D.C. Circuits

Classification of electrical energy sources:

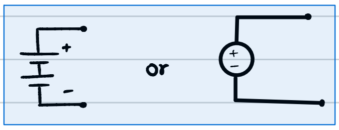
1) Voltage Source:
   i) Independent: a) Ideal
   b) Practical
   ii) Dependent

2) Current Source
   i) Independent: a) Ideal
   b) Practical
   ii) Dependent

Independent Voltage Source

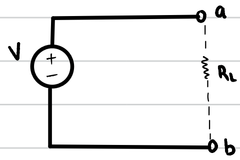
1) Ideal Voltage Source

• Symbol:



→ Case I:

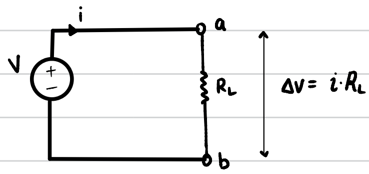
With $R_L = \infty$ (open circuit)



• Complete voltage 'V' is available across terminals 'a' & 'b'

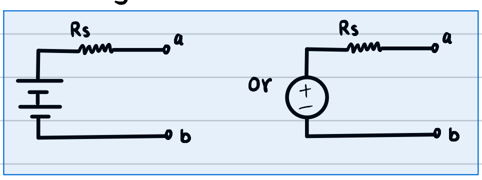
→ Case II

With load resistance = R_L



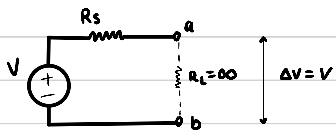
2) Practical Voltage Source

Symbol:



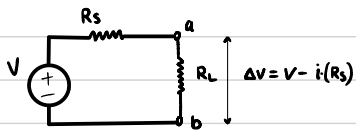
→ Case I:

With $R_L = \infty$ (open circuit)



→ Case II

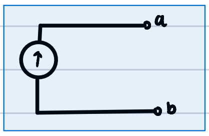
With load resistance = R_L



Independent Current Source

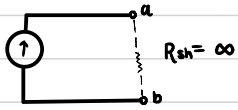
1) Ideal Current Source

Symbol:

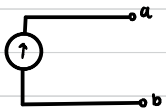


Case I:

→ With $R_L = \infty$ (open circuit)

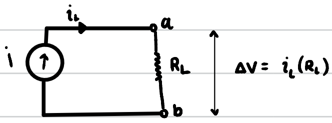


• Shunt resistance is ∞.



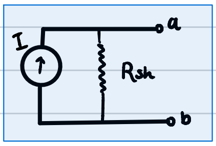
→ Case II

With load resistance $= R_L$



2) Practical Current Source

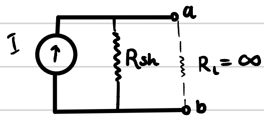
Symbol:



* R_{sh}: Internal shunt resistance

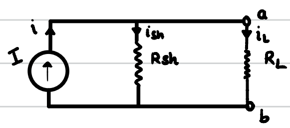
Case I:

→ With $R_L = \infty$ (open circuit)



→ Case II

With load resistance $= R_L$



• $i_L = i - i_{sh}$

Dependent Voltage Source

• Voltage or current of source depends on some other voltage or current

• 4 types —— Having 2 input & 2 output terminals

i) 'V' controlled 'V' source (VCVS)

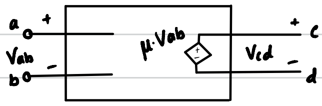
ii) 'V' controlled 'C' Source (VCCS)

iii) 'C' controlled 'V' source (CCVS)

iv) 'C' controlled 'C' source (CCCS)

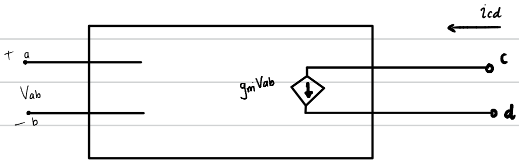
VCVS

Symbol:



$V_{cd} = \mu \cdot V_{ab}$

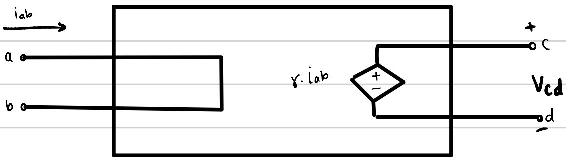
VCCS



$i_{cd} = g_m \cdot V_{ab}$

g_m = transconductance / mutual conduction

CCVS

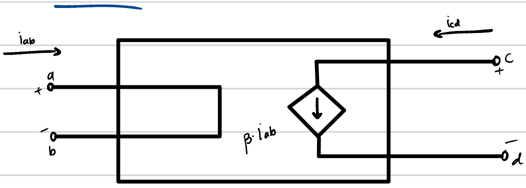


$V_{cd} = r \cdot i_{ab}$

r: mutual resistance / transresistance

unit: V/Amp or ohm.

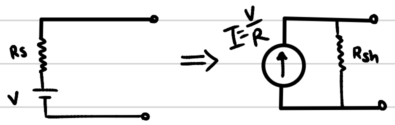
CCCS



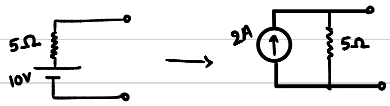
where β = current gain

Transformation of Sources

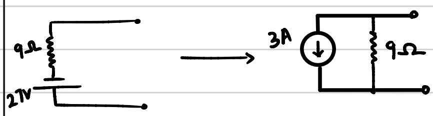
Case – I



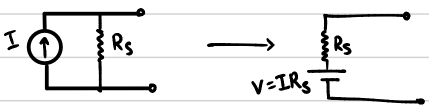
Example: i)



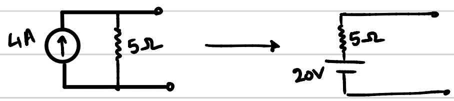
ii)



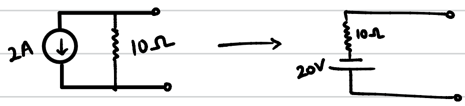
Case – II



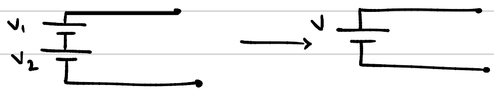
Example: i)



ii)



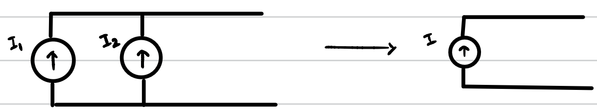
Case – III



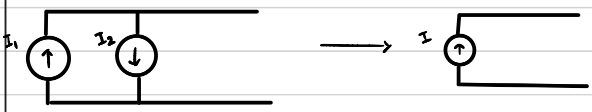
$\{V = V_1 + V_2\}$

$\{V = |V_1 - V_2|\}$

Case – IV



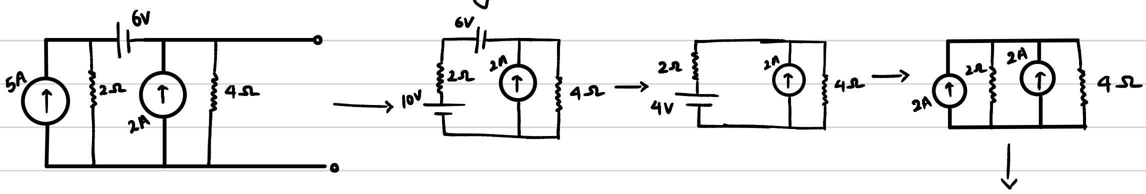
$\{I = I_1 + I_2\}$

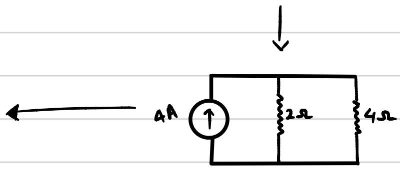


$\{I = |I_1 - I_2|\}$

Q. Find value of current in following circuits.

i)





4A ⟶ 6Ω

$= \frac{2}{3}$ parts

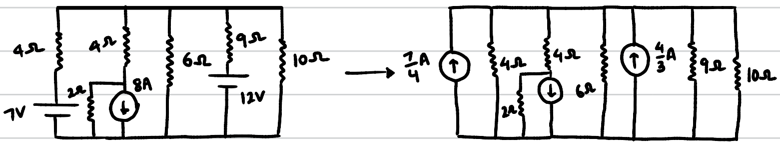
Current through 4Ω resistor

$= 2 \times \frac{2}{3} = \frac{4}{3}$ A

Ans: Current through 4Ω resistor is 1.334 A

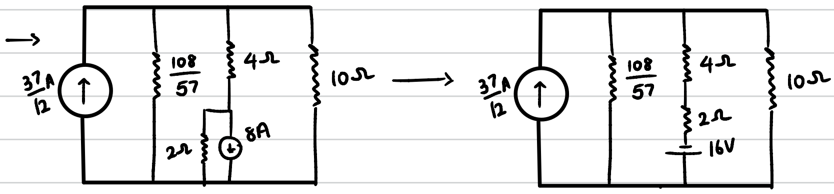
ii)

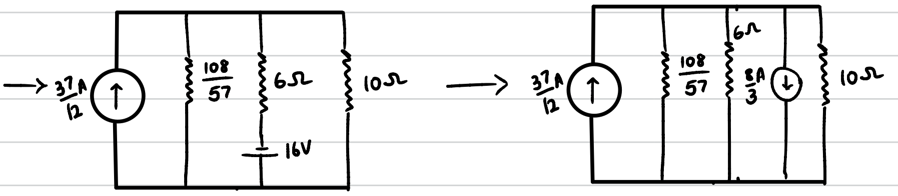
Check (0.0516A)



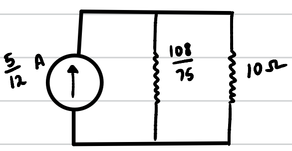
$\left(\frac{7}{4} + \frac{4}{3} = \frac{21+16}{12} = \left(\frac{37}{12}\right)\right)$

$\left(\frac{1}{9} + \frac{1}{6} + \frac{1}{4} \quad \frac{24+36+54}{216} = \frac{\cancel{114}^{57}}{\cancel{216}_{108}} = \frac{57}{108}\right)$





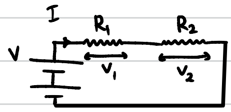
$\left\{ \frac{37}{12} - \frac{8}{3} = \frac{37-32}{12} = \left(\frac{5}{12}\right)A \qquad \frac{57}{108} + \frac{1}{6} = \frac{57+18}{108} = \left(\frac{75}{108}\right)\Omega \right\}$



$I_{10\Omega} = \frac{5}{\cancel{12}} \times \frac{\frac{\cancel{108}^{9}}{\cancel{75}_{15}}}{\frac{108}{75} + 10} = \frac{\frac{9}{15}}{\frac{858}{\cancel{75}_{5}}} = \frac{45}{858} = \boxed{0.0524 \text{ A}}$

Voltage Divider Circuit

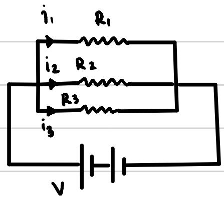
- $V_1 = I(R_1) = \frac{V}{(R_T)} \cdot (R_1)$



$V = V_1 + V_2$

- $V_2 = I(R_2) = \frac{V}{R_T} \cdot (R_2)$

Current Divider Circuit



- $i_1 = \frac{V}{R_1}$
- $i_2 = \frac{V}{R_2}$
- $i_3 = \frac{V}{R_3}$

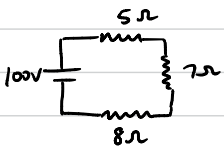
$i_T = i_1 + i_2 + i_3$

$i_T = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

$\boxed{i_T = \frac{V}{R_T}}$

Q. Determine the current through & voltages across 5Ω, 7Ω, 8Ω connected in series across 100V supply.

A.

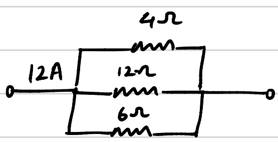


$R_T = 20\Omega$

$\frac{V}{R} = \frac{100}{20} = \boxed{5A}$

$V_1 = 5(5)$ $\boxed{= 25V}$, $V_2 = 5(7)$ $\boxed{= 35V}$, $V_3 = 5(8)$ $\boxed{= 40V}$

Q.



$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{12} + \frac{1}{6}$

$= \frac{3+1+2}{12} = \frac{6}{12}$

$(R_T = 2\Omega)$

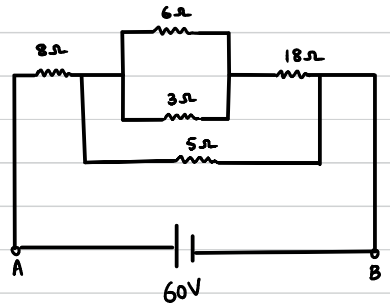
$V = iR$

$= 12(2)$

$= 24V$

$$\boxed{\begin{aligned} i_1 &= \frac{24}{4} = 6A \\ i_2 &= \frac{24}{12} = 2A \\ i_3 &= \frac{24}{6} = 4A \end{aligned}}$$

Q.



$\frac{1}{6}+\frac{1}{3}=\frac{3+6}{18}=\frac{1}{2} \Rightarrow 2\Omega+18$

$=20\Omega$

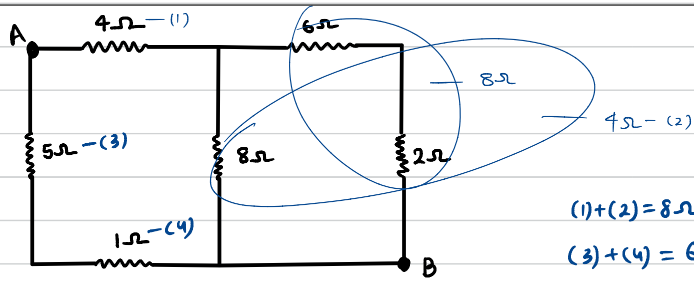
$\frac{1}{20}+\frac{1}{5}=\frac{5}{20} \Rightarrow \frac{1}{4}=4\Omega$

$4+8=(12\Omega)-R_T$

$V=IR$

$I=\frac{60}{12}=(5A) \longrightarrow$ $\boxed{\text{Current through } 8\Omega \text{ resistor } = 5A}$

Ans: 3·43Ω



$(1)+(2)=8\Omega$

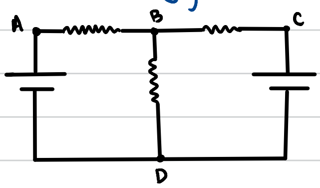
$(3)+(4)=6\Omega$

$\frac{1}{R}=\frac{1}{8}+\frac{1}{6}$

$\longrightarrow \frac{1}{R}=\frac{6+8}{48} \longrightarrow \frac{1}{R}=\frac{\cancel{14}}{\cancel{48}}\ \frac{7}{24}$

$\boxed{\therefore R=\frac{24}{7}=3{\cdot}42\ \Omega}$

Terminology



1] Node: Junction where two or more elements are connected.
(In above example: A, B, C, D)

2) Branches: Any element/s connected between two nodes.
(In above example: AB, BD, BC)

3) Loop: Any closed path in a circuit.
(In example: ABDA, BCDB, ABCDA)

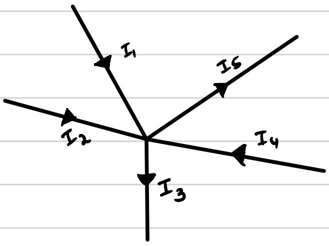
4) Mesh:

Kirchoff's Law

1) Kirchoff's current law (KCL)
2) Kirchoff's voltage law (KVL)

Kirchoff's Current Law

Algebraic sum of current meeting at a junction/node in an electric circuit is zero.



If incoming current = (+)ve
& outgoing current = (−)ve

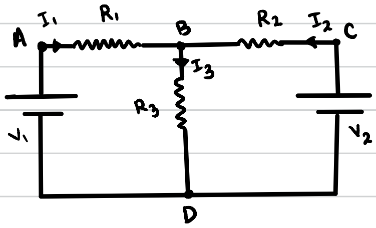
$$\boxed{I_1 + I_2 + I_4 = I_3 + I_5}$$

Kirchoff's Voltage Law

In any closed circuit / mesh, algebraic sum of electromotive forces & voltage drop equals zero.

Sign Conventions

1) Rise in potential — (+)ve
2) Fall in potential — (−)ve
3) ΔV along the flow of current across a resistor — (−)ve
4) ΔV opposite the flow of current across a resistor — (+)ve



Using KVL in ABDA

$V_1 - i_1R_1 - i_3R_3 = 0$

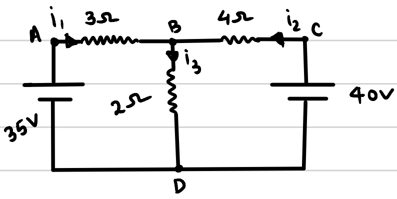
Using KVL in BCDB

$V_2 - i_2R_2 - i_3R_3 = 0$

Using KVL in ABCDA

$V_1 - i_1R_1 + i_2R_2 - V_2 = 0$

Q.



A.

$35 - i_1(3) - i_3(2) = 0$

$3i_1 + 2i_3 = 35 \quad — (1)$

$40 - i_2(4) - i_3(2) = 0$

$4i_2 + 2i_3 = 40$

$2i_2 + i_3 = 20 \quad — (2)$

$35 - i_1(3) + i_2(4) - 40 = 0$

$4i_2 - 3i_1 = 5 \quad — (3)$

$i_3 = (i_1 + i_2)$

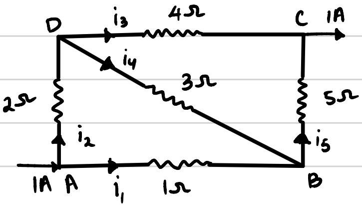
$2i_2 + i_1 + i_2 = 20$

$i_1 + 3i_2 = 20$

$5i_1 + 2i_2 = 35$

$$\boxed{\begin{aligned} \therefore\ i_1 &= 5A \\ i_2 &= 5A \\ i_3 &= 10A \end{aligned}}$$

Q.



$i_1 + i_2 = 1A$

$i_3 + i_5 = 1A$

$i_3 + i_4 = i_2$

$i_1 + i_4 = i_5$

$-i_2(2) - i_4(3) + i_1(1) = 0$

$i_1 = 2i_2 + 3i_4 \longrightarrow i_4 = \dfrac{i_1 - 2i_2}{3} \longrightarrow 1 - i_2 = 2i_2 + 3(i_2 - i_3)$

$1 = 6i_2 - 3i_3 \quad — (1)$

$-i_3(4) + i_5(5) + i_4(3) = 0$

$4i_3 = 3i_4 + 5i_5$

$4i_3 = \cancel{3}\left(\dfrac{i_1 - 2i_2}{\cancel{3}}\right) + 5i_5$

$4i_3 = i_1 - 2i_2 + 5i_5$

$4i_3 = 1 - 3i_2 + 5i_5$

$4i_3 = 6 - 8i_2 + 5i_4$

$4i_3 = 6 - 8i_2 + 5i_2 - 5i_3$

$9i_3 = 6 - 3i_2$

$3i_2 + 9i_3 = 6$

$i_2 + 3i_3 = 2 \quad — (1)$

$+\ 6i_2 - 3i_3 = 1 \quad —$ from (1)

$7i_2 = 3 \longrightarrow i_2 = 3/7\ A$ hence $i_3 = 11/21$

$i_3 + i_4 = i_2$

$i_4 = \dfrac{3}{7} - \dfrac{11}{21} \Rightarrow i_4 = -2/21$

$i_1 = 1 - i_2$

$\therefore i_1 = 4/7\ A$

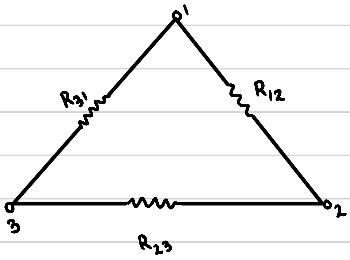
$\Rightarrow i_5 = i_1 + i_4$

$= \dfrac{4}{7} - \dfrac{2}{21} = \dfrac{10}{21} A$

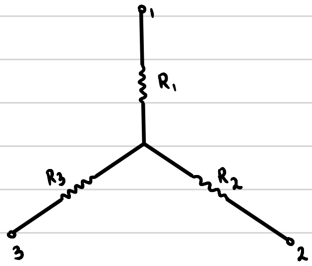
<u>Ans:</u> $i_1 = 4/7\ A = 0.571A$ $\quad i_3 = 11/21\ A = 0.523A$ $\quad i_5 = 10/21\ A = 0.476A$

$i_2 = 3/7\ A = 0.428A$ $\quad i_4 = -2/21\ A = -0.095A$

Star – Delta Transformation



— Delta connection (Fig 1.1)



— Star Connection (Fig 1.2)

(1) Delta (Δ) to Star (Y) transformation

In Fig 1.1, $R_{12} \parallel (R_{23} + R_{31})$

$$R_{eq} = \frac{(R_{12})(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})}$$

In Fig 1.2, R_{eq} between ① & ② is in series

$\therefore R_{eq} = R_1 + R_2$

Since both networks are electrically equivalent

$$\Rightarrow \frac{(R_{12})(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \quad \text{——— (between ① \& ②) — i}$$

$$\frac{(R_{23})(R_{12}+R_{31})}{R_{23}+(R_{12}+R_{31})} = R_2 + R_3 \quad \text{——— (between ② \& ③) — ii}$$

$$\frac{(R_{31})(R_{12}+R_{23})}{R_{31}+(R_{12}+R_{23})} = R_3 + R_1 \quad \text{——— (between ③ \& ①) — iii}$$

iii − i

$$R_1 - R_3 = \frac{(R_{12})(R_{31}) - (R_{23})(R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{— iv}$$

iii + iv

$$\boxed{R_1 = \frac{(R_{12})(R_{31})}{R_{12} + R_{23} + R_{31}}} \quad \text{— v}$$

$$\boxed{R_3 = \frac{(R_{31})(R_{23})}{R_{12} + R_{23} + R_{31}}} \quad \text{— vii}$$

$$\boxed{R_2 = \frac{(R_{23})(R_{12})}{R_{12} + R_{23} + R_{31}}} \quad \text{— vi}$$

Star (Y) to Delta (Δ) transformation

v × vi

$$R_1 R_2 = \frac{(R_{12})^2 (R_{23})(R_{31})}{(R_{12} + R_{23} + R_{31})^2} \quad \text{— viii}$$

Similarly,

$$R_2 R_3 = \frac{(R_{12})(R_{23})^2 (R_{31})}{(R_{12} + R_{23} + R_{31})^2} \quad \text{— ix} \quad \text{and} \quad R_3 R_1 = \frac{(R_{12})(R_{23})(R_{31})^2}{(R_{12} + R_{23} + R_{31})^2} \quad \text{— x}$$

viii + ix + x

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}(R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

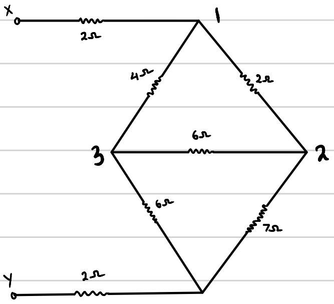
$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} \cdot R_{23} \cdot R_{31}}{(R_{12} + R_{23} + R_{31})} \quad \text{— xi}$$

Using vii in xi

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12} \cdot R_3$$

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}} \qquad \boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}} \qquad \boxed{R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}}$$

Q.



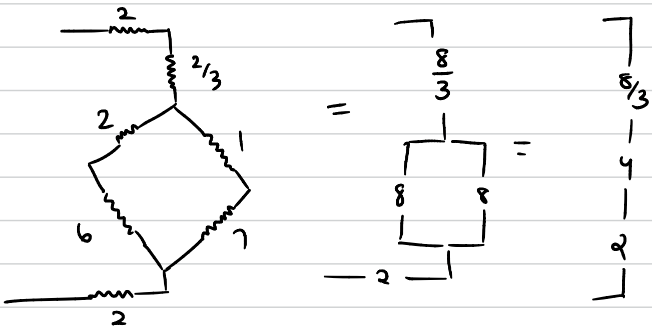
Find eq. resistance

Ans:

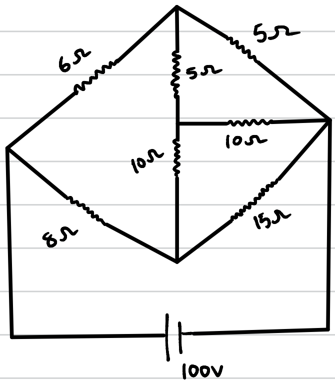
$$R_1 = \frac{4\times 2}{12} = \frac{8}{12} = \frac{2}{3}$$

$$R_2 = \frac{6\times 2}{12} = \frac{12}{12} = 1$$

$$R_3 = \frac{4\times 6}{12} = \frac{24}{12} = 2$$



$$= 6 + \frac{8}{3} = \frac{18+8}{3} = \boxed{\frac{26}{3}\,\Omega = 8.667\,\Omega}$$



Find current

$R_1 = 5\Omega$ $R_2 = 10\Omega$ $R_3 = 10\Omega$

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} = 5 + 10 + \frac{5\times \cancel{10}}{\cancel{10}}$$

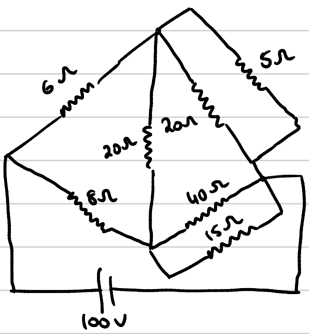
$R_{12} = 20\Omega$

$$R_{23} = 10 + 10 + \frac{10\times \cancel{10}^{2}}{\cancel{5}_{1}} = 20 + 20$$

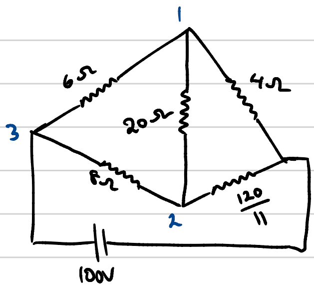
$R_{23} = 40\Omega$

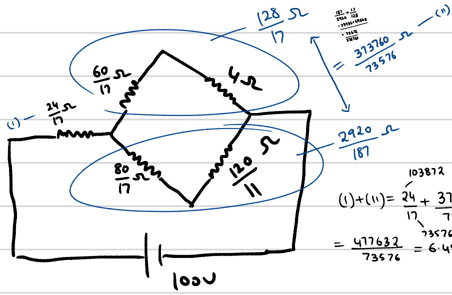
$$R_{31} = 10 + 5 + \frac{\cancel{10}\times 5}{\cancel{10}}$$

$R_{31} = 20\Omega$



$\Rightarrow$



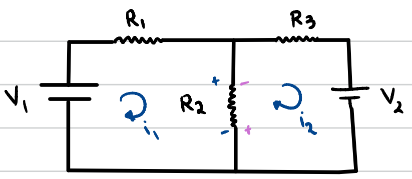


$$(I)+(II) = \frac{24}{17} + \frac{373760}{73576}$$

$$= \frac{477632}{73576} = 6.49\,\Omega$$

$$\boxed{V = iR \rightarrow i = \frac{100}{6.49} = 15.40\text{ A}}$$

Mesh Analysis Method



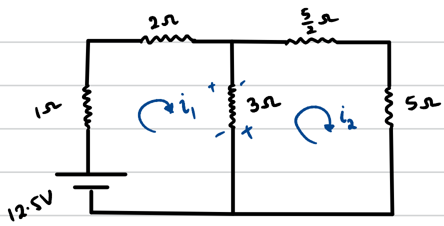
Using Kirchoffs Voltage Law:

$V_2 + i_2 R_3 + (i_2 - i_1) R_2 = 0$

$V_2 = i_1 R_2 - i_2 R_2 - i_2 R_3$

$V_2 = i_1 R_2 - i_2 (R_2 + R_3)$

Q.



$12.5 - i_1(1) - i_1(2) - (i_1 - i_2)(3) = 0$

$\rightarrow 12.5 - 6i_1 + 3i_2$

$\rightarrow (2i_1 - i_2) = \frac{12.5}{3} \rightarrow 2i_1 - i_2 = 4.166$

$2.5(i_2) - 5(i_2) - (i_2 - i_1)(3) = 0$

$= 3i_1 - 5.5i_2 = 0$

$i_1 = \frac{5.5}{3} i_2$ — (2)

$2\left(\frac{5.5}{3} i_2\right) - i_2 = 4.166$

$\rightarrow 11i_2 - 3i_2 = 4.166 \times 3$

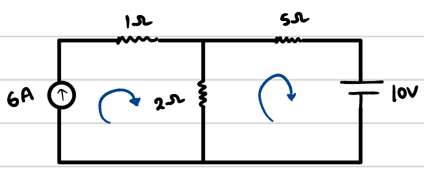
$\rightarrow 8i_2 = 12.498$

$\boxed{i_2 = 1.56\ A}$

$i_1 = \frac{5.5 \times 1.56}{3}$

$\boxed{i_1 = 2.86\ A}$

Q. Find current through 2Ω resistor.



$-5i_2 - 10 - (i_2 - i_1)(2) = 0$

$\rightarrow i_1 = 6$

$-5i_2 - 10 - 2i_2 + 2(6) = 0$

$-7i_2 + 2 = 0$

$\rightarrow i_2 = 2/7\ A$

Ans: In mesh 1 KVL is not applicable.

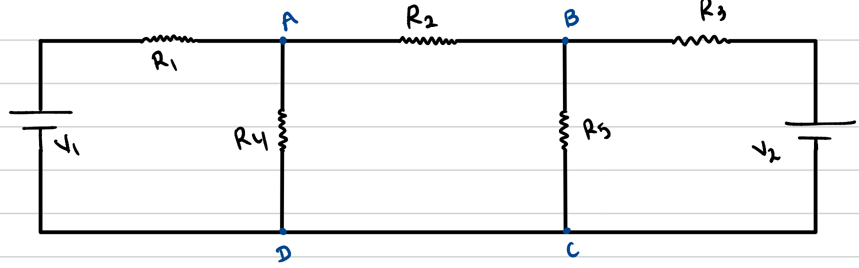
$i_1 = 6A$

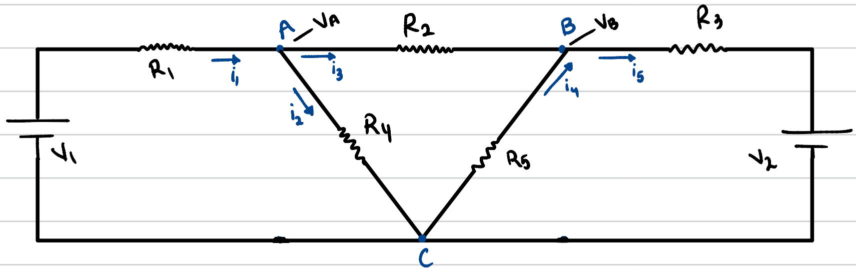
∴ Current through 2Ω

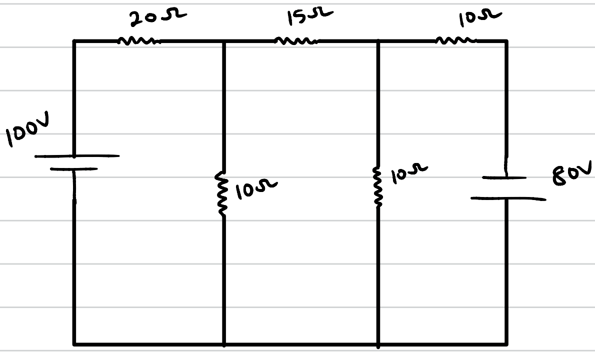
$= \frac{2}{7} - 6 = \frac{2-42}{7} = \frac{-40}{7} = \boxed{5.71\ A}$

Nodal Analysis

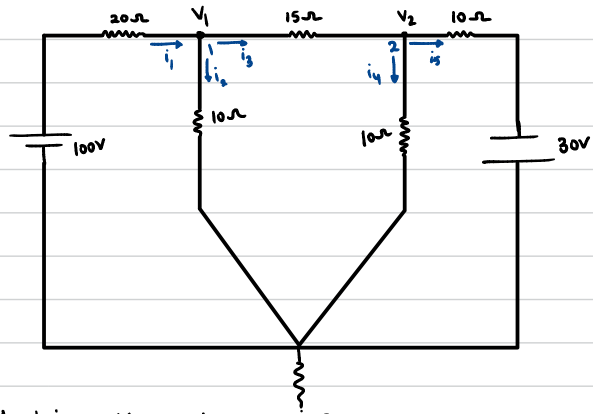
- Based on KCL.







Using nodal analysis find current through 15Ω resistor.



Applying KCL at node 1

$$I_1 = I_2 + I_3$$

$$\frac{100 - V_1}{20} = \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{15}$$

$$\frac{100 - V_1}{2\cancel{0}} = \frac{15V_1 + 10V_1 - 10V_2}{15\cancel{0}}$$

$$\Rightarrow 1500 - 15V_1 = 30V_1 + 20V_1 - 20V_2$$

$$\Rightarrow 1500 = 65V_1 - 20V_2$$

$$\Rightarrow 300 = 13V_1 - 4V_2 \quad \text{—(1)}$$

Applying KCL at node 2

$$I_3 = I_4 + I_5$$

$$\frac{V_1 - V_2}{15} = \frac{V_2 - 0}{10} + \frac{V_2 - (-80)}{10}$$

$$\frac{V_1 - V_2}{15} = \frac{2V_2 + 80}{10} \Rightarrow 10V_1 - 10V_2 = 30V_2 + 1200$$

$$10V_1 - 40V_2 = 1200$$

$$V_1 - 4V_2 = 120 \quad \text{—(2)}$$

(1) − (2)

$$300 = 13V_1 - \cancel{4V_2}$$

$$-\ 120 = -V_1 \mp \cancel{4V_2}$$

$$12V_1 = 180$$

$$V_1 = \frac{180}{12} = \frac{30}{2} \rightarrow \boxed{V_1 = 15\text{ V}}$$

$$15 - 4V_2 = 120$$

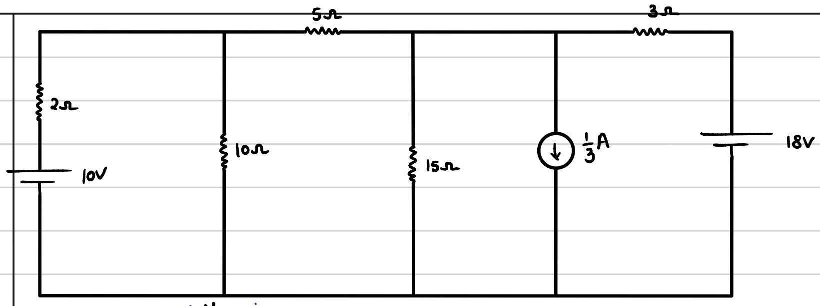
$$4V_2 = 15 - 120$$

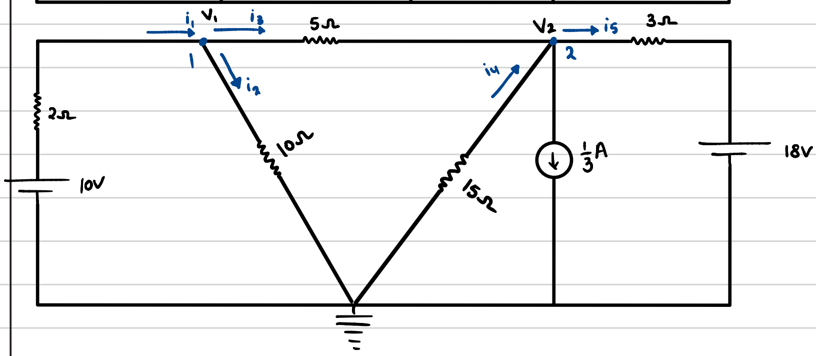
$$\boxed{V_2 = -26.25\text{ V}}$$

$$\therefore i_3 = \frac{15 + 26.25}{15}$$

$$\boxed{= 2.75\text{ A}}$$

Q.





At node ①

$$\frac{10-V_1}{2} = \frac{V_1-0}{10} + \frac{V_1-V_2}{5}$$

$$\Rightarrow \frac{10-V_1}{\cancel{2}\,1} = \frac{V_1+2V_1-2V_2}{\cancel{10}\,5}$$

$$\Rightarrow 50-5V_1 = 3V_1-2V_2$$

$$8V_1-2V_2 = 50$$

$$\Rightarrow 4V_1 - V_2 = 25 \quad \text{— (1)}$$

At node ②

$$\frac{V_1-V_2}{5} = \frac{V_2-0}{15} + \frac{V_2-18}{3} + \frac{1}{3}$$

$$\frac{V_1-V_2}{\cancel{5}} = \frac{V_2+5V_2-90+5}{\cancel{15}\,3} \rightarrow 3V_1-3V_2 = 6V_2-85$$

$$\Rightarrow 9V_2-3V_1 = 85 \quad \text{— (2)}$$

$$4V_1 - V_2 = 25 \quad \times 9$$

$$3V_1 - 9V_2 = -85$$

$$\rightarrow \quad 36V_1 - \cancel{9V_2} = 225$$

$$- \quad 3V_1 \underset{+}{-} \cancel{9V_2} = \underset{+}{-}85$$

$$33V_1 = 310$$

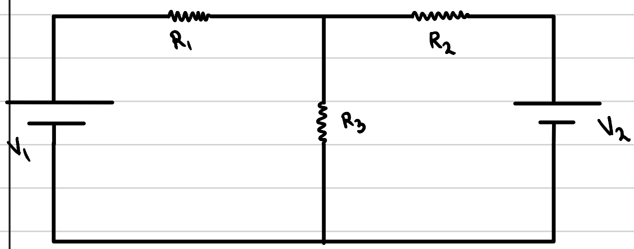
$$V_1 = \frac{310}{33} \quad \text{— (1)}$$

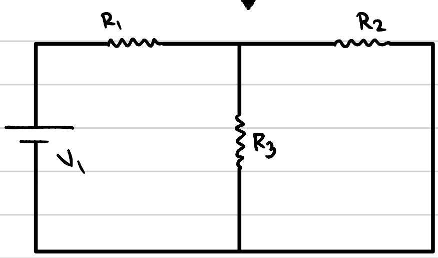
4

$$i_2 = \frac{\overset{5}{\cancel{10}} - \frac{\overset{155}{\cancel{310}}}{33}}{\cancel{2}\,1} = \frac{165-155}{33} = \frac{10}{33} \boxed{= 0.303A}$$

Superposition Theorem

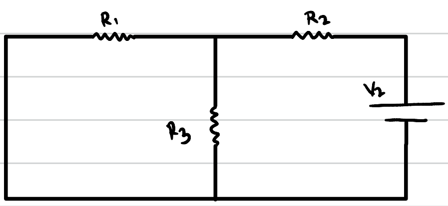
- Applicable for linear & bilateral networks.
- Ideal type sources will be replaced by:
  - voltage — short circuited wire
  - current — open circuit
- Non-ideal sources are replaced by their internal resistance.





Replacing V_2 with a short wire.

Calculate i through R_3 due to V_1

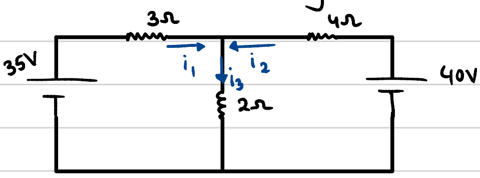


Replacing V_1 with a short wire.

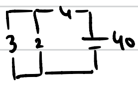
Calculate i through R_3 due to V_2.

$$\boxed{I_{3\,total} = I_{R3|V_1} + I_{R3|V_2}}$$

Q. Find current through each branch using superposition theorem.



Replacing 35V with short wire:



$\Rightarrow R_{eq} = 26/5$

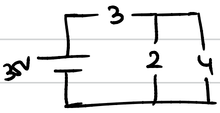
$V = IR \longrightarrow \quad i = \frac{V}{R} = \frac{\cancel{40}^{20}}{\cancel{26}_{13}/5} = \frac{100}{13}\ A$

$\frac{100}{13} \times \frac{3}{2+3} = \frac{\cancel{100}^{20}}{13} \times \frac{3}{\cancel{5}}$

$= \frac{60}{13}$ through 2Ω

$= \frac{40}{13}$ through 3Ω

Replacing 40V with short wire:



$\Rightarrow R_{eq} = 13/3\ \Omega$

$V = IR$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \rightarrow \frac{4}{3}\ \Omega$

$\frac{4}{3} + 3 = \frac{13}{3}$

$\therefore \quad i = \frac{V}{R} = \frac{35}{13/3} = \frac{105}{13}\ A$

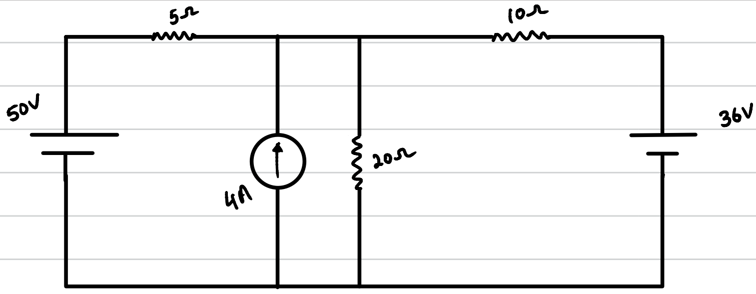
- Total current through $R_3 \Rightarrow \frac{100 + 105}{13} = \frac{205}{13}\ A$

$\frac{105}{13} \times \frac{2}{2+4}$ $\quad \frac{70}{13}\ A$ through 2Ω

$= \frac{\cancel{105}}{13} \times \frac{1}{\cancel{3}} = \frac{35}{13}$ through 4Ω

$$\boxed{\text{Hence: } i_1 = \frac{65}{13} = 5A \qquad i_2 = \frac{65}{13} = 5A \qquad i_3 = \frac{130}{13} = 10A}$$

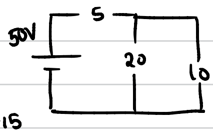
Q.



Determine current in 5Ω resistor using superposition theorem.

Ans:

Considering 50V source



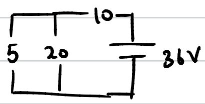
$\frac{1}{20}+\frac{1}{10}=\frac{3}{20} \rightarrow \frac{20}{3}+5=\frac{20+15}{3}$

$R_{eq}=\frac{35}{3}\,\Omega \implies V=IR \longrightarrow i=\frac{V}{R}=\frac{\cancel{50}^{10}}{{}_7\cancel{35}/3}=\frac{30}{7}\,A$ through $5\,\Omega\,(\rightarrow)$

$\frac{\cancel{30}^{10}}{7}\times\frac{\cancel{20}}{\cancel{30}}=\frac{20}{7}\,A$ through $10\Omega\,(\rightarrow)$

$\frac{30}{7}\times\frac{10}{30}=\frac{10}{7}\,A$ through $20\Omega\,(\downarrow)$

Considering 36V source



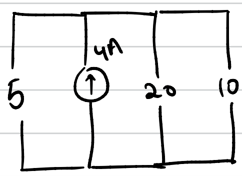
$\frac{1}{5}+\frac{1}{20}=\frac{5}{20}=\frac{1}{4}=4\Omega+10 \Rightarrow R_{eq}=14\,\Omega$

$V=iR \rightarrow i=\frac{V}{R} \rightarrow i=\frac{36}{14}=\frac{18}{7}A$ through $10\Omega\,(\leftarrow)$

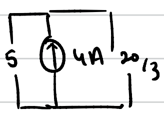
$\frac{18}{7}\times\frac{\cancel{20}^{4}}{\cancel{25}_5}=\frac{72}{35}\,A$ through $5\Omega\,(\leftarrow)$

$\frac{18}{7}\times\frac{\cancel{5}\,1}{\cancel{25}\,5}=\frac{18}{35}A$ through $20\Omega\,(\downarrow)$

Considering 4A current Source:



$\frac{1}{20}+\frac{1}{10}=\frac{3}{20} \Rightarrow \frac{20}{3}\,\Omega$

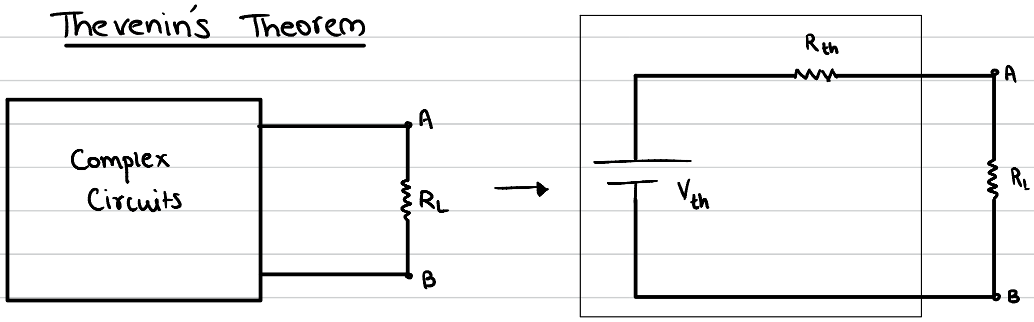


$4\times\frac{5/}{{}_7\cancel{35}/3}=\frac{4\times3}{7}=\frac{12}{7}A$ through $\frac{20}{3}\Omega\,(\rightarrow)$

$\frac{4\times4\cancel{20}/\cancel{3}}{{}_7\cancel{35}/\cancel{3}}=\frac{16}{7}A$ through $5\Omega\,(\leftarrow)$

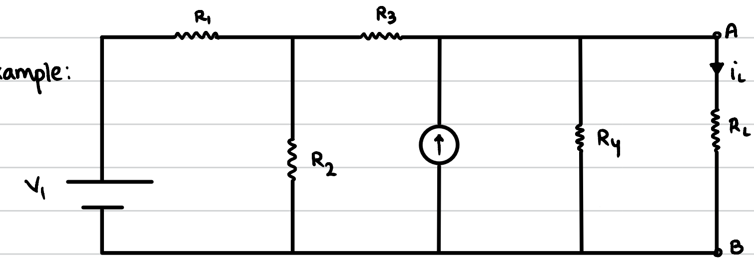
Through 5Ω resistor : $\frac{30}{7}{}^{\times5}_{\times5}-\frac{72}{35}-\frac{16}{7}{}^{\times5}_{\times5}=\frac{150-72-80}{35}=\boxed{\frac{2}{35}A = 0.057A}$

Thevenin's Theorem



Thevenin's equivalent circuit

Example:



Calculation of V_{th}

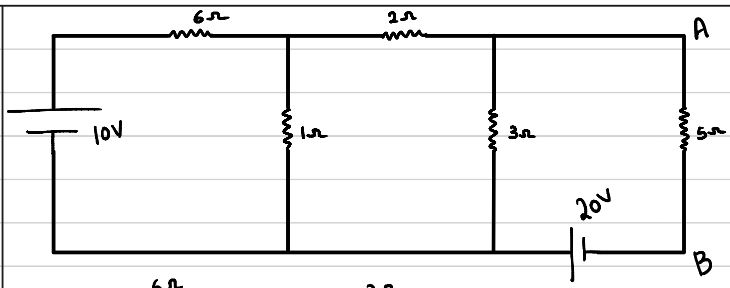
- → Remove R_L & open the circuit at A & B.
- → Calculate V_{th} using any method.

Calculation of R_{th}

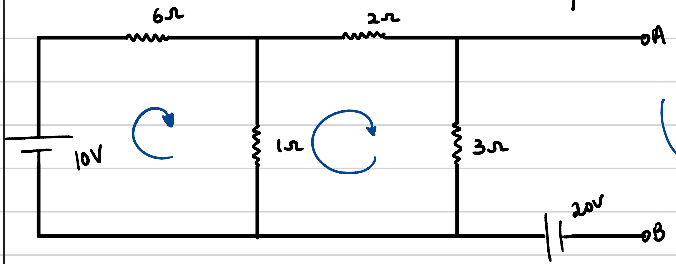
- → Remove R_L & open the circuit at A & B.
- → Replace voltage source by equivalent resistance or short wire.
- → Replace current source by equivalent resistance or open circuit.

$$\boxed{i_{load} = \frac{V_{th}}{R_{th} + R_L}}$$

Q.



Ans:



(mesh analysis)

Applying KVL

$$10 - 6(i_1) - (1(i_1) - (i_2)) = 0$$

$$7i_1 - i_2 = 10 \quad —(1)$$

$$-2(i_2) - 3(i_2) - (i_2 - i_1) = 0$$

$$i_1 - 6i_2 = 0 \quad —(2)$$

$\therefore i_1 = 6i_2$ — Putting in (1)

$$42i_2 - i_2 = 10$$

$$i_2 = \frac{10}{41} = 0.244\text{ A}$$

$$i_1 = \frac{6 \times 10}{41} = 1.46\text{A}$$

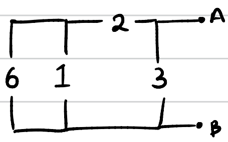
$$\Delta V_{AB} = V_{th}$$

$$V_{th} = 20 + 3(I_{3\Omega})$$

$$= 20 + 3(0.24)$$

$$\boxed{= 20.732}$$

$R_{th} =$



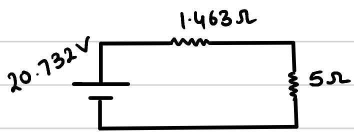
$$\frac{1}{1} + \frac{1}{6} = \frac{7}{6} \rightarrow \frac{6}{7}\Omega$$

$$\frac{6}{7} + 2 = \frac{6+14}{7} = \frac{20}{7}\Omega$$

$$\frac{7}{20} + \frac{1}{3} = \frac{21+20}{60}$$

$$= \frac{41}{60}$$

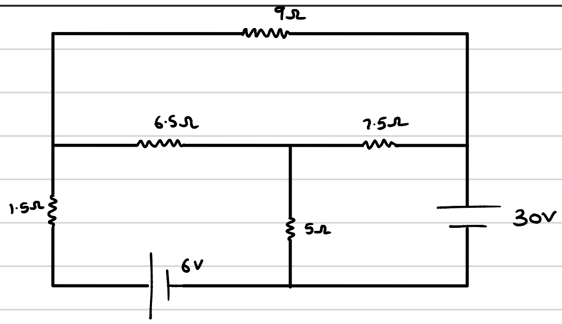
$$= 60/41 \boxed{= 1.463\Omega}$$



← Thevenin's equivalent circuit.

$$i = \frac{V}{R} \rightarrow \frac{20.732}{6.463} \boxed{= 3.207\text{A}}$$

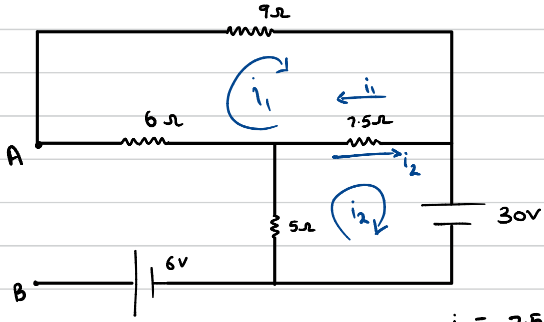
Q.



i) Calculate V_{th}

ii) Calculate R_{th}

iii) Draw Thevenin's circuit.



Applying KVL

$$6(i_1) + 7.5(i_1 - i_2) + 9i_1 = 0$$

$$22.5\, i_1 = 7.5\, i_2 \quad \text{—(1)}$$

$$-30 - 5i_2 - 7.5(i_2 - i_1) = 0$$

$$7.5 i_1 - 12.5 i_2 = 30$$

$$i_1 = \frac{7.5\, i_2}{22.5} \rightarrow 7.5\left(\frac{7.5 i_2}{22.5}\right) - 12.5 i_2 = 30$$

$$56.25\, i_2 - 281.25\, i_2 = 675$$

$$-225 i_2 = 675$$

$$i_2 = -\frac{27}{9} = -3A$$

$$i_1 = \frac{\cancel{7.5}}{\cancel{22.5}} \times -3 = -1A$$

$i_1 = -1A$

$i_2 = -3A$

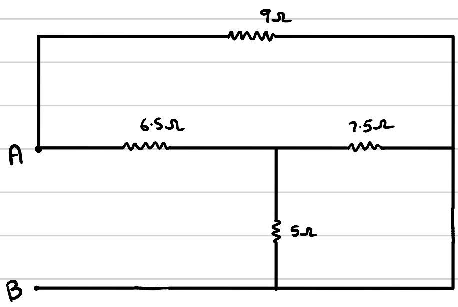
$$V_A + 6i_1 + 5i_2 + 6 = V_B$$

$$V_A - V_B = -6i_1 - 5i_2 - 6$$

$$V_{th} = -6(-1) - 5(-3) - 6$$

$$= 6 + 15 - 6$$

$$V_{th} = 15V$$



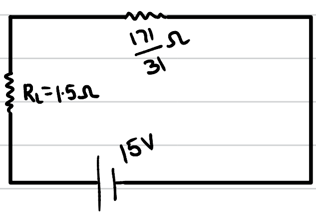
$$\frac{1}{5} + \frac{1}{7.5} = \frac{\cancel{12.5}\ 2.5}{\cancel{37.5}\ 7.5} = \frac{1}{3} = 3Ω$$

$$[9 \,/\, 6.5 - 3] \Rightarrow [9 \,/\, 9.5Ω]$$

$$\Rightarrow \frac{1}{9} + \frac{1}{9.5} = \frac{\cancel{18.5}\ 3.7}{\cancel{85.5}\ 17.1} \Rightarrow \frac{171}{37}Ω = 4.622Ω$$

✗ Check Soln.

Hence, thevenin's circuit:



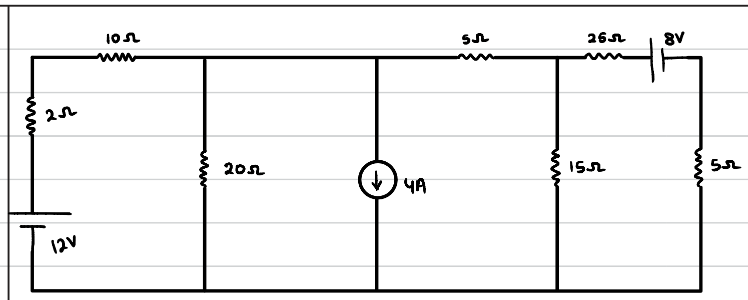
$4.622 + 1.5$

$= 6.122$

$V = iR$

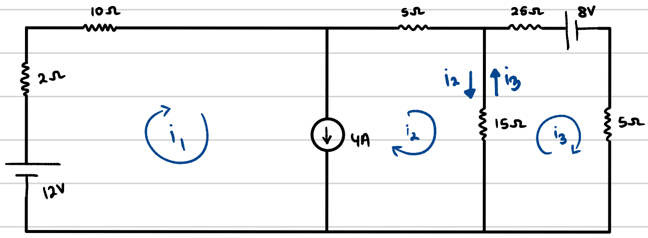
$$\boxed{i = \frac{15}{6.122} = 2.45A}$$

Q.



Calculate current through 20Ω resistor using Thevenin's theorem.

Ans:



$$i_1 - i_2 = 4A \quad —(1)$$

Now applying KVL to supermesh:

$$12 - 2i_1 - 10i_1 - 5i_2 - 15(i_2 - i_3) = 0$$

→ $$12 - 12i_1 - 20i_2 + 15i_3 = 0$$

→ $$12i_1 + 20i_2 - 15i_3 = 12 \quad —(2)$$

Applying KVL in mesh-3:

$$-25i_3 - 8 - 5i_3 - 15(i_3 - i_2) = 0$$

$$15i_2 - 45i_3 = 8 \quad —(3)$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 12 & 20 & -15 \\ 0 & 15 & -45 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix}$$

, $$\Delta = 1(20\times-45 - 15\times-15) + 1(12\times-45)$$

$$= -900 + 225 - 540$$

$$= -675 - 540$$

$$\boxed{\Delta = -1215}$$

$$\Delta_1 = \begin{bmatrix} 4 & -1 & 0 \\ 12 & 20 & -15 \\ 8 & 15 & -45 \end{bmatrix} = 4(20\times-45 - 15\times-15) + 1(12\times-45 - 8\times-15)$$

$$= 4(-900 + 225) + (-540 + 120)$$

$$= -4(675) - 420$$

$$= -2700 - 420$$

$$\boxed{\Delta_1 = -3120}$$

Using Cramer's rule:

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{-3120}{-1215} = 2.568\ A$$

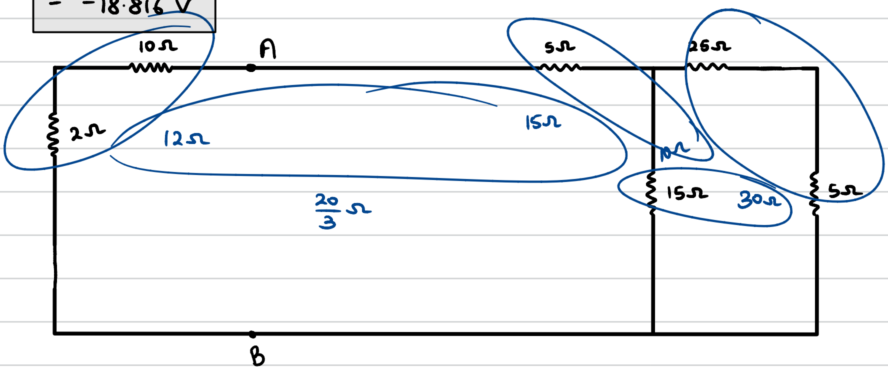
$V_{th} = V_{AB}$

$= 12 - 2i_1 - 10i_1$

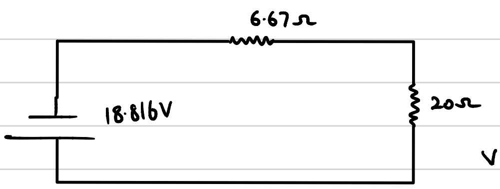
$= 12 - 12(2.568)$

$= 12 - 30.816$

$\boxed{= -18.816\ V}$



$\boxed{R_{th} = \frac{20}{3}\,\Omega = 6.66\,\Omega}$



$R_{eq} = 26.67\,\Omega$

$V = IR$

$I = \frac{18.816}{26.67} = 0.705\ A$

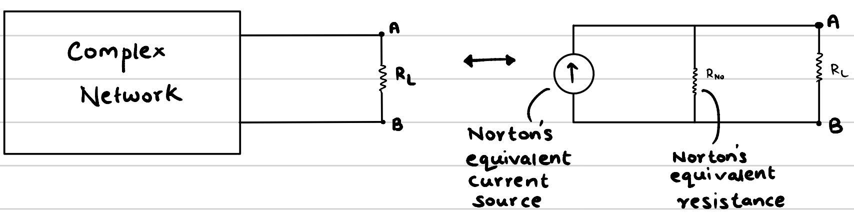
$\therefore$ Power $= i^2R$

$= (0.705)^2(20)$

$= 0.497 \times 20$

$\boxed{= 9.94\ W}$

Norton's Theorem



Calculation of Norton's eq. current:

i) Short the branch resistance through which current is to be calculated.

ii) Obtain the Norton's equivalent current from resultant circuit using any method.

Calculation of Norton's Resistance

i) Remove R_L

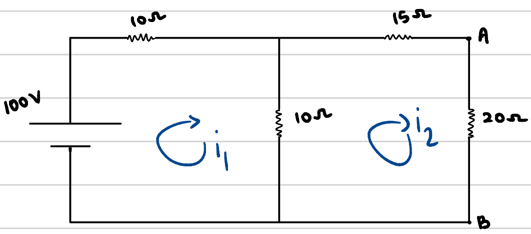
ii) Replace voltage source by internal resistance / short-circuit if ideal.

Replace current source by internal resistance / open-circuit if ideal.

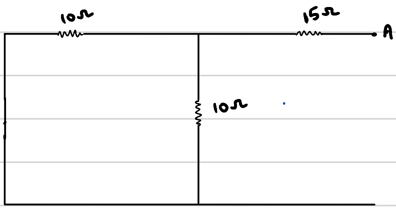
iii) Draw Norton's equivalent circuit along with R_L at load terminal.

$$\boxed{\text{Calculate} \quad I_L = I_N \cdot \frac{R_N}{R_L + R_N}}$$

Q. Find current through 20Ω resistor.



Ans:



$\frac{1}{10} + \frac{1}{10} = \frac{10+10}{100} = \frac{20}{100} \Rightarrow 5Ω$

$R_{eq} = 5 + 15 = 20Ω$

$V = 100V$

$100 - 10i_1 - 10(i_1 - i_2) = 0$

$-20i_1 + 10i_2 = -100$

$2i_1 - i_2 = 10$

$2i_1 - \frac{2}{5}i_1 = 10$

$8i_1 = 50$

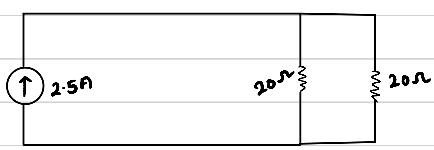
$-15i_2 - 10i_2 + 10i_1 = 0$

$10i_1 = 25i_2$

$2i_1 = 5i_2$

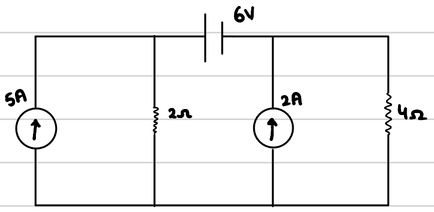
$i_2 = \frac{2}{5}i_1$

$\frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{50}}{8} \frac{10}{4} \rightarrow 2.5A$



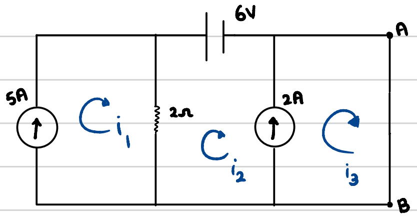
$i = 2.5 \times \frac{20}{40} = 1.25A$

$\boxed{\text{Current flowing through 20Ω resistor} = 1.25A}$

Q. 

Find current through 4Ω resistor.

Ans:



In mesh-1

$i_1 = 5A$

In mesh 2 & 3

$i_3 - i_2 = 2A$

$i_3 = 2+2$

$\boxed{i_3 = 4A}$

By applying KVL to mesh 2 & 3:

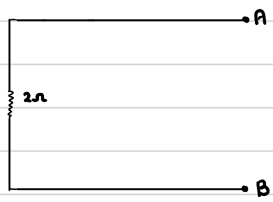
$-6 - 2(i_2 - i_1) = 0$

$-6 - 2i_2 + 2i_1 = 0$

$i_1 - i_2 = 3$

$5 - 3 = i_2$

$i_2 = 2A$



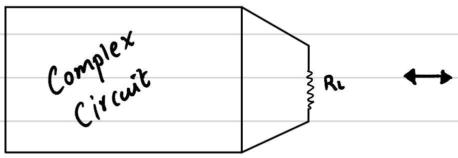
$R_L = 2\Omega$

$i_L = \cancel{4}^{2} \times \frac{2}{\cancel{6}\,3} \longrightarrow \boxed{i_L = \frac{4}{3} = 1.33A}$

$\boxed{\text{Hence current flowing through } 4\Omega \text{ resistor} = 1.33A}$

Max Power Transfer Theorem

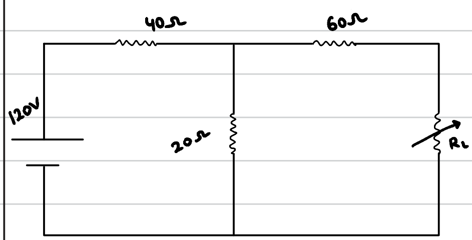
$\boxed{R_{eq} = R_L}$ for max. power transfer.



For P_{max} : $R_L = R_{eq} = R_{th}$

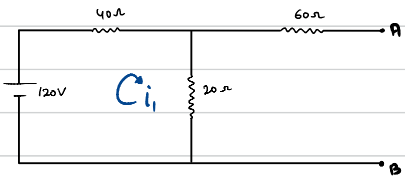
$$\boxed{P_{max} = \frac{(V_{th})^2}{4(R_{th})} \text{ W}}$$

Q.



Find R_L for maximum power transfer.

Ans:



$\frac{1}{20} + \frac{1}{40} = \frac{3}{40} \rightarrow \frac{40}{3}\,\Omega$

$\frac{40}{3} + 60 = \frac{40+180}{3} \boxed{= \frac{220}{3}\,\Omega = R_{th}}$

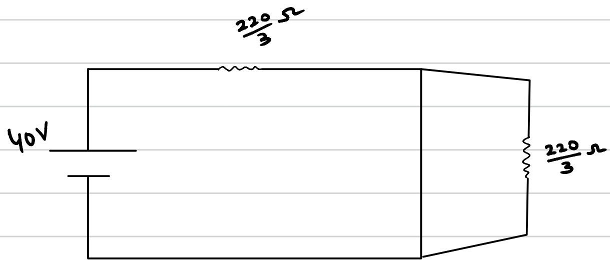
$120 - 60i_1 = 0$

$i_1 = 2A$

$\therefore V = iR \rightarrow V = 2(20)$

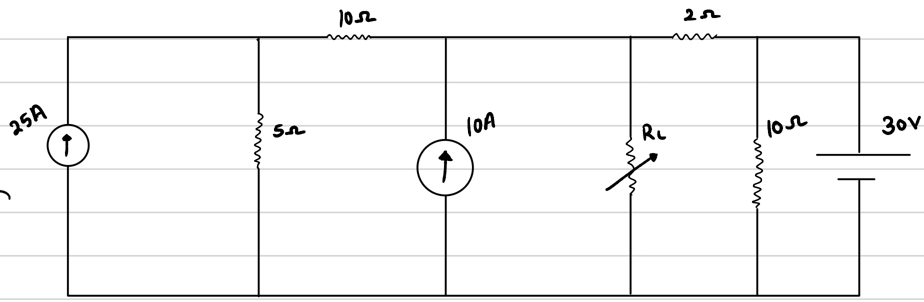
$= 40V$

$\underline{V = 40V}$

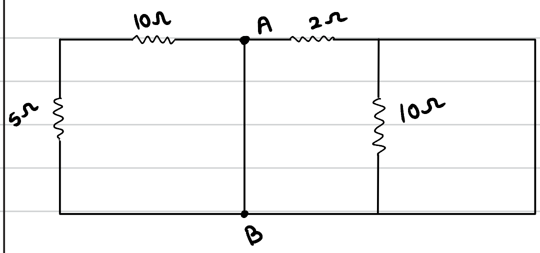


Hence Power $= \frac{V^2}{4R} = \frac{\cancel{40}^{10} \times 40}{\cancel{4}_1 \times \frac{22\cancel{0}}{3}} = \frac{\cancel{120}^{60}}{\cancel{22}_{11}}$ $\boxed{= 5.45 \text{ W}}$

Q. Determine max. power that can be delivered.



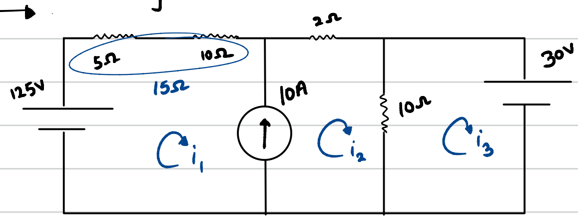
Ans:



$10+5=15\,\Omega$

$\frac{1}{15}+\frac{1}{2} = \frac{2+15}{30} \rightarrow \frac{30}{17}\,\Omega = 1.764\,\Omega$

Converting 25A || 5Ω current source into voltage source:



$i_2 - i_1 = 10 \quad -(1) \longrightarrow i_2 - 10 = i_1$

$125 - 15i_1 - 2i_2 - 10(i_2 - i_3) = 0$

$125 - 15i_1 - 12i_2 + 10i_3 = 0$

$15i_1 + 12i_2 - 10i_3 = 125 \quad -(2)$

$-30 - 10(i_3 - i_2) = 0$

$-30 - 10i_3 + 10i_2 = 0$

$10i_2 - 10i_3 = 30$

$i_2 - i_3 = 3 \quad -(3)$

$i_2 - 3 = i_3$

$15(i_2 - 10) + 12i_2 - 10(i_2 - 3) = 125$

$15i_2 - 150 + 12i_2 - 10i_2 + 30 = 125$

$17i_2 = 245$

$\boxed{i_2 = 14.41A}$

$\therefore V = iR = 14.41(2)$

$= 28.82V + 30V$

$\boxed{V_{th} = 58.82V}$

$P = \frac{V^2}{4R} = \frac{(58.82)^2}{4 \times 1.764} = \frac{3459.79}{7.056}$

$\boxed{P = 490.33W}$