MODULE - 3

Q. Towner Transform of e-5/2

Z- TRANSFORM

7- Transform of a sequence {f(K)} denoted by

 $\frac{2\left\{f(k)\right\}}{2} = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$

Z = Complex no.

LINEARITY PROPERTY

 $\mathcal{Z}\left\{a\left(k\right)+bg(k)\right\}=a\mathcal{Z}\left\{f(k)\right\}+b\mathcal{Z}\left[g(k)\right]$

HANGE OF SCALE PROPERTY

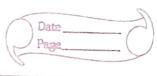
If I f(k) = F(Z) then

 $Z\left[a^{k}\cdot f(k)\right] = F\left(\frac{2}{a}\right)$

and $\frac{1}{2}\left[a^{-k}, f(k)\right] = F(a \cdot t)$

2(1) = 3 SHIFTING PROPERTY If 7/f(x) = F(7)

then $\frac{1}{2}\left(f(k\pm n)\right) = 2^{\pm n}F(\frac{1}{2})$



&. Find Z-Transform of eak for k70

 $Z\left[e^{\alpha k}\right] = Z\left[e^{\alpha k}z^{-\kappa}\right]$

San:

 $= \underbrace{\frac{2}{2}(e^a)^k}_{k=0}$

 $= 1 + e^{q} + (e^{q})^{2} + (e^{q})^{3} + \dots$

Sum $e1 - e^{9/2}$ $ie(e^{9/21}) < 1$ > @ < |z|

 $|Z|e^{ak} = 2 \quad \text{for } |Z| > e^{a}$ $|Z| = e^{a}$

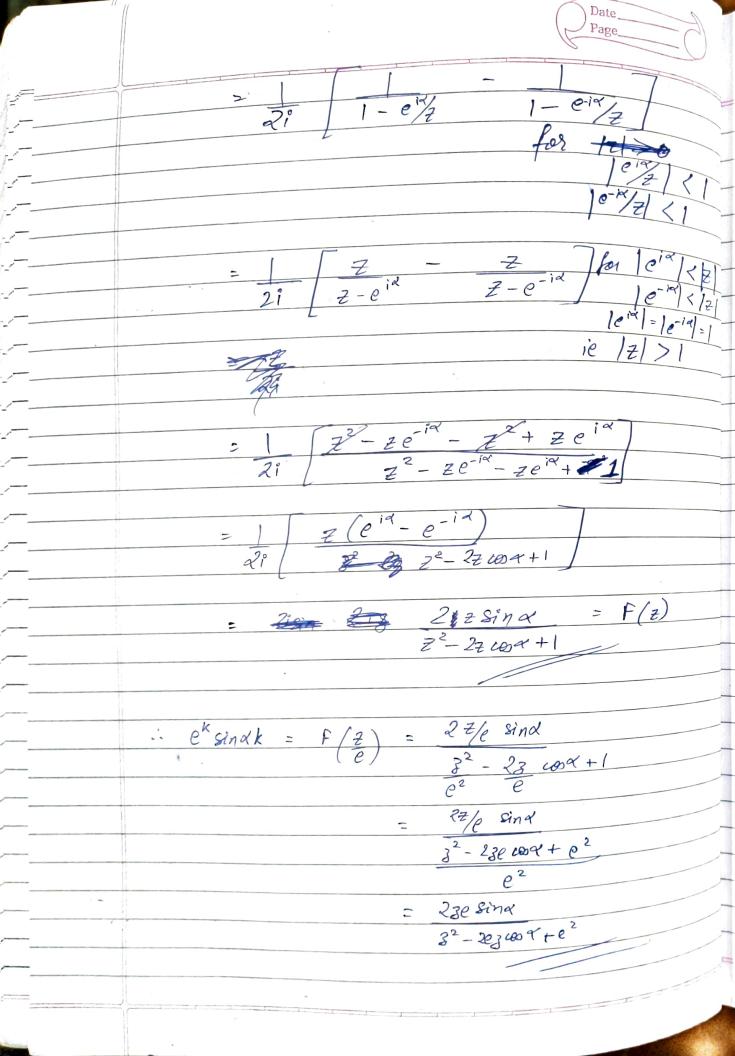
Find Z-transform of singk and here find Z-transform of [e*singk] . (k) 8)

Ans: $Z\left[\sin\alpha k\right] = Z\sin\alpha k \frac{3}{3}z^{-k}$

 $=\frac{2}{k=0}\left(\frac{e^{i\alpha k}-e^{-i\alpha k}}{2i}\right)$

= 1 \(\frac{2}{2} \) \(\left(\frac{e^{-1}}{2} \) \(\frac{e^{-1}}

 $= \int \left(1 + e^{iX} + \left(e^{iX} \right)^{2} + \dots \right) dx$ $\left(1+\frac{e^{-14}}{2}+\left(\frac{e^{-14}}{2}\right)^2+\cdots\right)$





8. Find Z[a k]

2) Z[k"] = 3 -8 d Z(k"-1)

John: 177 [alki] = 2 alki z-k

 $= \underbrace{\underbrace{\underbrace{2}}_{k=-n} a^{-k} \underbrace{2}_{k} \underbrace{$

 $= \underbrace{\underbrace{\xi}_{k=-m} (a \cdot \overline{z})^{-k}}_{k=0} + \underbrace{\underbrace{\xi}_{k} a^{k} \cdot \overline{z}^{-k}}_{k=0}$

 $= \left[a_3 + (a_3)^2 + (a_3)^3 + \dots \right] + \left[1 + \frac{2}{2} + \left(\frac{2}{2} \right)^2 + \dots \right]$

= a3 [1 + a3 + (a3) + ...] + [1 + 1/2 + (a/3) 2 + ...]

19/2/ < 1

12/1/2 121 >a iff o < a < 1

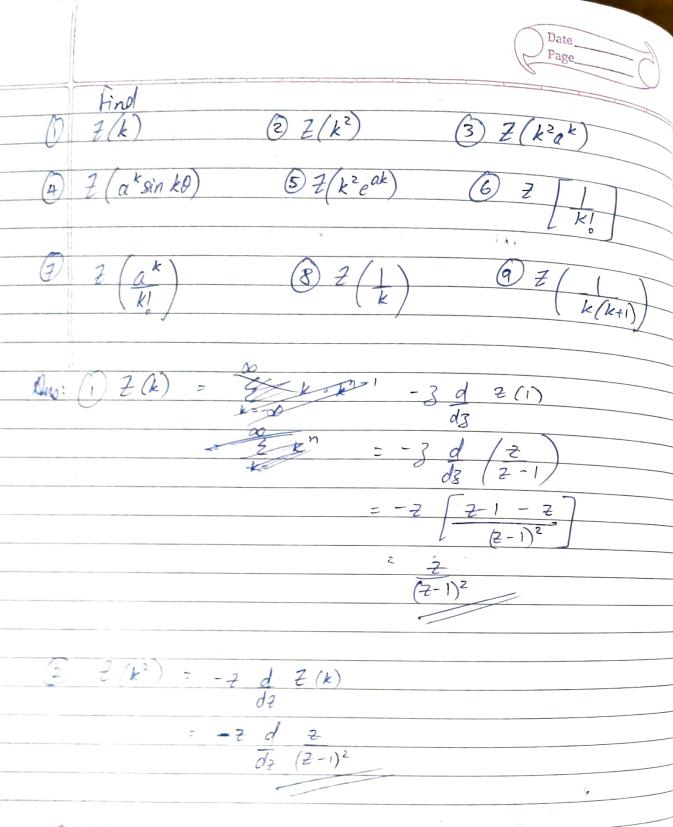
 $2 > Z \left[\chi^{n} \right] = -Z d Z \left(\chi^{n-1} \right)$

Z[kn-1] = Z kn-1 2-k

d [2(kn-1)] = E Kn-1 d (2-k)

= 2 kn-1 (-k) 7-k-1

-3 d z(xn-1) = 2 kn3-k = 2[xn]/



$$\frac{3}{2}(+^{2}o^{2}) = F\left(\frac{2}{a}\right)$$

$$= \frac{3}{a}\left(\frac{3}{a}+1\right)^{2}$$

$$\frac{3}{a}(-1)^{2}$$

Date _____

$$S(k^2 e^{ak}) = Z(k^2 (e^a)^k)$$

$$= \frac{3/e^{\alpha}(^{2}/e^{\alpha}+1)}{(3/e^{\alpha}-1)^{2}}$$

$$(7)$$
 (7)

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \cdots$$

$$\log (1-2) = -2 - 2^2 - 2^3 - 2^4 = - \dots$$

$$-\log(1-z) = +\left(z+z^2+z^2+z^2+z^2+\ldots\right) -$$

$$\frac{1}{2} \left[\frac{1}{2} \right] = -\log\left(1 - \frac{1}{2}\right) = -\log\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \log \left(\frac{z}{z-1} \right)$$

$$\begin{array}{c|c}
9 & \overline{z} \left(\begin{array}{c} 1 \\ k \left(k+1 \right) \end{array} \right) = \overline{z} \left(\begin{array}{c} 1 - 1 \\ k & k+1 \end{array} \right)$$

$$=\frac{2}{2}\left(\frac{1}{k}\right)-\frac{2}{2}\left(\frac{1}{k+1}\right)$$

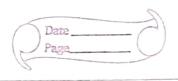
What
$$Z[f(k+n)] = 2^n Z[f(k)]$$

$$9f f(k) = 1 f(k+1) = 1$$
 $k+1$

$$= (1-3) \log \left(\frac{z}{z-1}\right)$$

of
$$Z[f(k)] = F(3)$$

Here $Z[kf(k)] = \begin{pmatrix} -3d \\ \overline{ds} \end{pmatrix} F(3)$



$$\frac{2\left(k^{2}e^{ak}\right) = \begin{pmatrix} -2d \\ \hline ds \end{pmatrix} \begin{pmatrix} -3d \\ \hline ds \end{pmatrix} \begin{pmatrix} 3\\ \hline 3-e^{a} \end{pmatrix}$$

$$\frac{2}{d3} \left[\frac{3}{3} - \frac{e^{a} - 3}{3} \right]$$

$$= -3 d \left[\frac{3 e^{a}}{(3 - e^{a})^{2}} \right]$$

$$= -\frac{e^{a_{3}}}{a_{3}} \left(\frac{3}{3 - e^{a}} \right)^{2}$$

$$2 - e^{a_3} \left(3 - e^{a} \right)^2 - 23(3 - e^{a})$$
 $(3 - e^{a})^4$

$$\frac{3(3+e^{4})e^{4}}{(3-e^{4})^{3}}$$

If
$$F(z) = Z [f(k)]$$

then $F(0) = \lim_{x \to \infty} F(3)$
 $Z = 0$

$$f(1) = \lim_{3 \to \infty} 3 \left[F(3) - f(0) \right]$$

$$f(2) = \lim_{3\to\infty} 3^2 \left[F(3) - f(0) - f(1) \right]$$

$$f(2) = \lim_{3 \to \infty} 3^{2} \left[F(3) - f(0) - f(1) \right]$$

$$f(3) = \lim_{3 \to \infty} 3^{3} \left[F(3) - f(0) - f(1) \right]_{3}$$

$$- f(2)$$

$$3^{2}$$

8. 9f
$$F(3) = 23^2 + 53 + 14$$

(3-1)4
Find $f(0)$, $f(1)$, $f(2)$, $f(3)$

Ano:
$$f(z) = 3^{2} \left(2 + 5 + 14\right)$$

$$= 2 + 5 + 14$$

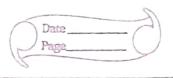
$$= 3$$

$$= 3^{2}$$

$$f(1) = \lim_{3\to\infty} \frac{3}{2} + \frac{15}{3} + \frac{14}{3^2}$$

$$= \lim_{3\to\infty} \left[\frac{2+15/3+14/3^2}{3(1-1/3)^4} \right]$$

-0



GONVOLUTION:

If {f(k)} and {g(k)} are two sequences

Then, their convolution, defined as

 $\{f(k)\} * \{g(k)\} = \{\{k\}\}$

= 2 f(m) g(k-m)

 $= \underbrace{2g(m)\cdot f(k-m)}$

 $= \{g(k)\} * \{f(k)\}$

THEOREM:

If {h(k)} is convolution of two sequences {f(k)}*{9(k)}

then $Z \{f(k)\} = Z \{f(k)\} \cdot Z \{g(k)\}$

eg: If $f(k) = 4^k U(k)$ $g(k) = 5^k U(k)$ find f(f*g)

Joh: V(k) = Unit Sequence.

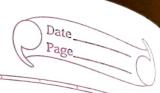
f(k) = 4k { 1,1,1,...}

= {4°, 4', 4², ...}

 $g(k) = 5^{k} \{1, 1, 1, \dots \}$ = $\{5^{0}, 5^{1}, 5^{2}, \dots \}$

 $\frac{1}{2} \left[f(k) \right] = \frac{2}{2} f(k) 3^{-k}$

= 4°3° + 4'3-1 + 423-2 + ...



$$= 1 + 4 + 4^{2} + 4^{3} + \dots$$

$$3 \quad 3^{2} \quad 3^{3}$$

$$= 3$$
 for $|z| > 4$

similarly,
$$Z[g(k)] = 3$$
 for $|z| > 5$

$$= 3^2$$
 for $|z| > 5$ $(3-4)(3-5)$

$$\begin{cases} g(k) = 1 & g(k) = 1 \\ 2^k & 3^k \end{cases}$$

$$= 1 + 1 + 1 + 1 + 1 + \cdots$$

$$= 1$$
 for $1/23$ < 1

$$= \frac{23}{23-1} = \frac{123}{3} > 1$$

$$= \frac{23}{3} = \frac{1}{3} > \frac{1}{2}$$



$$= \frac{6z^2}{(2z-1)(3z-1)} \quad \text{for } |z| > 1/2$$

$$= 4 \left(1 - \frac{a}{3}\right)^{-1}$$

$$= 4 \left[\frac{1+a+a^2+a^7+\cdots}{333^23^3} \right]$$

$$= 4 \left[\frac{2}{3} \right]$$



$$\frac{43}{3-a} = \frac{43}{-a(1-3)}$$

$$= -\frac{43}{a} \left(1 - \frac{3}{a} \right)^{-1}$$

$$= -\frac{43}{a} \left[\frac{1+3^2+3^2+3^3+...}{a^2 a^2 a^3} \right]$$

$$= -4 \left[\frac{3}{a} + \frac{3^2}{a^2} + \frac{3^3}{a^3} + \dots \right]$$

$$= -4 \frac{2}{k=01} \left(\frac{k3}{a} \right)^{k}$$

$$\frac{1}{3} - \frac{2}{3} - \frac{43}{3} = \frac{3}{4a} - \frac{4a}{3} = \frac{6n}{3} + \frac{13}{3} +$$

Il Inverse By Partial Fraction

Mw:
$$\frac{43^2-23}{3^3-53^2+83-4} = \frac{43^2-23}{(3-1)(3-2)^2} = \frac{23(3-1)}{(3-2)^2} = F(3)$$

$$\frac{23-1}{3^3-53^2+83-4} = \frac{1}{3-1} + \frac{1}{3-2} + \frac{1}{3-2} = \frac{1$$

$$= A(3-2)^{2} + B(3-1)(3-2) + C(3-1)$$
put $\{3=1\}$

$$C = 3$$

$$:. F(3) = 23 \left[\frac{1}{3-1} - \frac{1}{3-2} + \frac{3}{(3-2)^2} \right]$$

$$= 27^{-1} \begin{bmatrix} 3 \\ 3^{-1} \end{bmatrix} - 27^{-1} \begin{bmatrix} 3 \\ 3^{-2} \end{bmatrix} + 67^{-1} \begin{bmatrix} 3 \\ (8-2)^2 \end{bmatrix}$$

$$(1-3)^{-2} = 1+3+33^{2}+43^{3}+...$$

$$F(3) = \frac{1}{3-2} + \frac{1}{3-3}$$

$$\frac{13}{3-2} > 2$$

$$\frac{13}{3-2} > 1$$

$$\frac{13$$

