K. J. Somaiya College of Engineering, Mumbai-77
(Autonomous College Affiliated to University of Mumbai)

LAPLACE TRANSFORM

I.FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

1. $f(t)=(t-1)^4,\ \ t>4;\ \ f(t)=0,\ \ 0<t<4$
2. $f(t)=t,\ \ 0<t<1/2;\ \ f(t)=t-1, 1/2<t<1;\ \ \ f(t)=0,\ t>1$ [Ans: $\frac{1}{s^2}-\frac{e^{-s}}{s^2}-\frac{e^{-s/2}}{s}$]
3. $f(t)=0,\ 0<t<\pi;\ \ f(t)=\sin^2(t-\pi),\ t>\pi$ [Ans: $\frac{e^{-\pi s}}{2s}-\frac{s.e^{-\pi s}}{s^2+4}$]
4. $\cos t\cdot\cos 2t\cdot\cos 3t$ [Ans: $\frac{1}{4}\left(\frac{1}{s}+\frac{s}{s^2+2^2}+\frac{s}{s^2+4^2}+\frac{s}{s^2+6^2}\right)$]
5. $\left(\sqrt{t}-1\right)^2$ [Ans: $\frac{1}{s^2}-\frac{\sqrt{\pi}}{s^{3/2}}+\frac{1}{s}$]
6. $\frac{\cos\sqrt{t}}{\sqrt{t}}$ [Ans: $\sqrt{\frac{\pi}{s}}\cdot e^{-1/4s}$]
7. If $L\left[\sin\sqrt{t}\right]=\frac{\sqrt{\pi}}{2s\sqrt{s}}\cdot e^{-1/4s}$, find $L\left[\sin 2\sqrt{t}\right]$ [Ans: $\frac{\sqrt{\pi}}{s\sqrt{s}}\cdot e^{-1/s}$]
8. $\sinh(t/2)\cdot\sin(\sqrt{3}t/2)$ [Ans: $\frac{\sqrt{3}}{2}\cdot\frac{s}{(s^4+s^2+1)}$]
9. $e^{4t}\sin^3 t$ [Ans: $\frac{6}{(s^2-8s+17)(s^2-8s+25)}$]
10. $\frac{\cos 2t\cdot\sin t}{e^t}$ [Ans: $\frac{s^2+2s-2}{(s^2+2s+10)(s^2+2s+2)}$]
11. $e^{-4t}\sinh t\cdot\sin t$ [Ans: $\frac{2(s+4)}{(s^2+6s+10)(s^2+10s+26)}$]
12. $e^{2t}(1+t)^2$ [Ans: $\frac{1}{(s-2)}+\frac{2}{(s-2)^2}+\frac{2}{(s-2)^3}$]
13. If $L[f(t)]=\frac{s}{s^2+s+4}$, *find* $L[e^{-3t}f(2t)]$ [Ans: $\frac{s+3}{s^2+8s+10}$]
14. $\left(1+te^{-t}\right)^3$ [Ans: $\frac{1}{s}-\frac{3}{(s+1)^2}+\frac{6}{(s+2)^3}+\frac{6}{(s+3)^3}$]

15. $t\sin^3 t$ [Ans: $24\cdot\dfrac{s(s+5)}{(s^2+1)^2(s^2+9)^2}$]

16. $t^5\cosh t$ [Ans: $60\left(\dfrac{1}{(s-1)^6}+\dfrac{1}{(s+1)^6}\right)$]

17. $t\sqrt{1+\sin t}$ [Ans: $4\dfrac{(4s^2+4s-1)}{(4s^2+1)^2}$]

18. $t\left(\dfrac{\sin t}{e^t}\right)^2$ [Ans: $\dfrac{1}{2}\left(-\dfrac{1}{(s+2)^2}+\dfrac{s^2+4s}{(s^2+4s+8)^2}\right)$]

19. If $L[f(t)]=\dfrac{s+3}{s^2+s+1}$, *find* $L[t\,f(2t)]$ [Ans: $\dfrac{s^2+12s+8}{(s^2+2s+4)^2}$]

20. $t\,e^{-2t}\sinh 4t$ [Ans: $\dfrac{8(s+2)}{(s^2+4s-12)^2}$]

21. $t\cos(\omega t-\alpha)$ [Ans: $\dfrac{(s^2-\omega^2)\cos\alpha-2\omega s\sin\alpha}{(\omega^2+s^2)^2}$]

22. $(t\sinh 2t)^2$ Wrong answer [Ans: $\dfrac{1}{2}\left(\dfrac{1}{(s-4)^3}+\dfrac{1}{(s+4)^3}\right)$]

23. $(t+\sin 2t)^2$ [Ans: $\dfrac{2}{s^3}+\dfrac{8s}{(s^2+4)^2}+\dfrac{1}{2s}-\dfrac{s}{2(s^2+4^2)}$]

24. $\dfrac{1}{t}(1-\cos t)$ [Ans: $\dfrac{1}{2}\log\left(\dfrac{s^2+1}{s^2}\right)$]

25. $\dfrac{1}{t}e^{-t}\sin t$ [Ans: $\cot^{-1}(s+1)$]

26. $\dfrac{\sin^2 2t}{t}$ [Ans: $\dfrac{1}{4}\log\left(\dfrac{s^2+16}{s^2}\right)$]

27. $\dfrac{1-\cos t}{t^2}$ [Ans: $\dfrac{\pi}{2}-\dfrac{s}{2}\log\left(\dfrac{s^2+1}{s^2}\right)-\tan^{-1}s$]

28. Find the Laplace transform of $\dfrac{\sin at}{t}$. Does Laplace transform of $\dfrac{\cos at}{t}$ exist?

[Ans: $\cot^{-1}\dfrac{s}{a}$, does not exist]

29. $\dfrac{\cosh 2t\ \sin 2t}{t}$ [Ans: $\pi+\tan^{-1}\left(\dfrac{s-2}{2}\right)+\tan^{-1}\left(\dfrac{s+2}{2}\right)$]

30. $\dfrac{e^{-at}-\cos at}{t}$ [Ans: $\log\left(\dfrac{\sqrt{s^2+a^2}}{s+a}\right)$]

31. $\int_0^t u\,e^{-3u}\cos^2 u\,du$

32. $(i)\,t^3\delta(t-3)$ (ii) $t^3H(t-3)$

33. Given that $f(t)=t+1,\ 0\le t\le 2,\ \&\ f(t)=3,\ t>2$ find $L[f(t)],\ L[f'(t)] \,\&\, L[f''(t)]$

[Ans: $\frac{1}{s}+\frac{1}{s^2}\left(1-e^{-2s}\right),\ \frac{1}{s}\left(1-e^{-2s}\right),\ s^2\left[\frac{1}{s}+\frac{1}{s^2}\left(1-e^{-2s}\right)\right]-s-1$]

34. Find the Laplace transform of $\frac{d}{dt}\left(\frac{\sin 3t}{t}\right)$ [Ans: $s\cot^{-1}(s/3)-3$]

35. $erf\sqrt{t}$ [Ans: $\frac{1}{s\sqrt{s+1}}$]

36. $erf\,2\sqrt{t}$ [Ans: $\frac{2}{s\sqrt{s+4}}$]

37. $e^{3t}t\ erf\sqrt{t}$ [Ans: $\frac{3s-7}{2(s-3)^2(s-2)^{3/2}}$]

38. $\int_0^t\int_0^t\int_0^t t\sin t\,(dt)^3$ [Ans: $\frac{2}{s^2(s^2+1)^2}$]

39. $\int_0^t ue^{-3u}\cos^2 2u\,du$ [Ans: $\frac{1}{2s(s+3)^2}+\frac{s^2+6s-7}{2s(s^2+6s+25)^2}$]

40. $\int_0^t \frac{1-e^{-au}}{u}\,du$ [Ans: $\frac{1}{s}\log\left(\frac{s-a}{s}\right)$]

41. $t^{-1}\int_0^t e^{-u}\sin u\,du$ [Ans: $\frac{1}{4}\log\left(\frac{s^2+2s+2}{s^2}\right)-\frac{1}{2}\cot^{-1}(s+1)$]

42. $e^{-4t}\int_0^t u\sin 3u\,du$ [Ans: $\frac{6}{(s^2+8s+25)^2}$]

43. $\cosh t\int_0^t e^u\cosh u\,du$ [Ans: $\frac{1}{2}\left[\frac{s-2}{(s-1)^2(s-3)}+\frac{s}{(s+1)^2(s-1)}\right]$]

44. $\int_0^t ue^{-3u}\sin^2 u\,du$ [Ans: $\frac{1}{2s}\left[\frac{1}{(s+3)^2}+\frac{s^2+6s+5}{(s^2+6s+13)^2}\right]$]

45. $\frac{1}{t}(\cos at-\cos bt)$ [Ans: $\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$]

46. Find $L\{\cosh 2t\cdot erf\,3\sqrt{t}\}$ if $L\{erf\sqrt{t}\}=\frac{1}{s\sqrt{s+1}}$ [Ans:

$\frac{1}{2}\left[\frac{3}{(s+2)\sqrt{s+7}}+\frac{3}{(s-2)\sqrt{s+11}}\right]$]

47. If $L\left(2\sqrt{\frac{t}{\pi}}\right)=\frac{1}{s^{3/2}}$, show that $L\left(\frac{1}{\sqrt{\pi t}}\right)=\frac{1}{\sqrt{s}}$

48. A function $f(t)$ obeys the equation $f(t)+2\int_0^t f(t)\,dt = \cosh 2t$ find the Laplace transform of $f(t)$ [Ans: $\frac{s^2}{(s^2-4)(s+2)}$]

II. EVALUATE THE FOLLOWING INTEGRALS USING LAPLACE TRANSFORM:

49. $\int_0^\infty e^{-2t}\sin^3 t\,dt$ [Ans: 6/65]

50. If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)\,dt = 3/8$ *then find* α. [Ans: $\pi/4$]

51. $\int_0^\infty e^{-3t} t\sin t\,dt$ [Ans: 3/50]

52. If $L[J_0(t)] = \frac{1}{\sqrt{s^2+1}}$, prove that $\int_0^\infty e^{-3t} t\, J_0(4t)\,dt = 3/125$

53. $\int_0^\infty \frac{t^2\sin 3t}{e^{2t}}\,dt$ [Ans: 18/2197]

54. $\int_0^\infty \frac{\cos at-\cos bt}{t}dt$ [Ans: $\log\frac{b}{a}$]

55. $\int_0^\infty e^{-st}\frac{\sin^2(at/2)}{t}\,dt$ [Ans: $\frac{1}{2}\log\left(\frac{s^2+a^2}{s^2}\right)$]

56. Prove that $\int_0^\infty e^{-st}\frac{\sin t\sinh t}{t}\,dt = \frac{1}{2}\tan^{-1}\left(\frac{2a}{1+s^2-a^2}\right)$

57. $\int_0^\infty \frac{e^{-t}-\cos t}{t\,e^{4t}}\,dt$ [Ans: $\log\frac{\sqrt{17}}{5}$]

58. Prove that $\int_0^\infty \frac{\sin 2t+\sin 3t}{t\,e^t}\,dt = \frac{3\pi}{4}$

59. $\int_0^\infty e^{-2t}\sinh t\,\frac{\sin t}{t}\,dt$ [Ans: $\frac{1}{2}\tan^{-1}\frac{1}{2}$]

60. $\int_0^\infty e^{-t}\left(\int_0^t u^2\sinh u\cosh u\,du\right)dt$ [Ans: $-\frac{2}{125}$]

61. $\int_0^\infty e^{-4t}\left(\cosh t\int_0^t e^u\cosh u\,du\right)dt$ [Ans: 31/225]

62. Prove that $\int_0^\infty e^{-st}\frac{\sin bt+\sin at}{t}\,dt = \pi-\tan^{-1}\left(\frac{s(a+b)}{ab-s^2}\right)$

63. $\int_0^\infty e^{-t}\sin^5 t\,dt$ [Ans: $\frac{3}{8}$]

64. $\int_0^\infty \frac{\cos 4t-\cos 3t}{t}dt$ [Ans: $\log\frac{3}{4}$]

65. $\int_0^\infty e^{-t}t^3\sin t\,dt$ [Ans: 0]

66. $\int_{t=0}^{\infty}\int_{u=0}^{t}\frac{e^{-t}\sin u}{u}du\,dt$ [Ans: $\frac{\pi}{4s}$]

INVERSE LAPLACE TRANSFORM

Find the inverse laplace transform of following functions:

67 (a) $\frac{4s+12}{s^2+8s+12}$ [Ans: $e^{-4t}(4\cosh 2t-\sinh 2t)$]

67. (b) $\frac{s}{s^2+2s+2}$ [Ans: $e^{-t}(\cos t-\sin t)$]

68. (a) $\frac{s}{(2s+1)^2}$ [Ans: $e^{-1/2}(t-4)/16$]

68 (b) $\frac{s+1}{s^2-4}$ [Ans: $\frac{1}{4}\left(3e^{2t}+e^{-2t}\right)$]

69. $\frac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\frac{3}{2}e^{-t}\sin 2t-2e^{t}\sin t$]

70. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\frac{1}{a^2-b^2}(a\sin at-b\sin bt)$]

71. $\frac{s}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\frac{1}{b^2-a^2}(\cos at-\cos bt)$]

72. $\frac{5s^2+8s-1}{(s+3)(s^2+1)}$ [Ans: $2e^{-3t}+3\cos t-\sin t$]

73. $\frac{2s}{s^4+4}$ [Ans: $\sin t\,\sinh t$]

74. $\frac{1}{s^3+1}$ [Ans: $\frac{1}{3}e^{-t}-\frac{e^{t/2}}{3}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{e^{t/2}}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)$]

75. $\frac{1}{s^3(s-1)}$ [Ans: $1-t+\frac{t^2}{2}-e^{-t}$]

76. $\frac{s}{(s+1)^2(s^2+1)}$ [Ans: $\frac{1}{2}\left[\sin t-te^{-t}\right]$]

77. $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$ [Ans: $e^{-t}+4e^{2t}-7te^{2t}$]

78. $\frac{s+2}{s^2+6s+25}$

79. $\dfrac{s}{(s^2+1)\,(s^2+4)(s^2+9)}$ [Ans: $\dfrac{1}{24}\cos t-\dfrac{1}{15}\cos 2t+\dfrac{1}{40}\cos 3t$]

80. $\dfrac{s^2}{(s+1)^3}$ [Ans: $e^{-t}\left(1-2t+t^2\right)$]

81. $\dfrac{3s-2}{s^{5/2}}-\dfrac{7}{3s+2}$

82. $\log\left(\dfrac{s+a}{s+b}\right)$ [Ans: $-\dfrac{1}{t}\,(e^{-at}-e^{-bt})$]

83. $2\tanh^{-1} s$ [Ans: $\dfrac{2}{t}\ \sinh t$]

84. $\tan^{-1}\left(\dfrac{2}{s^2}\right)$ [Ans: $2\sin t\,\sinh t$]

85. $\tan^{-1}\left(\dfrac{s+a}{b}\right)$ [Ans: $-\dfrac{1}{t}\ e^{-at}\sin bt$]

86. $\log\sqrt{\dfrac{s^2+1}{s^2}}$ [Ans: $\dfrac{1}{t}\,(1-\cos t)$]

87. $\cot^{-1}(s+1)$ [Ans: $\dfrac{1}{t}\ e^{-t}\sin t$]

88. $\log\left[s^2+4\right]$ [Ans: $-\dfrac{2}{t}\cos 2t$]

FIND THE INVERSE OF THE FOLLOWING USING CONVOLUTION THEOREM:

89. $\dfrac{s^2}{(s^2+a^2)^2}$ [Ans: $\dfrac{1}{2a}\left[\sin at+at\cos at\right]$]

90. $\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\dfrac{e^{-t}}{3}(\sin 2t+\sin t)$]

91. $\dfrac{(s+2)^2}{(s^2+4s+8)^2}$ [Ans: $\dfrac{e^{-2t}}{4}(2t\cos 2t+\sin 2t)$]

92. $\dfrac{1}{(s+3)(s^2+2s+2)}$ [Ans: $\dfrac{1}{5}\left[e^{-t}\left(2\sin t-\cos t\right)+e^{-3t}\right]$

93. $\dfrac{1}{(s-2)^4(s+3)}$ [Ans: $\dfrac{e^{-3t}}{625}-e^{2t}\left[\dfrac{1}{625}-\dfrac{t}{125}+\dfrac{t^2}{50}-\dfrac{t^3}{30}\right]$]

94. $\dfrac{1}{s}\log\left(1+\dfrac{1}{s^2}\right)$ [Ans: $\displaystyle\int_0^t -\dfrac{2}{u}(\cos u-1)\,du$

95. $\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}$ [Ans: $\dfrac{1}{10}\left[e^{-t}(2\sin t-6\cos t)+(2\sin t+6\cos t)\right]$]

96. $\dfrac{s}{s^4+8s^2+16}$ [Ans: $\dfrac{1}{4}\,t\sin 2t$]

97. Find $\displaystyle\int_0^\infty \sin(tx^2)\,dx$ and hence find $\displaystyle\int_0^\infty \sin x^2\ dx$ [Ans: $\dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}$]

98. Find $\displaystyle\int_0^\infty \cos(tx^2)\,dx$ and hence find $\displaystyle\int_0^\infty \cos x^2\ dx$

99. Find $\displaystyle\int_0^\infty \cos(tx^2)\,dx$ and hence find $\displaystyle\int_0^\infty \cos x^2\ dx$

100. Find $\int_0^{\infty} e^{-tx^2}\,dx$

101. Using Convolution theorem prove that $L^{-1}\left[\frac{1}{s}\log\left(\frac{s+1}{s+2}\right)\right]=\int_0^t \frac{e^{-2u}-e^{-u}}{u}\,du$

102. Using Convolution theorem prove that

$$L^{-1}\left[\frac{1}{s}\log\left(a+\frac{b}{s^2}\right)\right]=\int_0^t \frac{2}{u}\left[1-\cos\left(\frac{b}{a}\right)u\right]du$$

Find the laplace transform of periodic function:

103. $f(t)=K\frac{t}{T}$ for $0<t<T$ and $f(t)=f(t+T)$ [Ans: $K\left[\frac{1}{Ts^2}-\frac{e^{-st}}{s(1-e^{-st)}}\right]$]

104. $f(t)=1,$ for $0\le t<a$ and $f(t)=-1,\ a<t<2a$ and $f(t)$ is periodic with period 2a.

[Ans: $\frac{1}{s}\tanh\left(\frac{as}{2}\right)$]

105. $f(t)=|\sin pt|,\ t\ge 0$ [Ans: $\frac{p}{s^2+p^2}\cdot\coth\left(\frac{\pi s}{2p}\right)$]

106. $f(t)=t,$ for $0<t<1$ and $f(t)=0, 1<t<2$ and $f(t+2)=f(t)$ for $t>0$

[Ans: $\frac{1}{s^2(1-e^{-2s})}\left(1-e^{-s}-se^{-s}\right)$]

107. $f(t)=\frac{t}{a},\ 0<t\le a;\ f(t)=\frac{1}{a}(2a-t),\ a<t<2a$ and $f(t)=f(t+2a)$

[Ans: $\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$]

HEAVISIDE'S UNIT-STEP FUNCTION FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

108. $t^2\ H(t-3)$ [Ans: $e^{-3s}\left[\frac{9}{s}+\frac{6}{s^2}+\frac{2}{s^3}\right]$]

109. $\sin t\cdot H\left(t-\frac{\pi}{2}\right)-H\left(t-\frac{3\pi}{2}\right)$ [Ans: $e^{-\pi s/2}\cdot\frac{s}{s^2+1}-e^{-3\pi s/2}\cdot\frac{1}{s}$]

110. $\left(1+2t-3t^2+4t^3\right)H(t-2)$ [Ans: $e^{-2s}\left[\frac{25}{s}+\frac{38}{s^2}+\frac{42}{s^3}+\frac{24}{s^4}\right]$]

111. Express following function as Heaviside Unit Step function and find its Laplace transform

$$f(t)=\begin{cases}\cos t, 0<t<\frac{\pi}{2}\\ \sin t,\ t>\frac{\pi}{2}\end{cases}$$

112. Express following function as Heaviside Unit Step function and find its Laplace transform $f(t)=\begin{cases}t^2, 0<t<4\\ 4t,\ t\ge 4\end{cases}$

113. Using Laplace transform evaluate

$\int_0^{\infty} e^{-t}\left(1+2t-3t^2+4t^3\right)H(t-2)\,dt$ [Ans: $\frac{e^{-2}}{129}$]

FIND THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

114. $\dfrac{e^{-a s}}{(s+b)^{5/2}}$ [Ans: $\dfrac{4}{3\sqrt{\pi}} \cdot e^{b(t-a)} \cdot (t-a)^{3/2} \cdot H(t-a)$]

115. $\dfrac{(s+1)\, e^{-s}}{s^2+s+1}$ [Ans: $e^{-t/2}\left[\cos\left(\sqrt{3}\ (t-1)/2\right)+\dfrac{1}{\sqrt{3}}\sin\left(\sqrt{3}\ (t-1)/2\right)\right] \cdot H(t-1)$]

116. $\dfrac{e^{-\pi s}}{s^2-2s+2}$ [Ans: $e^{(t-\pi)} \cdot \sin(t-\pi) \cdot H(t-\pi)$]

117. $e^{-s}\left(\dfrac{1-\sqrt{s}}{s^2}\right)^2$ [Ans: $\left[\dfrac{(t-1)^3}{6}-\dfrac{16}{15\sqrt{\pi}}(t-1)^{5/2}+\dfrac{(t-1)^2}{2}\right] \cdot H(t-1)$]

USING LAPLACE TRANSFORM SOLVE THE FOLLOWING DIFFERENTIAL EQUATIONS WITH THE GIVEN CONDITION:

118. $\left(D^2-4\right) y=3e^t, \quad y(0)=0,\ y'(0)=3$ [Ans: $y=-e^t+\dfrac{3}{2}e^{2t}-\dfrac{1}{2}e^{-2t}$]

119. $\left(D^2+D\right) y=t^2+2t, \quad y(0)=4,\ y'(0)=-2$ [Ans: $y=2+2e^{-t}+\dfrac{t^3}{3}$]

120. $\left(D^2+2D+1\right) y=3t\, e^{-t}, \quad y(0)=4,\ y'(0)=-2$ [Ans: $y=e^{-t}\left(4+6t+\dfrac{t^3}{2}\right)$]

121. $\left(D^2-2D-8\right) y=4, \quad y(0)=0,\ y'(0)=1$ [Ans: $y=-\dfrac{1}{2}+\dfrac{1}{6}e^{-2t}+\dfrac{1}{3}e^{4t}$

122. $\dfrac{d^2 y}{dt^2}+4y=H(t-2)$ *with conditions* $y(0)=0,\ y'(0)=1$

[Ans: $y=\dfrac{1}{2}\sin 2t+\dfrac{1}{4}H(t-2)-\dfrac{1}{4}\cos 2(t-2)\, H(t-2)$]

123. $\dfrac{dy}{dt}+2y+\displaystyle\int_0^t y\, dt=\sin t$, *given that* $y(0)=1$ [Ans: $y=e^{-t}-\dfrac{3}{2}t\, e^{-t}+\dfrac{1}{2}\sin t$]

124. $\dfrac{d^2 y}{dt^2}+9y=18\, t$ *with conditions* $y(0)=0,\ y(\pi/2)=0$ [Ans: $y=2t+\pi\sin 3t$]

125. $\dfrac{d^2 y}{d x^2}+2\dfrac{d y}{d x}-3y=0$, *where* $y(0)=0,\ y'(0)=4$ [Ans: e^x-e^{-3x}]

126. $\dfrac{d^2 y}{dt^2}+4y=f(t)$ *with conditions* $y(0)=0,\ y'(0)=1$ *and* $f(t)=1$, *when* $0<t<1$

$=0$, *when* $t>1$

[Ans: $y=\dfrac{1}{2}\sin 2t+\dfrac{1}{4}(1-\cos 2t)-\dfrac{1}{4}\{1-\cos(t-1)\}\, H(t-1)$]