SOMAIYA
VIDYAVIHAR UNIVERSITY

Maximum Marks: 30	Semester: July 2023 – Dec 2023 Examination: In-Semester Examination	Duration: 1 Hr 15 Min
Programme code: Programme: B.Tech	Class: FY	Semester: I (SVU R-2023_24-07-2023)
Name of the Constituent College: K. J. Somaiya College of Engineering		Name of the department: All Branches
Course Code: 216U06C101	Name of the Course: Applied Mathematics I	

Q.No.		Max. Marks
Q1	**Attempt any Three questions out of the following.**	18
(a)	Express the matrix $A=\begin{bmatrix}2 & 3-i & 2+i\\ i & 0 & 1-i\\ 1+2i & 1 & 3i\end{bmatrix}$ as $P+iQ$ where P and Q are Hermitian matrices.	6

Solution:

Ans.

As we know the unique representation, $A=\frac{1}{2}(A+A^{\theta})+i\frac{1}{2i}(A-A^{\theta})$ say,

A = P + iQ,

Where, $P=\frac{1}{2}(A+A^{\theta})$ *and* $Q=\frac{1}{2i}(A-A^{\theta})$

Now, Consider $A^{\theta}=\begin{bmatrix}2 & -i & 1-2i\\ 3+i & 0 & 1\\ 2-i & 1+i & -3i\end{bmatrix}$

$$\therefore P=\frac{1}{2}\left\{\begin{bmatrix}2 & 3-i & 2+i\\ i & 0 & 1-i\\ 1+2i & 1 & 3i\end{bmatrix}+\begin{bmatrix}2 & -i & 1-2i\\ 3+i & 0 & 1\\ 2-i & 1+i & -3i\end{bmatrix}\right\}=$$

$$\frac{1}{2}\begin{bmatrix}4 & 3-2i & 3-i\\ 3+2i & 0 & 2-i\\ 3+i & 2+i & 0\end{bmatrix}$$

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$$\therefore Q=\frac{1}{2i}\left\{\begin{bmatrix}2 & 3-i & 2+i\\ i & 0 & 1-i\\ 1+2i & 1 & 3i\end{bmatrix}-\begin{bmatrix}2 & -i & 1-2i\\ 3+i & 0 & 1\\ 2-i & 1+i & -3i\end{bmatrix}\right\}=$$

$$\frac{1}{2i}\begin{bmatrix}0 & 3 & 1+3i\\ -3 & 0 & -i\\ -1+3i & -i & 6i\end{bmatrix}$$

For all elements P & Q, $a_{ij}=\overline{a_{ji}}$. Hence P and Q are Hermitian.

Hence we get the unique expression,

$$A=\begin{bmatrix}2 & 3-i & 2+i\\ i & 0 & 1-i\\ 1+2i & 1 & 3i\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}4 & 3-2i & 3-i\\ 3+2i & 0 & 2-i\\ 3+i & 2+i & 0\end{bmatrix}+i\frac{1}{2i}\begin{bmatrix}0 & 3 & 1+3i\\ -3 & 0 & -i\\ -1+3i & -i & 6i\end{bmatrix}$$

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(b) Reduce the following matrix to Normal Form. Find the rank of following matrices. 6

$$\begin{bmatrix} 4 & 3 & 0 & -2 \\ 3 & 4 & -1 & -3 \\ 7 & 7 & -1 & -5 \end{bmatrix}$$

Ans

$$R_1 - R_2 \quad \sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 3 & 4 & -1 & -3 \\ 7 & 7 & -1 & -5 \end{bmatrix}$$

$$\left.\begin{matrix} R_2 - 3R_1 \\ R_3 - 7R_1 \end{matrix}\right\} \sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 7 & -4 & -6 \\ 0 & 14 & -8 & -12 \end{bmatrix}$$

$$\left.\begin{matrix} C_2 + C_1 \\ C_3 - C_1 \\ C_4 - C_1 \end{matrix}\right\} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & -4 & -6 \\ 0 & 14 & -8 & -12 \end{bmatrix}$$

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$$\frac{C_2}{7} \quad \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & -6 \\ 0 & 2 & -8 & -12 \end{bmatrix}$$

$$R_3 - 2R_1 \quad \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{matrix} C_3 + 4C_2 \\ C_4 + 6C_2 \end{matrix}\right\} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, the rank of matrix is 2.

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(c) Test the consistency of the system of equations and solve if consistent. 6

$x_1 - 2x_2 + x_3 - x_4 = 2$, $x_1 + 2x_2 + 2x_4 = 1$ $4x_2 - x_3 + 3x_4 = -1$

Ans Let's write given system of equations in matrix form as AX = B

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 2 & 0 & 2 \\ 0 & 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$By R_2 \to R_2 - R_1,$$

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

$$By R_3 \to R_3 - R_2,$$

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

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$$\therefore Rank A = Rank[A:B] = 2 < No. of Unknowns = 4$$

$\therefore$ System has infinitely many solutions.

No. of parameters = 4 − 2 = 2

$$\therefore x_1 - 2x_2 + x_3 - x_4 = 2 \ldots\ldots\ldots\ldots(1)$$

$$4x_2 - x_3 + 3x_4 = -1 \ldots\ldots\ldots\ldots(2)$$

Let $x_2 = s, x_4 = t$

Equation (2) $\Rightarrow x_3 = 1 + 4x_2 + 3x_4 = 1 + 4s + 3t$

Equation (1) $\Rightarrow x_1 = 2 + 2x_2 - x_3 + x_4 = 2 + 2s - (1 + 4s + 3t) + s$

$= 1 - 2s - 2t$

$\therefore$ Solution set is $\{x_1 = 1 - 2s - 3t, x_2 = s, x_3 = 1 + 4s + 3t, x_4 = t\}$

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(d)	Examine linear dependence or independence of the following vectors. Find relation between them if they are dependent. $[2,-1,3,2]$, $[1,3,4,2]$, $[3,-5,2,2]$	6

consider the equation $K_1X_1 + K_2X_2 + K_3X_3 = 0$

$\therefore 2K_1 + K_2 + 3K_3 = 0, \; -K_1 + 3K_2 - 5K_3 = 0$

$3K_1 + 4K_2 + 2K_3 = 0 \quad 2K_1 + 2K_2 + 2K_3 = 0$

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & -5 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{R_{12}} \begin{bmatrix} -1 & 3 & -5 \\ 2 & 1 & 3 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$-R_1 \begin{bmatrix} 1 & -3 & 5 \\ 2 & 1 & 3 \\ 3 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow[R_4 - 2R_1]{R_2 - 2R_1,\; R_3 - 3R_1} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 7 & -7 \\ 0 & 13 & -13 \\ 0 & 8 & -8 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} \frac{1}{7}R_2 \\ \frac{1}{13}R_3 \\ \frac{1}{8}R_4 \end{array} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{array}{l} R_3 - R_2 \\ R_4 - R_3 \end{array} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$K_1 - 3K_2 + 5K_3 = 0 \quad K_2 - K_3 = 0$

$K_3 = t \quad K_2 = t \Rightarrow K_1 = -2t$

vectors are linearly dependent $2X_1 - X_2 - X_3 = 0$

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(e)	Solve the following equations by Gauss-Jacobi's Method (Three Iteration only)	6

$$15x + y - z = 14 \;, x + 20y + z = 23 \;, 2x - 3y + 18z = 35$$

Rewrite given equations as,

$$x = \frac{1}{15}(14 - y + z)$$
$$y = \frac{1}{20}(23 - x - z)$$
$$z = \frac{1}{18}(35 - 2x + 3y)$$

(i) First iteration:

start with $x_0 = 0,\ y_0 = 0\ and\ z_0 = 0$

$$x_1 = \frac{1}{15}(14 - y + z) = \frac{14}{15} = 0.9333$$
$$y_1 = \frac{1}{20}(23 - x - z) = \frac{23}{20} = 1.15$$
$$z_1 = \frac{1}{18}(35 - 2x + 3y) = \frac{35}{18} = 1.9444$$

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(ii) Second iteration:

Use $x_1 = 0.9333,\ y_1 = 1.15\ and\ z_1 = 1.9444$

$$x_2 = \frac{1}{15}(14 - 1.15 + 1.9444) = 0.9863$$
$$y_2 = \frac{1}{20}(23 - 0.9333 - 1.9444) = 1.0061$$
$$z_2 = \frac{1}{18}(35 - 2(0.9333) + 3(1.15)) = 2.0324$$

(iii) **Third iteration:**

Use $x_2 = 0.9863,\ y_2 = 1.0061 and\ z_2 = 2.0324$

$$x_3 = \frac{1}{15}(14 - 1.0061 + 2.0324) = 1.0018$$
$$y_3 = \frac{1}{20}(23 - 0.9863 - 2.0324) = 0.9991$$
$$z_3 = \frac{1}{18}(35 - 2(0.9863) + 3(1.0061)) = 2.0025$$

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Q2	**Attempt any Two questions out of the following.**	12
(a)	If $u = x^3y + e^{xy^2}$, verify that $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$	6

Differentiating u partially w.r.t. x we get,

$$\frac{\partial u}{\partial x} = 3x^2y + e^{xy^2}y^2$$

Differentiating this partially w.r.t. y we get,

$$\frac{\partial^2 u}{\partial y\,\partial x} = 3x^2 + e^{xy^2}\,2y + y^2e^{xy^2}\,2xy \qquad ....(i)$$

03

Now, differentiating u partially w.r.t. y, we get,

$$\frac{\partial u}{\partial y} = x^3 + e^{xy^2}\,2xy$$

Differentiating this again partially w.r.t. x, we get,

$$\frac{\partial^2 u}{\partial x\,\partial y} = 3x^2 + e^{xy^2}\,2y + 2xy\,e^{xy^2}y^2 \qquad ....(ii)$$

From (i) and (ii) $\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x}$

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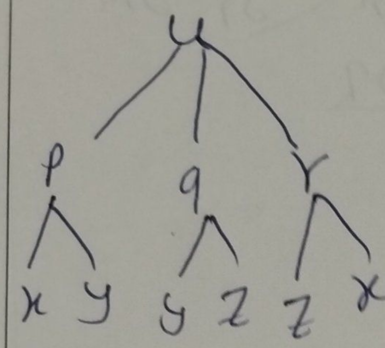
(b) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 6

Sol. $\cancel{X = e^{x-y} \quad Y = e^{y-z} \quad Z = e^{z}}$

let $p = e^{x-y} \quad q = e^{y-z} \quad r = e^{z-x}$



$\cancel{\frac{\partial u}{\partial x} = \frac{\partial x}{\partial x}}$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial y}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}$$ 02

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} e^{x-y} + \frac{\partial u}{\partial r} e^{z-x}(-1) \quad \text{— } (1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p} e^{x-y}(-1) + \frac{\partial u}{\partial q} e^{y-z}(1) \quad \text{- - } (2)$$

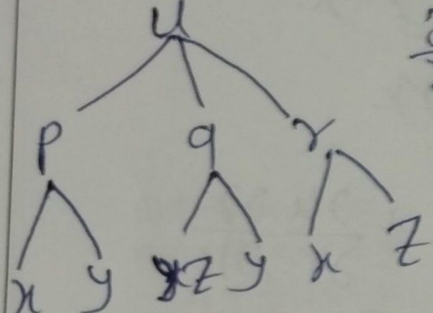
$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial q} e^{y-z}(-1) + \frac{\partial u}{\partial r} e^{z-x}(1) \quad \text{- - - } (3)$$

(1) + (2) + (3)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$ 06

(c) If $u = f(ax - by, by - cz, cz - ax)$, prove that $\frac{1}{a}\frac{\partial u}{\partial x} + \frac{1}{b}\frac{\partial u}{\partial y} + \frac{1}{c}\frac{\partial u}{\partial z} = 0$ 6

$p = ax - by \quad q = by - cz \quad r = cz - ax$



$$\cancel{\frac{\partial u}{\partial p} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}}$$

$$\cancel{\frac{\partial u}{\partial q} = \frac{\partial u}{\partial q}\frac{\partial q}{}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial y}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}$$

02

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}a + \frac{\partial u}{\partial r}(-a) \quad — (1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}(-b) + \frac{\partial u}{\partial q}(b) \quad — (2)$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}(-c) + \frac{\partial u}{\partial r}(c) \quad — (3)$$

Multiply (1), (2), and (3) by $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ then add

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$$\frac{1}{a}\frac{\partial u}{\partial x} + \frac{1}{b}\frac{\partial u}{\partial y} + \frac{1}{c}\frac{\partial u}{\partial z} = 0$$