

$$\text{tr}(B) = \text{tr}(A(P^{-1}P)) \quad [\because P^{-1}P = I_n]$$

$$\boxed{\text{tr}(B) = \text{tr}(A)}$$

$$(iii) \underline{\text{Rank}(A) = \text{Rank}(B)}$$

Since it is given that both the matrices are similar therefore, the dimensions are also to be same $n \times n$, $\dim A = \dim B$.

~~The for~~

Therefore the base variables are equal also.

Thus the number of free variables are equal.

If no of free variables are same.

$$\text{Nullity}(B) = \dim N(B) = K = \dim N(A) = \text{Nullity}(A)$$

Thus both will have same number of base variables as pivot elements.

$$\therefore \text{The Rank}(A) = N - \text{Nullity}(A) = N - \text{Nullity}(B) = \text{Rank}(B).$$

$$\boxed{\therefore \text{Rank}(A) = \text{Rank}(B)}$$

Hence Proved