EXPERIMENT – VIII

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Vertical deflection in swimming pool diving

Aim:

Finding the vertical deflection in a cantilever beam subjected to variable load and material properties and visualization of it.

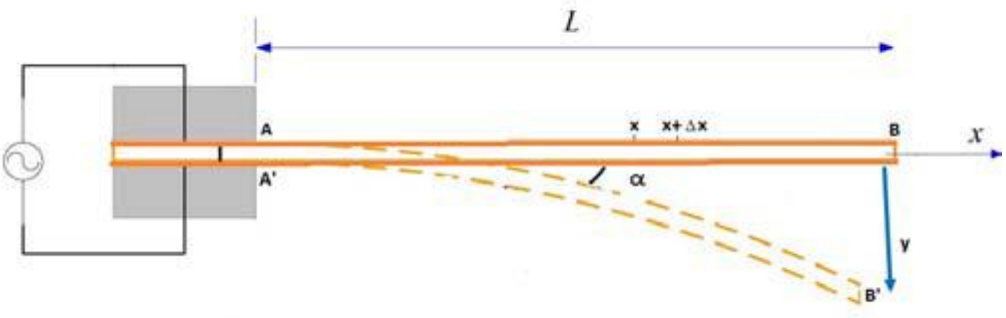
Problem Statements:

Solving governing equation of vertical deflection in swimming pool diving board using Laplace transform.

Matlab Commands:

- laplace(f) – Returns the Laplace transform of f using the default independent variable t and the default transformation variable s.
- laplace(f, transVar) – Uses the specified transformation variable transVar instead of s.
- laplace(f, var, transVar) – Uses the specified independent variable var and transformation variable transVar instead of t and s respectively.
- ilaplace(F) – Returns the inverse Laplace transform of F using the default independent variables for the default transformation variable t. If F does not contain s, ilaplace uses symvar.
- ilaplace(F, transVar) – Uses the specified transformation variable transVar instead of t.

Description of Physical Experiment:



The following assumptions are undertaken in order to derive a differential equation of elastic curve for the loaded beam:

- Stress is proportional to strain. Thus, the equation is valid only for beams that are not stressed beyond the elastic limit.

- The curvature is always small.

- Any deflection resulting from the shear deformation of the material or shear stresses is neglected.

- For the deflected shape of the beam, the slope α at any point is defined as tan α = $\frac{dy}{dx}$. Assuming tan α = α we can write α = $\frac{dy}{dx}$.

- The curvature of a plane curve at a point can be expressed as

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left(1 + \frac{dy}{dx}^2\right)^{3/2}}$$

- In the elastic curve of beam $\frac{dy}{dx}$ is very small so we can neglect its higher order terms.

$$\frac{1}{\rho} = \frac{d^2y}{dx^2}$$

- From the theory of elasticity, if x is the distance of the section from the left end of the beam then

$$\frac{1}{\rho} = \frac{M(x)}{El}$$

where M-Bending moment E-Modulus of Elasticity I-Moment of inertia of the cross section.

- $\frac{d^2y}{dx^2} = \frac{-M(x)}{El}$ is the governing equation for an elastic curve.

- When a beam supports a distributive load w(x) then $\frac{dM}{dx}$ = V(Shear force) and , $\frac{dV}{dx} = -w$

  Therefore, $\frac{d^4y}{dx^4} = \frac{W(x)}{El}$.

Vertical deflection in a swimming pool diving board subjected to a distributed load can be seen as the deflection in a cantilever beam of length L subjected to a distributed load w(x) which is the solution of the differential equation:

$$\frac{d^4y}{dx^4} = \frac{W(x)}{El}$$

subjected to the boundary conditions y(0) = y'(0) = y''(L) = y'''(L) = 0.

Note: y''(L) = 0 because there is no bending moment and y'''(L) = 0 because there is no shear at that point.

Connection to Mathematics:

Laplace Transform

The Laplace Transform of a function f(t), defined for all real numbers t > 0 is the function F(s) defined by,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s)$$

Inverse Laplace Transform

The inverse Laplace Transform of F(s) is defined by

$$f(t) = L^{-1}[f(t)] = \int_0^\infty e^{-st} f(s)dt$$

Code:

1. Example 1

```
clear all
clc
syms t
f=input('Enter the function in terms of t:');
F=laplace(f);
F=simplify(F)
```

```
Enter the function in terms of t:
t^2*(heaviside(t-0)-heaviside(t-2))+(t-1)*(heaviside(t-2)-
heaviside(t-3))+7*(heaviside(t-3))

F =

(7*exp(-3*s)*(s - 2*exp(-2*s) + 1/s + 2/s^3 -(exp(-3*s)* (2*s-
exp(s) - s*exp(s)+1))/s^2
```

2. Example 2

```
clc
clear all
syms t s Y
y2=diff(sym('y(t)'),2);
y1=diff(sym('y(t)'),1);
y0=sym('y(t)');
a = input('The Coefficient of D2y = ');
b = input('The Coefficient of Dy = ');
c = input('The Coefficient of y = ');
nh = input('Enter the non homogenous part = ');
eqn=a*y2+b*y1+c*y0-nh;
LTY=laplace(eqn,t,s);
continued
if (a==0)
d = input('The initial value at 0 is ');
LTY=subs(LTY,'laplace(y(t), t, s)','y(0)',Y,d)
else
d = input('The initial value at 0 is ');
e = input('The initial value at 0 is ');
LTY=subs(LTY,'laplace(y(t), t, s)','y(0)','D(y)(0)',Y,d,e)
end
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,t))
```

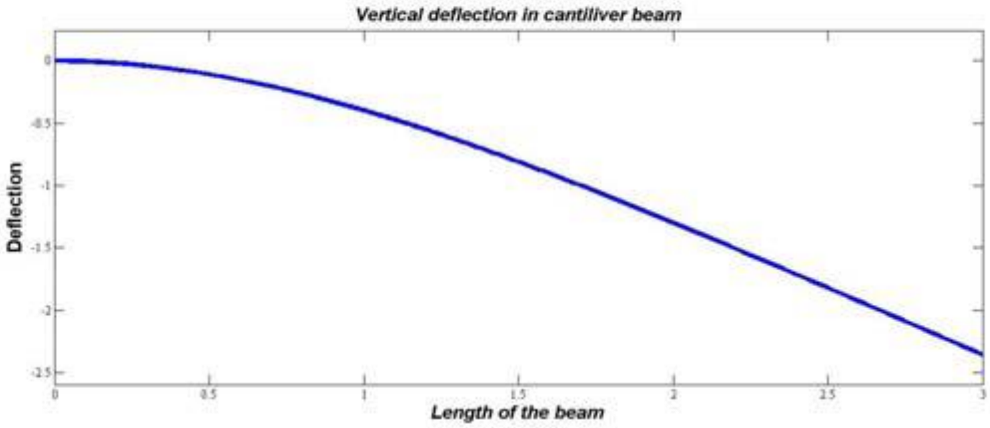
```
The Coefficient of D2y = 1
The Coefficient of Dy = 2
The Coefficient of y = 10
Enter the non homogenous part = 1 + 5*dirac(t-5)
The initial value at 0 is 1
The initial value at 0 is 2

LTY = 10 * Y - s - 5/exp(5 * s) + 2 * Y * s + Y * s
2 - 1/s - 4

y = (cos(3 * t) - sin(3 * t)/3)/exp(t) - (cos(3 * t) + sin(3 *
t)/3)/(10 * exp(t)) + (4 * sin(3 * t))/(3 * exp(t)) + (5 *
heaviside(t - 5) * exp(5 - t) * sin(3 * t - 15))/3 + 1/10
```

3. Find the deflection in a cantilever beam subjected to the following conditions y(0) = y'(0) = y''(L) = y'''(L) = 0 by taking L = 3, E = 2.1 ∗ 10^{11} N/mm^2 , I = 4.5 ∗ 10−11mm^4 and w(x) = x.

```
clc
clear all
syms x s C D Y
y4=diff(sym('y(x)'),4);
y0=sym('y(x)');
L=input('Enter the length of the beam: ');
E=input('Enter Modulus of elasticity: ');
I=input('Enter Moment of inertia of the cross section: ');
w=input('Enter distributive load w(x): ');
eqn=E*I*y4-w;
LTY=laplace(eqn,x,s);
a=input('Enter y(0): ');
b=input('Enter Dy(0): ');
c=input('Enter D2y(L): ');
d=input('Enter D3y(L): ');
LTY=subs(LTY,{'laplace(y(x),x,s)','y(0)','D(y)(0)','D(D((y)))(0)','D(D(D((y
))))(0)'},{Y, a, b, C, D})
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,x));
eq1=subs(diff(y,x,2),x,L);
eq2=subs(diff(y,x,3),x,L);
[C, D]=solve(eq1, eq2)
gen=subs(y);
def=subs(def,heaviside(x-L),0)
ezplot(-def,[0,L])
title('Vertical deflection in cantilever beam')
xlabel('Length of the beam')
ylabel(Deflection')
```

```
Enter the length of the beam: 3

Enter Modulus of elasticity: 2.1 ∗ 1011

Enter Moment of inertia of the cross section: 4.5 ∗ 10−11

Enter distributive load w(x): x*(heaviside(x-L)-heaviside(x))

Enter y(0):0

Enter Dy(0):0

Enter D2y(L):0

Enter D3y(L):0

LTY =

-189/20*D-189/20*s*C+(1-(1-(3*s+1)*exp(-3*s))/s^2+189/20*s^4*Y


C = -20/21

D = 10/21
```

```
def = 5/63*x^3-10/21*x^2-1/1134*x^5
```



Deflection v/s length graph of the beam

<u>My Work:</u>

The following code can be applied in data sciences where we store the data solving the differential equations using Laplace transform techniques.

For example, we can easily detect the vertical deflection using the above code. Then we can covert that to big data where we can then analyze that to determine the perfect technique for Olympic divers. This would be an excellent training tool for coaches. Then it can be converted into software where the system can undergo *Machine Learning* and can detect, analyze and correct athletes on its own.

It can be extremely useful for diving board manufacturers as well who can easily calculate appropriate load distribution in the beam, reducing manufacturing defects.

This can be useful for aeroplanes as well which uses Information Technology tools for taking off a plane.

We can put the length of the plane, its moment of inertia, its load taking into account the additional entity that is air resistance.

We can simulate the perfect takeoff tool which can be then implemented and would make take-offs extremely safer.