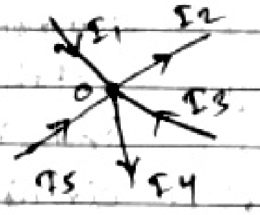
2) Branch :- An element or number of elements connected between two nodes constitutes a branch. In example, 3 branches BAD, BCD and BD

3) Loop :- Any closed part of the circuit. ABDA, BCDB, ABCDA

4) Mesh :- Elementary form of a loop & cannot be

Kirchhoff's Current Law (KCL)

The algebraic sum of currents meeting at a junction or node in electric circuit is zero.



Assuming incoming currents to be positive and outgoing currents negative

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

The sum of currents flowing towards any junction in an electric circuit = sum of currents flowing away from the junction.

or

Incoming currents = Outgoing currents

Kirchhoff's Voltage Law:- (KVL)
- Related to electromotive forces and voltage drops in a circuit.

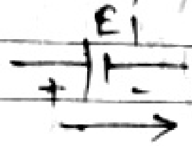
KVL is stated as:

In any closed circuit or mesh, the algebraic sum of the electromotive forces and the voltage drops is equal to zero.

Sign Conventions

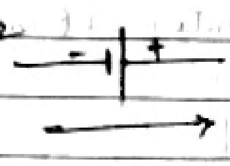
1. • Rise in potential can be assumed positive while a fall in potential can be considered negative.

• Going from positive terminal of source to negative terminal is a fall in potential so emf is assigned -ve sign.



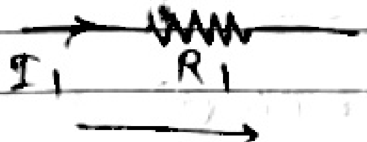
$\Rightarrow -E_1$

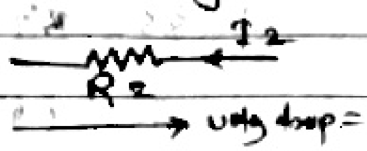
• Going from negative terminal of source to positive terminal, there is a rise in potential & so emf is given positive sign.



$\Rightarrow +E_2$

2. When current flows thr' a resistor, there is a voltage drop across it.
If we go thr' resi. in the same direction as the current, there is a fall in the potential & so for potential drop sign is -ve



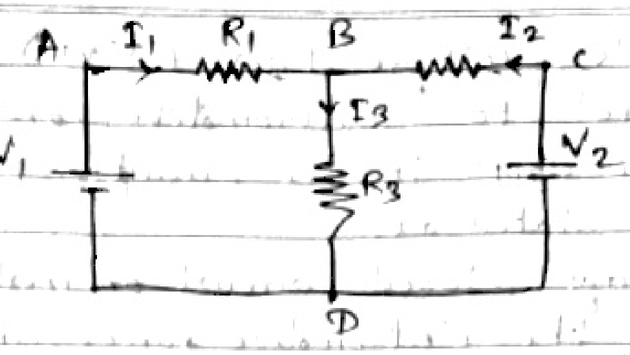


vltg drop = $+I_2R_2$

Volg. drop = $-I_1R_1$

If we go opposite to current dirn there is rise in pot. so sign is +ve.

e.g.



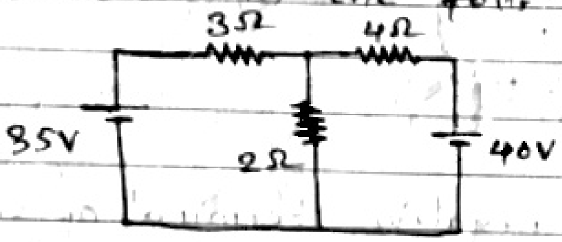
Applying KVL to above ckt
for loop ABDA

$$V_1 - I_1R_1 - I_3R_3 = 0$$

Applying KVL to Loop ABCDA

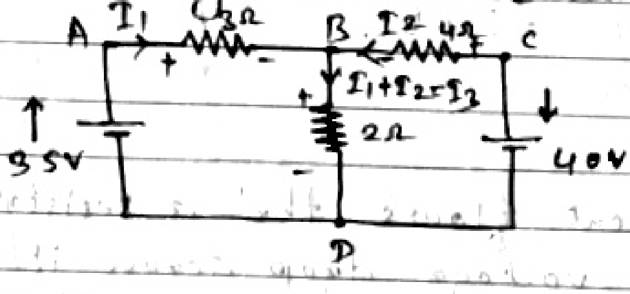
$$V_1 - I_1R_1 + I_2R_2 - V_2 = 0$$

1 Using Kirchhoff's laws, calculate the branch currents in the foll. ckt.



steps

1. In the given ckt name the nodes.
2. Assuming unknown currents ckt will be



Applying KVL to Loop ABDA

$$35 - 3I_1 - 2(I_1 + I_2) = 0 \quad \text{—} (1)$$

$$= 35 - 3I_1 - 2I_1 - 2I_2 = 0$$

$$\therefore 35 = 5I_1 + 2I_2 \quad \text{—} (ii)$$

or

$$5I_1 + 2I_2 = 35 \quad (2)$$

Applying KVL to Loop ABCDA

$$35 - 3I_1 + 4I_2 - 40 = 0$$

$\therefore \quad 3I_1 - 4I_2 = -5$ —— (3)

From eqⁿ (2) and (3) we get

$I_1 = 5A$

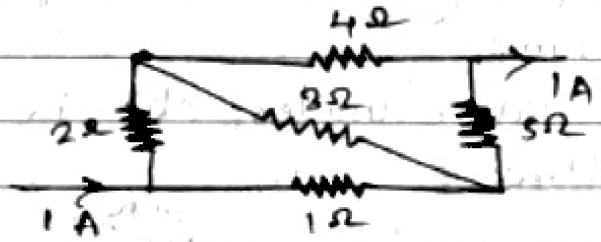
$I_2 = 5A$

$I_3 = I_1 + I_2$

$I_3 = 10A$

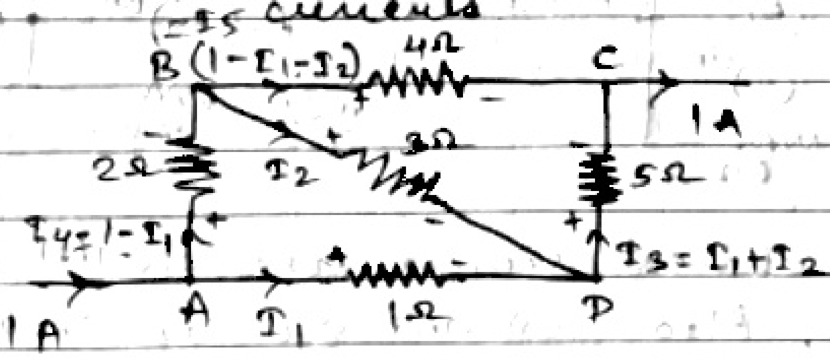
$\therefore \boxed{I_1 = I_2 = 5A, \ I_3 = 10A}$

2



Using Kirchhoff's laws

step 1 Naming nodes + assuming unknown currents



step 2 Applying KVL to loop ABDA

$I_1 - 2(1-I_1) - 3I_2 = 0$

$\therefore \quad 3I_1 - 3I_2 = 2$ —— (1)

Applying KVL to loop BCDB

$-4(1 - I_1 - I_2) + 5(I_1 + I_2) + 3I_2 = 0$

$-4 + 4I_1 + 4I_2 + 5I_1 + 5I_2 + 3I_2 = 0$

$9I_1 + 12I_2 = 4$ —— (2)

Solving eq^{n}s (1) and (2) we get

$I_1 = 0.571\,A$

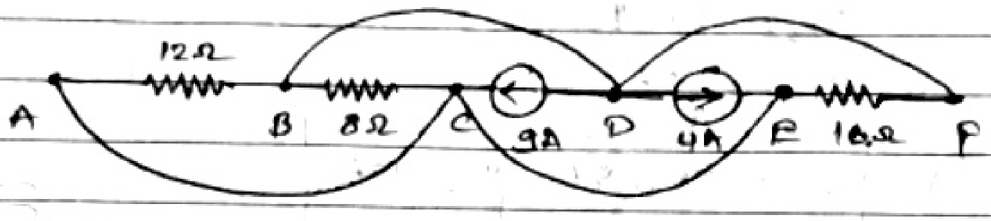
$I_2 = -0.0952\,A$

$I_1 + I_2 = I_3 = 0.495\,A$

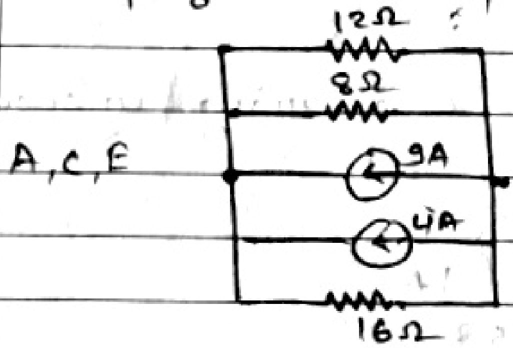
$1 - I_1 = I_4 = 0.429\,A$

$1 - I_1 - I_2 = I_5 = 0.524\,A.$

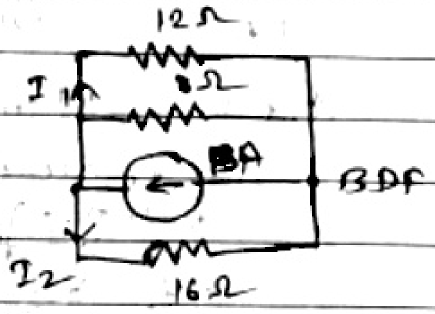
3 Calculate the power dissipation in each resistor in the network shown:



Nodes A, C and E are same, Also nodes B D and F are same.
By joining the same nodes, ckt is simplified as follows



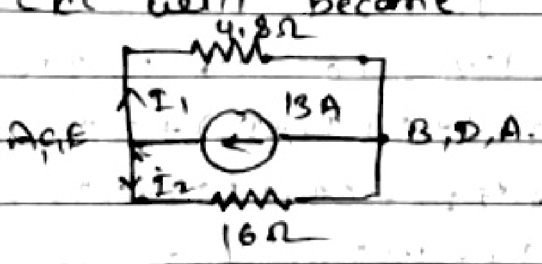
$\Rightarrow$



Both the current sources are in ||el so get added, Also 12Ω and 8Ω are in ||el.

$\therefore\ 12\Omega \parallel 8\Omega = R_{eq1} = \frac{12 \times 8}{12+8} = \frac{96}{20} = 4.8\,\Omega$

$\therefore$ Ckt will become



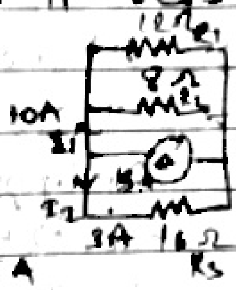
13A total current is divided in the ckt as I_1 + I_2 shown in the fig.

By current division rule

$$I_1 = \frac{13 \times 16}{16 + 4.8} = 10A$$

$$I_2 = \frac{13 \times 4.8}{16 + 4.8} = 3A$$

Current I_1 is divided in two $\|^{el}$ res's 12Ω + 8Ω.



∴ By current divi

∴ current in 8Ω res^r will be

$$I_{8\Omega} = \frac{I_1 R_1}{R_1 + R_2} = \frac{10 \times 12}{8 + 12} = 6A$$

$$I_{12\Omega} = \frac{I_1 R_2}{R_1 + R_2} = \frac{10 \times 8}{8 + 12} = 4A$$

Power dissipation in res^r = $I^2 R$

∴ Power dissipation in 12Ω res^r = $(I_{12\Omega})^2 \times 12$

$= 4^2 \times 12$

$= 192W$

Power dissipation in 8Ω res^r = $(I_{8\Omega})^2 \times 8$

$= (6)^2 \times 8$

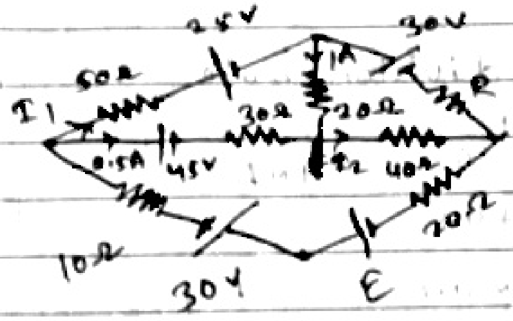
$= 288W$

Power dissipation in 16Ω res^r = $(I_{16\Omega})^2 \times 16$

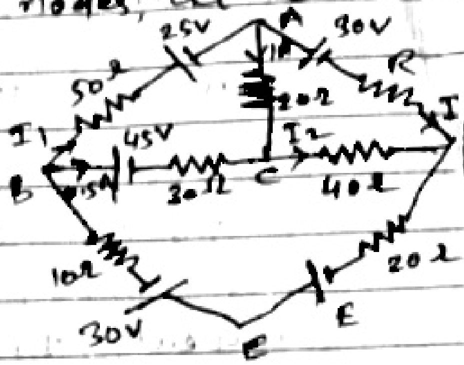
$= (3)^2 \times 16$

$= 144W$

4. For the n/w shown, determine (1) I_1, I_2 and I_3 (2) resistance R and (3) value of emf E



After naming the nodes, ckt will be



1) At node C, by KCL

$I_2 = 0.5 + 1$, $\therefore I_2 = 1.5A$

Applying KVL to Loop ACBA

$-20(1) + (30 \times 0.5) + 45 - (50 \times I_1) - 25 = 0$

$\therefore I_1 = 0.3A$

At node A, by applying KCL we get

$I_1 = 1 + I_3$ $\therefore 0.3 = 1 + I_3$ $\therefore I_3 = -0.7A$

2) Applying KVL to loop ADCA

$-30 - I_3 R + 40 I_2 + (20 \times 1) = 0$

putting values of I_2 & I_3 in above eq^n.

we get $R = -71.43\Omega$

3) At node D, by applying KCL

Current in branch DEB $= I_2 + I_3 = 1.5 - 0.7$

$= 0.8A$

Applying KVL to loop DEBCD we get

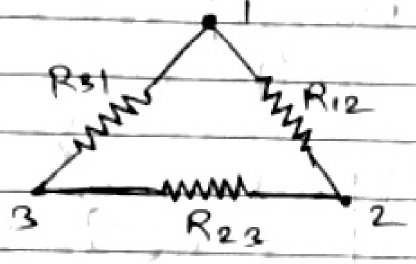
$(-20 \times 0.8) + E - 30 - (10 \times 0.8) - 45 - (30 \times 0.5) - (40 \times 1.5$

$\therefore$ After solving this eq^n. we get

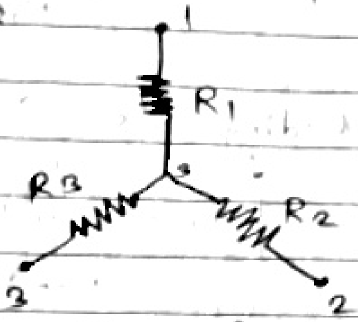
$E = 174V$.

Star - Delta (Y - Δ) Transformation:

- Some N/ws in which the resistances are neither in series nor in parallel and are connected in Y - or Δ connection
- N/w cannot be simplified by series/parallel ckt rules.
- So first Δ - connection into equivalent Y-connⁿ and vice-versa is done and then application of series/parallel ckt rules is made possible.



Delta Connection
Fig. a



Star Connection
fig b

1) Delta (Δ) to star (Y) Transformation

As per Delta connection n/w in fig a

Resistance betⁿ terminals 1 and 2 $= R_{12} || (R_{23} + R_{31})$

Term 1 & 2

$$= \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{---} \quad (1)$$

As per Y (star) connection n/w in fig b

Resistance betⁿ terminals 1 and 2 $= R_1 + R_2$ — (2)

Since two n/ws are electrically equivalent,

$$R_1 + R_2 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad \text{---} \quad (3)$$

Similarly in Δ n/w

Term 2 & 3

Resi. betⁿ terminals 2 and 3 $= R_{23} || (R_{12}+R_{31})$

$$= \frac{R_{23}(R_{12}+R_{31})}{R_{23}+R_{12}+R_{31}} \quad \text{---} \quad (4)$$

In Y n/w

Resi. betⁿ 2 & 3 $= R_3 + R_2$ — (5)

$$\therefore R_2 + R_3 = \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad (6)$$

Similarly replacing Δ and Y n/ws for resistance bet pts 3 and 1 we get

$$R_1 + R_3 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad \text{— (7)}$$

Subtracting eq^n (6) from eq^n (3)

$$R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{— (8)}$$

Adding eq^ns. (7) and (8)

$$\boxed{R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}} \quad \text{— (9)}$$

Similarly obtaining

$$\boxed{R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}} \quad \text{— (10)}$$

&

$$\boxed{R_3 = \frac{R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}} \quad \text{— (11)}$$

Star (Y) to Delta (Δ) Transformation.

Multiplying eq^ns (9) and (10)

$$R_1R_2 = \frac{R_{12}^2 R_{23} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \quad \text{— (12)}$$

Multiplying eq^ns (10) and (11)

$$R_2R_3 = \frac{R_{23}^2 R_{31} R_{12}}{(R_{12} + R_{23} + R_{31})^2} \quad \text{— (13)}$$

Multiplying eq^n's (9) and (11) we get

$$R_3 R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{——} \quad (14)$$

Adding eq^n's (12) (13) and (14) we get

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

or

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

or using $R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$

eq^n can be rewritten as

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12}R_3$$

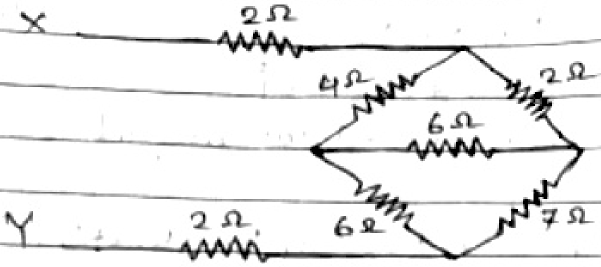
$$\therefore R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} \quad \text{——} \quad (15)$$

Similarly we can obtain

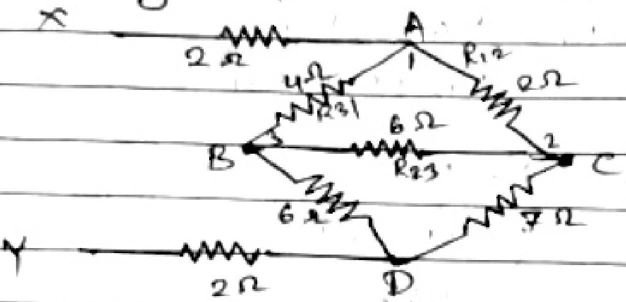
$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} \quad \text{——} \quad (16)$$

$$\& \quad R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} \quad \text{——} \quad (17)$$

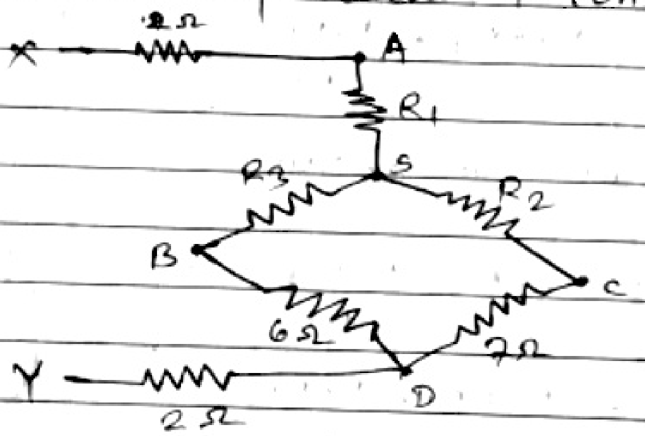
1 Find the equivalent resistance betⁿ the terminals X and Y in the foll. n/w



Naming different nodes in the given ckt



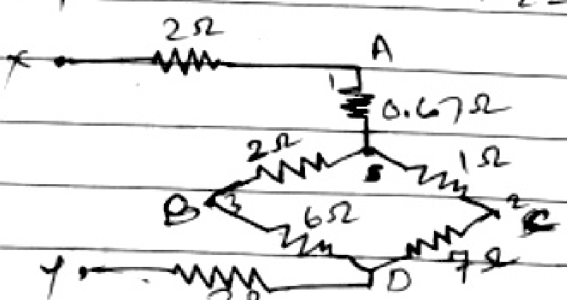
ΔABC is Δ connection, so converting this into equivalent Y-connection. $\Delta(ABC) = Y(ABC)$



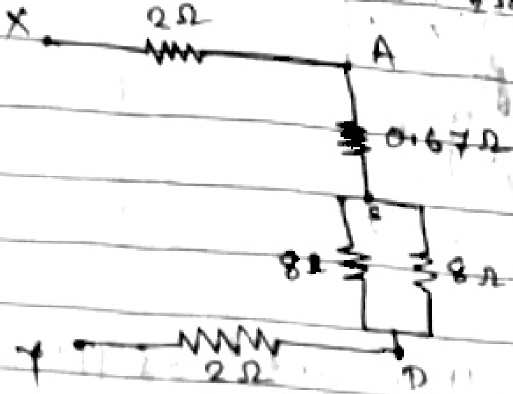
$\therefore R_1 = \dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$, $R_2 = \dfrac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$, $R_3 = \dfrac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$

$= \dfrac{4\times2}{4+2+6}$ $\qquad = \dfrac{6\times2}{4+2+6}$ $\qquad = \dfrac{6\times4}{4+2+6}$

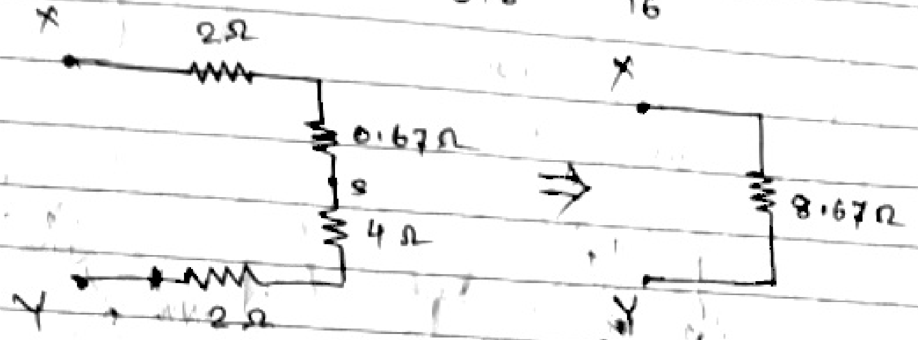
$R_1 = 0.67\,\Omega$ $\qquad R_2 = 1\,\Omega$ $\qquad R_3 = 2\,\Omega$



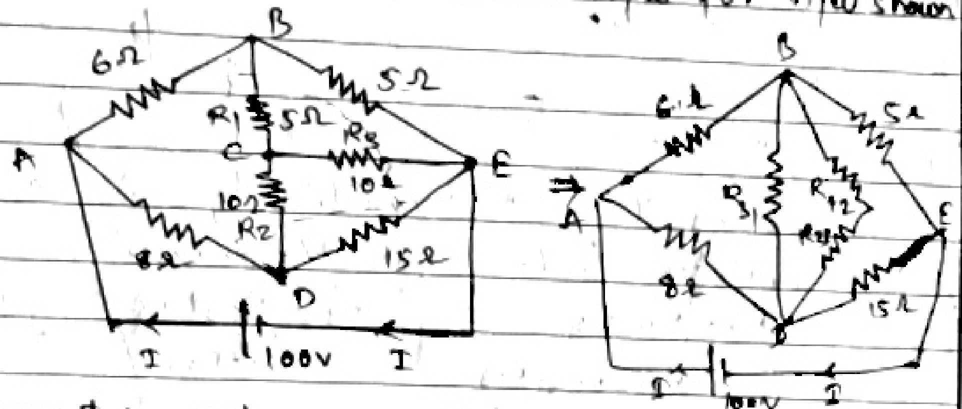
Now resisters 6 Ω and 2 Ω are in series.
Also resistors 1 Ω and 7 Ω are in series.



Now 8 Ω ∥ 8 Ω ∴ $\frac{8\times8}{8+8} = \frac{64}{16} = 4\,\Omega$



2 Find the current I in the n/w for n/w shown

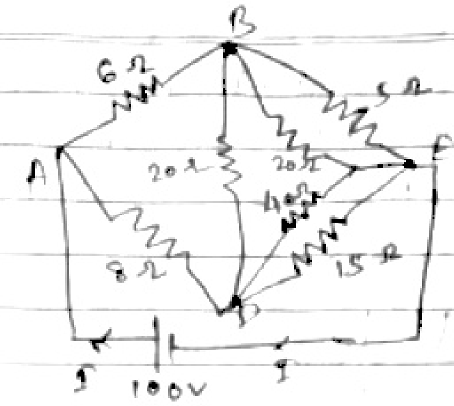


Converting star connection of 5 Ω, 10 Ω & 10 Ω into delta connection. i.e. Y(BED) to Δ(BED)

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} = 5 + 10 + \frac{(5\times10)}{10} = 20\,\Omega$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} = 10 + 10 + \frac{(10\times10)}{5} = 40\,\Omega$$

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} = 5 + 10 + \frac{(5\times10)}{10} = 20\,\Omega$$

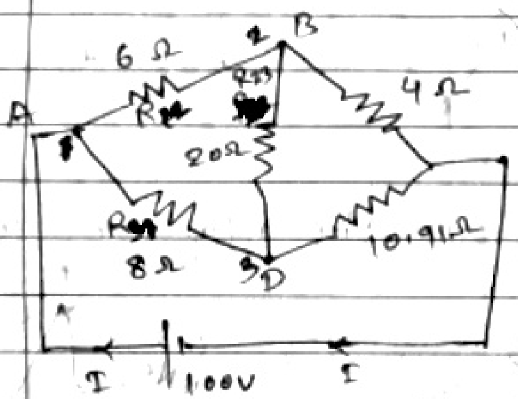


Resistors $20\Omega \parallel 5\Omega$ and $40\Omega \parallel 15\Omega$

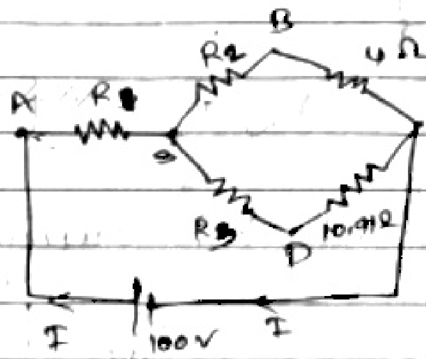
$$\frac{20 \times 5}{20+5} = \frac{100}{25} \qquad \frac{40 \times 15}{40+15} = \frac{600}{55}$$

$$= 4\Omega \qquad\qquad = 10.91\Omega$$

∴ n/w will be



⇒

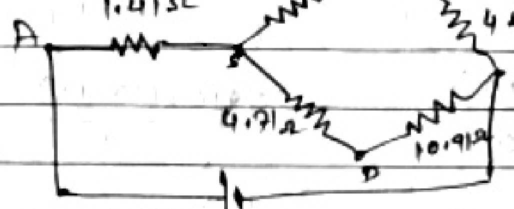


converting Δ-connection of 6Ω, 20Ω and 8Ω resistors into equi. Y-connection.

$$R_1 = \frac{R_{12} R_{31}}{R_{12}+R_{23}+R_{31}}, \quad R_2 = \frac{R_{23} R_{12}}{R_{12}+R_{23}+R_{31}}, \quad R_3 = \frac{R_{31} R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$= \frac{6 \times 8}{6+20+8} \qquad = \frac{20 \times 6}{6+20+8} \qquad = \frac{8 \times 20}{6+20+8}$$

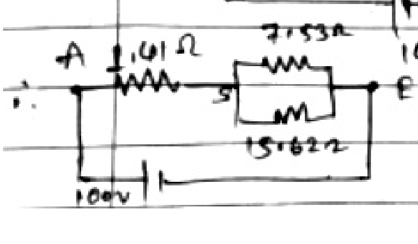
$$= 1.41\Omega \qquad = 3.53\Omega \qquad = 4.71\Omega$$



Now 3.53Ω & 4Ω are in series and 4.71Ω and 10.91Ω are in series. ∴ $3.53 + 4\Omega = 7.53\Omega$

$4.71 + 10.91\Omega = 15.62\Omega$

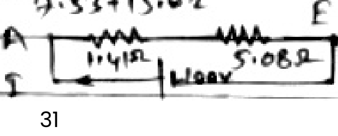
∴



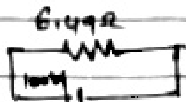
⇒

$7.53\Omega \parallel 15.62\Omega$

$$\frac{7.53 \times 15.62}{7.53 + 15.62} = 5.08$$



⇒



$$I = \frac{100}{6.49} = 15.41A.$$

1.2 Mesh and Nodal Analysis :-

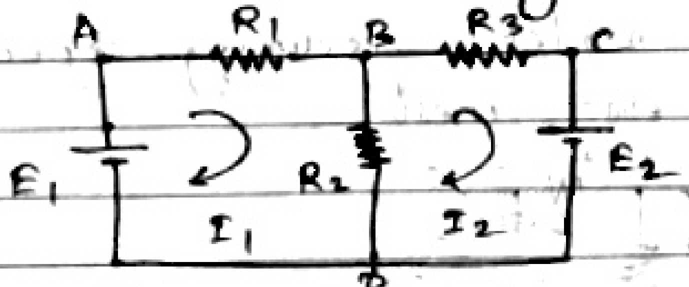
Maxwell's Mesh Current Method :-

. Kirchhoff's voltage law is applied to each mesh in terms of mesh currents instead of branch currents.

Steps for Maxwell's Mesh current method :

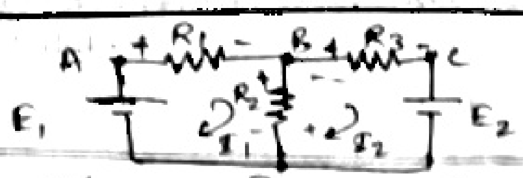
- Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in clockwise direction or anticlockwise direction.
- If two mesh currents are flowing the a ckt element, the actual current in the ckt element is the algebraic sum of two.

e.g.



This ckt consists of two meshes ABDA & BCDB I_1 & I_2 mesh currents are assigned respectively. If we assume clockwise direction & move from B to D, then current is $I_1 - I_2$. If we go from D to B in anticlockwise direction, current is $I_2 - I_1$.

- Kirchhoff's voltage law is applied to write eqⁿ for each mesh in terms of unknown mesh currents.



- If value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise.

Applying KVL to mesh ABDA

$$E_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$$

$$I_1(R_1 + R_2) - I_2R_2 = E_1 \quad \text{--- (1)}$$

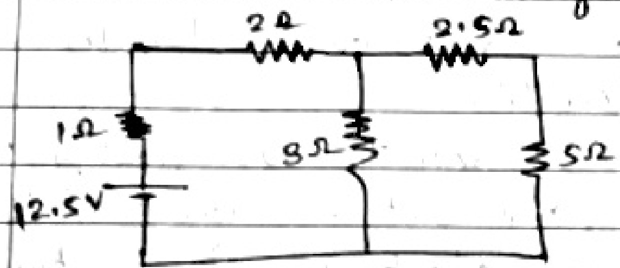
Applying KVL to mesh BCDB

$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

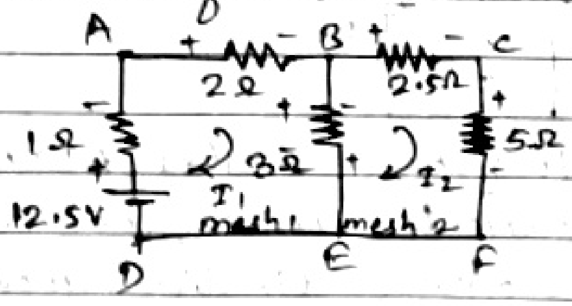
$$-I_1R_2 + (R_2 + R_3)I_2 = -E_2 \quad \text{--- (2)}$$

solving eq's (1) and (2) I_1 & I_2 can be found.

#1 Using mesh current method, find magnitude and direction of the current flowing thr' 1Ω resistor in the foll. n/w



Naming the nodes & assuming seperate mesh current for each mesh



Applying KVL to mesh 1

$$12.5 - I_1 - 2I_1 - 3(I_1 - I_2) = 0$$

$$-6I_1 + 3I_2 = -12.5 \quad \text{--- (1)}$$

Applying KVL to mesh 2

$$-3(I_2 - I_1) - 2.5I_2 - 5I_2 = 0$$

$$3I_1 - 10.5I_2 = 0 \quad \text{--- (2)}$$

solving eqⁿ's (1) + (2)

$$-6I_1 + 3I_2 = -12.5$$
$$3I_1 - 10.5I_2 = 0$$

$$-3I_1 - 6.5I_2 = -12.5$$

$$\therefore -6.5I_2 + 12.5 = 3I_1$$
$$\therefore I_1 = \frac{-6.5I_2 + 12.5}{3}$$

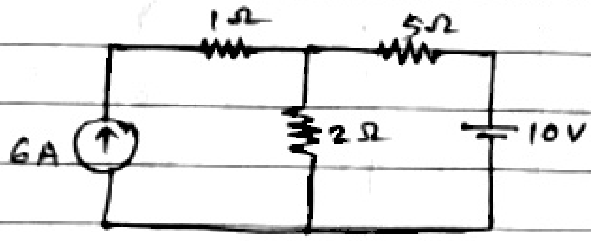
putting this in eqⁿ (2)

$$\cancel{3}\left(\frac{-6.5I_2+12.5}{\cancel{3}}\right) - 10.5I_2 = 0$$

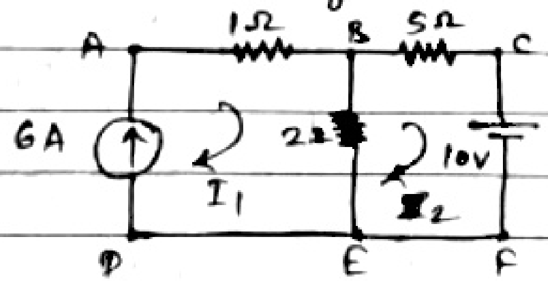
$I_1 = 2.43$ A

∴ current flowing thr' 1Ω resⁿ is 2.43A.

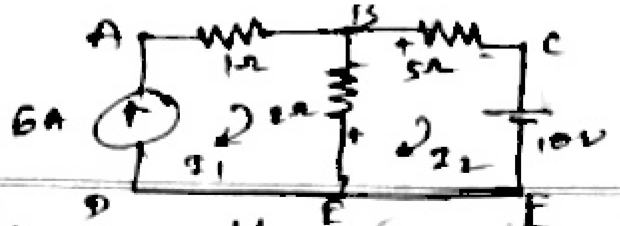
#2 By mesh analysis method, find current thr' 2Ω resistor in the ckt shown.



Naming the nodes and assigning separate mesh current for each mesh we get



If a current source exists in any mesh of a ckt KVL cannot be applied to such mesh.



In the given ckt, current source of 6A is present in mesh ABDEA, so KVL cannot be applied

$\therefore \quad I_1 = 6$

For mesh BCFEB KVL can be applied

$-2(I_2 - I_1) - 5I_2 - 10 = 0$

$2I_1 - 7I_2 = 10$

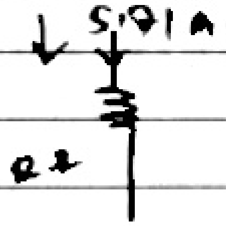
$2(6) - 7I_2 = 10$

$-7I_2 = -2$

$\therefore I_2 = 2/7 = 0.285$ A.

Current flowing thr' 2Ω resi. $= I_1 - I_2$

$6 - 0.285 = 5.71$ A

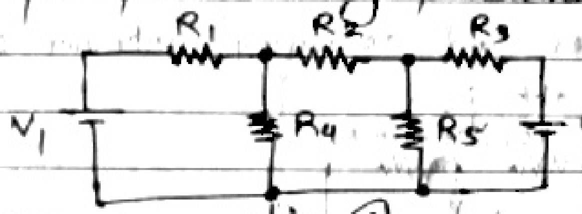


Nodal Analysis Method:

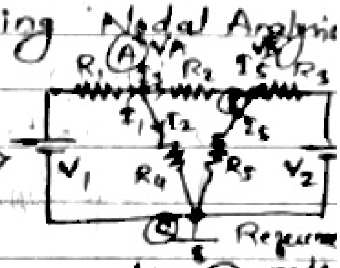
- Based on Kirchhoff's current law (KCL)
- Used to determine voltages of different nodes with respect to reference node.
- After determining node voltage, currents in all branches can be determined.
- This method is useful where no. of loops is large and so mesh analysis become lengthy.
- In this method advantage is a min. no. of eq^ns need to be written to determine the unknown quantities.

Steps for solving problems using Nodal Analysis

e.g.



dia. (a)



dia. (b)

<u>step 1</u> :- Mark nodes.

In the above ckt dia.(a) - 4 nodes

But Lower two nodes are same, so by joining them, we get only three nodes as shown in dia. (b)

<u>step 2</u> :- Select one of nodes as reference node.

For convenience, a node where max. elements are connected or max. branches are meeting is selected.

In above example, node (c) is selected as reference node.

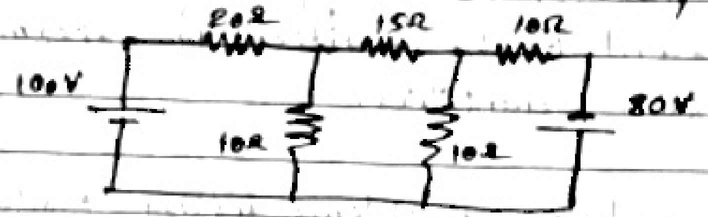
<u>step 3</u> :- Assign the unknown potentials of all nodes w.r.t. the reference node.

In above example at node A potential V_A and at node B, potential V_B.

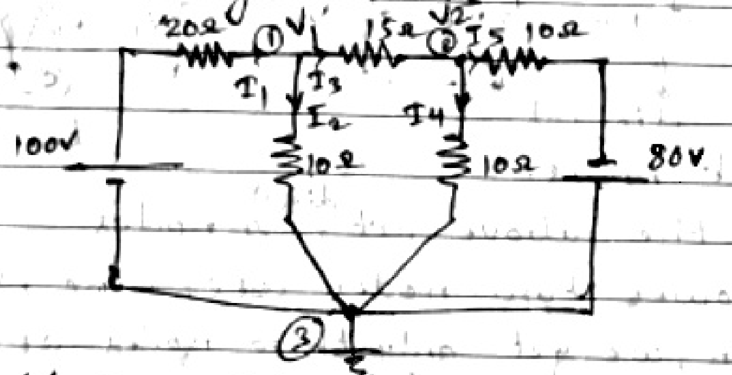
<u>step 4</u> :- At each node, excluding reference node, assume unknown currents and mark their direction.

Step 5 : Apply KCL at each node and write eq^n's in terms of node voltages. By solving eq^n's, determine the node voltages.
From node voltages, current in any branch can be determined.

#1 Using node voltage analysis method, find the current thr' 15Ω res^n in the foll. ckt



Naming nodes and assigning unknown currents and voltages ckt is



Now applying KCL at node 1

$$I_1 = I_2 + I_3$$

or, $$\frac{100 - V_1}{20} = \frac{V_1 - V_0}{10} + \frac{V_1 - V_2}{15}$$

$$\therefore 13V_1 - 4V_2 = 300 \quad \text{--- (1)}$$

Applying KCL at node 2

$$I_3 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{15} = \frac{V_2 - 0}{10} + \frac{V_2 - (-80)}{10}$$

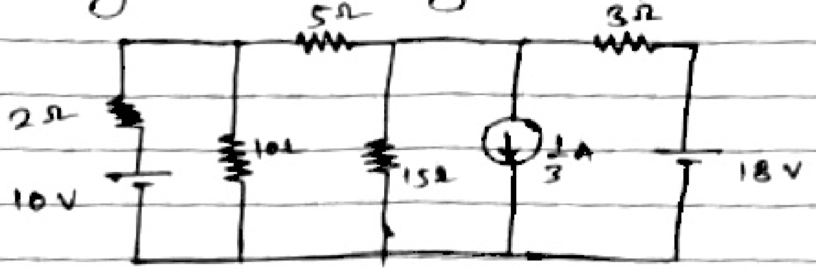
$$\therefore V_1 - 4V_2 = 120 \quad \text{--- (2)}$$

From eq^n's (1) and (2) we get

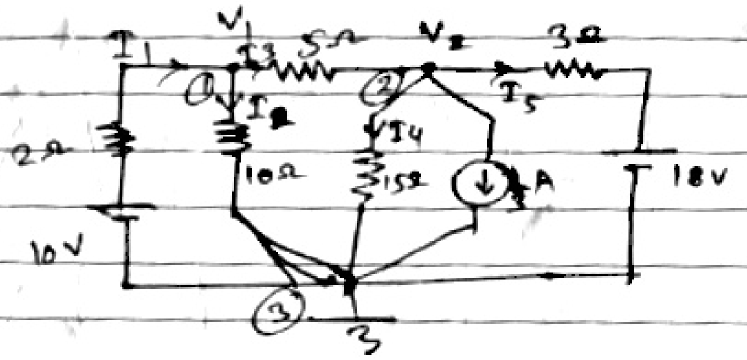
$$V_1 = 15V, \quad V_2 = -26.25V$$

Current thr' 15Ω resi = $I_3 = \frac{V_1 - V_2}{15} = 2.75A.$

2 Find the current thr' 2Ω and 3Ω resistances using nodal analysis in foll. ckt.



Naming the nodes, assigning unknown currents & voltages to the ckt. By joining same node ckt will be



Applying KCL at node 1

$$I_1 = I_2 + I_3$$

or $$\frac{10 - V_1}{2} = \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{5}$$

or $8V_1 - 2V_2 = 50$ —— (1)

Applying KCL at node 2

$$I_3 = I_4 + I_5 + \frac{1}{3}$$

or $$\frac{V_1 - V_2}{5} = \frac{V_2 - 0}{15} + \frac{V_2 - 18}{3} + \frac{1}{3}$$

$3V_1 - 9V_2 = -85$ —— (2)

Solving eq^{n}s (1) & (2)

$V_1 = 9.394V$, $V_2 = 12.576V$

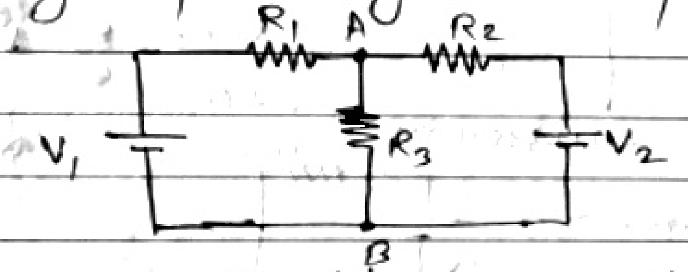
Current thr' 2Ω res^r = $\frac{10 - V_1}{2} = 0.303A \uparrow$, Current thr' 3Ω res^r = $\frac{V_2 - 18}{3}$ $= -1.8A \rightarrow$

Superposition Theorem:

- Linear n/w :- If current in the n/w is linearly related to voltage as per Ohm's law.
- Bilateral n/w :- Volg. - current relation is the same for the current flowing in either direction.

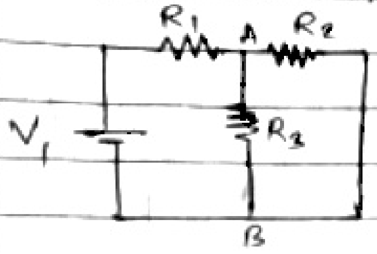
- This theorem is applicable for linear and bilateral n/ws.
- If there are no. of sources acting simultaneously in any linear bilateral n/ws, then each source acts independently of the others i.e. as if other source did not exit.
- This theorem is stated as, in a linear n/w containing more than one sources, the resultant current in any branch is the algebraic sum of the currents that would be produced by each source, acting alone, all other sources of emf being replaced by their respective internal resistance

e.g.

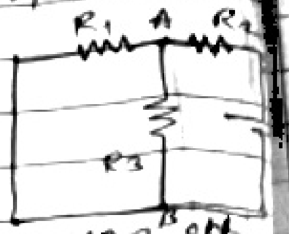


Voltage source - short
Current source - Openckt

Case 1: Consider source V_1 acting independently. Other sources must be replaced by internal resistances. But as internal resi. of V_2 is not given, it is assumed to be zero. So V_2 will be replaced by a short circuit, to obtain the current thr' AB due to source V_1 alone.



case 1 ckt



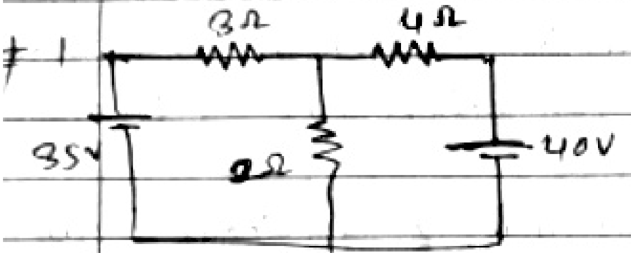
case 2 ckt

Case 2 :- Consider source V_2 alone. V_1 will be replaced by a short ckt to obtain current thr' AB. using any method

case 3 :- As per superposition th^m, the resultant current thr' branch AB is the sum of the currents thr' branch AB produced by each source acting independently.

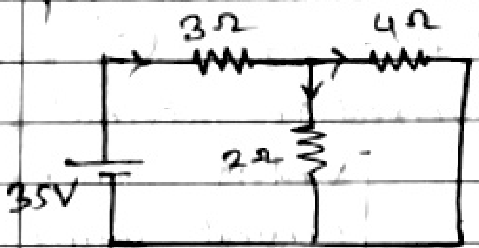
$\therefore$ $I_{AB} = I_{AB}$ due to V_1 + I_{AB} due to V_2

1



In the ckt shown, find the different branch currents by using Superposition th^m.

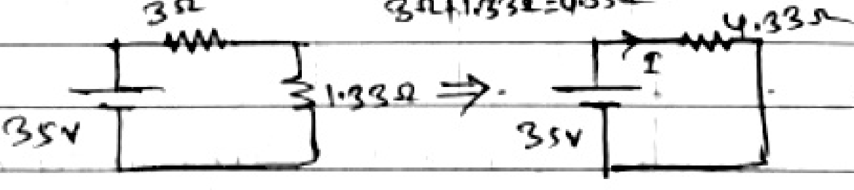
(1) Considering only 35V supply and replacing 40V supply by short ckt. We will get foll. ckt



In this ckt, resistors 2Ω||4Ω.

$$\therefore \frac{2 \times 4}{2+4} = \frac{8}{6} = 1.33\Omega$$

3Ω+1.33Ω=4.33Ω



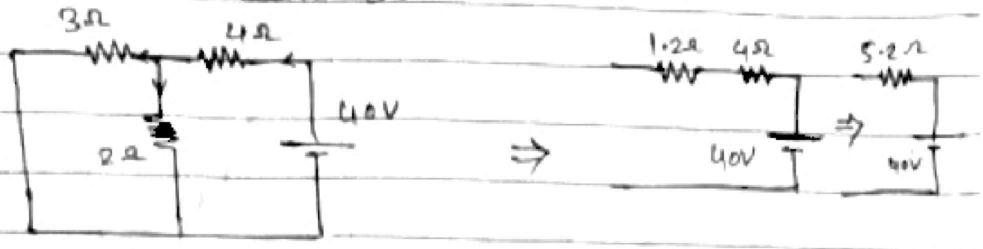
$$\therefore \text{current } I = \frac{35}{4.33} = 8.08\text{A}.$$

$\therefore$ current in 3Ω = 8.08 A →

current in 4Ω = $8.08 \times \frac{2}{2+4} = 2.69$ A →

current in 2Ω = $I_{3\Omega} - I_{4\Omega} = 8.08 - 2.69 = 5.39$ ↓

Step 2 : Considering 40V source acting alone, Replacing 35V source by short circuit ckt will be.



$$3\Omega \| 2\Omega = \frac{3\times 2}{3+2} = \frac{6}{5} = 1.2\Omega$$

It will be in series with the res^r.

Using Ohm's law current $I = \frac{40}{5.2} = 7.69A$.

$\therefore$ current in 4Ω resi^r $= 7.69A$ ($\leftarrow$) direction.

By current division rule, current in 3Ω res^r

$$I_{3\Omega} = 7.69 \times \frac{2}{3+2} = 3.08A\ (\leftarrow)$$

Current in 2Ω resi^r $= 7.69 - 3.08 = 4.61A\ (\downarrow)$

$\therefore$ Step 3 :- Total current thr' all required branches is summation of currents due to source 1 & source 2.

$$\therefore I_{3\Omega}(Tot.) = I_{3\Omega}\big|_{35V} + I_{3\Omega}\big|_{40V}$$

$$= 8.08(\rightarrow) + (3.08)(\leftarrow)$$

$$= 8.08 - 3.08$$

$$= 5A. \rightarrow$$

$$I(2\Omega)_{Tot.} = I_{2\Omega}\big|_{35V} + I_{2\Omega}\big|_{40V}$$

$$= 5.39(\downarrow) + 4.61(\downarrow)$$

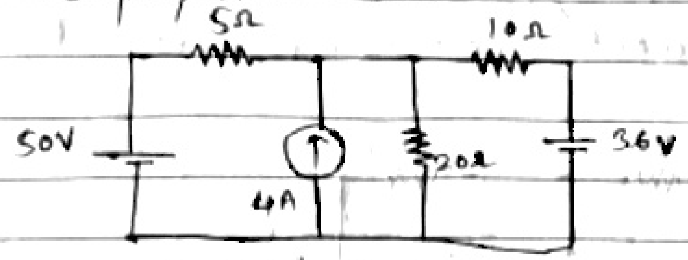
$$= 10A\ (\downarrow)$$

$$I(4\Omega)_{Tot.} = I_{4\Omega}\big|_{35V} + I_{4\Omega}\big|_{40V}$$

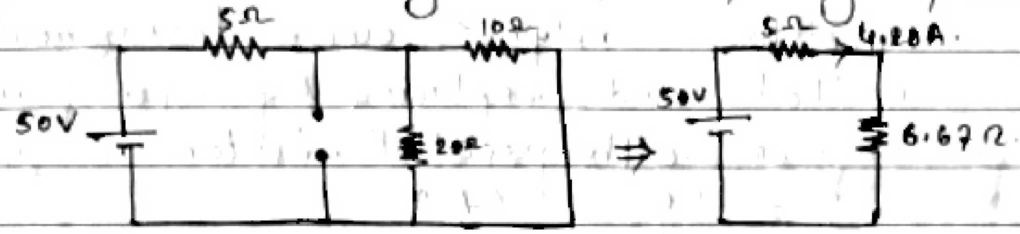
$$= 2.69(\rightarrow) + 7.69(\leftarrow)$$

$$= 2.69 - 7.69 = -5A \text{ or } 5A(\leftarrow)$$

Determine the current in 5Ω res^n in the n/w by superposition th^m.



Step1 :- Consider 50V source acting alone, and replacing 4A source by open circuit & 36V source by short ckt, we get foll.ckt

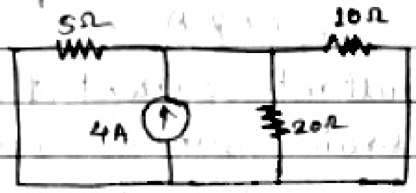


In this fig. resistors 20Ω||10Ω

$$\therefore \frac{20\times10}{20+10} = \frac{200}{30} = 6.67\,\Omega$$

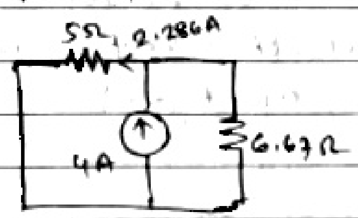
Current in 5Ω res^r = $I_{5\Omega} = \frac{50}{11.67} = 4.28A \rightarrow$

Step 2 :- Considering 4A source acting alone, replaced 50V & 36V volg. sources by short ckt we get foll. ckt



In this dia. resistors 20Ω||10Ω $\frac{20\times10}{20+10} = 6.67\Omega$

$\therefore$



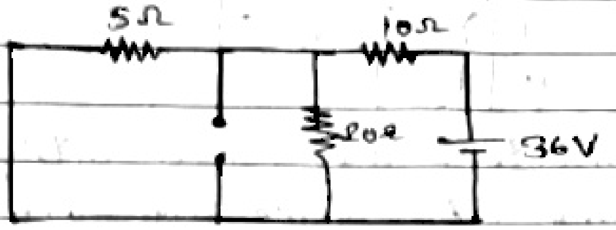
5Ω || 6.67 ∈

$\therefore$ By current division rule,

$$I_{5\Omega} = 4\times\frac{6.67}{5+6.67}$$

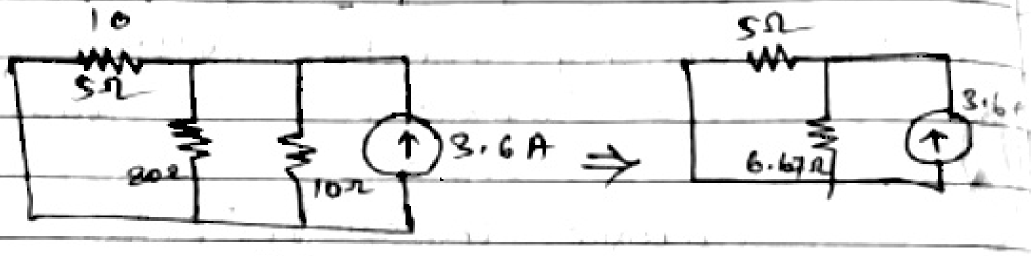
$$= 2.286\text{ A} \leftarrow$$

Step 3 : Considering 36V source acting alone, replacing 50V by short ckt and 4A current source by open ckt, ckt will be



For 36V volg. source with 10Ω series resistor, applying source transformation method and replacing by equi current source with parallel resistor we get

$$I = \frac{36}{10} = 3.6A \uparrow$$



$$20\Omega \| 10\Omega = 6.67\Omega$$

$I_{5\Omega}$ using current division rule

$$I_{5\Omega} = 3.6 \times \frac{6.67}{5+6.67}$$

$$= 2.057\ A \leftarrow$$

Step 4 :- Resultant current thr' 5Ω will be algebraic sum of currents due to individual sources acting alone.

$$\therefore \quad I_{5\Omega} = 4.28(\rightarrow) + 2.286A(\leftarrow) + 2.057A(\leftarrow)$$

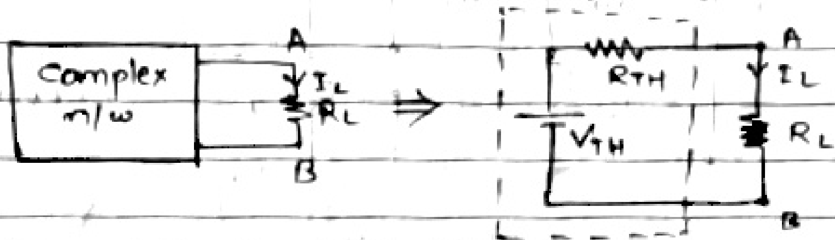
$$= 4.28A - 2.286A - 2.057A$$

$$= 0.063\ A\ (\leftarrow)$$

Thevenin's Theorem :- (French Engr M.L. Thevenin - 1883)

- It is used to simplify a complex problems and obtain the ckt. solution quickly.
- It is useful to find the current in a particular branch of a n/w as the resi. of that branch is varied by keeping all other resistances and sources constant.

As per this theorem, any two terminal complex n/w. can be replaced by a single source of emf V_{TH} known as Thevenin Voltage in series with a single resi. R_{TH} known as Thevenin resistance.

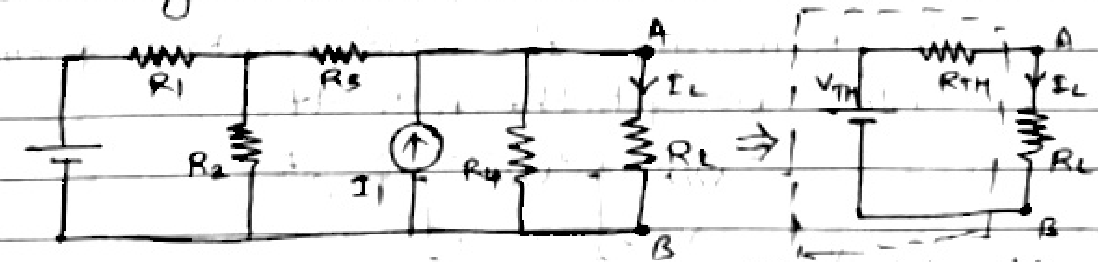


Thevenin's Equivalent ckt.

V_{TH} is the voltage that appears across terminals A and B with Load removed. The resi. R_{TH} is the resi. obtained with load removed and looking back into the terminals A and B when all the sources in the ckt are replaced by their internal resistances.

Once Thevenin's equ. ckt is obtained, current thr' any Load R_L connected across AB can be easily obtained.

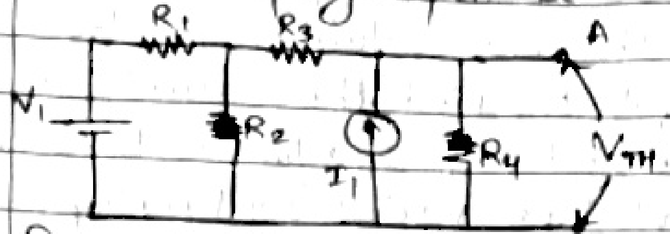
e.g.



Thevenin's ckt

V_{TH} :- Open ckt Thevenin's Voltage

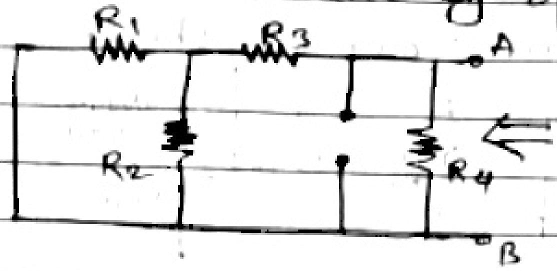
- V_{TH} is calculated betn A & B by removing R_L & keeping open ckt.



Thevenin's volg.

For calculating V_{TH} any method can be used.

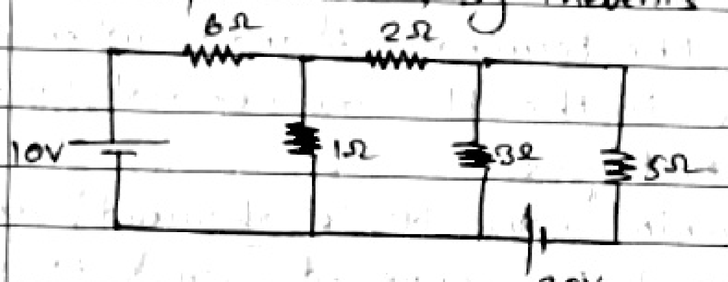
- R_{TH} is equivalent resi. obtained as viewed thr' terminals A and B with R_L removed and replacing volg. source with short ckt and current source by open ckt.



Thevenin's resistance

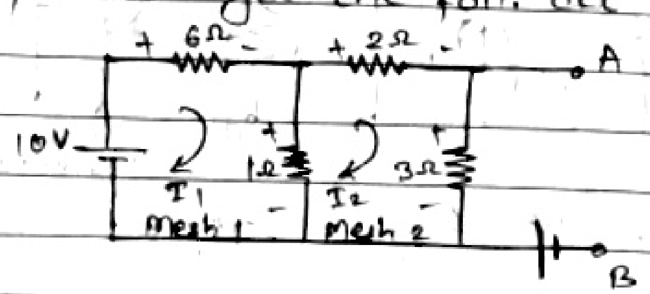
$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

\#1 Determine the current thr' 5Ω resistor in the n/w shown, by Thevenin's theorem.



Step 1 :- Calculation of V_{TH}

Removing the load resistance from the n/w we get the foll. ckt



Voltage across load terminals A and B, VAB is Thevenin's voltage. V_{TH}.

As shown in dotted line in dia. any path from A to B is selected. 3Ω resistor is present in the selected path and for V_{TH} calculation current thr' 3Ω res^r is required.
Using mesh analysis method, $I_{3Ω}$ can be calculated
Applying KVL to mesh 1

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$

$$\therefore 7I_1 - I_2 = 10 \quad \text{——— (1)}$$

Applying KVL to mesh 2

$$-2I_2 - 3I_2 - (I_2 - I_1) = 0$$

$$I_1 - 6I_2 = 0 \quad \text{——— (2)}$$

Solving (1) and (2)

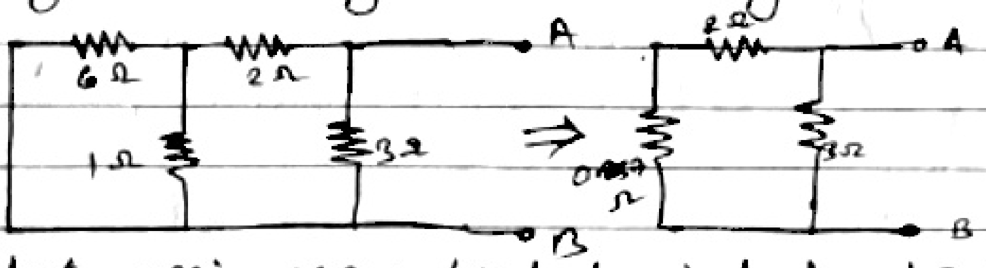
$$I_2 = 0.244A$$

$$\therefore I_{3Ω} = 0.244A \downarrow$$

Thus

$$V_{TH} = V_{AB}$$
$$= 20 + 3I_2$$
$$= 20 + 3(0.244)$$
$$= 20.732\ V$$

Step 2 :- Calculation of R_{TH}
Removing the load resi. from n/w and replacing the volt. sources by short ckt we get

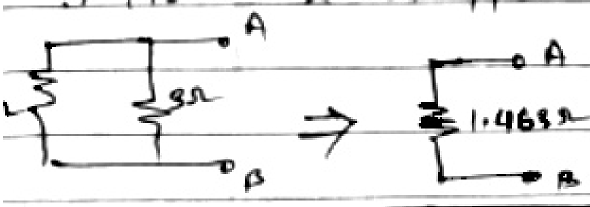


Equivalent resi. across load terminals A and B is called Thevenin's res. R_{TH}.

$$6Ω // 1Ω = \frac{6 \times 1}{6+1} = \frac{6}{7} = 0.857\,Ω$$

0.857Ω is in series with 2Ω = 2.857 Ω

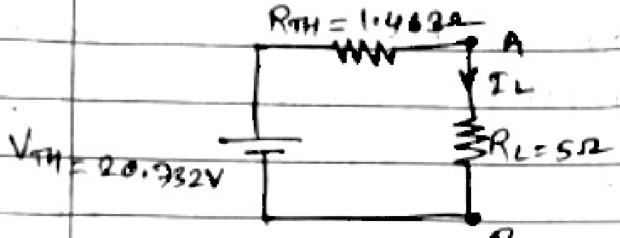
$$\therefore \text{Now } 2.857Ω // 3Ω = \frac{2.857 \times 3}{2.857 + 3} = 1.463\,Ω$$



$\therefore R_{TH} = R_{AB} = 1.463\Omega$

Step 3 :- Calculation of Load current

With V_{TH} & R_{TH}, Thevenin's equi. ckt can be drawn as :

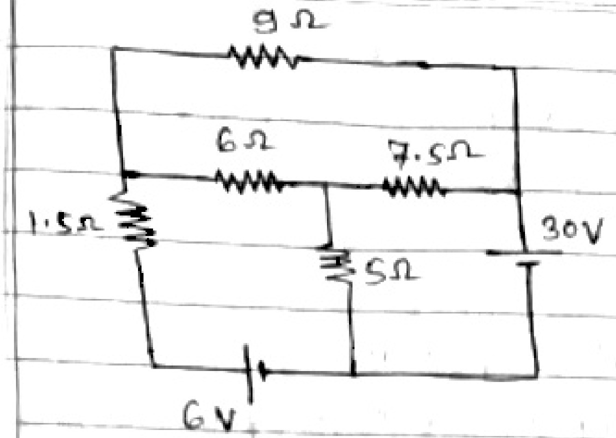


Using Ohm's law:

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

$$= \frac{20.732}{5 + 1.463}$$

$$\boxed{I_L = 3.21\,A}$$



Determine the current thr' 1.5Ω resistor in the network shown in fig. using Thevenin's theorem.

1.5Ω resistor is load resistor R_L.
Its terminals will be named as A & B as load terminals.

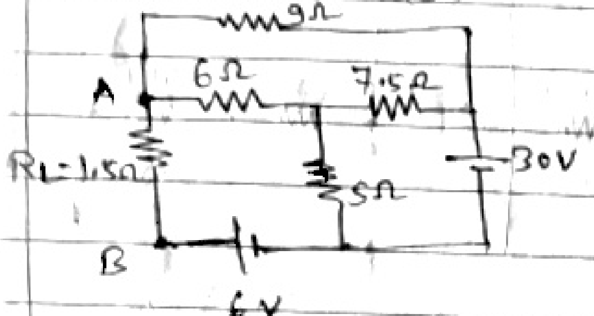
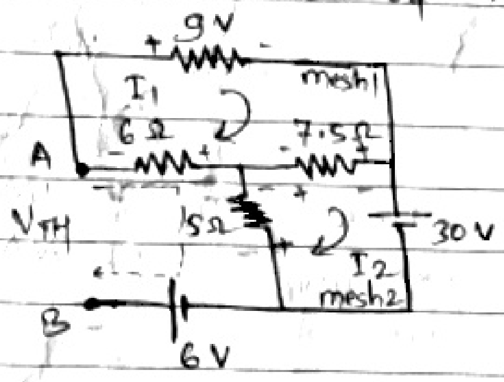


Diagram showing 1.5Ω res^r as R_L at load terminals A & B

step 1 :- Calculation of V_{TH} (Removing R_L from ckt)



Voltage across A & B will be V_{TH} (Thevenin's volt.)

Using mesh analysis method $V_{AB} = V_{TH}$ can be calculated.

Applying KVL to mesh 1

$$-9I_1 - 7.5(I_1 - I_2) - 6I_1 = 0$$

$$\therefore \quad -22.5I_1 + 7.5I_2 = 0 \quad \text{—— (1)}$$

Applying KVL to mesh 2

$$-7.5(I_2 - I_1) - 30 - 5I_2 = 0$$

$$\therefore \quad 7.5I_1 - 12.5I_2 = 30 \quad \text{—— (2)}$$

solving eq^n's (1) & (2) we get

$$\boxed{\begin{aligned} I_1 &= -1\,\text{Amp} \\ I_2 &= -3\,\text{Amp} \end{aligned}}$$

$\therefore V_{AB} = V_{TH}$

$= -6 - 5I_2 - 6I_1$

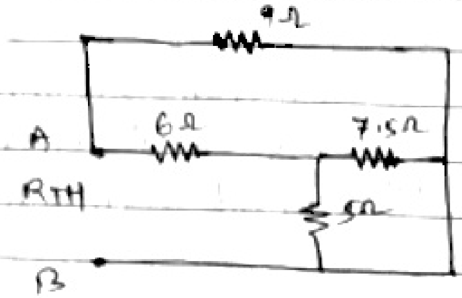
$= -6\,(-5\times-3)-6(-1)$

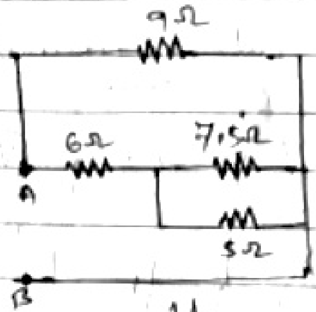
$\boxed{V_{TH} = 15V}$

Step 2 :- Calculation of R_{TH}

Remove R_L & replace voltage sources by short circuit.

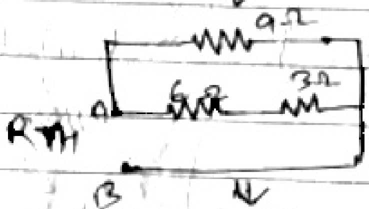
Now ckt will be



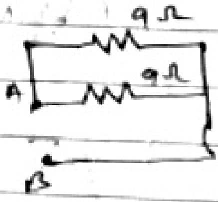


$7.5\Omega \parallel 5\Omega$

$\frac{7.5\times5}{7.5+5} = 3\Omega$

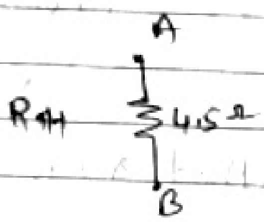


6+3Ω
=9Ω



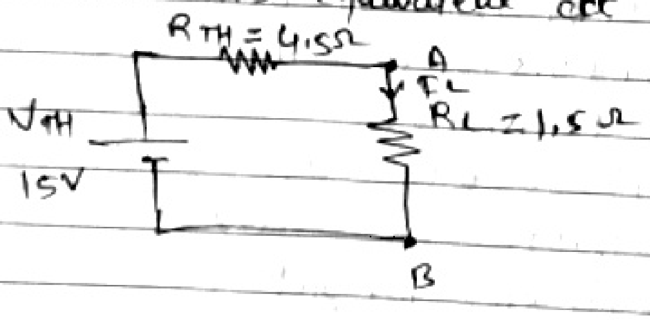
$9\Omega \parallel 9\Omega$

$\frac{9\times9}{9+9} = 4.5\Omega$



Step 3 :- Calculation of load current

Thevenin's equivalent ckt will be now



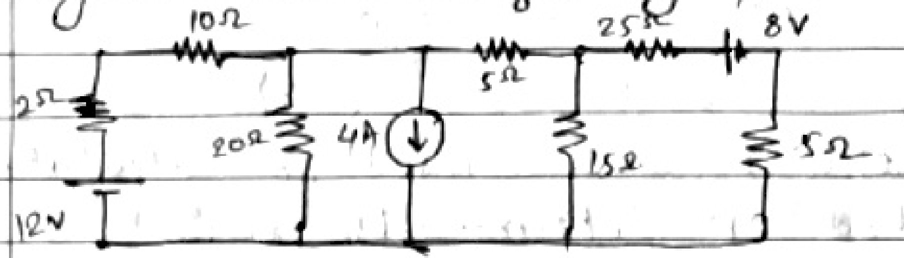
$I_L = \frac{V_{TH}}{R_L + R_{TH}}$

$= \frac{15}{1.5+4.5}$

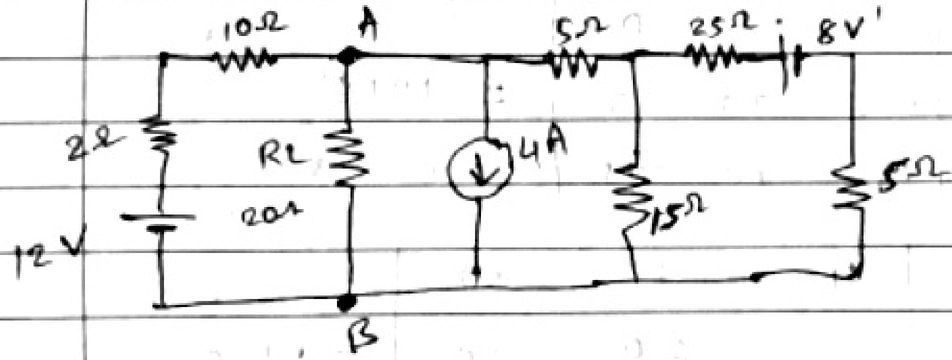
$= 2.5A \downarrow$

$\therefore$ Current flowing thr' 1.5Ω = 2.5A ↓

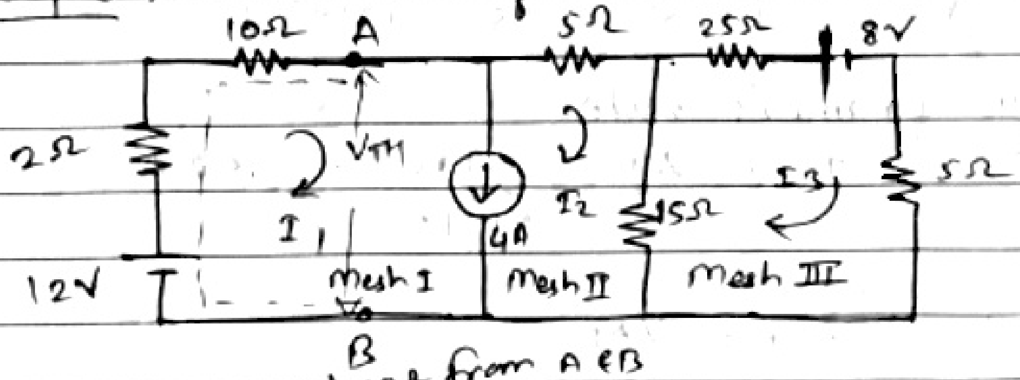
#2 Using Thevenin's theorem, obtain the power drawn by 20Ω resⁿ in the following n/w.



Power across 20Ω resⁿ is $P_{20\Omega} = I_{20\Omega}^2 \times 20$.
so, we can assume current thr' 20Ω resⁿ as load current and 20Ω resⁿ as load resistor.
Its terminals can be named as A & B.



Step I : calculation of V_{TH}



Removing load 20Ω from A & B or open circuiting at terminals A & B we get above circuit.
Volg. betⁿ A & B is V_{TH}.

Mesh I and Mesh II ~~are~~ as supermesh as shown in fig.

$\therefore \quad I_1 - I_2 = 4 \quad —— (1)$

Now by applying KVL to the supermesh we get

$$12 - 10I_1 - 5I_2 - 15(I_2 - I_3) - 2I_1 = 0$$

or $\quad -12I_1 - 20I_2 + 15I_3 = -12 \quad —— (2)$

Now by applying KVL to mesh III

$$-8 - 25I_3 - 5I_3 - 15(I_3 - I_2) = 0$$

or $\quad 15I_2 - 45I_3 = 8 \quad —— (3)$

Eqⁿs (1), (2) & (3) can be used to calculate I_1 by method of determinants as:

$$\begin{bmatrix} 1 & -1 & 0 \\ -12 & -20 & 15 \\ 0 & 15 & -45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -12 \\ 8 \end{bmatrix}$$

So $\quad \Delta = \begin{vmatrix} 1 & -1 & 0 \\ -12 & -20 & 15 \\ 0 & 15 & -45 \end{vmatrix} = 1215$

$$\Delta_1 = \begin{vmatrix} 4 & -1 & 0 \\ -12 & -20 & 15 \\ 8 & 15 & -45 \end{vmatrix} = 3120$$

By Cramer's rule

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{3120}{1215} = 2.57\,A$$

$$\therefore \quad V_{TH} = V_{AB}$$

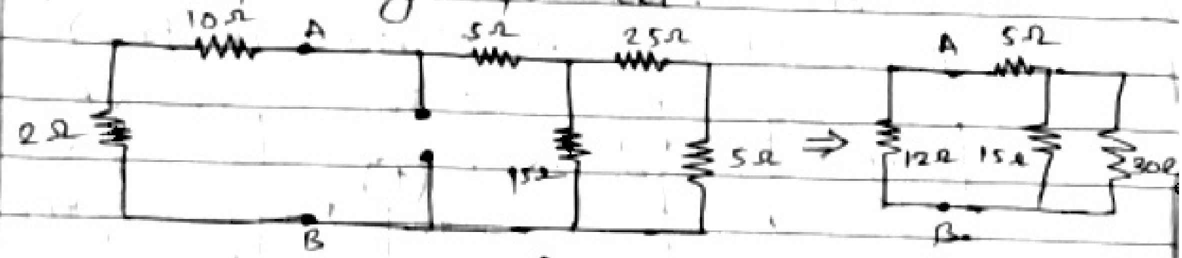
$$= 12 - 2I_1 - 10I_1$$

$$= 12 - 2(2.57) - 10(2.57)$$

$$\boxed{V_{TH} = -18.84\,V}$$

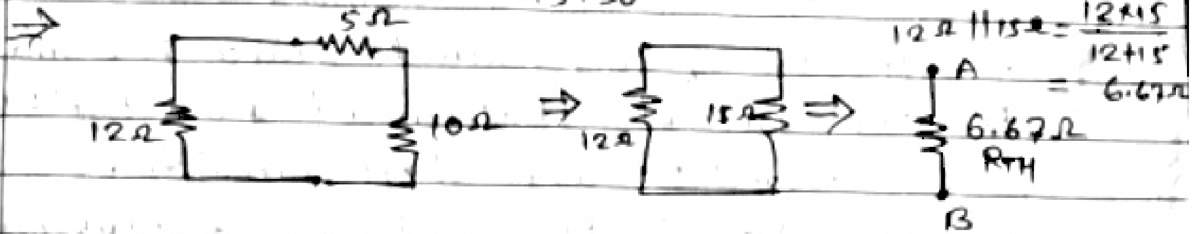
Step 2 :- Calculation of R_{TH}

Removing R_L from A & B and replacing volg source by short ckt and current source by open ckt, we get foll. ckt:



$$15\Omega \parallel 30\Omega = \frac{15 \times 30}{15+30} = \frac{450}{45} = 10\Omega$$

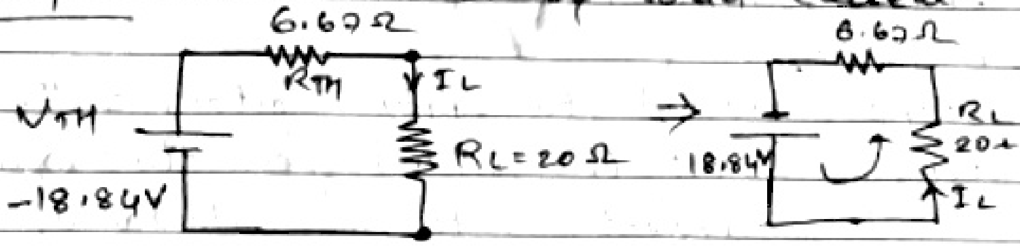
$\Rightarrow$



$$12\Omega \parallel 15\Omega = \frac{12 \times 15}{12+15} = 6.67\Omega$$

$\therefore R_{TH} = 6.67\Omega$

Step 3 :- Calculation of load current:



Using Ohm's law

$$I_L = \frac{V_{Th}}{R_{TH}+R_L} = \frac{18.84}{6.67+20} = 0.706\ A(\uparrow)$$

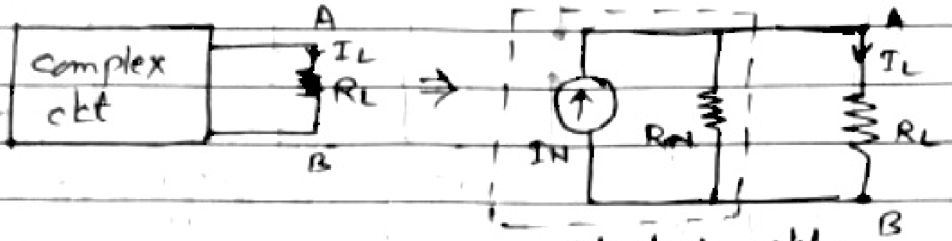
$\therefore$ Power drawn by 20Ω Load res^r will be

$$P_{20\Omega} = I_{20\Omega}^2 \times 20$$
$$= (0.706)^2 \times 20$$

$$\boxed{P_{20\Omega} = 9.97\ \text{Watts}}$$

Norton's Theorem:-

- Any two-terminal n/w can be replaced by a single current source of magnitude I_N (Norton's current) in parallel with a single resistance R_N (Norton's resistance)



Norton's ckt.

- o/p I_N of the current source is equal to the current that would flow thr' AB when A and B are short circuited.
- Resi. R_N is the resi. of the n/w measured betn A and B with the load removed and replacing the source with their internal resistances.

Steps involved to apply Norton's thm.

step 1: Short the branch resi. thr' which current is to be calculated.

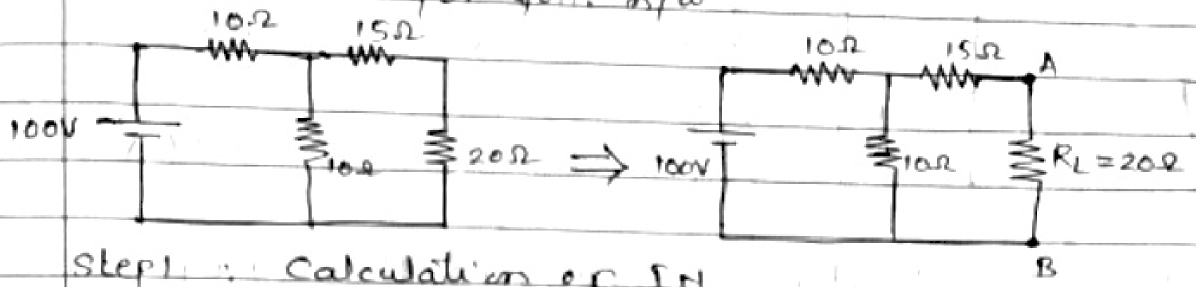
step 2: Obtain the current thr' the short circuited branch using any method. This current is Norton's current I_N.

step 3: Calculate R_N as viewed thr' the two terminals of the branch from which current is to be calculated by removing that branch res. & replacing all sources by their internal resistances.

step 4: Draw Norton's equi. ckt

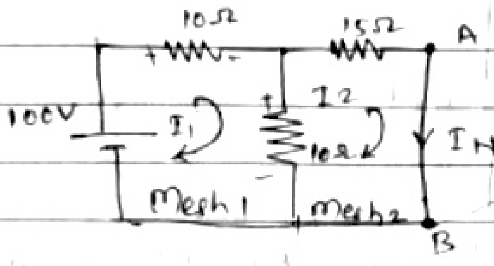
step 5: Reconnect the branch resi. R_L & calculate $I_L = I_N \frac{R_N}{R_L + R_N}$

1. Using Norton's theorem method, find the current in 20Ω res' for foll. n/w



Step 1 : Calculation of I_N

Removing R_L & replacing by short ckt, n/w will be



Current flowing thr' short ckt placed bet" A & B is called Norton current I_N.

By mesh analysis
Applying KVL to mesh 1

$$100 - 10I_1 - 10(I_1 - I_2) = 0$$

or $-20I_1 + 10I_2 = -100$ ——— (1)

Applying KVL to mesh 2

$$-10(I_2 - I_1) - 15I_2 = 0$$

or $10I_1 - 25I_2 = 0$ ——— (2)

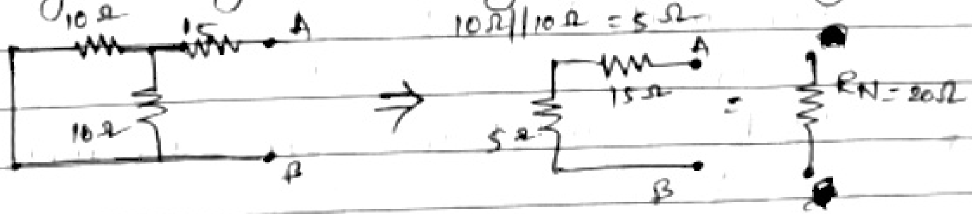
Solving eq^{n}s (1) and (2)

$$I_2 = 2.5A.$$

$\therefore I_N = 2.5A \downarrow$

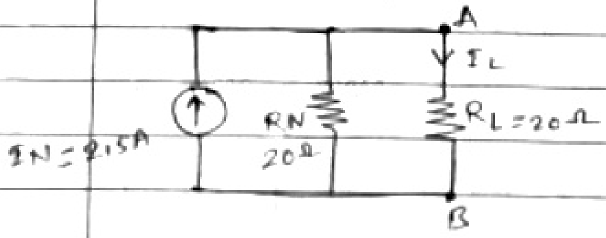
Step 2 : Calculation of R_N

Removing R_L from n/w & keeping open ckt.
Replacing voltage source by short ckt, we get n/w:



Step 3 :- Calculation of load current

Norton's equivalent ckt is

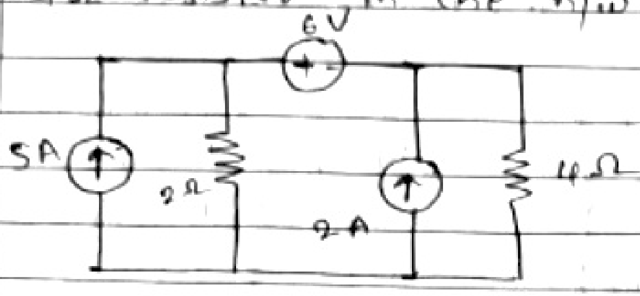


By current division rule

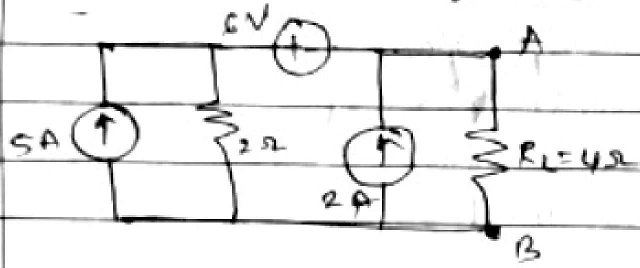
$$I_L = I_{20\Omega} = 2.5 \times \frac{20}{20+20}$$

$$= 1.25A \downarrow$$

\# 2 By Norton's theorem, find the current in 4Ω resistor in the n/w shown.

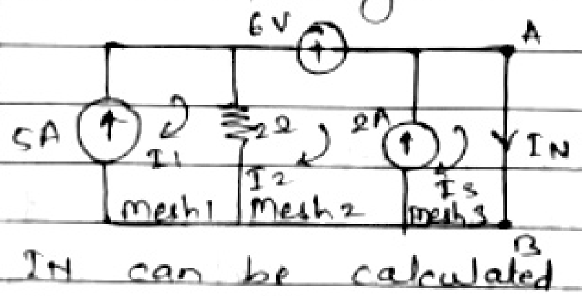


4Ω resistor is load resr R_L. It's terminals A & B will be load terminals.



Step1 Calculation of I_N

Remove load resistance & short ckt the load terminals, we get



I_N can be calculated by mesh analysis method:

For mesh 1

$I_1 = 5A$ —— (1)

Mesh 2 and Mesh 3 are supermesh.

$I_3 - I_2 = 2$ —— (2)

By applying KVL to the supermesh we get

$-6 - 2(I_2 - I_1) = 0$ —— (5)

or $-2I_1 - 2I_2 = 6 \;=\; -I_1 - I_2 = 3 \longrightarrow$ (3)

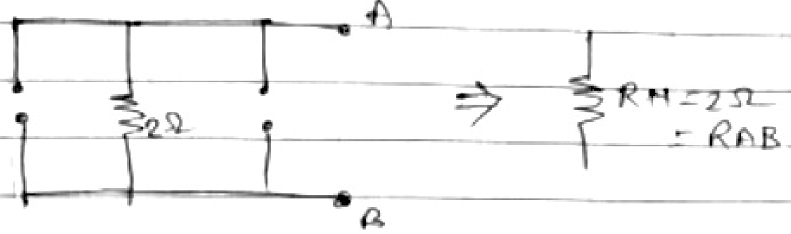
Solving eqⁿ's ①, ② & ③ we get

$I_3 = 4A$

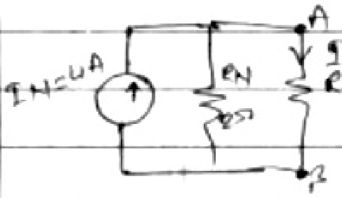
$\therefore I_N = 4A \downarrow$

step 2 Calculation of R_N

Removing load resi from A & B & replacing volt. source by short ckt and current source by open ckts we get foll. n/w



step 3 Calculation of load current



(Norton's equivalent ckt)

By current division rule,

$$I_L = I_{4\Omega} = 4 \times \frac{2}{2+4}$$

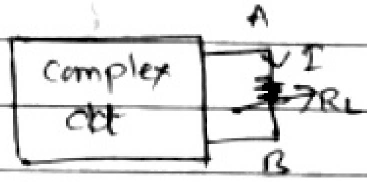
$$= 1.33A \downarrow$$

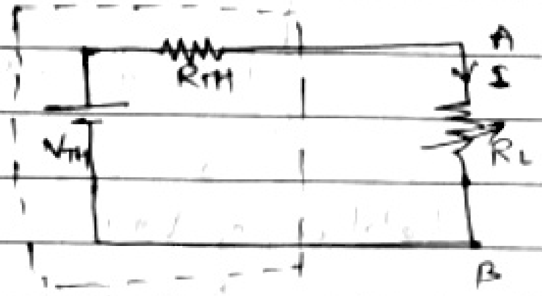
Maximum Power Transfer Theorem

Transfer of max. power from a source to load is required.

Theorem:

In dc ckts, max. power is transferred from a source to a load when the load resistance is made equal to the equivalent res. of the n/w as viewed from the load terminals, with load removed and replacing all sources with their internal resistances.





Thevenin's circuit

Complex ckt supplies power to the load R_L. ckt enclosed in box can be replaced by Thevenin's eq^t ckt consisting of single source of emf V_{TH} (Thevenin's voltage) in series with a single res^n R_{TH}, res. measured bet^n terminals A and B with R_L removed and replacing the sources with their internal resistances.

As per Max. Power Transfer theorem, max. power will be transferred from the ckt to the load when R_L is made equal to R_{TH} ($R_L = R_{TH}$)

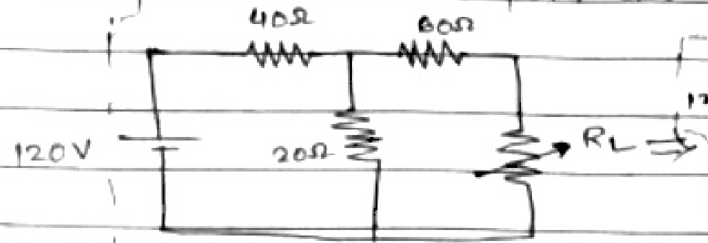
$$P = \frac{V_{TH}^2 R_L}{(R_L + R_{TH})^2}$$

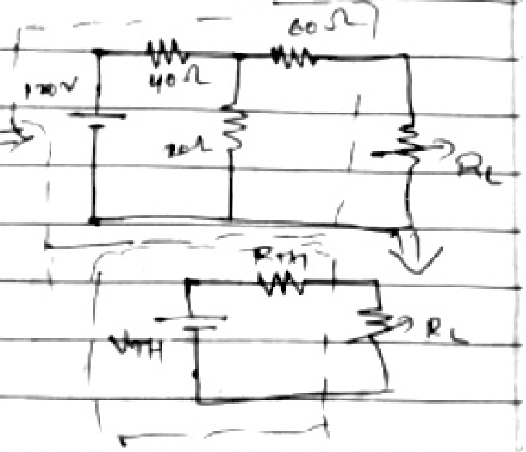
$$\boxed{P_{max} = \frac{V_{TH}^2}{4R_{TH}} \text{ watts}}$$

when $R_L = R_{TH}$ $P = P_{max}$

$$\therefore P_{max} = \frac{V_{TH}^2 R_{TH}}{(R_L + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}} \text{ W}$$

1 Calculate the value of R_L so that it will absorb the max. power and find out the max. power in the foll. ckt. Thevenin's ckt.

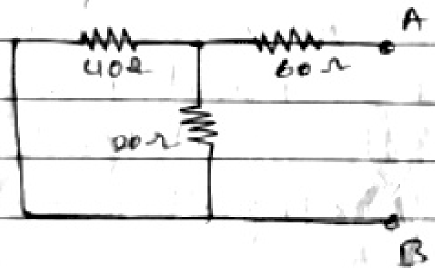




Step 1 Calculation of R_{TH}

Removing load resi. from n/w & replacing volg. source by short ckt.

So n/w becomes:

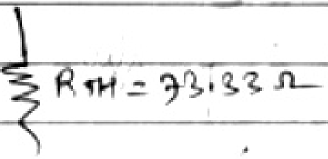


$$40 \parallel 20 = \frac{40 \times 20}{40+20} = \frac{800}{60}$$

$$= 13.33\Omega$$

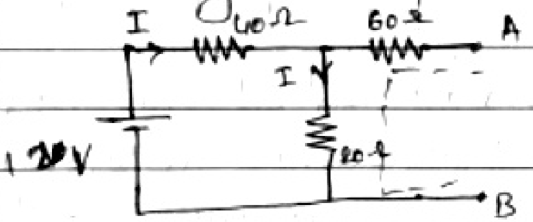
13.33Ω in series with $60\Omega = 13.33 + 60$

$$= 73.33\Omega$$



Step 2 : Calculation of V_{TH}

Removing R_L from the n/w, we will get



Volg. across terminals A & B will be Thevenin's volg. V_{TH}. To calculate V_{AB}, the selected path from A to B is marked by dotted line as shown in the dia. As this path contains 20Ω resi^r, current thr' this res. is required.

Using Ohm's law, circuit current

$$I = \frac{120}{40+20} = 2A$$

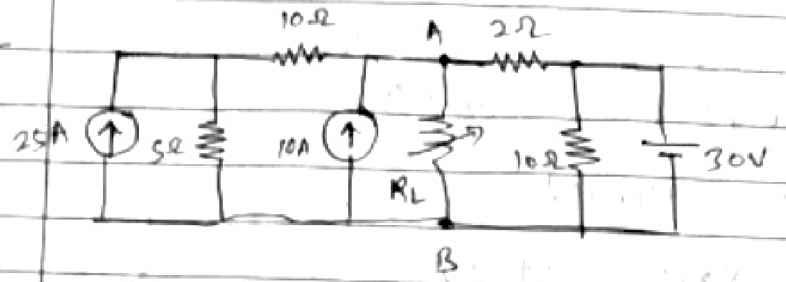
$$\therefore V_{TH} = V_{AB} = (20 \times 2) + (60 \times 0) = 40V$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} \qquad V_{TH} = 40V, \quad R_{TH} = 73.33\Omega$$

$$= \frac{(40)^2}{4(73.33)}$$

$$P_{max} = 5.45W$$

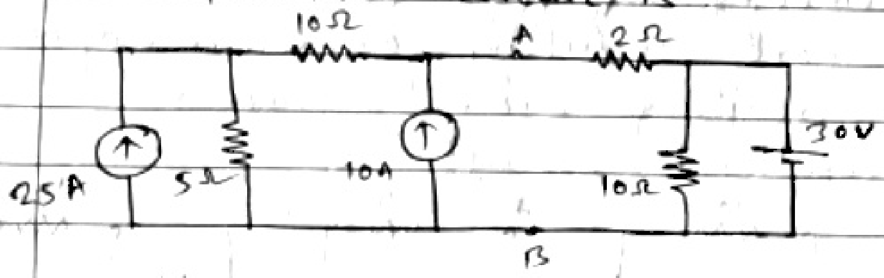
2



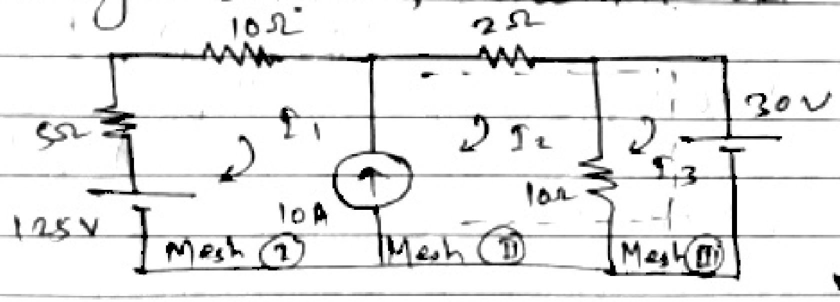
Determine the max power delivered to R_L.

Step 1 :- Calculation of V_{TH}

- Removing R_L & making open ckt at load terminals A & B
- Resultant circuit is :



Converting 25A, 5Ω current source into equi. voltage source, resultant ckt is



Selected path from A to B is marked by dotted line for V_{TH} calculⁿ.

From above dia. mesh (I) & mesh (II) are forming Supermesh.

$$\therefore \quad I_2 - I_1 = 10 \quad \text{--- (1)}$$

Applying KVL to supermesh we get eqⁿ as

$$+125 - 5I_1 - 10I_1 - 2I_2 - 10(I_2 - I_3) = 0$$

Simplifying and rearranging eqⁿ.

$$-15I_1 - 12I_2 + 10I_3 = -125 \quad \text{--- (2)}$$

Applying KVL to mesh (III)

$$-10(I_3 - I_2) - 30 = 0$$

$$-10I_3 + 10I_2 = 30$$

$$\therefore I_2 - I_3 = 3 \quad \text{--- (3)}$$

Solving eqⁿs (1), (2) and (3)

$$I_2 = 14.41\,A$$

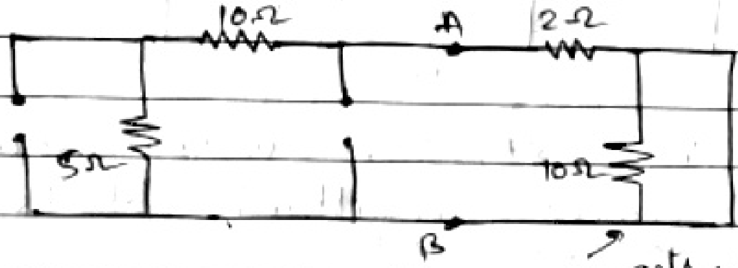
$$V_{AB} = V_{TH} = 30 + I_2(2\Omega)$$

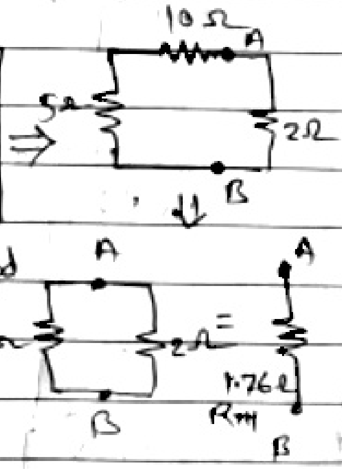
$$= 30 + 14.41(2)$$

$$\boxed{V_{TH} = 58.82\,V}$$

Step2 :- Calculation of R_{TH}

- Removing R_L from load terminal A & B
- Short circuiting volg. source & open circuiting current sources
- Resultant circuit is





10Ω gets short circuited ∴ =0

$$\boxed{R_{TH} = 1.76\,\Omega}$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

$$= \frac{(58.82)^2}{4 \times (1.76)}$$

$$\boxed{P_{max} = 491.45\,W}$$