

Review: Matrices

Matrix: 2D array of numbers (n rows, m columns → **n x m** matrix)

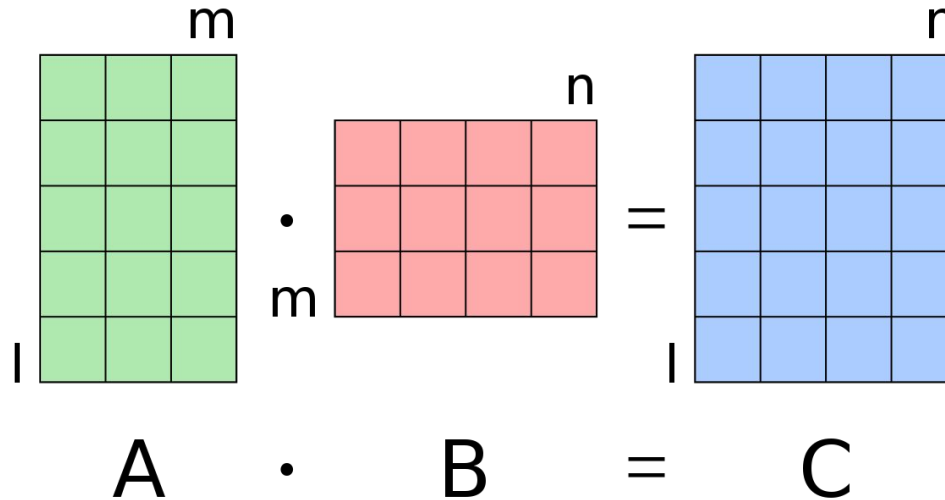
$$A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix}$$

Diagram illustrating the dimensions of matrix A :

- 3 columns (indicated by three orange arrows pointing down to the columns)
- 2 rows (indicated by two blue arrows pointing left to the rows)

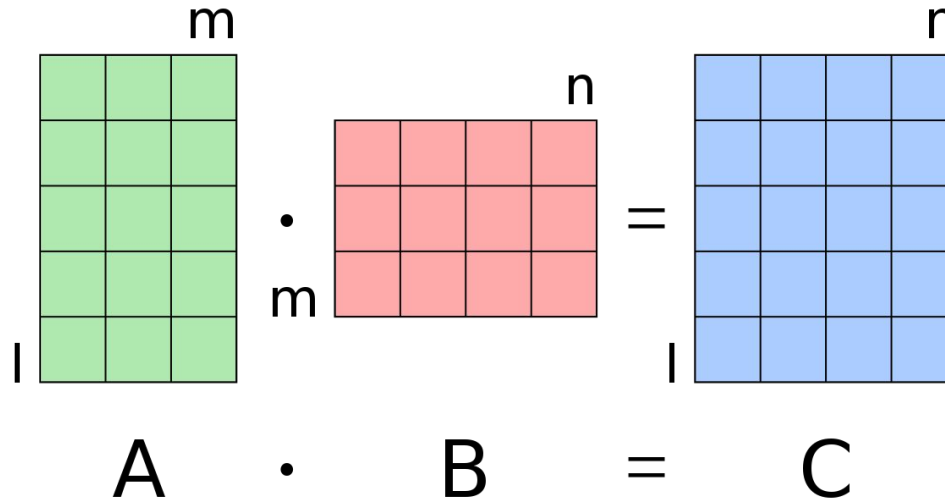
Matrix multiplication

To multiply matrices A and B, the second dimension of A must be the first dimension of B.



Matrix multiplication

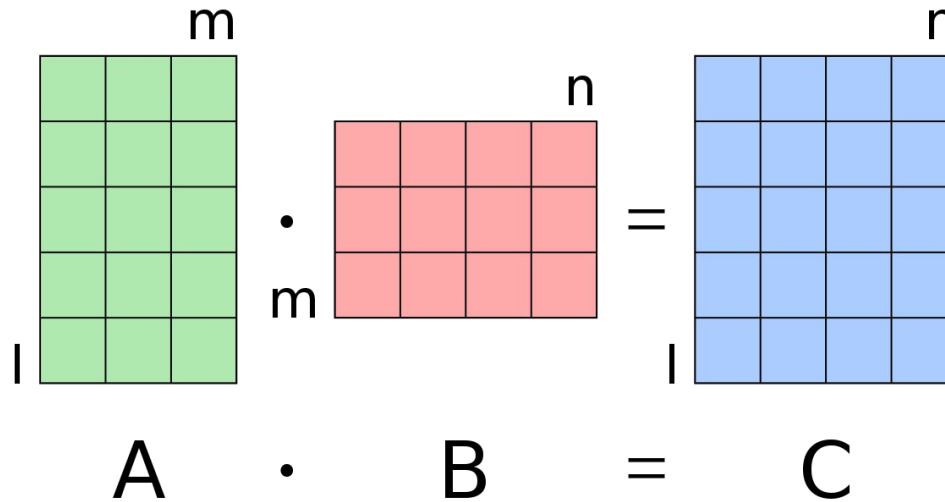
Usages: Many modern data-intensive processes, such as machine learning and dataset management, rely on matrix multiplication.



Matrix multiplication

Let $C = A \times B$. The entry $C[i][j] = \text{sum}(A[i][k] * B[k][j])$ for $k = 0 \dots m-1$.

- **dot product** of i th row of A , j th column of B



Matrix multiplication: Example

$$\begin{bmatrix} 1 & 8 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

Matrix multiplication: Example

$$\begin{bmatrix} 1 & 8 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1*4 + 8*3 \\ 3*4 + 6*3 \end{bmatrix}$$
$$= \begin{bmatrix} 28 \\ 30 \end{bmatrix}$$

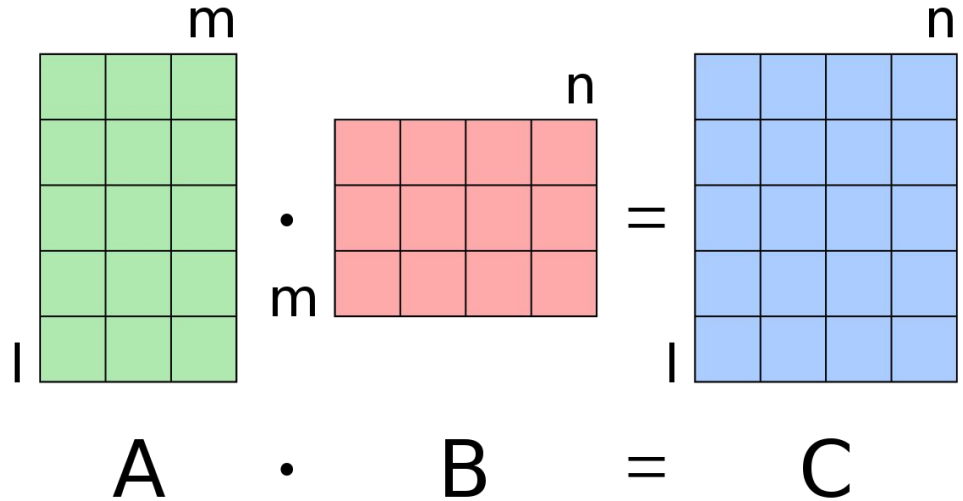
Time complexity of matrix multiplication

For each entry in C ($l \times n$):

Need to compute the summation of m different product terms.

Therefore, overall complexity is **$O(lmn)$** .

For square matrices, is **$O(n^3)$** .



Matrix multiplication is slow...

Naive method: $O(n^3)$

Strassen's method: $O(n^{\log_2 7}) \approx O(n^{2.8})$

Williams et al (2020): $O(n^{2.3728596})$

Duan et al (2022): $O(n^{2.371866})$

Williams et al (2023): $O(n^{2.371552})$