SOMAIYA VIDYAVIHAR UNIVERSITY

Batch: C3-1 Roll No. 16010123155

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Course: AM-I

Experiment / assignment / tutorial No 9

Grade: Signature of the Faculty with date

Q1} The given equation is $x^2-2x+2=0$

$$x = \frac{2 \pm \sqrt{(-2)^2-4(1)(2)}}{2(1)} = \frac{2\pm\sqrt{-4}}{2} = 1 \pm i$$

$$\therefore \alpha = 1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$\beta = 1-i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

$$\therefore \alpha^n+\beta^n = \left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^n + \left[\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\right]^n$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right) + 2^{n/2}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}+\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$$

$$= (\sqrt{2})^n\left(2\cos\frac{n\pi}{4}\right)$$

$$= 2\cdot 2^{n/2}\cos\frac{n\pi}{4}$$

Putting $n=8$, $\alpha^8+\beta^8 = 2\cdot 2^4\cos 2\pi$

$$= 2^5$$

$$= 32$$

Hence, proved.

Q3} (i) $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

$$\therefore e^u = \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{1+\tan\theta/2}{1-\tan\theta/2}$$

$$e^{-u} = \frac{1-\tan\theta/2}{1+\tan\theta/2}$$

$$\cosh u = \frac{e^u+e^{-u}}{2}$$

$$= \frac{1}{2}\left[\frac{(1+2\tan\frac{\theta}{2}+\frac{\theta}{2})+(1-2\tan\frac{\theta}{2}+\frac{\theta}{2})}{(1-\frac{\theta}{2})}\right]$$

$$= \frac{1}{2}\left(\frac{2+2\frac{\theta}{2}}{1-\frac{\theta}{2}}\right)$$

$$= \frac{1+\frac{\theta}{2}}{1-\frac{\theta}{2}} = \frac{1}{\cos\cos\theta} = \sec\sec\theta$$

(ii) $\sinh u = \sqrt{u-1} = \sqrt{\theta-1} = \sqrt{\theta} = \tan\theta$

(iii) $\tanh u = \dfrac{\sinh u}{\cosh u} = \dfrac{\tan\theta}{\sec\theta} = \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \sin\theta$

(iv) $\tanh\left(\frac{u}{2}\right) = \dfrac{\sinh(u/2)}{\cosh(u/2)} = \dfrac{2\sinh(u/2)\cdot\cosh(u/2)}{2\cosh(u/2)\cdot\cosh(u/2)}$

$$= \frac{\sinh u}{1+\cosh u} = \frac{\tan\theta}{1+\sec\theta}$$

{By (i) and (ii)}

$\therefore \tanh\left(\frac{u}{2}\right) = \dfrac{\sin\theta/\cos\theta}{(\cos\theta+1)\cos\theta} = \dfrac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$

$$= \frac{\sin(\theta/2)}{\cos(\theta/2)}$$

$$= \tan\frac{\theta}{2}$$

Q5} Let $\cosh^{-1}\sqrt{1+x^2} = y$

$\therefore \sqrt{1+x^2} = \cos\cosh y$

$\therefore 1+x^2 = \cos\cosh^2 y$

$x^2 = \cos\cosh^2 y - 1$

$x^2 = \sin\sinh^2 y$

$x = \sin\sinh y$

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$$tantanhy = \frac{sinsinhy}{coscoshy} = \frac{x}{\sqrt{1+x^2}}$$

$$\therefore y = \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \cosh^{-1}(\sqrt{1+x^2}) = \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Q6} We have $sinsin(x+iy) = sinsinx\ coscoshy + i\ cosx\ sinhy$

$$\therefore loglog\ sin\ sin(x+iy) = \frac{1}{2}\log(\sin^2 x\ cos\cosh^2 y + \cos^2 x\ sinsinh^2 y) + i\tan^{-1}\left(\frac{coscosx\ sin\ sinhy}{sinsinx\ coscoshy}\right)$$

Now, $\sin^2 x\ cos\cosh^2 y + \cos^2 x \sinh^2 y = (1-\cos^2 x)cos\cosh^2 y + \cos^2 x(cos\cosh^2 y - 1)$

$$= \cosh^2 y - \cos^2 x$$

$$= \left(\frac{1+coscosh2y}{2}\right) - \left(\frac{1+coscos2x}{2}\right)$$

$$= \frac{1}{2}(coscosh2y - cos\cos 2x)$$

$$\therefore loglog\ sinsin(x+iy) = \frac{1}{2}\log\left(\frac{coscosh2y - coscos2x}{2}\right) + i\tan^{-1}(cotcotx\ tantanhy).$$

Q 2} For $(1+i)$, $r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

We can express $(1+i)$ in polar form,

$$(1+i) = \sqrt{2}\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1+i)^{3/4} = \sqrt[4]{2^3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= \sqrt[4]{2^3}\left(\cos\left(\frac{3\pi}{4} + K\cdot\frac{\pi}{2}\right) + i\sin\left(\frac{3\pi}{4} + K\cdot\frac{\pi}{2}\right)\right)$$

Putting $K = 0, 1, 2, 3.$

$$K_0 = \sqrt[4]{2^3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$K_1 = \sqrt[4]{2^3}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

$$K_2 = \sqrt[4]{2^3}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$K_3 = \sqrt[4]{2^3}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$$

The required product is,

$$= \sqrt[4]{2^3}\left[\cos\left(\frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} + \frac{9\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} + \frac{9\pi}{4}\right)\right.$$

$$= \sqrt[4]{2^3}\left(\cos\frac{24\pi}{4}\right) + i\sin\left(\frac{24\pi}{4}\right)$$

$$= \sqrt[4]{2^3}\left(\cos 6\pi + i\sin 6\pi\right)$$

$$= \sqrt[4]{2^3}\,(1)$$

$$= \sqrt[4]{2^3}$$

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Q4)

cos cos (x+iy) = cos cos α + i sin sin α

cos cos x cos cos(iy) − sin sin x sin sin(iy) = cos cos α + i sin sin α

cos cos x cosh cosh y − i sin sin x sinh sinh y ⇒ cos cos α + i sin sin α

∴ cos cos x cosh cosh y = cos cos α and − sin sin x sinh sinh y = sin sin α.

(i) α + α = 1

xy + xy = 1

xy + (1−x)(1+y) = 1

xy + 1 + y − x − xy = 1

1 + y − x = 1

y − x = 0

∴ y = x ---- (i)

∴ sinh sinh y = ± sin sin x

sin sin α = − sin sin x sinh sinh y = − sin sin x

(± sin sin x) = ± x

(ii) cos cos 2x + cosh cosh 2y = 1 − 2x + 1 + 2y

= 2 − 2x + 2x

= 2