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Batch:- C3

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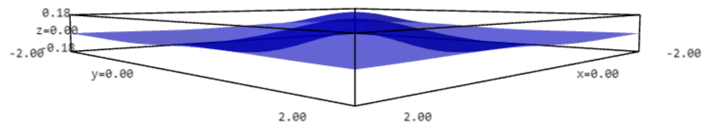
1 Plot the graph of the function

$$f(x,y) = xye^{-x^2-y^2} for - 2 \leq x, y \leq 2.$$

In [1]:
```
var('x y')
f(x,y) = x * y * exp(-x^2 - y^2)
p = plot3d(f, (x,-2,2), (y,-2,2), color='blue', opacity=0.6, scale=2)
show(p)
```



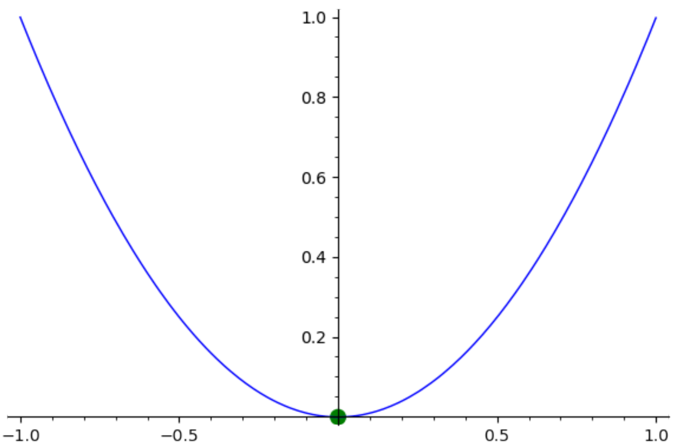
2 Find the following limits (plot graph of function):

$$\lim_{x \to 0} \left(x^2 - \frac{2^x}{1000} \right)$$

In [2]:
```
var('x')
f=(x^2-(2^x/1000))
limit_value = f.limit(x=0)
print("The limit is:")
limit_value.show()
r = plot(f,x)
r+=point((0, limit_value), color='green', size=100)
r.show()
```

The limit is:

$$-\frac{1}{1000}$$



3 Find the 1st four derivative of $f(t) = log(1 + t^2)$ and plot them along with the graph of f(t).

In [3]:
```
var('t')
f = log(1+t^2)
print("f(t) = log(1+t^2)")
print("The first derivative:")
d1 = f.diff()
d1.show()
print("The second derivative:")
d2 = d1.diff()
d2.show()
print("The third derivative:")
d3 = d2.diff()
d3.show()
print("The fourth derivative:")
d4 = d3.diff()
d4.show()

plot(f,(t,-3,3),color='green',legend_label="f(t)")+plot(d1,(t,-3,3),color='red',legend_label="f'(t)")+plot(d2,(t,-3,3),
color='orange',legend_label="f''(t)")+plot(d3,(t,-3,3),color='yellow',legend_label="f'''(t)")+plot(d4,(t,-3,3),color='blue',
legend_label="f''''(t)")
```

```
f(t) = log(1+t^2)
The first derivative:
```

$$\frac{2\,t}{t^2+1}$$

```
The second derivative:
```

$$-\frac{4\,t^2}{\left(t^2+1\right)^2}+\frac{2}{t^2+1}$$

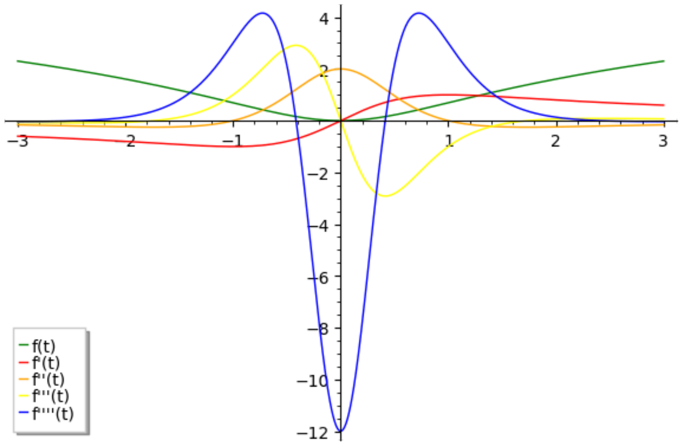
```
The third derivative:
```

$$\frac{16\,t^3}{\left(t^2+1\right)^3}-\frac{12\,t}{\left(t^2+1\right)^2}$$

```
The fourth derivative:
```

$$-\frac{96\,t^4}{\left(t^2+1\right)^4}+\frac{96\,t^2}{\left(t^2+1\right)^3}-\frac{12}{\left(t^2+1\right)^2}$$

Out[3]:



4 What is the n^{th} derivative of x^x for various values of n?

In [4]:
```
var('x')
f = x^x
for n in range(5):
    print(f"The {n}th derivative of x^x is :")
    show(f.diff(n))
```

```
The 0th derivative of x^x is :
```

$$x^x$$

```
The 1th derivative of x^x is :
```

$$x^x(\log(x)+1)$$

```
The 2th derivative of x^x is :
```

$$x^x(\log(x)+1)^2+\frac{x^x}{x}$$

```
The 3th derivative of x^x is :
```

$$x^x(\log(x)+1)^3+\frac{3\,x^x(\log(x)+1)}{x}-\frac{x^x}{x^2}$$

```
The 4th derivative of x^x is :
```

$$x^x(\log(x)+1)^4+\frac{6\,x^x(\log(x)+1)^2}{x}-\frac{4\,x^x(\log(x)+1)}{x^2}+\frac{3\,x^x}{x^2}+\frac{2\,x^x}{x^3}$$

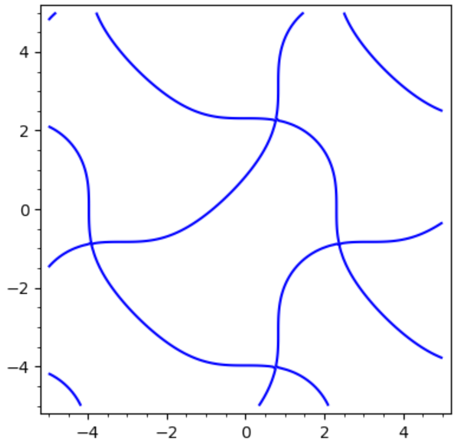
5 Consider the function implicitly defined cos(x-sin(y)) = sin(y - sinx)

i) Plot the curve represented by the given function.

ii) Find dy/dx and d^2y/dx^2

In [5]:
```
var('x y')
f = cos(x-sin(y)) - sin(y-sin(x))
print("Plot of  cos(x-sin(y)) - sin(y-sin(x))")
show(implicit_plot(f,(x,-5,5),(y,-5,5)))
dyx = f.implicit_derivative(y,x)
print("dy/dx = ")
show(dyx)
dyx2 = dyx.implicit_derivative(y,x)
print("d^2y/dx^2 =")
show(dyx2)
```

```
Plot of  cos(x-sin(y)) - sin(y-sin(x))
```




```
dy/dx =
```

$$-\frac{\cos(x)\cos(y-\sin(x))-\sin(x-\sin(y))}{\cos(y)\sin(x-\sin(y))-\cos(y-\sin(x))}$$

```
d^2y/dx^2 =
```

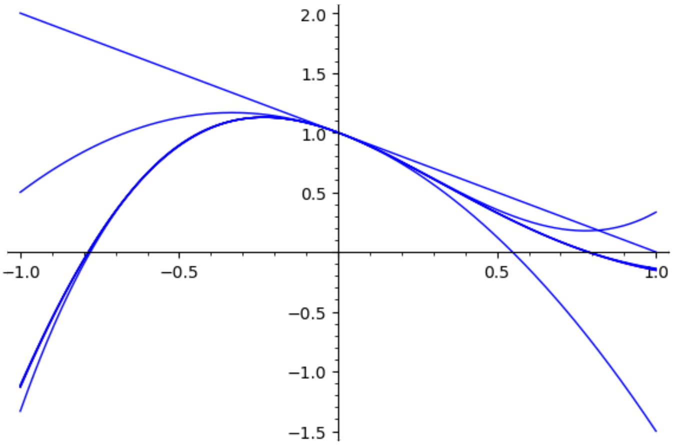
$$\frac{\frac{(\cos(x)\cos(y-\sin(x))-\sin(x-\sin(y)))(\cos(x-\sin(y))\cos(y)-\cos(x)\sin(y-\sin(x)))}{(\cos(y)\sin(x-\sin(y))-\cos(y-\sin(x)))^2}-\frac{\cos(x)^2\sin(y-\sin(x))-\cos(y-\sin(x))\sin(x)-\cos(x-\sin(y))}{\cos(y)\sin(x-\sin(y))-\cos(y-\sin(x))}}{\frac{\left(\cos(x-\sin(y))\cos(y)^2+\sin(x-\sin(y))\sin(y)-\sin(y-\sin(x))\right)(\cos(x)\cos(y-\sin(x))-\sin(x-\sin(y)))}{(\cos(y)\sin(x-\sin(y))-\cos(y-\sin(x)))^2}+\frac{\cos(x-\sin(y))\cos(y)-\cos(x)\sin(y-\sin(x))}{\cos(y)\sin(x-\sin(y))-\cos(y-\sin(x))}}$$

6 Consider f(x) = e^-x cos2x

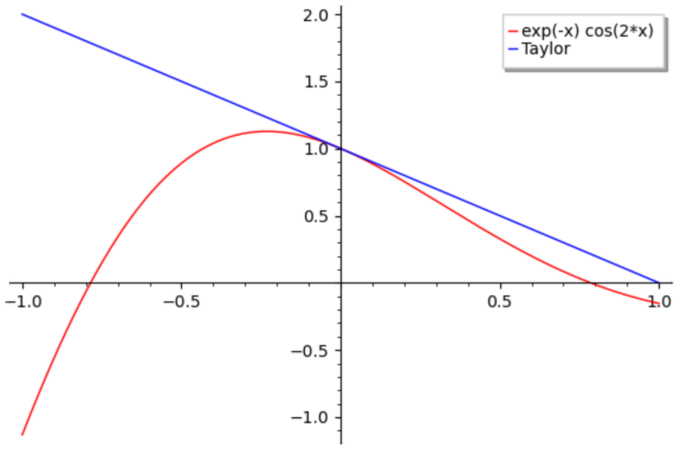
(i) Plot the graph of the function along with Taylor’s polynomial of degree 1, 2, 3, 6, 7, 9, 10.

(ii) Use sage interacts to create interactive plot to plot Taylor’s polynomial along with the curve

In [6]:
```
var('x')
f = exp(-x) * cos(2*x)
degrees = [1,2,3,6,7,9,10]
p = plot(f, color = 'red')
allPlots = p
for deg in degrees:
 allPlots += plot(f.taylor(x,0,deg))
show(allPlots)
@interact
def interactive(degree=slider([1,2,3,6,7,9,10])):
 f = exp(-x) * cos(2*x)
 p = plot(f, color='red', legend_label = "exp(-x) cos(2*x)")
 taylor = plot(f.taylor(x, 0, degree), legend_label = "Taylor")
 show(p+taylor)
```



degree 1



7 Evaluate the following indefinite integrals.

i) $\int \frac{-4}{\sqrt{1-x^2}} dx$

In [7]:
```
var('x')
f = -4/sqrt(1-x^2)
answer = integral(f,x)
c = " + c"
show(answer, c)
print("where c is constant of integration")
```

$-4 \arcsin(x)$ + c

```
where c is constant of integration
```

ii) $\int sin^5 xdx$

In [8]:
```
var('x')
f = (sin(x))^5
answer = integral(f, x)
c = " + c"
show(answer, c)
print("where c is constant of integration")
```

$-\frac{1}{5} \cos(x)^5 + \frac{2}{3} \cos(x)^3 - \cos(x)$ + c

```
where c is constant of integration
```

8 Evaluate the following definite integrals

i)$\int_1^4 \frac{3x}{\sqrt{3x-1}} dx$

In [9]:
```
var('x')
f = (3*x)/(sqrt(3*x-1))
integral(f,x,1,4).show()
```

$$\frac{28}{9}\sqrt{11} - \frac{10}{9}\sqrt{2}$$

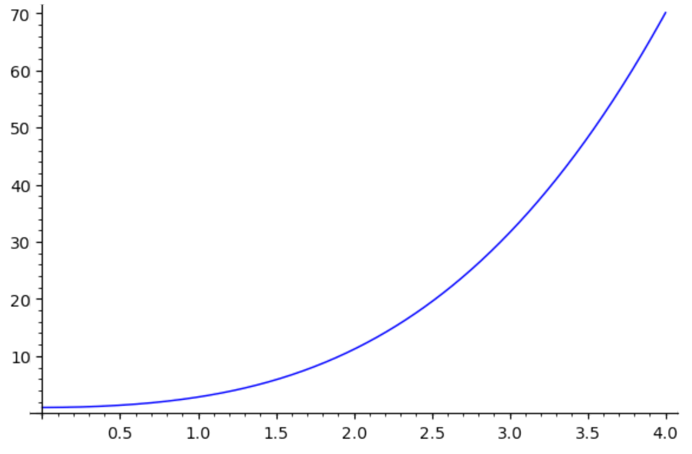
ii) $\int_{\pi/3}^{\pi/2} \frac{1}{1+sinx-cosx}\,dx$

In [10]:
```
f = 1/(1+sin(x)-cos(x))
integral(f,x,pi/3,pi/2).show()
```

$$\frac{1}{2}\log(3) - \log(2) + \log\left(\frac{1}{3}\sqrt{3} + 1\right)$$

9 Graph the curve $y = (1 + x^2)^{\frac{3}{2}}$ for 0 ≤ x ≤ 4 and hence find its arc length.

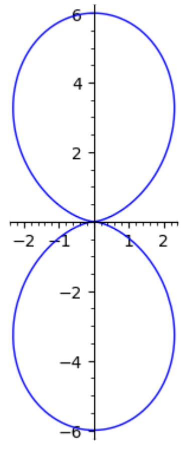
In [11]:
```
var('x')
y = (1+x^2)^(3/2)
plot(y, (x, 0, 4)).show()
dy_dx = diff(y,x)
result = integral(sqrt(1+(dy_dx)^2),(x, 0, 4))
print(f"The length of the arc is {result.n()}")
```




```
The length of the arc is 69.55759264684866
```

10 Find the area that the curve r = 3(1 − cos 2θ), 0 ≤ θ ≤ 2π encloses.

In [12]:
```
var('theta')
r = 3*(1-cos(2*theta))
p = polar_plot(r, theta, 0, 2*pi)
show(p)
area = integral((r*r)/2, theta, 0, 2*pi)
print("Area under the curve:")
show(area)
```




```
Area under the curve:
```

$$\frac{27}{2}\pi$$

11 Find roots of $x^3 - 2x^2 - 5x + 6 = 0$ for x

In [13]:
```
var('x')
f = x^3-2*x^2-5*x+6
print(f"Roots are:-{solve(f==0,x)}")
```

```
Roots are:-[
x == 3,
x == -2,
x == 1
]
```

12 Solve the system of nonlinear equations $x^2 + y^2 = 4$ and $y = x^2 - 2$ for x and y.

In [14]:
```
var('x y')
show(solve([x^2+y^2==4,y==x^2-2],x,y))
```

$$\left[\left[x = -\sqrt{3}, y = 1\right], \left[x = \sqrt{3}, y = 1\right], \left[x = 0, y = (-2)\right]\right]$$