

Assignment – 1

Partial Differentiation

Questions

1. If $u = e^{xyz}$, Prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$

2. If $z = x^2 \tan^{-1} \left(\frac{y}{x} \right)$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$

3. If $u = f \left(\frac{x^2}{y} \right)$, prove that $x \frac{\partial u}{\partial x} + 2y \frac{\partial u}{\partial y} = 0$

And $x^2 \frac{\partial^2 u}{\partial x^2} + 3xy \frac{\partial^2 u}{\partial x \partial y} + 2y^2 \frac{\partial^2 u}{\partial y^2} = 0$

4. If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, prove that $u_x + u_y + u_z = 0$

5. If $z = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = \left(\frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta} \right)^2$$