DISCRETE MATHEMATICS

DIGITAL ASSIGNMENT III

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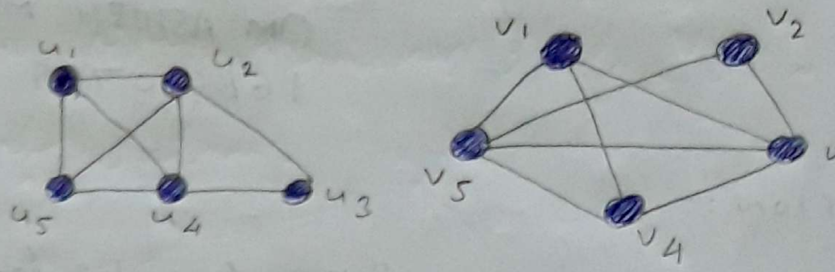
16BCE0789

① Show that the given graphs are isomorphic.



Ans :

The no of vertices (5) and no of edges (8) are shown.

Define a mapping :-

$u_1 \to v_1$, $u_2 \to v_5$, $u_3 \to v_2$, $u_4 \to v_3$,
$v_5 \to v_4$.

<u>In Graph G_1 :</u>

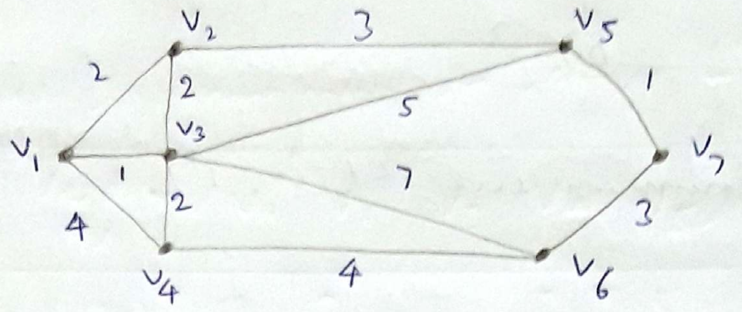
$$A(G_1) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

<u>In Graph G_2 :</u>

$$A(G_2) = \begin{array}{c} \\ v_1 \\ v_5 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_5 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Therefore $A(G_1)$ and $A(G_2)$ are same. Therefore they are isomorphic.

⑤ using Dijkstra's algorithm to find the shortest path from v_1 to v_7.



Ans :

Vertex V	v_1	V_2	V_3	V_4	V_5	V_6	V_7
L (V)	0	∞	∞	∞	∞	∞	∞
T	v_1	v_2	v_3	v_4	v_5	v_6	v_7

Iteration ①:

$u = v_1$ has $L\{u\} = 0$, T becomes $T - \{v_1\}$.

There are 3 edges incident with v_1, i.e,

$v_1 v_2$, $v_1 v_3$ and $v_1 v_4$, where $v_2, v_3, v_4 \in T$

$L(v_2) = \min\{ \text{old } L(v_2), L(v_1) + w(v_1, v_2)\}$
$= \min\{\infty, 0 + 2\} = 2$

$L(v_3) = \min\{ \text{old } L(v_3), L(v_1) + w(v_1, v_3)\}$
$= \min\{\infty, 0 + 1\} = 1$

$L(v_4) = \min\{ \text{old } L(v_4) + L(v_1) + w(v_1, v_4)\}$
$= \min\{\infty, 0 + 4\} = 4$

Hence minimum label is $L(v_3) = 1$.

Vertex v	v_1	v_2	v_3	v_4	v_5	v_6	v_7
$L(v)$	0	2	1	4	∞	∞	∞
T	—	v_2	v_3	v_4	v_5	v_6	v_7

Iteration 2:

$u = v_3$ has $L\{u\} = 1$, T becomes $T - \{v_3\}$.

There are 4 edges incident with v_3, i.e,

v_3v_2, v_3v_5, v_3v_6, v_3v_4 where $v_2, v_5, v_6, v_4 \in T$.

$L(v_2) = \min\{\text{old } L(v_2), L(v_3) + w(v_3, v_2)\}$

$= \min\{2, 1+2\} = 2$

$L(v_5) = \min\{\text{old } L(v_5), L(v_3) + w(v_3, v_5)\}$

$= \min\{\infty, 1+5\} = 6$

$L(v_6) = \min\{\text{old } L(v_6), L(v_3) + w(v_3, v_6)\}$

$= \min\{\infty, 1+7\} = 8$

$L(v_4) = \min\{\text{old } L(v_4), L(v_3) + w(v_3, v_4)\}$

$= \min\{4, 1+2\} = 3$

Hence minimum label is $L(v_2) = 2$.

Vertex v	v_1	v_2	v_3	v_4	v_5	v_6	v_7
$L(v)$	0	2	1	3	6	8	∞
T	—	v_2	—	v_4	v_5	v_6	v_7

Iteration 3:

$u = v_2$ has $L\{u\} = 2$, T becomes $T - \{v_2\}$

There are only 1 edge incident with v_2, i.e,

v_2v_5 where $v_5 \in T$.

$$L(V_5) = \min\{\text{old } L(V_5),\ L(V_2) + w(V_2, V_5)\}$$
$$= \min\{6,\ 2+3\} = 5$$

Hence the minimum label is $L(V_5) = 5$.

Vertex V	V_1	V_2	V_3	V_4	V_5	V_6	V_7
L(V)	0	2	1	3	5	8	∞
T	—	—	—	V_4	V_5	V_6	V_7

Iteration ④:

$u = V_5$ has $L\{u\} = 5$, T becomes $T - \{V_5\}$.

There are ~~only~~ only 1 edge incident with V_5, i.e, V_5V_7 where $V_7 \in T$.

$$L(V_7) = \min\{\text{old } L(V_7),\ L(V_5) + w(V_5, V_7)\}$$
$$= \min\{\infty,\ 5+1\} = 6$$

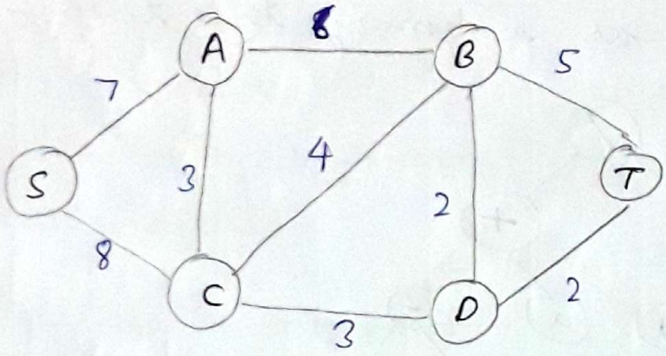
Hence the minimum label is $L(V_7) = 6$.

Vertex V	V_1	V_2	V_3	V_4	V_5	V_6	V_7
L(V)	0	2	1	3	5	8	6
T	—	—	—	V_4	—	V_6	V_7

Hence the shortest distance between V_1 and V_7 is 6 units.

Hence the shortest path is $V_1 - V_3 - V_2 - V_5 - V_7$.

③ Find the minimum spanning tree of the following graph by using Kruskal's algorithm and Prim's algorithm.

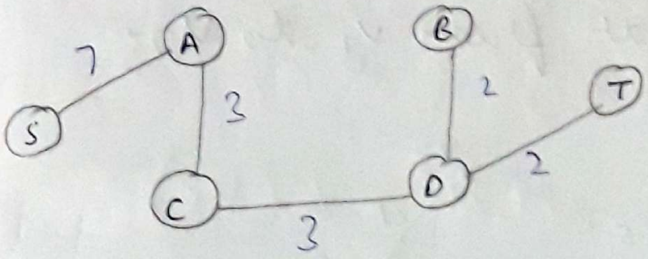


Ans : (a) Krushal's algorithm

List all the edges with respect to their weight

List of edges	Weight	Selection
(B, D)	2	✓
(D, T)	2	✓
(A, C)	3	✓
(C, D)	3	✓
(B, C)	4	✗
(B, T)	5	✗
(A, B)	~~7~~ 6	✗
(A, S)	~~8~~ 7	✓
(C, S)	8	✗

The graph becomes:-



Therefore minimum weight is 17.

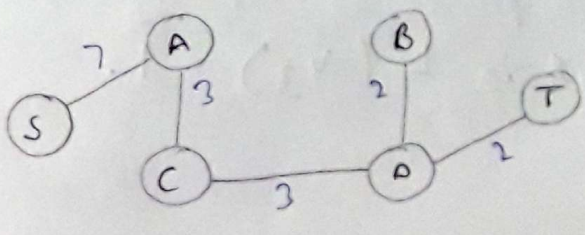
(b) Prim's algorithm

The matrix of the given weighted graph is :

	A	B	C	D	S	T
A	–	86	3	∞	7	∞
B	86	–	4	2	∞	5
C	3	4	–	3	8	∞
D	∞	2	3	–	∞	2
S	7	∞	8	∞	–	∞
T	∞	5	∞	2	∞	–

Iteration	Selected edge
1	B D
2	D T
3	A C
4	C D
5	A S

The graph becomes:-

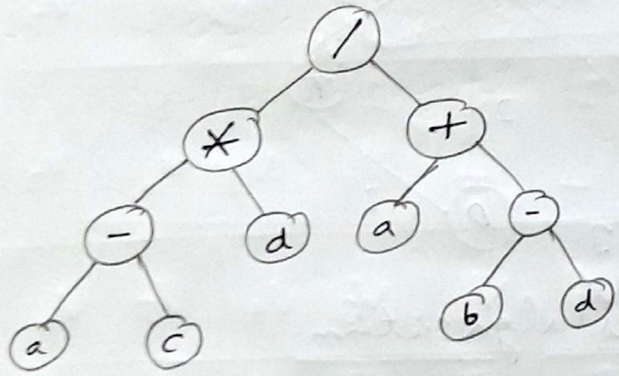


Therefore minimum weight is 17.

(4) Use a binary tree to represent the following expressions.

(i) $((a-c)*d)/(a+(b-d))$

Ans:- Representation of the expression $((a-c)*d)/(a+(b-d))$ as a binary tree then:-



(ii) Draw the binary tree when inorder and postorder traversals is given below:

Inorder	m	k	n	j	o	l	u	s	v	q	t	p	r
Postorder	m	n	k	o	u	v	s	t	q	r	p	l	j

Solⁿ:- We know that the last node in post-order is the root node hence J is the root.