EXPERIMENT – III

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Determining the Shape of a Suspension Bridge Cable

Aim:

The purpose of this experiment is to determine the shape of the cable subject to different load functions and tension in the cable.

Problem Statements:

Finding the shape of a cable/bridge subject to different load functions and tension in the cable using differential equations.

Matlab Commands:

Subs – substitute into an object

Solve – solve equations and inequalities

Real – real part of complex number

Imag – imaginary part of complex number

Description of Physical Experiment:

- We consider a main cable to be inextensible, hanging between the two given fixed ends A and B.

- The vertical distance between the highest and the lowest points of the cable is called Sag, while the horizontal distance between two supports A and B is called Span.

- We assume that the cable is of length S and of constant self-weight per unit length $W = mg$, where m is the mass per unit length of the cable, and g is the acceleration due to gravity.

- Determination of the static equilibrium shape of a cable can, in some cases, be simplified if the downwardly directed external loads, which are approximated as point forces in Fig. a, can be further approximated by a continuously distributed load, as shown in Fig. b.

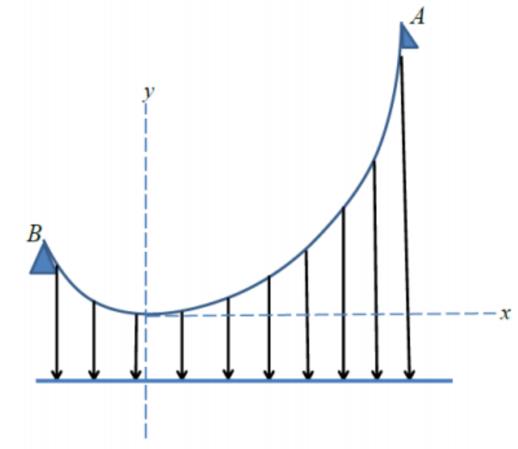


Fig a. Downwardly directed external loads.

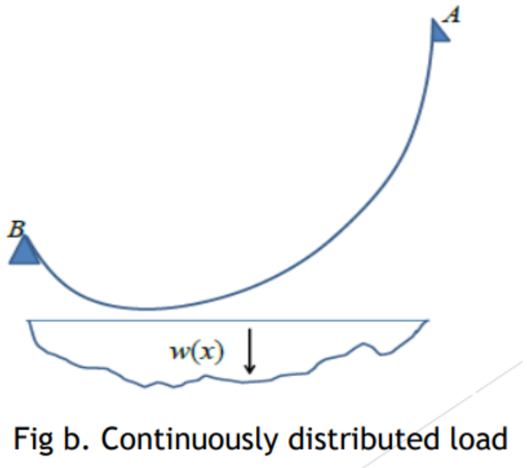
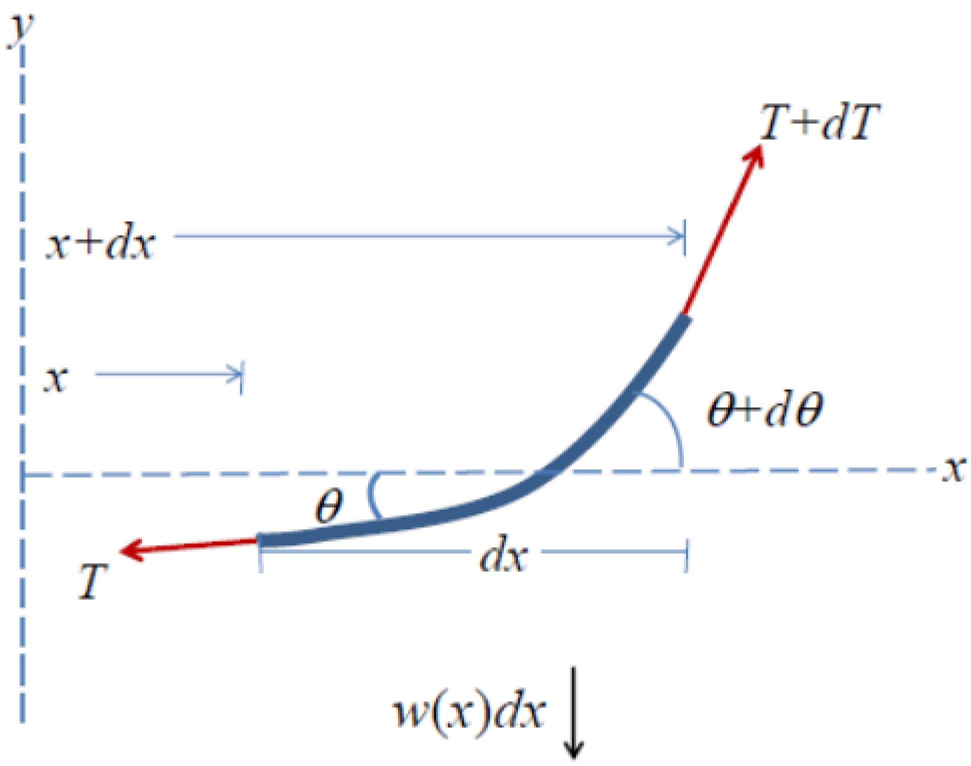


Fig b. Continuously distributed load

We assume that the cable is loaded by the distributed vertical external load w(x). Now to derive the equation of the cable, we consider a cable element as shown in the figure below:



We now determine the equation for a flexible cable.

- From the figure, it is clear that the elongations in the vertical direction are $y1 = T \sin \theta$ and $y2 = (T + dT) \sin(\theta + d\theta)$.
- For static equilibrium, the sum of the forces must be equal to zero. That is, theυ sum of the vertical forces $\Sigma Fy = 0$.

$$\Rightarrow -y1 + y2 - w\ x\ dx = 0$$

$\Rightarrow T + dT \sin(\theta + d\theta) = T \sin \theta + w\ x\ dx$ (1)

The sum of the horizontal forces $\Sigma Fx = 0 \Rightarrow T + dT \cos(\theta + d\theta) = T \cos \theta$. (2)

Assuming that $d\theta \longrightarrow 0$, we have $sin\ d\theta \backsimeq d\theta$, and $cos\ d\theta = 1$. Thus Eq. (1) and (2) read as:

$T + dT\ [\sin \theta + \cos \theta\ d\theta] = T \sin \theta + w\ (x)\ dx$ (3)

$T + dT\ [\cos \theta - \sin \theta\ d\theta] = T \cos \theta$ (4)

On simplification, we get:

$dT \sin\theta + T \cos\theta\, d\theta + dT \cos\theta\, d\theta = w(x)\, dx$

$dT \cos\theta - T \sin\theta\, d\theta - dT \sin\theta\, d\theta = 0$

Ignoring the second order terms $dTd\theta$ in these equations, we get

$dT \sin\theta + T \cos\theta\, d\theta = w(x)\, dx$ (5)

$dT \cos\theta - T \sin\theta\, d\theta = 0$ (6)

Equations (5) and (6) can be written as

$d\, T \sin\theta = w(x)\, dx$ (7)

$d\, T \cos\theta = 0 \Rightarrow T \cos\theta = T_H \quad (a\ constant) \Longrightarrow T = \frac{T_H}{\cos\ \theta}$ (8)

Substituting the value of in (7)

we get $d\,(TH \tan\theta) = w(x)\, dx$

But we know that $\tan\theta = \frac{dy}{dx}$. Hence we have

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_H}$$

This is the differential equation for a flexible cable.

Connection to Mathematics:

Solution by method of variation of parameters:

Method of variation of parameters enables to find solution of any linear non homogeneous differential equation of second order, provided its complimentary function (C.F.) is given / known. The particular integral of the non-homogeneous equation is obtained by varying the parameters, i.e. by replacing the arbitrary constants in the C.F. by variable functions.

Consider a linear non-homogeneous second order differential equation with constant coefficients

$\frac{d^2y}{dx^2} + \text{p}\frac{dy}{dx} + qy = f(x)$, where p, q are constants. (13)

Let the complimentary function be of the form $y_c = C_1y_1(x) + C_2y_2(x)$, where $C_1, C_2$are arbitrary constants. This is the solution of the homogeneous equation

$$\frac{d^2y}{dx^2} + \text{p}\frac{dy}{dx} + qy = 0$$

In the method of variation of parameters, the arbitrary constants C_1, $C_2$are replaced with two unknown functions u(x) and v(x) .

Let us assume that the particular integral is of the form $y_p = u(x)\, y_1 x + v(x)\, y_2(x)$ (14)

where $u(x) = -\int \frac{y_2(x) f(x)}{y_1 y'_2}$ and $v(x) = -\int \frac{y_1(x) f(x)}{y_1 y'_2}$

On putting the values of u(x) and v(x) in (14), we get the particular integral y_p. Hence the required solution $y(x) = y_c + y_p$

Code:

```
clear all
close all
clc
syms A B x m
p=input('Enter the coefficients a,b,c') ;
f=input('Enter the RHS function f(x)');
a=p(1);b=p(2);c=p(3);
disc=b^2-4*a*c;
m=subs(solve('a*m^2+b*m+c') );
if(disc>0)
 CF= A*exp(m(1)*x)+B*exp(m(2)*x)
 u=exp(m(1)*x);v=exp(m(2)*x);
elseif (disc==0)
 CF=(A+B*x)*exp(m(1)*x)
 u=exp(m(1)*x);v=x*exp(m(1)*x);
else
 alfa=real(m(1));
 beta=imag(m(1));
```

```
 CF=exp(alfa*x)*(A*cos(beta*x)+B*sin(beta*x))
 u=exp(alfa*x)*cos(beta*x);v=exp(alfa*x)*sin(beta*x);
end
```

Questions:

1) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$

Solution:

MATLAB input:

```
Enter the coefficients a,b,c[1 3 -10]

Enter the RHS function f(x)0
```

MATLAB output:

```
CF =


A*exp(-5*x) + B*exp(2*x)
```

2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 0$

Solution:

MATLAB input:

```
Enter the coefficients a,b,c [1 2 4]

Enter the RHS function f(x) 0
```

MATLAB output:

```
CF =


exp(-x)*(A*cos((12^(1/2)*x)/2) - B*sin((12^(1/2)*x)/2))
```

3) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$

Solution:

MATLAB input:

```
Enter the coefficients a,b,c [1 8 16]

Enter the RHS function f(x) 0
```

MATLAB output:

```
CF =

exp(-4*x)*(A + B*x)
```

4) $\frac{d^2y}{dx^2} + 4y = 0$

Solution:

MATLAB input:

```
Enter the coefficients a,b,c [1 0 4]

Enter the RHS function f(x) 0
```

MATLAB output:

```
CF =

A*cos(2*x) - B*sin(2*x)
```

Question:

Consider the problem of suspension cable $\frac{d^2 y}{dx^2} = \frac{w(x)}{T_H}$ with the conditions y(0) = 0, $\left(\frac{dy}{dx}\right)_{x=0} = 0$.

Code:

```
clear all
close all
clc
syms A B x m
W=input('Enter the external load: ') ;
T=input('Enter the horizontal tension: ') ;
f=W/T;
a=1;b=0;c=0;
disc=b^2-4*a*c;
m=subs(solve('a*m^2+b*m+c') );
if(disc>0)
 CF= A*exp(m(1)*x)+B*exp(m(2)*x);
 u=exp(m(1)*x);v=exp(m(2)*x);
elseif (disc==0)
 CF=(A+B*x)*exp(m(1)*x);
 u=exp(m(1)*x);v=x*exp(m(1)*x);
else
 alfa=real(m(1));
 beta=imag(m(1));
 CF=exp(alfa*x)*(A*cos(beta*x)+B*sin(beta*x));
 u=exp(alfa*x)*cos(beta*x);v=exp(alfa*x)*sin(beta*x);
end
f1=f/a;
jac=u*diff(v,x)-diff(u,x)*v;
P=int(-v*f1/jac,x);
Q=int(u*f1/jac,x);
PI=P*u+Q*v;
y_gen=CF+PI;
dy_gen=diff(y_gen);
cond=[0 0 0];
eq1=(subs(y_gen,x,cond(1))-cond(2));
eq2=(subs(dy_gen,x,cond(1))-cond(3));
A=solve(eq1);
B=solve(eq2);
y=subs(CF+PI)
```

Ouput:

MATLAB input:

```
Enter the external load: 1

Enter the horizontal tension: 1
```

MATLAB ouput:

```
y =

x^2/2
```

My Work:

We can use this method that determines the shape of bridges using differential equations for game development!

Let's say we have a video game, we need to determine the way a character should behave in a game when a certain command is given.

For example, we make a basic game where the main character has to chase down a thief. He should be able to run from walk when we press the button *Shift*. We can use differential equations to find the rate of acceleration, the distance that is travelled by the character and the time constraint after which the character will get tired and need a certain time to rest. All of this can be determined and implemented in the game using differential equations exactly like the given code.

It can also be used to predict human behaviour in a way that would use machine learning where the system would be fed the input of activities by the user, let's assume running just like the game. We can easily determine the speed, the calories he burnt in his running, his stamina etc. and then process the data and give the user suggestions and his progress in his activity. This can be vital for Artificial Intelligence development for future machines.