MATHEMATICS DIGITAL ASSIGNMENT 2

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Batch : 10 (COMPUTER SCIENCE (CORE))

Slot : D2 + TD2

Subject Code : MAT1011

) Use convolution theorem to find

$$L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$$

Ans:

$$F(s) = \frac{s^2}{(s^2+4)(s^2+9)} = \left(\frac{s}{(s^2+4)}\right)\left(\frac{s}{(s^2+9)}\right)$$

$$L^{-1}\left\{\frac{s}{s^2+4}\right\} = \cos 2t = g(t)$$

$$L^{-1}\left\{\frac{s}{s^2+9}\right\} = \cos 3t = f(t)$$

$$L^{-1}\{F(s)\} = L^{-1}\{f(t) * g(t)\} =$$

$$= \int_0^t \cos 3u \cos 2(t-u)\,du = \frac{1}{2}\int_0^t 2\cos 3u \cos 2(t-u)\,du$$

$$= \frac{1}{2}\left[\sin(u+2t) + \frac{\sin(5u-2t)}{5}\right]_0^t$$

$$= \frac{1}{2}\left[\sin(t+2t) + \frac{\sin(5t-2t)}{5} - \sin(0+2t) - \frac{\sin(0-2t)}{5}\right]$$

$$= \frac{1}{2}\left[\sin 3t + \frac{\sin 3t}{5} - \sin 2t + \frac{\sin 2t}{5}\right]$$

$$= \frac{1}{2}\left[\frac{6}{5}\sin 3t - \frac{4}{5}\sin 2t\right]$$

$$= \frac{1}{5}\left[3\sin 3t - 2\sin 2t\right]$$

$$\therefore L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\} = \frac{1}{5}\left[3\sin 3t - 2\sin 2t\right]$$

) Discuss the continuity of 2 variables function at any point (take any function) based on function definition.

Ans:

Let $f(x,y) = \frac{2xy}{x^2+y^2}$ be the function

and lets check the continuity of the function at $(0,0)$.

The function f is continuous at any point $(x,y) \neq (0,0)$ because its value are then given by a rational function of x and y and the limiting value is obtained by substituting the values of x and y into the functional expression.

At $(0,0)$, the value of f is defined, but f, we claim has no limit as $(x,y) \to (0,0)$. The reason is that different paths of approach to the origin can lead to different results, as we can see.

For every value of m, the function f has a constant value on the punctured line $y = mx$, $x \neq 0$, because

$$f(x,y)\Big|_{y=mx} = \frac{2xy}{x^2+y^2}\Big|_{y=mx} = \frac{2x(mx)}{x^2+(mx)^2}$$

$$= \frac{2mx^2}{x^2+m^2x^2} = \frac{2m}{1+m^2}$$

Therefore, f has this number as its limit as (x,y) approaches $(0,0)$ along the line:

$$\lim_{\substack{y\to 0\\ x\to 0\\ \text{along } y=mx}} f(x,y) = \lim_{\substack{y\to 0\\ x\to 0}} \left[f(x,y)\Big|_{y=mx} \right] = \frac{2m}{1+m^2}$$

This limit changes with each value of the slope m. There is therefore no single number we may call the limit of f as (x,y) approaches the origin. The limit fails to exist, and the function is not continuous at $(0,0)$.

) Use taylor formula for $f(x,y) = \sin(x^2+y^2)$ to find cubic approximation.

Ans:

$f(x,y) = \sin(x^2+y^2)$

$f(0,0) = 0$

$f_x'(x,y) = \cos(x^2+y^2)(2x)$ $\quad f_x'(0,0) = 0$

$f'_y(x,y) = \cos(x^2+y^2)(2y)$ $\quad f'_y(0,0) = 0$

$f'_{xx}(x,y) = 2\cos(x^2+y^2) - [\sin(x^2+y^2)](4x^2)$.

$f'_{xx}(0,0) = 2$

$f'_{yy}(x,y) = 2\cos(x^2+y^2) - [\sin(x^2+y^2)]4y^2$

$f'_{yy}(0,0) = 2$

$f'_{xy}(x,y) = 2x(-\sin(x^2+y^2))(2y) = -4xy\sin(x^2+y^2)$

$f'_{xy}(0,0) = 0$

$f'_{xxx}(x,y) = -4x\sin(x^2+y^2) + 8x\sin(x^2+y^2) + 8x^3\sin(x^2+y^2)$

$= 4x\sin(x^2+y^2) + 8x^3\sin(x^2+y^2)$

$f'_{xxx}(0,0) = 0$

$f'_{yyy}(x,y) = -4y\sin(x^2+y^2) - 8x^3\sin(x^2+y^2)$

$f'_{yyy}(0,0) = 0$

$f'_{xyy}(x,y) = -4x\sin(x^2+y^2) - 8xy^2\cos(x^2+y^2)$

$f'_{xyy}(0,0) = 0$

$f'_{xxy}(x,y) = -4y\sin(x^2+y^2) + [\cos(x^2+y^2)](8x^2y)$

$f'_{xxy}(0,0) = 0$

$$\therefore F(x,y) = f(0,0) + \frac{[x f_x(0,0) + y f_y(0,0)]}{1!} +$$

$$\frac{[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)]}{2!}$$

$$+ \frac{[x^3 f_{xxx}(0,0) + 3x^2 y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0) + y^3 f_{yyy}(0,0)]}{3!}$$

$$\Rightarrow F(x,y) = 0 + \frac{[x(0) + y(0)]}{1!} + \frac{[x^2(2) + 2xy(0) + y^2(2)]}{2!}$$

$$+ \frac{[x^3(0) + 3x^2y(0) + 3xy^2(0) + y^3(0)]}{3!}$$

$$\Rightarrow F(x,y) = \frac{2(x^2+y^2)}{2} = x^2+y^2$$

Ans: The cubic approximation of the function $f(x,y) = \sin(x^2+y^2)$ is x^2+y^2.

$$F(x,y) = x^2+y^2$$

) Find the extreme values of function $f(x,y,z) = xy+z^2$ on the circle in which the plane $y-x=0$ intersects the sphere $x^2+y^2+z^2=4$.

Ans: Let $g_1(x,y,z) = y-x=0$ and $g_2(x,y,z) = x^2+y^2+z^2-4=0$

Then $\nabla f = y\hat{i} + x\hat{j} + 2z\hat{k}$ $\nabla g_1 = -\hat{i}+\hat{j}$ and $\nabla g_2 = 2x\hat{i}+2y\hat{j}+2z\hat{k}$ so that $\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$

$y\hat{i} + x\hat{j} + 2z\hat{k} = \lambda(-\hat{i}+\hat{j}) + \mu(2x\hat{i}+2y\hat{j}+2z\hat{k})$

$y = -\lambda + 2x\mu$, $x = \lambda + 2y\mu$ and $2z = 2z\mu$

$\Rightarrow z=0$ or $\mu = 1$

CASE I: $z=0 \Rightarrow x^2+y^2-4=0 \Rightarrow 2x^2-4=0$ $(\because x-y=0)$

$\Rightarrow x = \pm\sqrt{2}$ and $y = \pm\sqrt{2}$

$\therefore$ The points are $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$.

CASE II: $\mu = 1 \Rightarrow y = -\lambda + 2x$ and $x = \lambda + 2y \Rightarrow x+y = 2(x+y)$

$\Rightarrow 2x = 2(2x)$ $(\because x-y=0) \Rightarrow x=0$ $\therefore y=0$

$\Rightarrow z^2 - 4 = 0 \Rightarrow z = \pm 2$.

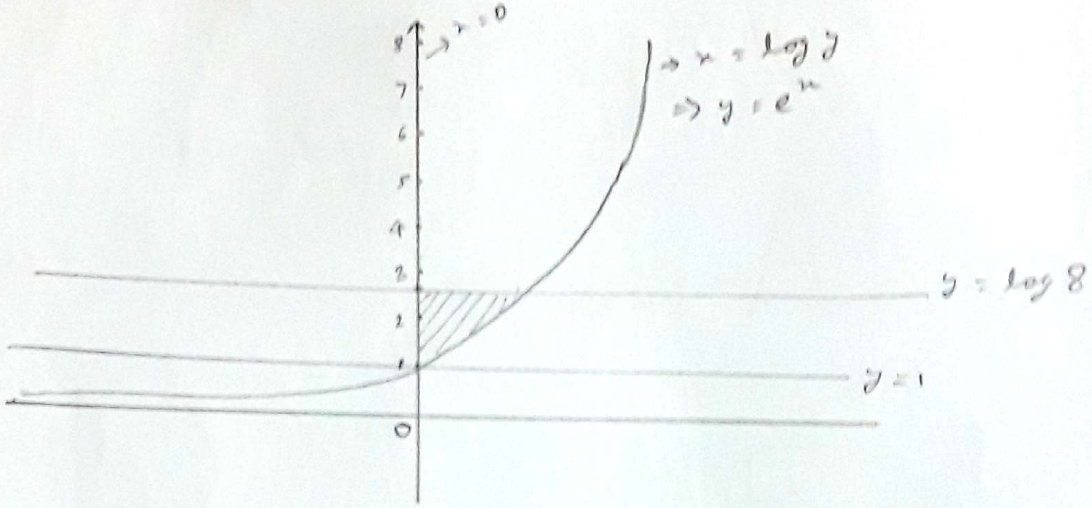
$\therefore$ The points are $(0,0,\pm 2)$.

Now, $f(0,0,\pm 2) = 4$ and $f(\pm\sqrt{2}, \pm\sqrt{2}, 0) = 2$.

Therefore the maximum value of f is 4 at $f(0,0,\pm 2)$ and minimum value of f is 2 at $f(\pm\sqrt{2}, \pm\sqrt{2}, 0)$.

) Sketch the region of integration for $\int_1^{\log 8}\int_0^{\log y} e^{x+y}\,dx\,dy$ and find the value of given integration.

Ans: The region is



$$\int_1^{\log 8}\int_0^{\log y} e^{x+y}\,dx\,dy$$

$$\Rightarrow \int_1^{\log 8} e^y\,[e^x]_0^{\log y}\,dy$$

$$\Rightarrow \int_1^{\log 8} e^y\,[y-1]\,dy$$

$$\Rightarrow \int_1^{\log 8} ye^y - e^y\,dy$$

$$\Rightarrow [ye^y - e^y - e^y]_1^{\log 8}$$

$$\Rightarrow [(\log 8)e^{(\log 8)} - e^{\log 8} - e^{\log 8}] - [e - e - e]$$

$$\Rightarrow [8\log 8 - 2\times 8] - [e - 2e] = 8\log 8 - 16 + e$$

Ans: Therefore the value of given integration is $8\log 8 - 16 + e$.

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