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Div: C3

Laplace Transform

Q.1 Find the Laplace Transform of the following functions

(i) $(t + e^{-t} + sint)^2$

In [25]:
```
from sympy import symbols, laplace_transform, exp, sin
t, s = symbols('t s')
f = (t + exp(-t) + sin(t))**2
phi = laplace_transform(f, t, s)
print("Laplace of f(t) =")
phi[0]
```

```
Laplace of f(t) =
```

Out[25]:

$$\frac{4s}{\left(s^2+1\right)^2} + \frac{2}{(s+1)^2+1} + \frac{2}{(s+1)^2} + \frac{1}{2\left(\frac{s}{2}+1\right)} + \frac{1}{2s\left(\frac{s^2}{4}+1\right)} + \frac{2}{s^3}$$

(ii) $\frac{e^{-t}\sin t}{t}$

In [20]:
```
t,s = var('t s')
f(t) = (exp(-t)*sin(t))/(t)
show("f(t) =", f(t))
show("Laplace of f(t) =", f.laplace(t,s))
```

f(t) $= \frac{e^{(-t)}\sin(t)}{t}$

Laplace of f(t) $= t \mapsto \frac{1}{2}\pi - \arctan(s+1)$

(iii) t sin2t cosht

In [21]:
```
t, s = var('t s')
f = t * sin(2*t) * cosh(t)
phi = laplace(f, t, s)
show("Laplace of f(t) =",phi)
```

Laplace of f(t) $= \frac{8\left(s^3+3\,s\right)\left(s^2+5\right)}{\left(s^4+6\,s^2+25\right)^2} - \frac{4\,s}{s^4+6\,s^2+25}$

Q.2 Find the Inverse Laplace Transform of the following functions

(i) $\frac{1}{s^3+s}$

In [22]:
```
s, t = var('s, t')
phi(s) = 1/(s^3 + s)
show("Laplace inverse of phi(s) =",inverse_laplace(phi(s),s,t))
```

Laplace inverse of phi(s) $= -\cos(t) + 1$

(ii) $\frac{(s+2)^2}{(s^2+4s+8)^2}$

In [23]:
```
s, t = var('s, t')
phi(s) = (s + 2)^2 / (s^2 + 4*s + 8)^2
show("Laplace inverse of phi(s) =",inverse_laplace(phi(s), s, t))
```

Laplace inverse of phi(s) $= \frac{1}{2}\,t\cos(2\,t)e^{(-2\,t)} + \frac{1}{4}\,e^{(-2\,t)}\sin(2\,t)$

Q.3 Solve the following differential equation using Laplace Transform

$$x''(t) + 2x'(t) + 5x(t) = e^{-t}sint$$

with x(0) = 0, x'(0) = 1

In [6]:
```
s, t = var('s, t')
x = function('x')(t)
de = diff(x, t, t) + 2*diff(x, t) + 5*x == exp(-2*t) * sin(2*t)
solution = desolve_laplace(de, x, ics=[0, 1, 0])
show(solution)
```

$$\frac{1}{34}\,(26\,\cos(2\,t) + 19\,\sin(2\,t))e^{(-t)} + \frac{1}{17}\,(4\,\cos(2\,t) + \sin(2\,t))e^{(-2\,t)}$$

Fourier Series

Question 1) Find all the Fourier Coefficients and Fourier Series for the following functions. Also plot the graph of the function and the Fourier

(i) f(x)= $(\frac{\pi - x}{2})^2$ in (0,2π) for n=10 and n=20

In [7]:
```
var('x')
var('n')
assume(n,'integer')
f(x)= ((pi - x)/2)^2
L = pi
an=(1/L)*integrate(f*cos(n*pi*x/L),x,0,2*L)
a0=(1/L)*integrate(f,x,0,2*L)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,0,2*L)
s_10 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,10)
show("value of a0 is: ", a0)
show("value of an is: ", an)
show("value of bn is: ", bn)
show("Fourier series with 10 terms is: ",s_10)
plot(f,0,2*L,legend_label="f(x)") + plot(s_10,0,2*L,color = "red",legend_label="fourier series(10 terms)")
```

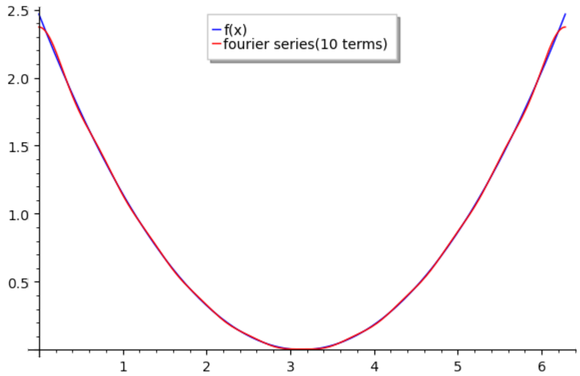
value of a0 is: $\frac{1}{6}\pi^2$

value of an is: $\frac{1}{n^2}$

value of bn is: 0

Fourier series with 10 terms is: $\frac{1}{12}\pi^2 + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$

Out[7]:

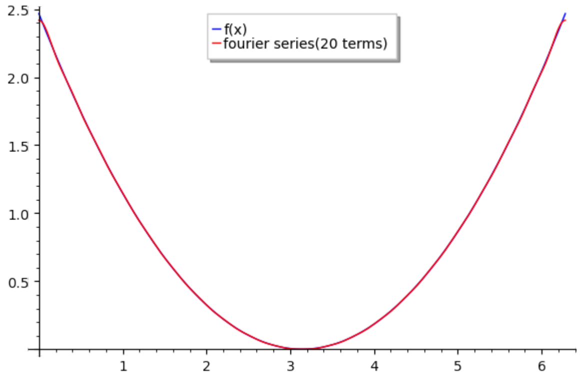


for n = 20

In [8]:
```
s_20 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,20)
show("\n Fourier series with 20 terms: \n",s_20)
plot(f,0,2*L,legend_label="f(x)") + plot(s_20,0,2*L,color = "red",legend_label="fourier series(20 terms)")
```

Fourier series with 20 terms: $\frac{1}{12}\pi^2 + \frac{1}{400}\cos(20x) + \frac{1}{361}\cos(19x) + \frac{1}{324}\cos(18x) + \frac{1}{289}\cos(17x) + \frac{1}{256}\cos(16x) + \frac{1}{225}\cos(15x) + \frac{1}{196}\cos(14x) + \frac{1}{169}\cos(13x) + \frac{1}{144}\cos(12x) + \frac{1}{121}\cos(11x) + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$

Out[8]:



(ii) f(x)= x^5 in (-π,π) for n=5 and n=15

for n = 5

In [9]:

```
var('x')
var('n')
assume(n,'integer')
L = pi
f(x)= x^5
an=(1/L)*integrate(f*cos(n*pi*x/L),x,-pi,pi)
a0=(1/L)*integrate(f,x,-pi,pi)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,-pi,pi)
s =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,5)
print("The function is odd")
show("value of a0 is ", a0)
show("value of an is ", an)
show("value of bn is ", bn)
show("Fourier series with 5 terms is: ",s)
plot(f,-pi,pi,legend_label="f(x)") + plot(s,-pi,pi,color = "yellow",legend_label="fourier series(5 terms)")
```

The function is odd

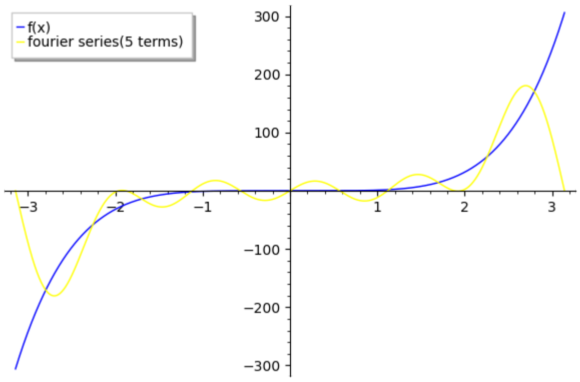
value of a0 is0

value of an is0

value of bn is $-\frac{2\left(120\pi+\pi^5n^4-20\pi^3n^2\right)(-1)^n}{\pi n^5}$

Fourier series with 5 terms is:$\frac{2}{625}\left(125\pi^4-100\pi^2+24\right)\sin(5x)-\frac{1}{64}\left(32\pi^4-40\pi^2+15\right)\sin(4x)+\frac{2}{81}\left(27\pi^4-60\pi^2+40\right)\sin(3x)-\frac{1}{2}\left(2\pi^4-10\pi^2+15\right)\sin(2x)+2\left(\pi^4-20\pi^2+120\right)\sin(x)$

Out[9]:



for n = 15

In [24]:

```
var('x')
var('n')
assume(n,'integer')
L = pi
f(x)= x^5
an=(1/L)*integrate(f*cos(n*pi*x/L),x,-L,L)
a0=(1/L)*integrate(f,x,-L,L)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,-L,L)
s =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
print("The function is odd")
show("value of a0 =", a0)
show("value of an =", an)
show("value of bn =", bn)
show("Fourier series with 15 terms is: ",s)
plot(f,-L,L,legend_label="f(x)") + plot(s,-L,L,color = "yellow",legend_label="fourier series(15 terms)")
```

The function is odd

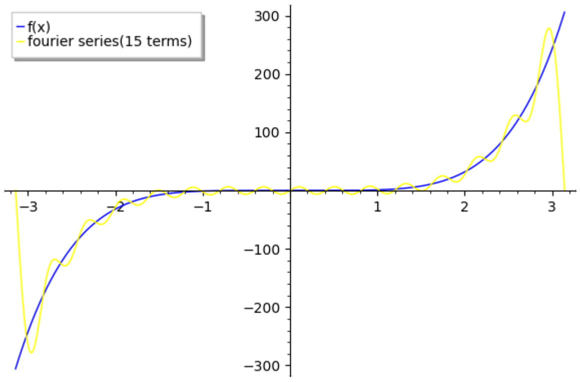
value of a0 =0

value of an =0

value of bn = $-\frac{2\left(120\pi+\pi^5n^4-20\pi^3n^2\right)(-1)^n}{\pi n^5}$

Fourier series with 15 terms is:$\frac{2}{50625}\left(3375\pi^4-300\pi^2+8\right)\sin(15x)-\frac{1}{33614}\left(4802\pi^4-490\pi^2+15\right)\sin(14x)+\frac{2}{371293}\left(28561\pi^4-3380\pi^2+120\right)\sin(13x)-\frac{1}{5184}\left(864\pi^4-120\pi^2+5\right)\sin(12x)+\frac{2}{161051}\left(14641\pi^4-2420\pi^2+120\right)\sin(11x)-\frac{1}{1250}\left(250\pi^4-50\pi^2+3\right)\sin(10x)+\frac{2}{19683}\left(2187\pi^4-540\pi^2+40\right)\sin(9x)-\frac{1}{2048}\left(512\pi^4-160\pi^2+15\right)\sin(8x)+\frac{2}{16807}\left(2401\pi^4-980\pi^2+120\right)\sin(7x)-\frac{1}{162}\left(54\pi^4-30\pi^2+5\right)\sin(6x)+\frac{2}{625}\left(125\pi^4-100\pi^2+24\right)\sin(5x)-\frac{1}{64}\left(32\pi^4-40\pi^2+15\right)\sin(4x)+\frac{2}{81}\left(27\pi^4-60\pi^2+40\right)\sin(3x)-\frac{1}{2}\left(2\pi^4-10\pi^2+15\right)\sin(2x)+2\left(\pi^4-20\pi^2+120\right)\sin(x)$

Out[24]:



Q.2 Find the Half range cosine series for f(x) = x 0 < x < 2 for n=20 . Also plot the graph of the function and the cosine series.

In [11]:

```
var('x')
var('n')
assume(n,'integer')
L = 2
f = piecewise([[[0,L],x]])
an=(2/L)*integrate((x)*cos(n*pi*x/L),x,0,L)
a0=(2/L)*integrate(x,x,0,L)
s =a0/2+sum(an*cos(n*pi*x/L),n,1,20)
show("Value of a0 = ", a0)
show("Value of an = ", an)
show("Fourier series with 20 terms is: ",s)
plot(f,0,L,legend_label="f(x)") + plot(s,0,L,color = "yellow",legend_label="fourier series(20 terms)")
```

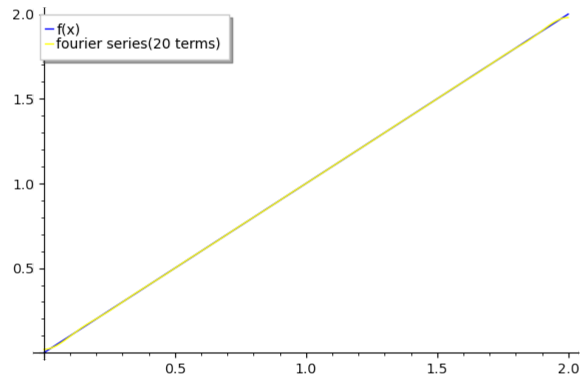
Value of a0 $=2$

Value of an $=\frac{4\,(-1)^n}{\pi^2 n^2} - \frac{4}{\pi^2 n^2}$

Fourier series with 20 terms is:

$$-\frac{8\left(586396035225\,\cos\left(\frac{19}{2}\,\pi x\right) + 732487781025\,\cos\left(\frac{17}{2}\,\pi x\right) + 940839860961\,\cos\left(\frac{15}{2}\,\pi x\right) + 1252597448025\,\cos\left(\frac{13}{2}\,\pi x\right) + 1749495609225\,\cos\left(\frac{11}{2}\,\pi x\right) + 2613444058225\,\cos\left(\frac{9}{2}\,\pi x\right) + 4320183035025\,\cos\left(\frac{7}{2}\,\pi x\right) + 8467558748649\,\cos\left(\frac{5}{2}\,\pi x\right) + 23520996524025\,\cos\left(\frac{3}{2}\,\pi x\right) + 211688968716225\,\cos\left(\frac{1}{2}\,\pi x\right)\right)}{211688968716225\,\pi^2} + 1$$

Out[11]:



Q.3 Find the Half range sine series for (x)= 1-x^2 in (0,1) for n=15 . Also plot the graph of the function and the sine series.

In [12]:

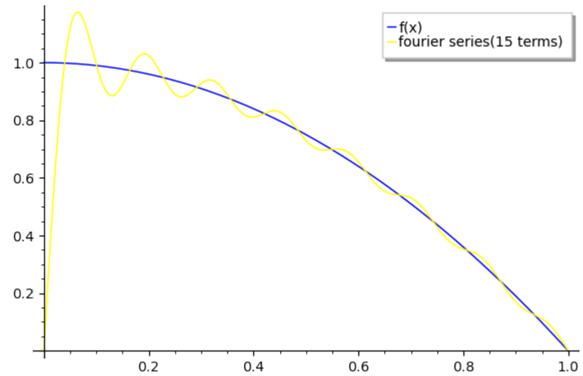
```
var('x')
var('n')
assume(n,'integer')
L = 1
f = piecewise([[[0,L],1-x^2]])
bn=(2/L)*integrate((1-x^2)*sin(n*pi*x/L),x,0,L)
s =sum(bn*sin(n*pi*x/L),n,1,15)
show("Value of bn = ", bn)
show("Fourier series with 15 terms is: ",s)
plot(f,0,L,legend_label="f(x)") + plot(s,0,L,color = "yellow",legend_label="fourier series(15 terms)")
```

Value of bn $=\frac{2\left(\pi^2 n^2 + 2\right)}{\pi^3 n^3} - \frac{4\,(-1)^n}{\pi^3 n^3}$

Fourier series with 15 terms is:

$$\frac{52227799123500\,\pi^2\sin(14\,\pi x) + 60932432310750\,\pi^2\sin(12\,\pi x) + 73118918772900\,\pi^2\sin(10\,\pi x) + 91398648466125\,\pi^2\sin(8\,\pi x) + 121864864621500\,\pi^2\sin(6\,\pi x) + 182797296932250\,\pi^2\sin(4\,\pi x) + 365594593864500\,\pi^2\sin(2\,\pi x) + 216648648216\left(225\,\pi^2+4\right) + 332812557000\left(169\,\pi^2+4\right)\sin(13\,\pi x) + 549353259000\left(121\,\pi^2+4\right)\sin(11\,\pi x) + 1003003001000\left(81\,\pi^2+4\right)\sin(9\,\pi x) + 21317 \left(49\,\pi^2+4\right)\sin(7\,\pi x) + 5849513501832\left(25\,\pi^2+4\right)\sin(5\,\pi x) + 27081081027000\left(9\,\pi^2+4\right)\sin(3\,\pi x) + 731189187729000\left(\pi^2+4\right)}{365594593864500\,\pi^3}$$

Out[12]:



Q.4 Find the Fourier series (n=15) , a10 and b15 for f(x)=x(π-x) in (-π,π).

In [13]:
```
var('x')
var('n')
assume(n,'integer')
L = pi
f(x)= x*(pi -x)
an=(1/L)*integrate(f*cos(n*pi*x/L),x,-L,L)
a0=(1/L)*integrate(f,x,-L,L)
a10=(1/L)*integrate(f*cos(10*pi*x/L),x,-L,L)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,-L,L)
b15=(1/L)*integrate(f*sin(15*pi*x/L),x,-L,L)
s =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
show("Value of a0 =", a0)
show("Value of an =", an)
show("Value of bn =", bn)
show("Fourier series with 15 terms is: ",s)
show("Value of a10 =", a10)
show("Value of b15 =", b15)
```

Value of a0 $= -\frac{2}{3}\pi^2$

Value of an $= -\frac{4(-1)^n}{n^2}$

Value of bn $= -\frac{2\left(\frac{(\pi^2 n^2-1)(-1)^n}{n^3} + \frac{(-1)^n}{n^3}\right)}{\pi}$

Fourier series with 15 terms is: $-\frac{1}{3}\pi^2 + \frac{2}{15}\pi\sin(15x) - \frac{1}{7}\pi\sin(14x) + \frac{2}{13}\pi\sin(13x) - \frac{1}{6}\pi\sin(12x) + \frac{2}{11}\pi\sin(11x) - \frac{1}{5}\pi\sin(10x) + \frac{2}{9}\pi\sin(9x) - \frac{1}{4}\pi\sin(8x) + \frac{2}{7}\pi\sin(7x) - \frac{1}{3}\pi\sin(6x) + \frac{2}{5}\pi\sin(5x) - \frac{1}{2}\pi\sin(4x) + \frac{2}{3}\pi\sin(3x) - \pi\sin(2x) + 2\pi\sin(x) + \frac{4}{225}\cos(15x) - \frac{1}{49}\cos(14x) + \frac{4}{169}\cos(13x) - \frac{1}{36}\cos(12x) + \frac{4}{121}\cos(11x) - \frac{1}{25}\cos(10x) + \frac{4}{81}\cos(9x) - \frac{1}{16}\cos(8x) + \frac{4}{49}\cos(7x) - \frac{1}{9}\cos(6x) + \frac{4}{25}\cos(5x) - \frac{1}{4}\cos(4x) + \frac{4}{9}\cos(3x) - \cos(2x) + 4\cos(x)$

Value of a10 $= -\frac{1}{25}$

Value of b15 $= \frac{2}{15}\pi$