POISSON DISTRIBUTION

Poisson distribution was discovered by the French Mathematician Poisson in 1837. Poisson distribution is the limiting case of the binomial distribution under the following conditions:

(i) n, the number of trails is infinitely large i.e $n \to \infty$

(ii) p, the probability of success in each trial is constant and infinitely small i.e $p \to 0$

(iii) np, the average success is finite say $m, i, e\ np = m$

1. To derive Poisson Distribution:

Consider $p(x) = {}^nC_x p^x q^{n-x} = {}^nC_x \left(\frac{p}{q}\right)^x q^n = {}^nC_x \left(\frac{p}{1-p}\right)^x (1-p)^n$ $\quad [\because\ q = 1-p]$

Putting $p = \frac{m}{n}$

$$P(x) = \frac{n(n-1)(n-2)\ldots.(n-x+1)}{x!} \frac{(m/n)^x}{[1-(m/n)]^x}\left[1-\frac{m}{n}\right]^n \quad = \frac{\left[1-\frac{1}{n}\right]\left[1-\frac{2}{n}\right]\ldots.\left[1-\frac{x-1}{n}\right]m^x\left[1-\frac{m}{n}\right]^n}{x!\left[1-\frac{m}{n}\right]^x}$$

Since $\lim\limits_{n\to\infty}\left[1-\frac{m}{n}\right]^n = e^{-m}\ and\ \lim\limits_{n\to\infty}\left[1-\frac{m}{n}\right]^x = 1$

Taking the limits of both sides as $n \to \infty$ we get $\quad p(x) = \frac{m^x.e^{-m}}{x!}$

Thus, the limit of the Binomial random variable is the Poisson random variable

2. Definition: A random variable X is said to follow Poisson distribution if probability of x is given by

$P(X = x) = \frac{e^{-m}m^x}{x!}, x = 0,1,2 \ldots \ldots \ldots$ and $m(> 0)$ is called the parameter of the distribution.

Remarks:

(i) The sum of the probabilities is 1.

$$\sum_{x=0} P(X = x) = \sum_{x=0} \frac{e^{-m}m^x}{x!} = e^{-m}\sum\frac{m^x}{x!} = e^{-m}\left[1 + m + \frac{m^2}{2!} + \ \ldots.\right] = e^{-m}.e^m = 1$$

(ii) Poisson distribution occurs where the probability of occurance p is very small and the number of trials n is very large and where the probability of occurance only can be known

(iii) Since the Poisson distribution is the limiting case of Binomial distribution, we can calculate binomial probabilities approximately by using Possion distribution whenever n is large and p is small.

3. Mean and Variance: The mean and variance of the Poisson's distribution are both equal to m.

4. Mode Of Poisson Distribution:

Case (i): If m is not an integer then the mode is the integer between $m - 1\ and\ m$.

Case (ii): If m is an integer then there two modes, $m\ and\ m-1$.

Note: If m is an integer then the mean and one mode coincide.

5. Additive property of independent Poisson distributions:

(i) If two independent variates have a Poisson distribution with means $m_1\ and\ m_2$ then their sum is also a Poisson distribution with mean $m_1 + m_2$

(ii) The sum of two Poisson variates is a Poisson variate, but the difference between two Poisson variates is not a Poisson variates.

(iii) If X_1 and X_2 are independent Poisson variates with parameter m_1, m_2 then $Y = a_1X_1 + a_2X_2$ is not a Poisson variate.

6. Recurrence Relation For the Probabilities of Poisson distribution:

(fitting of Poisson distribution)

For Poisson distribution $p(x+1) = \frac{m}{x+1} . p(x)$

If we know $p(0) = e^{-m}$, we can find the probabilities of $x = 1,2,3$ ....

Since, expected frequency of x i.e $f(x)$ is $Np(x)$, we have from the above relation $f(x+1) = \frac{m}{x+1} f(x)$

This relation can be used to find expected frequencies.

EXERCISE

1. The mean and the variance of a probability distribution is 2. Write down the distribution.

2. In a Poisson distribution the probability $P(x = 3)$ is $2/3$ of $P(X = 4)$. Find the mean and the Standard deviation.

3. If a random variable X follows Poisson distribution such that $P(X = 2) = 9\, P(X = 4) + 90P(X = 6)$, find the mean and the variance of X.

4. In a Poisson distribution the probability p(x) for $x = 0$ is 20 percent. Find the mean of the distribution.

5. A variable X follows a Poisson distribution with variance 3. Calculate **(i)** $P(X = 2)$, **(ii)** $P(X \geq 4)$.

6. If X is a Poisson variate with mean 4 and Y is a Poisson variate with mean 5, what is the mean of the variate $X + Y$?

7. If X and Y are Poisson variate with mean 2 and 4 respectively, find $P[(X + Y) \geq 4]$.

8. If the mean of the Poisson distribution is 2. Find the probabilities of $x = 1, 2, 3, 4$, from the recurrence relation of Poisson distribution.

9. Using Poisson distribution, find the approximate value of ${}^{300}\mathrm{C}_2 (0.03)^2 (0.97)^{298} + {}^{300}C_3 (0.03)^3 (0.97)^{297}$

10. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which

(i) neither car is used, **(ii)** some demand is refused.

11. A hospital switch board receives an average of 4 emergency calls in a 10 minutes interval. What is the probability that **(i)** there are atleast 2 emergency calls,

(ii) There are exactly 3 emergency call an interval of 10 minutes?

12. An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that no more than two of its clients are involved such accident next year?

13. Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is know that 2 percent of the bulbs are defective .(Given $e^{-4} = 0.0183$)

14. In a certain manufacturing process 5% of the tools produced turn out to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective. (Given : $e^{-2} = 0.135$).

15. If the probability that an individual suffers a bad reaction from a particular injection is 0.001, determine the probability that out of 2,000 individuals **(i)** exactly three, **(ii)** more than two individuals will suffer a bad reaction . $(\text{Given}: \ e^{-2} = 0.1353)$.

16. The probability that a man aged 40 years will die within next year is 0.001. What is the probability that out of 100 such persons at least 99 will survive till next year ? $(\text{Given}: \ e^{-0.1} = 0.9048.)$.

17. Between the hours of 2 and 4 p.m. the average number of phone calls per minute coming into the switch board of a company is 2.5. Find the probability that during a particular minute there will be

(i) no phone call at all, **(ii)** 4 or less calls, **(iii)** more than 6 calls.

18. Which probability distribution is appropriate to describe the situation where 100 misprints are randomly distributed over 100 pages of a book. For this distribution find the probability that a page selected at random will contain atleast 3 misprints ? $(\text{Given}: \ e^{-1} = 0.3679)$.

19. A manufacture of pins knows that on an average 5% of his product is defective. He sells pins in boxes of 100 and guarantees that no more than 4 pins will be defective. What is the probability that a box will meet the guaranteed quality ?

20. Suppose that a local appliances shop has found from experience that the demand for tube lights is roughly distributed as Poisson with a mean of 4 tube lights per week. If the shop keeps 6 tube lights during a particular week what is the probability that the demand will exceed the supply during that week ? $(\text{Given}: \ e^{-4} = 0.0183)$.

21. Accidents occur on a particular stretch of highway at an average rate 3 per week. What is the probability that there will be exactly two accidents in a given week ? $(\text{Given}: \ e^{-3} = 0.0498)$.

22. The average number of customers who appear at a counter of a certain bank per minute is two. Find the probability that during a given minute **(i)** no customer appears, **(ii)** three or more customers appear.

23. A manufacture of electric bulbs sends out 500 lots each consisting of 100 bulbs. If 5% bulbs are defective in how many lots can we expect **(i)** 97 or more good bulbs, **(ii)** less than 96 good bulbs ?

24. A firm produces articles, 0.1 percent of which are defective. It packs them in cases containing 500 articles. If a wholesaler purchases 100 such cases, how many cases can be expected

(i) to be free from defective, **(ii)** to have one defective?

25. A manufacturer of certain articles knows that on an average 5% of the articles are defective. He sells them in boxes of 100 and guarantees that no more than 4 articles will be defective. In how many boxes out of 1000 he will meet the guaranteed quality ?

26. In a certain factory turning out blades there is a small chance $1/250$ for a blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing

(i) no defective, **(ii)** one defective, **(iii)** two defective blades

in a consignment of 10,000 packets using **(a)** Binomial Distribution ,

(b) Poisson approximation to Binomial distribution.

27. In a certain factory producing certain articles the probability that an article is defective is $1/400$. The articles are supplied in packets of 10. Find approximately the number of packets in a consignment of 20,000 packets containing **(i)** no defective, **(ii)** one defective and **(iii)** two defective blades using **(a)** Binomial Distribution, **(b)** Poisson approximation to Binomial distribution.

28. Fit a Poisson distribution to the following data.

X:	0	1	2	3	4	Total
f:	122	60	15	2	1	200

29. The following mistakes per page were observed in a book.

No. of mistakes:	0	1	2	3	4	Total
No. of pages:	211	90	19	5	0	325

Fit a poisson distribution .

ANSWERS

1. $P(X=x)=\frac{e^{-2}(2)^x}{x!}, x=0,1,2,\ldots\ldots$ **2.** $6,\sqrt{6}$ **3.** 1

4. 1.6 **5.** $0.224,\ 0.353$ **6.** 9 **7.** 0.8487

8. $P(0)=e^{-2},\ P(1)=2e^{-2}, P(2)=e^{-2}, P(3)=\frac{2}{3}e^{-2}, P(4)=\frac{1}{3}e^{-2}$

9. **10.** **(i)** 0.2231 **(ii)** 0.1912

11. **(i)** 0.238 **(ii)** 0.195 **12.** 0.9998 **13.** 0.6283

14. 0.675 **15.** **(i)** 0.1804 **(ii)** 0.3233

16. 0.9996 **17.** **(i)** 0.0821, **(ii)** 0.8909, **(iii)** 0.0145

18. **19.** **20.** **21.**

22. **23.** **24.** **25.**

26. **(a)** **(i)** 9607 **(ii)** 386, **(iii)** 7

(b) **(i)** 9608 **(ii)** 384 **(iii)** 8

27. **(a)** **(i)** 19505 **(ii)** 489 **(iii)** 6

(b) **(i)** 19506 **(ii)** 488 **(iii)** 6

28. $m=0.595$, frequencies : $121, 61, 15, 3, 0$. **29.** $m=0.44$, Frequencies : $209, 92, 20, 3, 1$.