

EXPERIMENT – IV

Anirban Mukherjee

16BIT0200

Functionality of Windshield Wipers

Aim:

Analyzing and solving the working mechanism of the windshield wipers by solving Differential equations.

Problem Statements:

Many automobiles are equipped with windshield wipers that can operate intermittently during a light rainfall. The operation of such wipers depends on the charging and discharging of a capacitor. The wipers are part of an RC circuit whose time constant can be varied by selecting different values of R through a multi position switch. As the voltage across the capacitor increases, the capacitor reaches a point at which it discharges and triggers the wipers. The circuit then begins another charging cycle. The time interval between the individual sweeps of the wipers is determined by the value of the time constant.

We divide this experiment in two parts:

- (i) Charging a Capacitor in an RC Circuit.
- (ii) Discharging a Capacitor in an RC Circuit

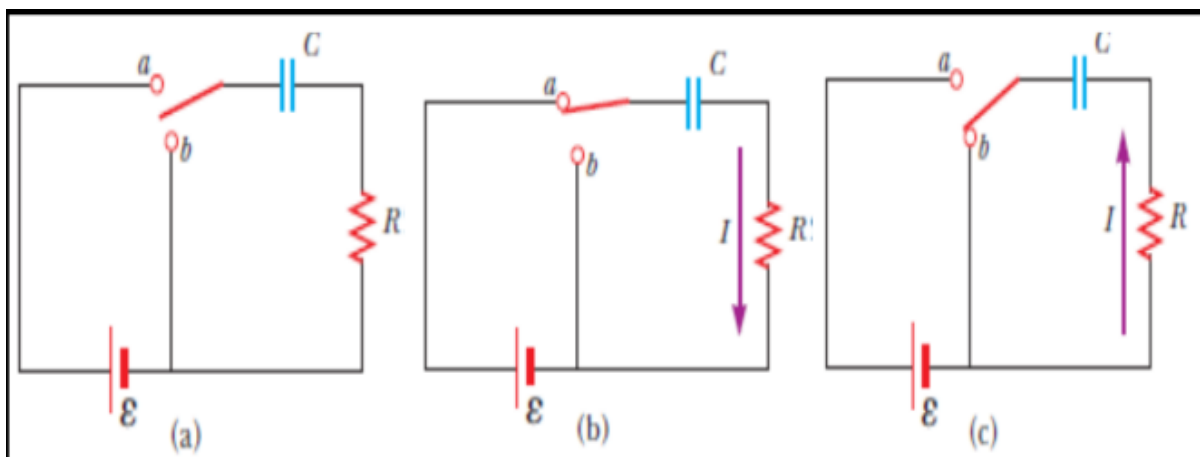
Matlab Commands:

Dsolve – solves the ordinary differential equation with the initial or boundary equation

Ezplot – plots the expression $f(x)$ over the default domain – $2\pi < 0 < 2\pi$, where $f(x)$ isn't an implicit function of only one variable.

Description of Physical Experiment:

Charging a Capacitor in an RC Circuit



- Assume the capacitor in this circuit is initially uncharged. There is no current while the switch is open in figure (a).
- If the switch is thrown to position a at $t = 0$ as shown in figure (b), however, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.
- During charging, charges do not jump across the capacitor plates because the gap between the plates represents an open circuit. Instead, charge is transferred between each plate and its

connecting wires due to the electric field established in the wires by the battery until the capacitor is fully charged.

- As the plates are being charged, the potential difference across the capacitor increases. The value of the maximum charge on the plates depends on the voltage of the battery.
- Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.
- To analyse this circuit quantitatively, apply Kirchhoff's loop rule to the circuit after the switch is thrown to position . Traversing the loop in Figure (b) clockwise gives

$$\epsilon - \frac{q}{C} - IR = 0 \quad (1)$$

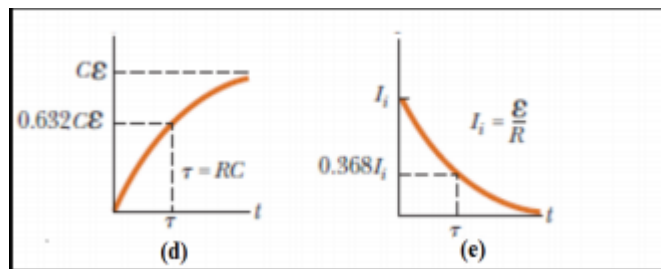
where $\frac{q}{C}$ is the potential difference across the capacitor and is the potential difference across the resistor. The capacitor is traversed in the direction from the positive plate to the negative plate, which represents a decrease in potential. Therefore, we used the negative sign for the potential difference.

- Using equation (1) we can find the initial current in the circuit and the maximum charge on the capacitor.
- At the instant the switch is thrown to position a($t = 0$) the charge on the capacitor is zero.
- The initial current is a maximum and is given by $I_i = \frac{\epsilon}{R}$.
- At this time, the potential difference from the battery terminals appears entirely across the resistor.
- When the capacitor is charged to its maximum value Q , charges cease to flow, the current in the circuit is zero and the potential difference from the battery terminals appears entirely across the capacitor.
- Substituting $I = 0$ into the equation (1) gives the maximum charge on the capacitor $Q = C \epsilon$.

- To determine analytical expressions for the time dependence of the charge and current, let us substitute $I = \frac{dq}{dt}$ and rearrange the equation (1), we will get

$$\frac{dq}{dt} = \frac{\epsilon}{R} - \frac{q}{RC} \quad (2)$$

- Solving the linear differential equation using variable separable method, we will get an expression for q as $q(t) = C \epsilon (1 - e^{-\frac{t}{RC}}) = Q (1 - e^{-\frac{t}{RC}})$ (3)
- The above equation denotes the charge as a function of time for a capacitor being charged. We can find an expression for the charging current by differentiating equation (3) with respect to time. We get $I(t) = \frac{\epsilon}{R} e^{-\frac{t}{RC}}$ (4)
- The above equation denotes the current as a function of time for a capacitor being charged. The quantity RC , which appears in the exponents of equations (3) and (4) is called time constant τ of the circuit i.e., $\tau = RC$.
- The time constant represents the time interval during which the current decreases to $\frac{1}{e}$ of its initial value; that is after a time interval τ , the current decreases to $I = e^{-1} I_i = 0.368 I_i$. After a time interval 2τ , the current decreases to $I = e^{-2} I_i = 0.135 I_i$ and so forth. Likewise in a time interval τ , the charge increases from zero to $C \epsilon [1 - e^{-1}] = 0.632 C \epsilon$.



- Figure (d): Plot of capacitor charge versus time for the above circuit. After a time interval equal to one time constant has passed, the charge is 63.2% of the maximum value $C \epsilon$. The charge approaches its maximum value as t approaches infinity.
- Figure (e): Plot of current versus time for the above circuit. The current has its maximum value $I_i = \frac{\epsilon}{R}$ at $t = 0$ and decays to zero exponentially as t approaches infinity. After a time interval equal to one time constant τ has passed, the current is 36.8% initial value.

Discharging a Capacitor in an RC Circuit

- If the capacitor in figure (b) is completely charged then a potential difference will exist across the capacitor and there will be zero potential difference across the resistor because $I = 0$.
- If the switch is now thrown to position b at $t = 0$ as shown in figure (c), the capacitor begins to discharge through the resistor.
- At some time during the discharge, the current in the circuit is I and the charge on the capacitor is q .
- The circuit in figure (c) is the same as the circuit in figure (b) except for the absence of the battery.
- We eliminate the e.m.f ϵ from equation (1), we obtain the appropriate loop equation for the circuit.

$$-\frac{q}{C} - IR = 0 \quad (5)$$

Then $-R \frac{dq}{dt} = \frac{q}{C}$ and solving the differential equation, we will get the charge as a function of time for a discharging capacitor.

$$q(t) = Q(e^{\frac{-t}{RC}}) \quad (6)$$

Differentiating the equation (6), we will get the current as a function of time for a discharging capacitor

$$I(t) = -\frac{Q}{RC} (e^{\frac{-t}{RC}}) \quad (7)$$

where $I_i = \frac{Q}{RC}$ is the initial current. The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.

Connection to Mathematics:

The general linear differential equation of the nth order is of the form

$$\frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + p_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + p_n y = X$$

where p_1, p_2, \dots, p_n and X are function of x .

Linear differential differential equation with constant co-efficient are of the form

$$\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1} y}{dx^{n-1}} + k_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + k_n y = X$$

where k_1, k_2, \dots, k_n are constants.

Code:

```
clc
clear all;
close all;
eqn=input('Enter the equation: ');
inits=input('Enter the conditions: ');
y=dsolve(eqn,inits,'t');
soln=['y(t)=',char(simplify(y))];
disp(soln)
ezplot(y)
```

Questions:

$$1) \frac{d^2 y}{dx^2} + 8 \frac{dy}{dx} + 10y = 0, y'(0) = 1, y(0) = 4$$

Solution:

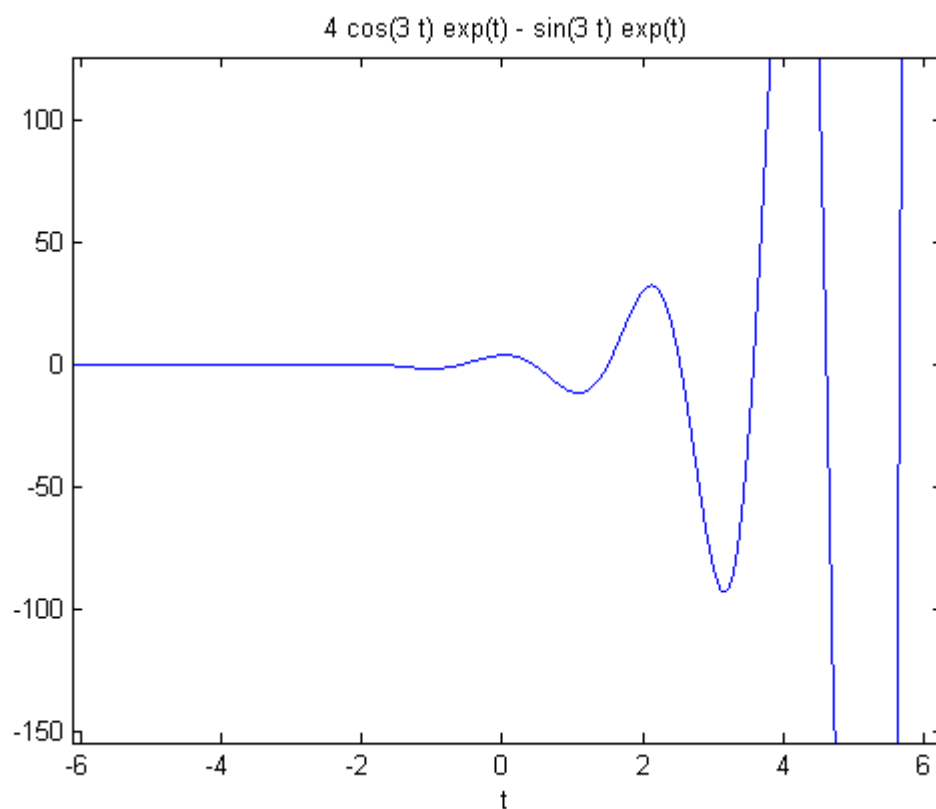
MATLAB input:

Enter the equation: 'D2y-2*Dy+10*y=0'

Enter the conditions: 'y(0)=4,Dy(0)=1'

MATLAB output:

$y(t) = \exp(t) * (4 * \cos(3*t) - \sin(3*t))$



$$2) \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 4y = 3 \sin(t) + 4 \cos(t), y'(0) = 0, y(0) = 1$$

Solution:

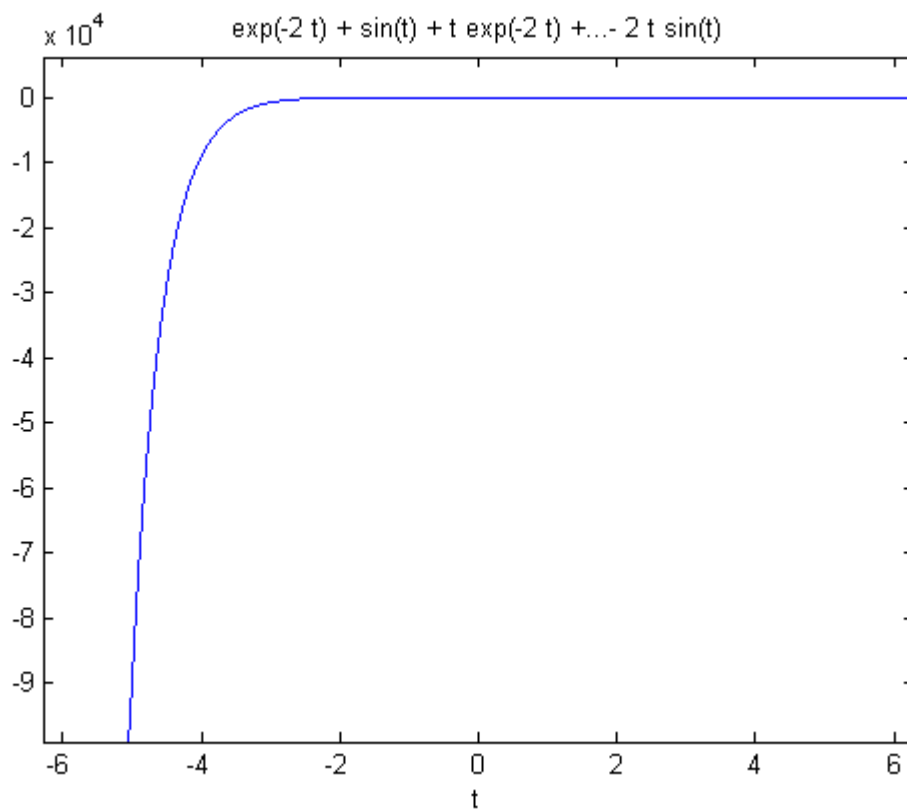
MATLAB input:

Enter the equation: 'D2y+4*Dy+4*y=3*sin(t)+4*cos(t) '

Enter the conditions: 'y(0)=1,Dy(0)=0'

MATLAB output:

$$y(t) = \exp(-2*t) + \sin(t) + t*\exp(-2*t)$$



$$3) \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0, y'(0) = 2, y(0) = 2$$

Solution:

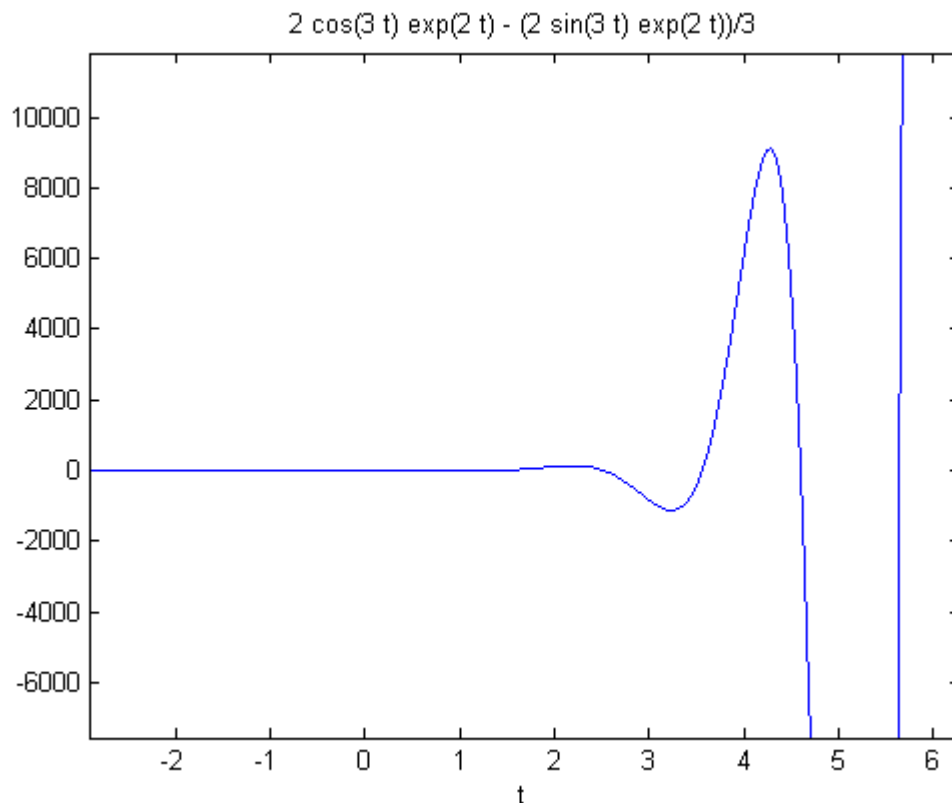
MATLAB input:

Enter the equation: 'D2y-4*Dy+13*y=0'

Enter the conditions: 'y(0)=2,Dy(0)=2'

MATLAB output:

$$y(t) = (2 \cdot \exp(2 \cdot t) \cdot (3 \cdot \cos(3 \cdot t) - \sin(3 \cdot t))) / 3$$



My Work:

This can be used for developing a machine that would create an Artificially Intelligent traffic signal.

What we can do is input the entry and exit of vehicles in matrix form and convert it into linear differential equation form, there we can input the constraints. The output would help us determine the no. of vehicles and distribution of vehicles and the type of the vehicles with respect to time. Over time it will learn which time interval has what kind of vehicles in most numbers, its average speeds and the time

taken to clear one side of the road and the time it takes to traffic to gather there requiring to clear that side again.

The AI machine will learn how to manage the traffic with less possibility of accidents and will change the traffic light after a given period of time depending on the time, type and the speeds and time it would take to clear any side of the road. This system can also be manually operated if needed and would reduce police workload immensely. The switching of signals closely resembles the to and fro motion of a windshield wiper as well.

Test Case:

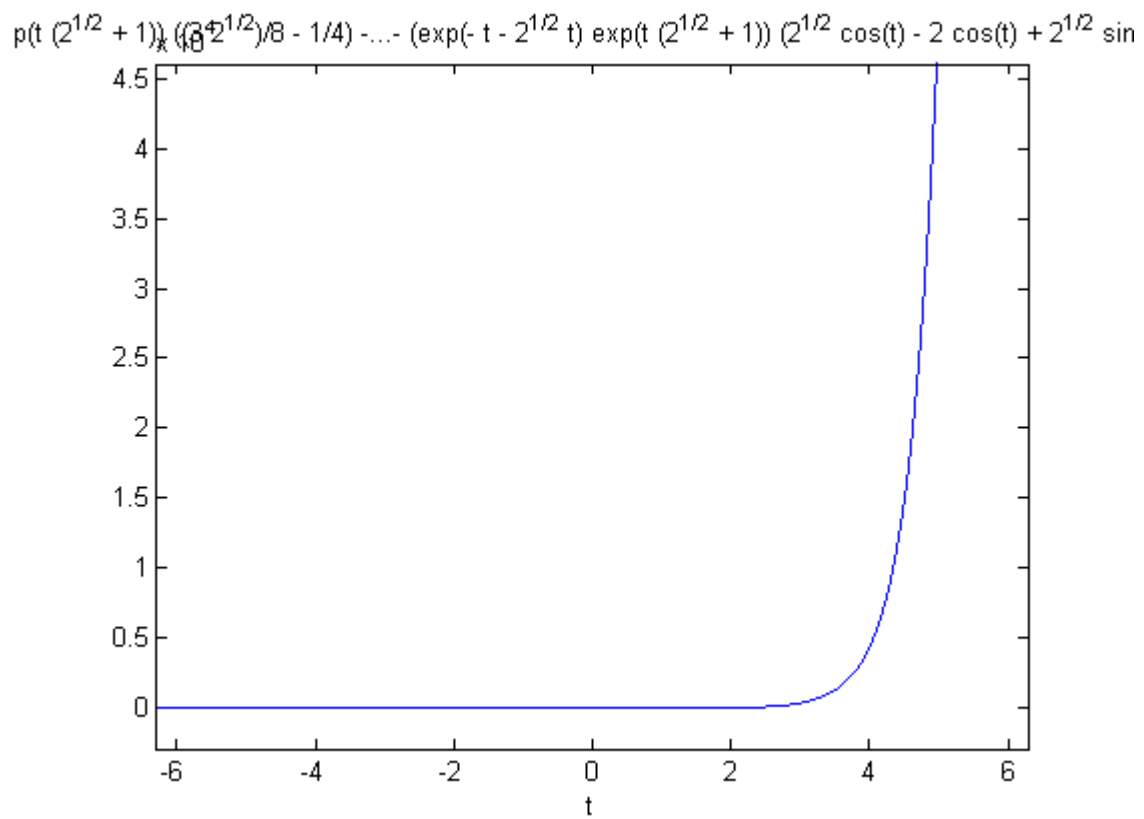
```
clc
clear all;
close all;
eqn=input('Enter the equation: ');
inits=input('Enter the conditions: ');
y=dsolve(eqn,inits,'t');
soln=['y(t)=',char(simplify(y))];
disp(soln)
ezplot(y)
```

We input a sample equation and constraints and find the output, where we can infer the traffic rate and other information already mentioned beforehand, the time period is assumed to be between 8 a.m. to 10 a.m., just approaching peak hours.

```
Enter the equation: 'D2y-2*Dy-y=sin(t)-cos(t)'
```

```
Enter the conditions: 'y(0)=0,Dy(0)=1'
```

```
y(t)=cos(t)/2 + exp(t + 2^(1/2)*t)*((3*2^(1/2))/8 - 1/4) - (2^(1/2)*exp(t - 2^(1/2)*t)*(2^(1/2) + 3))/8
```



This can determine the traffic rate