DISCRETE MATHEMATICS AND GRAPH THEORY

DIGITAL ASSIGNMENT I

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① (a) Without using truth table show that

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Ans: $\neg(p \vee (\neg p \wedge q))$

$\Rightarrow \neg p \wedge \neg(\neg p \wedge q)$ [By De Morgan's law]

$\Rightarrow \neg p \wedge (\neg(\neg p) \vee \neg(q))$ [By De Morgan's law]

$\Rightarrow \neg p \wedge (p \vee \neg q)$ [By Double negation law]

$\Rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ [By Distributive law]

$\Rightarrow F \vee (\neg p \wedge \neg q)$ [By Negation law]

$\Rightarrow (\neg p \wedge \neg q)$ [By Identity law]

(b) Obtain principle disjunctive normal form (PDNF) for $(p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r)$.

Ans: $(p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r)$

$\Rightarrow (p \wedge q \wedge T) \vee (\neg p \wedge r \wedge T) \vee (q \wedge r \wedge T)$
[By Identity law]

$\Rightarrow (p \wedge q \wedge (r \vee \neg r)) \vee (\neg p \wedge (q \vee \neg q) \wedge r)$
$\vee ((p \vee \neg p) \wedge q \wedge r)$ [By Negation law]

$\Rightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee$
$(\neg p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r)$
[By Distributive law]

$\Rightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee$
$(\neg p \wedge \neg q \wedge r)$ (PDNF)

Hence $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r)$
$\vee (\neg p \wedge \neg q \wedge r)$ is the required PDNF.

② (a) Obtain a Conjunctive normal form (C.N.F) of $\neg(P \vee Q) \leftrightarrow (P \wedge Q)$

Ans : We know that $(P \leftrightarrow Q) \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

Consider $\underbrace{\neg(P \vee Q)}_{P} \leftrightarrow \underbrace{(P \wedge Q)}_{Q}$

$$\equiv (\neg(P \vee Q) \rightarrow (P \wedge Q)) \wedge ((Q \wedge P) \rightarrow \neg(P \vee Q))$$

$$\equiv (\neg(\neg(P \vee Q)) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee \neg(P \vee Q))$$
[By Implication law]

$$\equiv ((P \vee Q) \vee (P \wedge Q)) \wedge ((\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q))$$
[By Double Negation law and De-Morgan law]

$$\equiv ((P \vee Q \vee P) \wedge (P \vee Q \vee Q)) \wedge ((\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \wedge \neg Q \wedge \neg Q))$$
[By Distributive law]

$$\equiv ((P \vee Q) \wedge (P \vee Q)) \wedge ((\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q))$$
[By Idempotent law]

$$\equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$$ [By Idempotent law]

Hence $(P \vee Q) \wedge (\neg P \vee \neg Q)$ is the C.N.F.

(b) Show that $R \vee S$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

Ans: The Truth Table

P	Q	R	S	$P \vee Q$	$P \to R$	$Q \to S$	$(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$ X	$R \vee S$ Y	$X \to Y$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	T	T
T	T	F	T	T	F	T	F	T	T
T	T	F	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F	T	T
T	F	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	T	T
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	F	F	T
F	F	T	T	F	T	T	F	T	T
F	F	T	F	F	T	T	F	T	T
F	F	F	T	F	T	T	F	T	T
F	F	F	F	F	T	T	F	F	T

Here $X : (P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

$Y : R \vee S$

Since $X \to Y$ is therefore tautology, $(R \vee S)$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

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③ (a) Show that the following premises are inconsistent.

(i) "If Jack misses many classes through illness, then he fails high school."

(ii) "If Jack fails high school, then he is uneducated."

(iii) "If Jack reads a lot of books, then he is not uneducated"

(iv) "Jack misses many classes through illness and reads a lot of books".

Ans: Let P: Jack misses many classes

Q: Jack fails high school

R: Jack reads a lot of books

S: Jack is uneducated.

Now the given statement becomes

$P \rightarrow Q$, $Q \rightarrow S$, $R \rightarrow \neg S$, $P \wedge R$

Steps	Reasons
1. $P \rightarrow Q$	By rule P
2. $Q \rightarrow S$	By rule P
3. $P \rightarrow S$	By rule T, from 1, 2 and $P \rightarrow Q$, $Q \rightarrow R \Rightarrow P \rightarrow R$
4. $R \rightarrow \neg S$	By rule P

5. $S \to \neg R$ By rule T and $P \to Q \equiv \neg Q \to \neg P$

6. $P \to \neg R$ By rule T, 3, 5 and $P \to Q$, $Q \to R$, $P \to R$

7. $P \wedge R$ By rule P

8. $\neg P \vee \neg R$ By rule T from 6

9. $\neg(P \wedge R)$ By rule T and $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

10. $(P \wedge R) \wedge \neg(P \wedge R)$ By rule T from 7, 9 and $P, Q \Rightarrow P \wedge Q$

Hence the inconsistency is proved.

(b) Show that $(\exists x) M(x)$ follows logically from the premises $(x)(H(x) \to M(X)) \wedge (\exists X) H(x)$.

Ans:

Steps	Reasons
1. $(x)(H(x) \to M(x))$	By rule P
2. $H(y) \to M(y)$	By rule US [Universal Specification]
3. $(\exists x) H(x)$	By rule P
4. $H(y)$	By rule ES [Existential Specification]
5. $M(y)$	By rule T from 2, 4 and $P, P \to Q \equiv Q$
6. $(\exists x) M(x)$	By rule EG [Existential Generalisation]

Hence $(\exists x) M(x)$ follows logically from the premises $(x)(H(x) \to M(X)) \wedge (\exists X) H(x)$ is proved.

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(4) Prove or disprove the validity of the following argument.

i) "All men are fallible", (ii) "All Kings are men" and the conclusion is "All kings are fallible."

Ans: Let $M(x)$: x is man

$K(x)$: x is king

$F(x)$: x is fallible

Then the premises are

$\forall x[M(x) \to F(x)]$, $\forall x[K(x) \to M(x)]$

$\therefore \forall x[K(x) \to F(x)]$

Steps	Reason
1. $\forall x[M(x) \to F(x)]$	By rule P
2. $M(y) \to F(y)$	By rule T and US
3. $\forall x[K(x) \to M(x)]$	By rule P
4. $K(y) \to M(y)$	By rule T and US
5. $K(y) \to F(y)$	By rule T from 2, 4 and $P \to Q$, $Q \to R \equiv P \to R$
6. $\forall x[K(x) \to F(x)]$	By rule T and UG

where US = Universal specification

UG = Universal Generalisation

"All Kings are fallible." Hence Proved.