S.Y. Comp. div A&B.
ITVS
Sem - III

Vector Integration
Practice Problems

Equation of line when two points are given

DIV-A & B	Questions
Line Integral	
1,48,D1	Find the work done of the moving partical in the force filed $\bar{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the line (0, 0,0) to (2,1,3) x-x1/x2-x1 = y-y1/y2-y1 = z-z1/z2-z1 = say t
2,49,D2	Evaluate $\int_A^B \left(3xy\,dx - y^2 dy\right)$ along the parabola $y = 2x^2$ from $A(0,0)$ to $B(1,2)$. subs. y = 2x^2 and dy = 4xdx What is the integral if the path is a straight line joining A to B ?
3,50,D3	Find the work done in moving a particle in the force field $\bar{F} = 3xyi - 5zj + 10xk$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. Just substitute values in terms of t
4,51,D4	Show that $\bar{F} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$ is conservative. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$ and hence, find the work done by $\bar{F}$ in displacing a particle from (0,0,1) to (1,π/4,2) along the straight line AB.
5,52,D5	Prove that $\bar{F} = (2xy + z)\, i + (x^2 + 2yz^3)\, j + (3y^2z^2 + x)\, k$ is irrotational. Find its scalar potential ϕ and evaluate $\int_{(1,2,0)}^{(2,2,1)} \bar{F}.d\bar{r}$ along the straight line .
6,53,D6	If $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ be the force field, find the work done in moving a particle from (1,2,-4) to (3,3,2). Same as 1
7,54,D7	Prove that $\bar{F} = 2xye^z i + x^2e^z j + x^2ye^z k$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$ and hence, find the work done by $\bar{F}$ in displacing a particle from (2,1,1) to (2,0,1) .
8,55,D8	Prove that $\bar{F} = 3x^2yi + (x^3 - 2yz^2)j + (3z^2 - 2y^2z)k$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$ and hence, find the work done by $\bar{F}$ in displacing a particle from (0,0,0) to (1,1,1) .
9,56,D9	A vector field is given by $\bar{F} = (x^2 - y^2 + x)i + (2xy + y)j$ is irrotational. Find its scalar potential and also find the line integral from (1,2) to (2,1)
10,57, D10	Prove that $\bar{F} = (3x^2yz - 3y)i + (x^3z - 3x)j + (x^3y + 2z)k$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$ and hence, find the work done by $\bar{F}$ in displacing a particle from (0,0,0) to (1,1,1) .
11,58, D11	Find scalar potential of $\bar{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ if exists. Also find the work done by $\bar{F}$ in displacing a particle from (1,0,2) to (0,1,1) .
12,59, D12	Evaluate $\int_c \bar{F}.d\bar{r}$ along the arc of the curve $\bar{r} = (e^t\cos t)i + (e^t\sin t\, j)$ from (1,0) to $(e^{2\pi}, 0)$ where $\bar{F} = \frac{xi+yj}{(x^2+y^2)^{3/2}}$.
13,60,95	Find the constants a, b, c such that $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is a conservative field . Find its scalar potential and work done in moving a particle from (1,20) to (11,-1).
14,61,96	Prove that $\int_{(1,2)}^{3,4}(6xy^2 - y^3)dx + (6x^2y - 3xy^2dy$ is independent of the path joining the points (1,2) and (3,4) and hence evaluate it.
15,62,97	Find the constant a so that $\bar{F} = (axy - z^3)i + (a-2)x^2j + (1-a)xz^2k$ is a conservative field . Find its scalar potential and work done in moving a particle from (1,2.-3) to (1,-4,2).
GREEN'S Theorem: State GREEN'S Theorem and hence evaluate the following integrals	
16,63,98	$\int_C \left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy$ where 'c' is the boundary of the surface enclosed by the lines $x = 0$, $y = 0$, $x = 2$, $y = 2$.

When field is conservative don't integrate subs limits directly

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17,64,99	$\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = x^2 i - xy\,j$ &'c' is the triangle having vertices (0,2),(2,0),(4,2).
18,65,100	$\int_C \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$ where 'c' is the boundary of the region defined by $x = 1, x = 4, y = 1, \& y = \sqrt{x}$.
19,66,101	$\int_C (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = 4, \& y = x^2$.
20,67,102	$\int_c \bar{F}\cdot d\bar{r}$ where $\bar{F} = -xy(x\,i - y\,j)$ and c is $r = a\,(1 + cos\theta)$
21,68,103	The work done by $\bar{F} = (4x - 2y)i + (2x - 4y)j$ in moving a particle once counter clockwise around the circle $(x-2)^2 + (y-2)^2 = 4$
22,69,104	$\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ around the boundary in the xy plane enclosed by the x-axis and the semi circle $y = \sqrt{1 - x^2}$.
23,70,105	$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the region bounded by $y = \sqrt{x}$ & $y = x$
24,71,106	$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the region bounded by $y = \sqrt{x}$ & $y = x^2$
25,72,107	$\int_c \bar{F}\cdot d\bar{r}$ where $\bar{F} = (x^2 - xy)i + (x^2 - y^2)j$ and c is the region bounded by $x^2 = 2y$ & $x = y$
26,73,108	$\int_C \left[(x^2 + y^2)i + (x^2 - y^2)j\right]\cdot d\bar{r}$ where 'c' is the boundary of the region enclosed by circles $x^2 + y^2 = 4, \quad x^2 + y^2 = 16$
27,74,109	$\int_C \bar{F}.d\bar{r}$ where $\bar{F} = x^2\,i + xy\,j$ and C is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$.
2875,110	$\int_C (xy\,dx + xy^2\,dy)$ where C is the square in the xy-plane with vertices $(1,0), (0,1), (-1,0), and\ (0,-1)$
STOKE'S Theorem: State STOKE'S Theorem and hence evaluate the following integrals	
29,76,111	$\int_C \bar{F}.d\bar{r}$ where $\bar{F} = y\,i + z\,j + x\,k$ and C is the boundary of the surface $x^2 + y^2 = 1 - z,\ z > 0$.
30,77,112	$\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (2x - y)i - yz^2\,j - y^2 z\,k$ and C is the boundary of the hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane.
31,78,113	$\int_C \bar{F}.d\bar{r}$ where $\bar{F} = -xy\,i + 2yz\,j + y^2\,k$ and C is the boundary of the sphere $x^2 + y^2 + z^2 = a^2, z = 0$.

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32,79,114	$\int_C \bar{F}.d\bar{r}$ where $\bar{F}=(x+y)i+(y+z)j-x\,k$ and s is the surface of the plane $2x+y+z=2$ in the first quadrant.
33,80,115	Evaluate $\iint_s (\nabla\times\bar{F})\cdot d\bar{s}$ where $\bar{F}=(2x-y+z)i+(x+y-z^2)j+(3x-2y+4z)\,k$ and s is the surface of the cylinder $x^2+y^2=4$ bounded by the plane $z=9$ and open at the other end.
34,81,116	$\int_C \bar{F}.d\bar{r}$ where $\bar{F}=(x^2+y^2)i+4xyj$ and c is the boundary of the region bounded by the parabola $y^2=4x$ and line $x=4$.
35,82,117	$\int_C \bar{F}.d\bar{r}$ where $\bar{F}=(x+y)i+(2x-z)j+(y+z)k$ and c is the boundary of the triangle cutoff by the plane x + 2y + 3z = 6 on the coordinate axes.
36,83,118	$\int_C \bar{F}.d\bar{r}$ where $\bar{F}=z^2i+x^2j+y^2k$ and C is the curved surface of the hemisphere $x^2+y^2+z^2=100, z\geq 0$
37,84,119	Work done in moving a particle once around the perimeter of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6) under the force $\bar{F}=(x+y)i+(2x-z)j+(y+z)k$
38,85,120	$\int_C \bar{F}.d\bar{r}$ where $\bar{F}=zi+xj+yk$ and C is the boundary of the hemisphere $z=\sqrt{1-x^2-y^2}$ in the xy plane
GAUSS'S DIVERGENCE Theorem: State GAUSS'S DIVERGENCE Theorem and hence evaluate the following	
39,86,121	$\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}=4xi-2y^2j+z^2k$ and S is the region bounded by $x^2+y^2=4, z=0, z=3$.
40,87,122	$\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}=(2x+3z^2)i-(xz^2+y)j+(y^2+2z)k$ and S is the surface of the sphere with centre (3,-14, -17) and radius 3.
41,88,123	$\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}=4xzi-y^2j+yzk$ and S is the surface of the cube bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$.
42,89,124	$\iint_S \bar{F}.d\bar{s}$ where $\bar{F}=xi+yj+zk$ and S is the triangle (1,0,0), (0,1,0) and (0,0,1).
43,90,125	$\iint_S \bar{F}.d\bar{s}$ where $\bar{F}=x^3i+y^3j+z^3k$ and S is the surface of the sphere $x^2+y^2+z^2=9$.
44,91,126	$\iint_S \bar{F}.d\bar{s}$ where $\bar{F}=xi+yj+zk$ and S is the cylindrical surface bounded by $x^2+y^2=a^2, z=0, z=h$
45,92,127	$\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}=2x^2yi-y^2j+4xz^2k$ and S is the region of first octant bounded by $y^2+z^2=9, x=2$
46,93,128	By expressing $\iint_S (y^2z^2i+z^2x^2j+x^2y^2k).d\bar{s}$ as volume integral and evaluate over the part of $x^2+y^2+z^2=1$ lying above the xy plane.
47,94	$\iiint_v \nabla.\bar{F}dv$ where $\bar{F}=4xi-2y^2j+z^2k$ over the cylindrical region $x^2+y^2=a^2, z=0, z=b$.