EXPERIMENT – X

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16BIT0200

Stability Analysis of Difference Equation Systems using z–transform

Aim:

Finding the stability analysis of a motor driven train using MATLAB

Problem Statements:

We have to find the poles, zeros and stability as well as impulse and unit step responses to perform stability analysis of a system.

Matlab Commands:

- [z p k] = tf2zpk(num,den)
  zplane(z,p)

→ The zero-pole-gain analysis and plotting of zeros-poles of the system

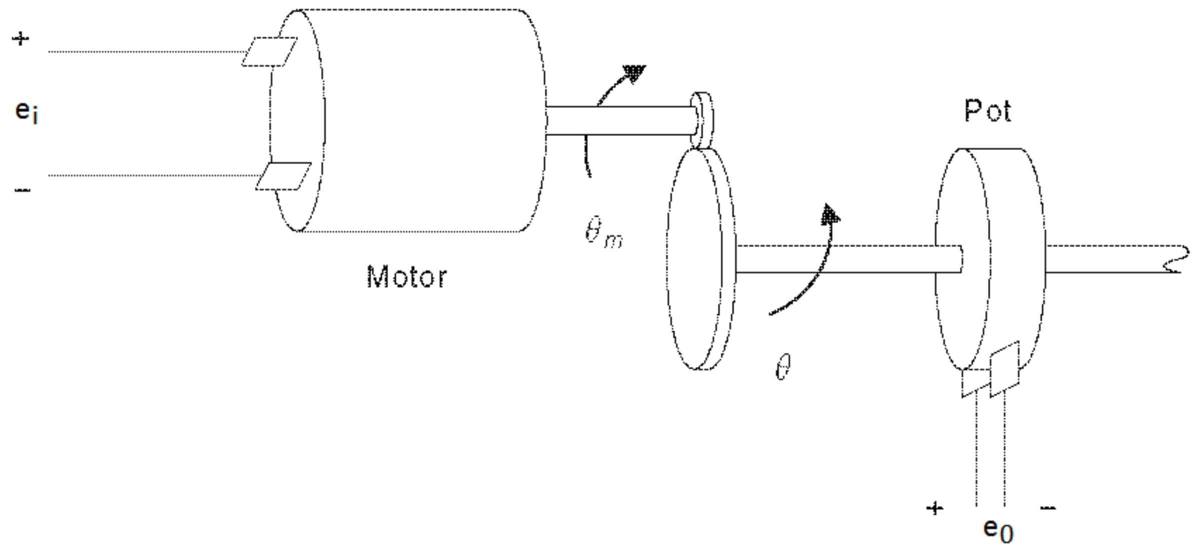
- residuez(num, den) – Finds partial fraction expansion of H(z)
- dimpulse(num,den,nooftimpesteps) – For impulse responcse
- filter(num,den,u) – For power series transforms
- dstep(num,den,nooftimesteps) – For unit step response

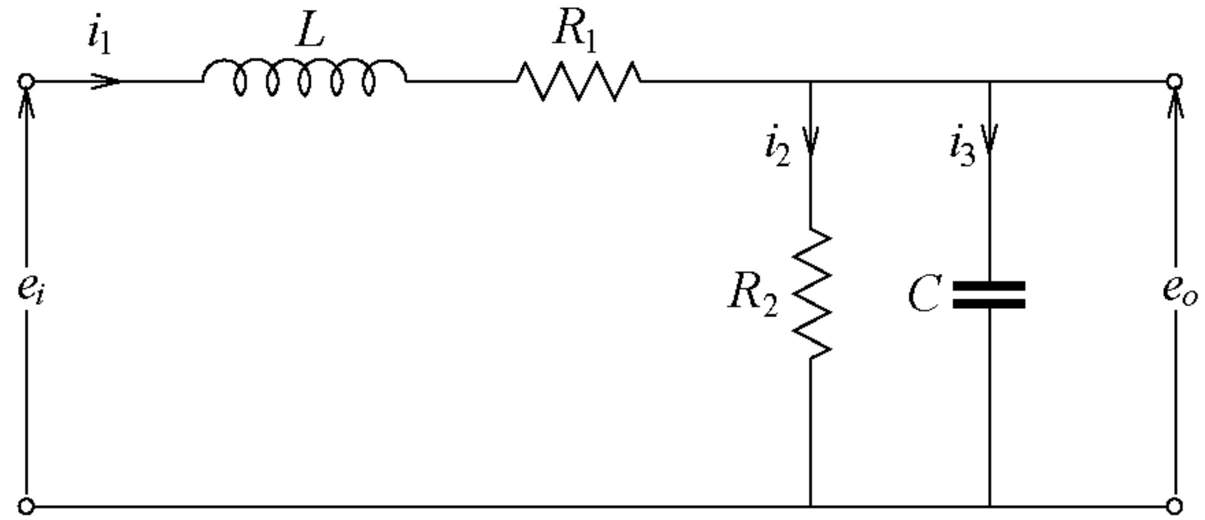
Description of Physical Experiment:

- A gear train is a mechanical system formed by mounting gears on a frame so that the teeth of the gears engage.
- Gear train system occurs in many of the automobile drivetrains, which generally have two or more major areas where gearing is used.
- Gearing is employed in the transmission, which contains a number of different sets of gears that can be changed to allow a wide range of vehicle speeds.
- Gearing is also used in the differential, which facilitates the splitting torque equally between two wheels while permitting them to have different speeds when travelling in a curved path.

Connection to Mathematics:

Consider a motor driving a rotary load through a gear train and the simple RLC circuit representing the motor connection, as shown in below Figures.





Applying the basic circuit laws for voltage and currents, we obtain:

$$e_i(t) = L\frac{di_1(t)}{dt} + R_1 i_1(t) + e_0(t) \tag{1}$$

and

$$e_0(t) = R_2 i_2(t) = \frac{1}{C}\int_0^t i_3(\tau)d\tau + v_c(0) \Rightarrow i_3(t) = C\frac{de_0(t)}{dt} \tag{2}$$

But, we have:

$$i_1(t) = i_2(t) + i_3(t) = \frac{1}{R_2}e_0(t) + C\frac{de_0(t)}{dt} \tag{3}$$

From which, we obtain the following second order differential equation, relating input and output of the system, representing mathematical model of the circuit given in the Figure 1.

$$\frac{d^2e_0(t)}{dt^2} + \left(\frac{L+R_1R_2C}{R_2LC}\right)\frac{de_0(t)}{dt} + \left(\frac{R_1+R_2}{R_2LC}\right)e_0(t) = \frac{1}{LC}e_i(t) \tag{4}$$

Denoting $e_0(t)$ by $x(t)$ and $e_i(t)$ by $u(t)$, and using the approximations $\frac{dx}{dt} \approx x_t - x_{t-1}$ and $\frac{d^2x}{dt^2} \approx x_t - 2x_{t-1}$, the above differential equation (4) reduces to the following second order difference equation,

$$x_t = (a+1)tx_{t-1} = ax_{t-2} + bu_{t-1} \tag{5}$$

where

$a = \frac{R_2 LC}{(R_1+L)(R_2 C+1)+R_2}$ and $b = \frac{R_2}{(R_1+L)(R_2 C+1)+R_2}$

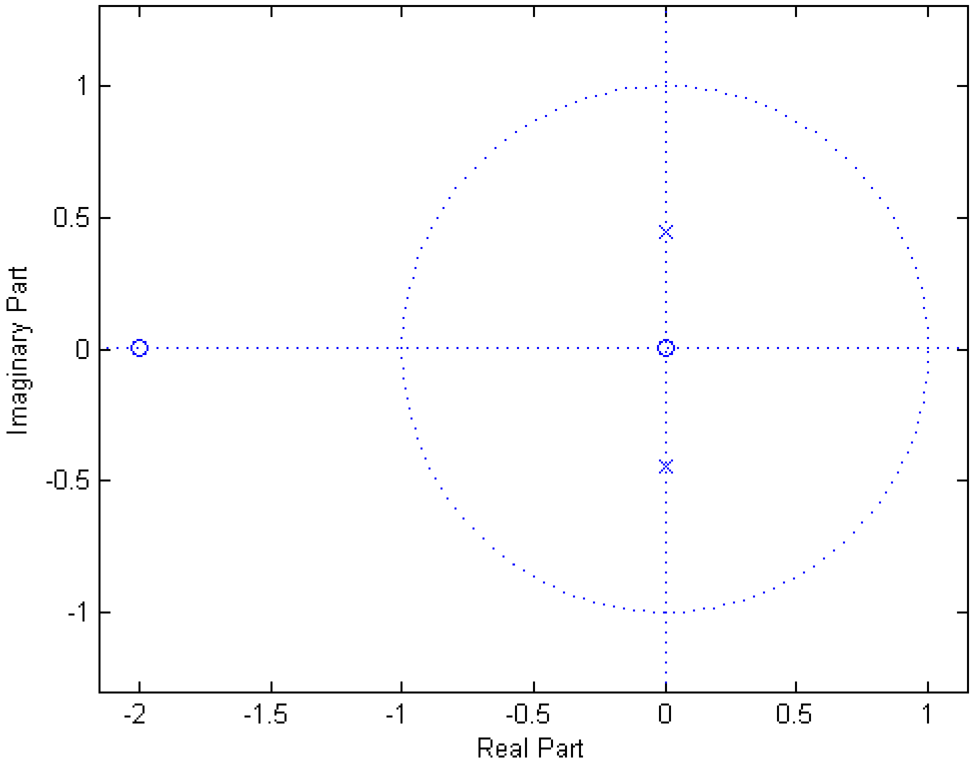
The transfer function for the above difference equation (5) is given by:

$$H(z) = \frac{bz}{z^2-(1+a)z+a} \qquad (6)$$

Code:

//Z-transform of $H(z) = \frac{z^2+2z}{z^2+0.2}$

```
num = [1 2 0];
den = [1 0 0.2];
[z p k] = tf2zpk(num,den)
zplane(z,p)
```



//Partial fraction form of $H(z) = \frac{18z^3}{18z^3+3z^2-4z-1}$

```
num = [18 0 0 0];
den = [18 3 -4 -1];
[r,p,k] = residuez(num,den);
disp('Residues'); disp(r');
disp('Poles'); disp(p');
disp('Constants'); disp(k)
```

```
Residues
    0.3600    0.2400    0.4000

Poles
    0.5000   -0.3333   -0.3333
```

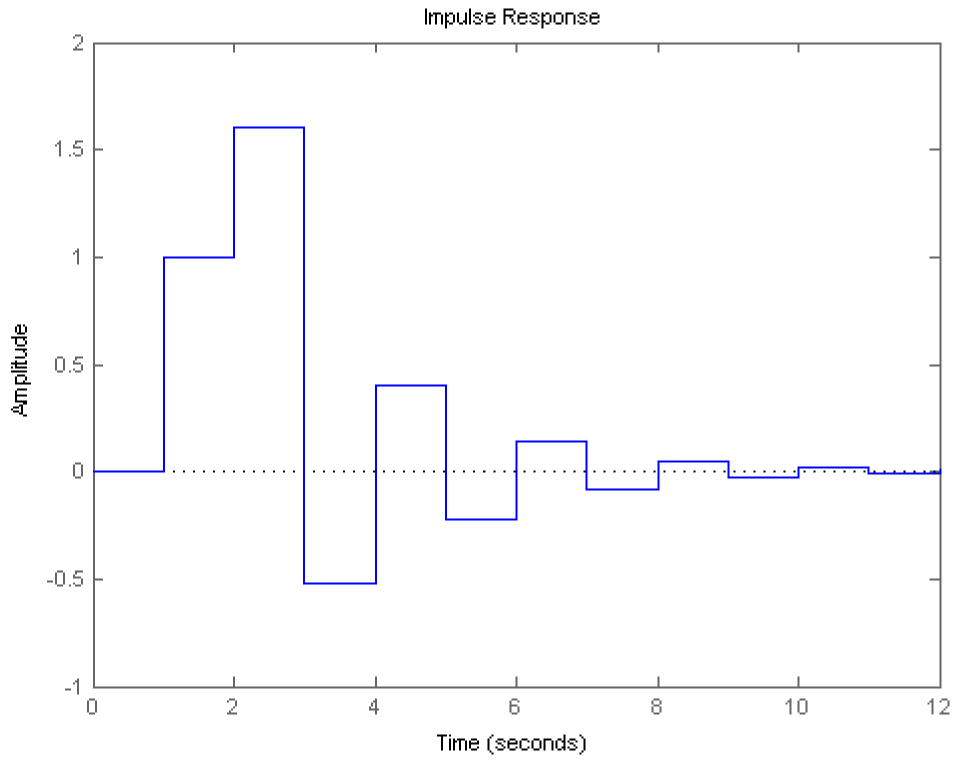
```
Constants
      0
```

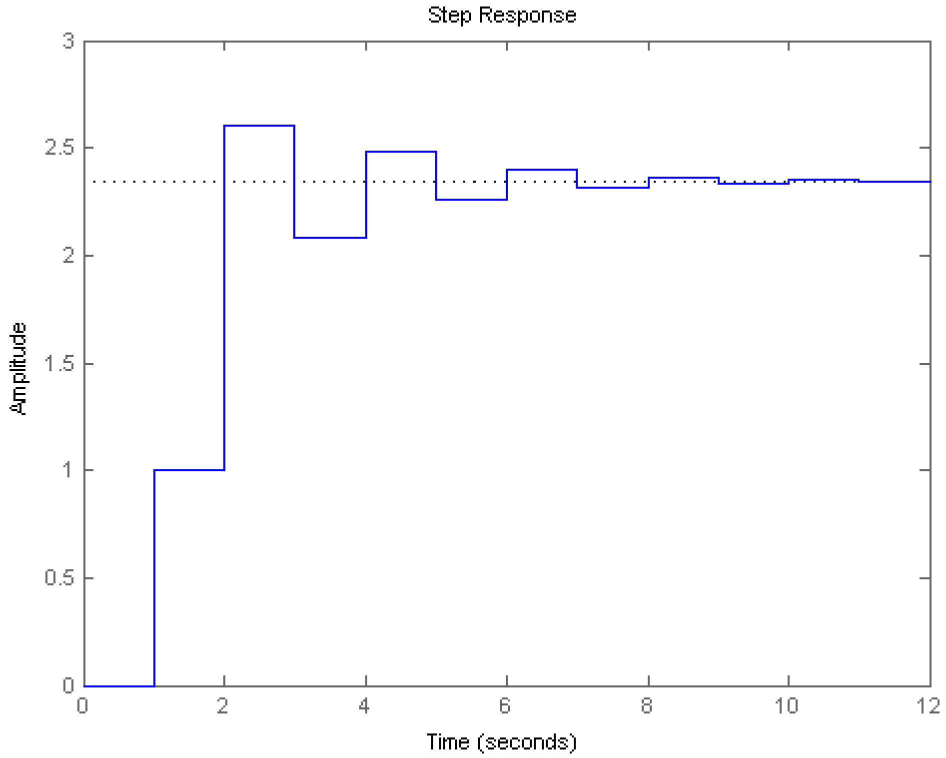
//Power series of $H(z) = \frac{1+2z^{-1}}{1+0.4z^{-1}-0.12z^{-2}}$

```
L = 11;
num = [1 2];
den = [1 0.4 -0.12];
u = [1 zeros(1,L-1)];
x = filter(num,den,u);
disp('Coefficients of the power series expansion: ');
disp(x);
```

```
Coefficients of the power series expansion:
  Columns 1 through 7

    1.0000    1.6000   -0.5200    0.4000   -0.2224    0.1370
-0.0815

  Columns 8 through 11

    0.0490   -0.0294    0.0176   -0.0106
```

```
//Impulse and Step Function

L = 11;
num = [1 2];
den = [1 0.4 -0.12];
u = [1 zeros(1,L-1)];
x = filter(num,den,u);
disp('Coefficients of the power series expansion: ');
disp(x);
figure; dimpulse(num,den,13);
figure; dstep(num,den,13)
```


Impulse Response
Amplitude
Time (seconds)
2
1.5
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12


Step Response
Amplitude
Time (seconds)
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12

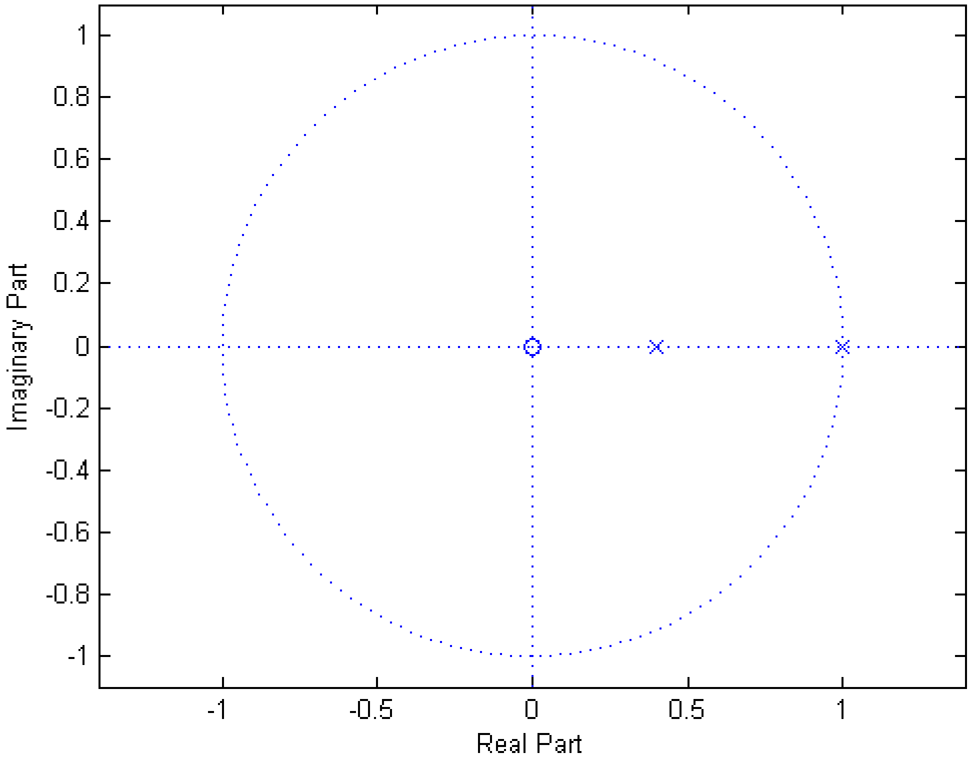
Exercise (MatLab code for Application oriented problems based on stability Analysis)

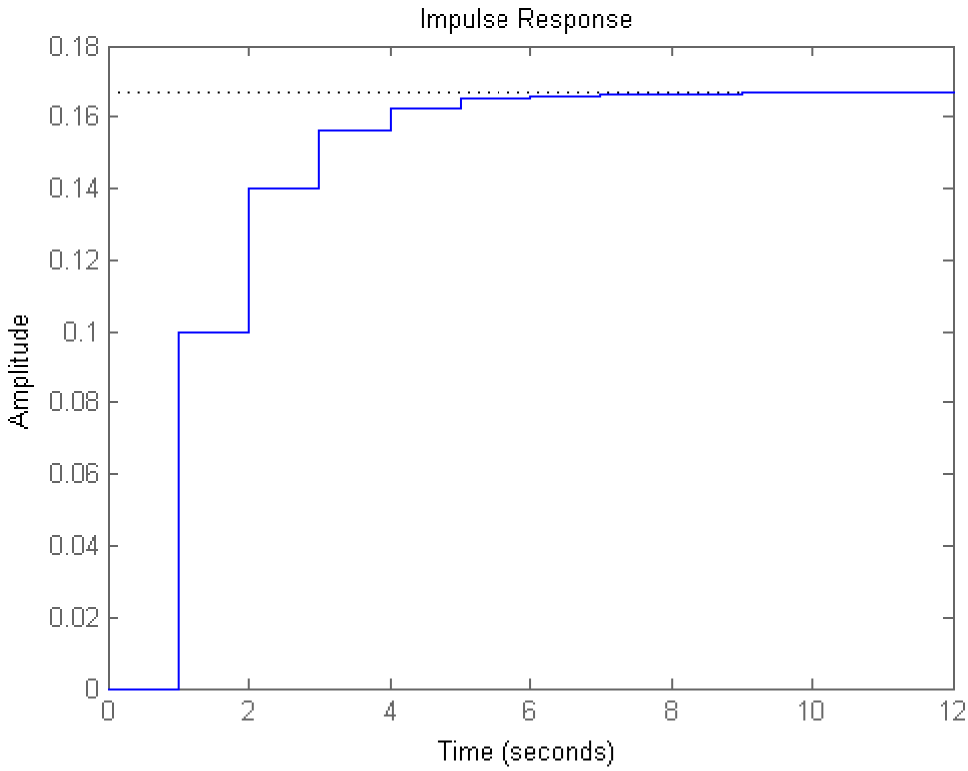
1. Determine the poles, zeros and stability of the above transfer function for R1 = 1 ohms, R2 = 1 ohms, L = 2 Henry and C = 2 farad. Also plot the impulse and unit step responses.

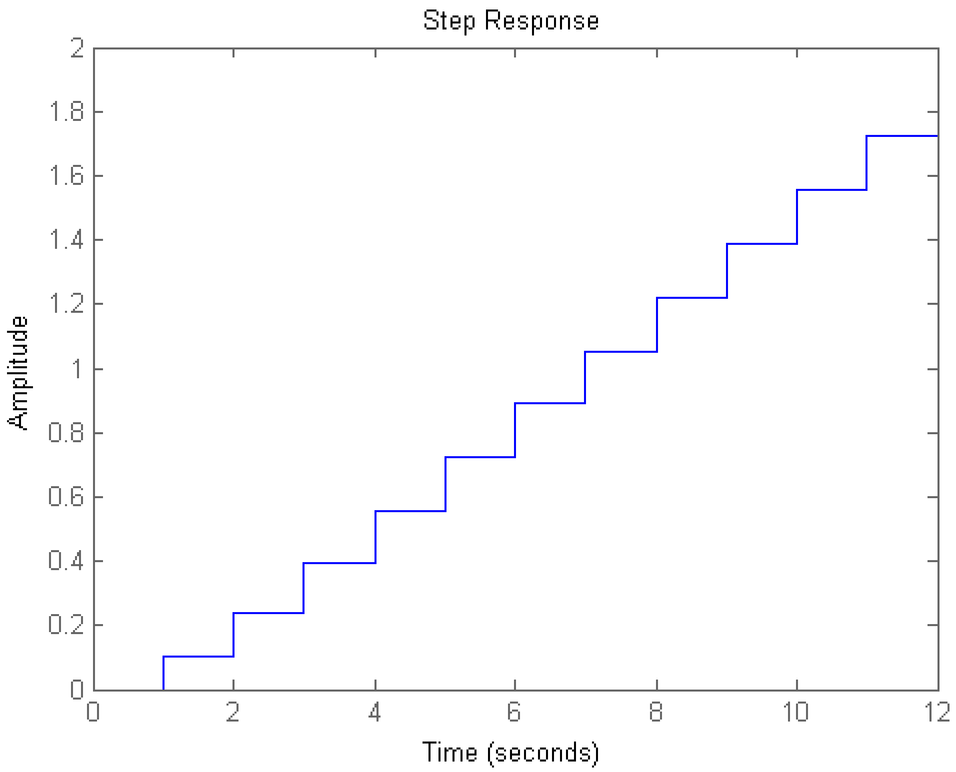
```
clc
clearvars
close all
syms z
R1 = input('Enter the value of R1: ');
R2 = input('Enter the value of R2: ');
L = input('Enter the value of L: ');
C = input('Enter the value of C: ');

a = (R2*L*C)/((R1+L)*(R2*C+1)+R2);
b = (R2)/((R1+L)*(R2*C +1)+R2);

disp('The value of a = ')
disp(a)
disp('The value of b = ')
disp(b)
num = input('Enter the numerator of the transfer function: ');
den = input('Enter the denominator of the transfer function: ');
Hnum = num(1)*z^2 + num(2)*z^1 + num(3);
Dnum = den(1)*z^2 + den(2)*z^1 + den(3);
disp('z = ');
Hnum = solve(Hnum,z);
disp(Hnum);
disp('p = ');
Dnum = solve(Dnum,z);
disp(Dnum)
[z,p,k] = tf2zpk(num,den);
disp('k = ');
disp(k)
L=11;
u = [1 zeros(1,L-1)];
x = filter(num,den,u);
disp('Coefficients of the power series expansion: ');
figure;
zplane(z,p)
hold on
figure;
dstep(num,den,10)
hold on
figure;
dimpulse(num,den,10)
```

Output

```
The value of a =
    0.4000

The value of b =
    0.1000

Enter the numerator of the transfer function: [0 0.1 0]
Enter the denominator of the transfer function: [1 -1.4 0.4]

z =
0

p =
1.0000
0.4000

k =
0.1000

Coefficients of the power series expansion:
  Columns 1 through 8

         0    0.1000    0.1400    0.1560    0.1624    0.1650
0.1660    0.1664

  Columns 9 through 11

    0.1666    0.1666    0.1666
```


Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


Impulse Response
Amplitude
Time (seconds)
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
12



<u>My Work:</u>

Z-transforms can be used for analysis of data in multiple sectors.

We can develop tools as I.T. engineers that can check for any disturbances in motor vehicles and trains which we can analyze from the generated output which will help a lot to reduce accidents.

We can also use this for making software that will track electrical appliances and its supply where the software would constantly store and analyze the data generated during usage and transmission. In case of any errors, sudden changes or emergencies, the software can automatically cut the power supply off and immediately notify the authorities and the owner of the house about this particular incident.

This software would reduce the risk of accidents a lot.