BINOMIAL DISTRIBUTION

This was discovered by James Bernoulli's in 1700 and it expresses probabilities of events of dichotomous (dicho + tomy = Two parts) nature i.e, which results in only two ways, success or failure.

1. Consider an experiment which results in either success or failure. Let it be repeated n time, the probability p of success remaining constant every time and let $q = 1 - p$, the probability of failure.

The probabilities of x successes in any order is $p(x) = {}^nC_x p^x q^{n-x}, x = 0,1,2 \ldots . n$.

This is the **Binomial Distribution.**

2. **Definition:** A random variable is said to follow the Binomial distribution if the probability of x is given by

$P(X = x) = {}^nC_x p^x q^{n-x}, x = 0,1,2,3 \ldots n \text{ and } q = 1 - p$

The two constants n and p are called the parameters of the distribution.

Remarks:

(i) The sum of the probabilities is 1. i.e. $\sum_{x=0}^{n} p(x) = 1$

(ii) Let the experiment of n trials be repeated N times. Then we except x successes to occur $N.\,({}^nC_x p^x q^{n-x})$ times. This is called **frequency function.**

The excepted frequencies of $0, 1, 2, \ldots n$ successes are the successive terms in the binomial expansion $N(q+p)^n$

(iii) If x is a binomial variate with parameters $n \text{ and } p$, it is denoted as $b(x, n, p)$

(iv) The distribution is called "Binomial Distribution" because the probabilities ${}^nC_x p^x q^{n-x}, x = 01,2, \ldots, n$ are the successive terms of the expansion of the binomial expression $(q+p)^n$

3. $Mean = np \quad and \quad variance = npq$

4. **Mode of the Binomial Distribution:**

Case (i): When $(n + 1)p$ is an integer, say, k then $k - 1 \leq x \leq k$.

Since x is an integer, in this case there are two modes $k \text{ and } k - 1$.

Case (ii): If $(n + 1)p$ is not an integer then the mode is the integral part of $(n + 1)p$.

5. **Additive property of Binomial Distribution:**

(i) If X_1 is a Binomial variate with parameter $n_1 \text{ and } p_1$ and X_2 is another Binomial variate with parameter $n_2 \text{ and } p_2$ then $X_1 + X_2$ in general is not a Binomial variate.

(ii) If $X_1 \text{ and } X_2$ are two Binomial variates with parameters n_1, p and n_2, p then $X_1 + X_2$ is a Binomial variate with parameters $(n_1 + n_2), p$

6. **Recurrence Relation for the Probability of Binomial Distribution:**

(Fitting of Binomial Distribution)

From a given set of observations of a variate following binomial distribution, $p(x + 1) = \frac{n-x}{x+1} \cdot \frac{p}{q} p(x)$

Further since the expected frequency of x i.e $f(x) = Np(x)$,

we have $f(x + 1) = N.\,p(x + 1) = N.\left(\frac{n-x}{x+1} \cdot \frac{p}{q} p(x)\right) = \left(\frac{n-x}{x+1} . \frac{p}{q}\right) . N.\,p(x)$

$\therefore f(x + 1) = \frac{n-x}{x+1} . \frac{p}{q} . f(x)$

EXERCISE

1. Find the fallacy if any in the statement 'The mean of a Binomial distribution is 6 and standard deviation is 4'

2. The mean and variance of a Binomial variate are 3 and 1.2 . Find 'n', 'p' and $P(X < 4)$.

3. Find the Binomial distribution if the mean is 4 and variance is 3. Find also its mode.

4. What is the mean and variance of the Binomial distribution $(0.3 + 0.7)^{10}, q = 0.3$

5. If the probability of a defective bulb is 0.2, find the mean and the standard deviation for the distribution of defective bulbs in a lot of 1000 bulbs.

6. In a Binomial distribution consisting of 5 independent trials, probability of 1 and 2 successes are 0.4096 $and\ 0.2048$ respectively. Find the parameter 'p' of the distribution.

7. What is the expectation of heads if an unbiased coin is tossed 12 times?

8. A factory turns out an article by mass production methods. From the past experience it is found that 20 articles on an average are rejected out of every batch of 100. Find the mean and the variance of the number of rejected articles.

9. If 10 fair coins are tossed simultaneously, what is the chance of getting atleast 7 heads?

10. The probability that a man aged 60 will live upto 70 is 0.65. What is the probability that out of 10 such men now at 60 atleast 7 will live upto 70?

11. The odds in favour of X's winning a game against Y are 4:3. Find probability of Y's winning 3 games out of 7 played.

12. The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers 4 or more will catch the disease?

13. On an average 3 out of ten student fail in an examination. What is the provability that out of 10 student that appear for the examination none will fail ?

14. If X is the random variable showing the number of boys in a family with 4 children, construct a table showing the probability distribution of X

15. Two unbiased dice are thrown three times. Find the probability that the sum nine would be obtained
(i) once **(ii)** twice

16. If 10% of the rivets produced by a machine are defective, find the probability that out of 5 randomly chosen rivets **(i)** none will be defective, **(ii)** at the most two will be defective.

17. In a multiple choice examination there are 20 questions. Each question has 4 alternative correct answers following it-and the student must select one correct answer. 4 marks are given for correct answer and 1 mark is deducted for wrong answer. A student must secure at least 50% of maximum possible marks to pass the examination. Suppose a student has not studied at all, so that he answers the questions by guessing only. What is the probability that he will pass the examination?

18. The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is $1/4$. What is the probability of 4 successes in 6 independent trials?

19. A communication system consists of n components, each of which functions independently with probability

p. The total system will be able to function effectively if atleast one – half of its components are functioning. For what value of p is a 5 – component system more likely to operate effectively then a 3 – component system ?

20. In a precision bombing attack there is a 50 % chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least 99 % chance of destroying the target ?

21. A lot contains 1% defective items. What should be the number of items in a lot so that the probability of finding at least one defective item in it, is at least 0.95.

22. The probability that a bomb will hit the target is 0.2. Two bombs are required to destroy the target. If six bombs are used, find the probability that the target will be destroyed.

23. A biased coin is tossed n times. Prove that the probability of getting even number of heads is $0.5\,[1 + (q - p)^n]$.

24. Assuming that half the population is female and assuming that 100 samples each of 10 individuals are taken, how many samples would you except to have 3 or less females ?

25. Assuming that half the population is vegetarian so that the chance of an individual being vegetarian is $1/2$ & assuming that 100 investigators can take a sample of 10 individuals to see whether they are vegetarians, how many investigators would you expect to report that three people or less were vegetarians ?

26. An irregular six faced die is thrown. The probability that in 10 throws it will give five even numbers is twice as likely that it will give four even numbers. How many times in 10,000 sets of 10 throws, would you expect to give no even number ?

27. The probability of failure in Physics practical examination is 20%. If 25 batches of 6 students each take the examination, in how many batches 4 or more students would pass ?

28. In a sampling of a large number of parts produced by a machine the mean number of defectives in a sample of 20 is 2. Out of 1000 such sample how many sample would you expect to contain atleast 3 defectives.

29. Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five ?

30. Out of 1000 families of 3 Children each, how many would you expect to have 2 boys and 1 girl ?

31. Let X, Y be two independent binomial variates with parameters $(n_1 = 6, p = 1/2)$ and $(n_2 = 4, p = 1/2)$ respectively. Find $(X + Y = 3)$, $P(X + Y \geq 3)$

32. Five fair coins are tossed 3200 times, find the frequency distribution of number of heads obtained. Also find mean and standard deviation

33. Five dice are thrown together 96 times. The number of times 4, 5 or 6 was obtained is given below.

No of times 4,5 OR 6 was obtained	0	1	2	3	4	5
Frequency	1	10	24	35	18	8

Fit a Binomial distribution if **(i)** dice are unbiased **(ii)** the nature of the dice is not known.

ANSWERS

1. False, $q = 8/3$ is impossible.

2. $n = 5, p = 0.6,\ 2072/3125$

3. ${}^{16}C_x(0.25)^x(0.75)^{16-x}$, Mode $= 4$

4. $7, 2.1$

5. 200 , 12.648

6. $p = 0.2$

7. 6

8. 20, 16

9. $\frac{11}{64}$

10. $\sum_{x=7}^{10} {}^{10}C_x(0.65)^x(0.35)^{10-x}$

11. ${}^{7}C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^4 = 0.294$

12. 0.01696

13. $(0.7)^{10}$

14.

X	0	1	2	3	4
P(X)	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

15. **(i)** 0.2633 **(ii)** 0.0329

16. **(i)** 0.59 **(ii)** 0.99

17. $\sum_{x=12}^{20} {}^{80}C_x\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{20-x}$

18. ${}^{6}C_4\left(\frac{1}{5}\right)^4.\left(\frac{4}{5}\right)^2 = 6.444 \times 10^{-4}$

19. $p \geq \frac{1}{2}$

20. $Minimum\ 11$

21. 299

22. 0.3447

24. 17

25. 17

26. $p = 5/8\,; 0.5499$

27. 23

28. 323

29. 126.3

30. 375

31. 0.1172 , 0.945

32. Mean = 5.2 , S.D. = 1.118

X :	0	1	2	3	4	5
f :	100	500	1000	1000	500	100

33. **(i)**

No. of successes	0	1	2	3	4	5
Frequencies	3	15	30	30	15	3

(ii)

No. of successes	0	1	2	3	4	5
Frequencies	1	9	25	33	22	6