DIGITAL ASSIGNMENT I

MAT 2002

APPLICATION OF DIFFERENTIAL EQUATIONS

F1 + TF1

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① Diagonalize $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and hence find A^4.

Ans: The eigen values are given by

$$|A - \lambda I| = \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow (8-\lambda)\{(7-\lambda)(3-\lambda)-16\} + 6(-18+6\lambda+8) + 2(24-14+2\lambda) = 0$

$\Rightarrow (8-\lambda)(21-10\lambda+\lambda^2-16) + 36\lambda - 60 + 4\lambda + 20 = 0$

$\Rightarrow (8-\lambda)(\lambda^2-10\lambda+5) + 40\lambda - 40 = 0$

$\Rightarrow 8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 40 - 40 = 0$

$\Rightarrow -\lambda^3 + 18\lambda^2 - 45\lambda = 0$

$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\Rightarrow \lambda(\lambda^2 - 18\lambda + 45) = 0$

$\Rightarrow \lambda(\lambda-3)(\lambda-15) = 0$

$\lambda = 0, 3, 15$ are the eigen values.

Eigen vectors are given by

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$$(8-\lambda)x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + (7-\lambda)x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 + (3-\lambda)x_3 = 0$$

when $\lambda = 0$, we get

$$8x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + 7x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 + 3x_3 = 0$$

From the first two equation,

$$\frac{x_1}{24-14} = \frac{x_2}{-12+32} = \frac{x_3}{56-36}$$

$$\Rightarrow \frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20} \Rightarrow \frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$

The eigen vector is $X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

when $\lambda = 3$, we get

$$5x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 + 4x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 + 0x_3 = 0$$

From the first two equations

$$\frac{x_1}{24-8} = \frac{x_2}{-12+20} = \frac{x_3}{20-36}$$

$$\Rightarrow \frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16} \Rightarrow \frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

The giver vector is $X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

when $\lambda = 15$ we get,

$$-7x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 - 8x_2 - 4x_3 = 0$$
$$2x_1 - 4x_2 + 12x_3 = 0$$

From the first two equations

$$\frac{x_1}{24+16} = \frac{x_2}{-12-28} = \frac{x_3}{56-36}$$

$$\Rightarrow \frac{x_1}{40} = \frac{x_2}{-40} = \frac{x_3}{20} \Rightarrow \frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

The given vector is $X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

Let $B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & -2 \\ 2 & 1 & 1 \end{bmatrix}$ be the modal matrix.

To find B^{-1}

$$|B| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = 1(1-4) - 2(2+4) + 2(-4-2)$$
$$= -3 - 12 - 12 = -27$$

Cofactors and minors of the B matrix =

$$\begin{bmatrix} \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{bmatrix}$$

$$\text{Adj } B = \begin{bmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj } B}{|B|} = \frac{-1}{27} \begin{bmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$B^{-1}AB = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix} = D$$

To find A^4,

$$B^{-1}AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix} = D$$

$$(B^{-1}AB)^4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3^4 & 0 \\ 0 & 0 & 15^4 \end{bmatrix} = D^4$$

$$\Rightarrow B^{-1}A^4B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 50625 \end{bmatrix}$$

$$\therefore A^4 = B \begin{bmatrix} 0 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 50625 \end{bmatrix} B^{-1}$$

$$\Rightarrow A^4 = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 50625 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 162 & 81 & -162 \\ 101250 & -101250 & 50625 \end{bmatrix}$$

$$\Rightarrow A^4 = \frac{1}{9} \begin{bmatrix} 324+202500 & 162-202500 & -324+101250 \\ 162-202500 & 81+202500 & -162-101250 \\ -324+101250 & -162-101250 & 324+50625 \end{bmatrix}$$

$$\Rightarrow A^4 = \frac{1}{9} \begin{bmatrix} 202824 & -202338 & 100926 \\ -202338 & 202581 & -101412 \\ 100926 & -101412 & 50949 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 22536 & -22482 & 11214 \\ -22482 & 22509 & -11268 \\ 11214 & -11268 & 5661 \end{bmatrix}$$

(Ans).

$$\text{and } D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix} \text{ (Ans).}$$

② Identify the nature, order and signature of the given quadratic form (Q). Also write the canonical form of the same.

$$Q = -3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

Ans: The matrix of the quadratic form is

$$Q = \begin{bmatrix} -3 & -1 & -1 \\ -1 & -3 & 1 \\ -1 & 1 & -3 \end{bmatrix}$$

The eigen values are given by $|Q - \lambda I| = 0$

$$\therefore \begin{vmatrix} -3-\lambda & -1 & -1 \\ -1 & -3-\lambda & 1 \\ -1 & 1 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-3-\lambda)\{(-3-\lambda)(-3-\lambda) - 1\} - (-1)\{(-1)(-3-\lambda) - (-1)\}$$
$$+ (-1)\{(-1)(1) - (-3-\lambda)(-1)\} = 0$$

$$\Rightarrow (-3-\lambda)\{\lambda^2 + 6\lambda + 9 - 1\} + \{3 + \lambda + 1\}$$
$$- \{-1 - 3 - \lambda\} = 0$$

$$\Rightarrow (-3-\lambda)\{\lambda^2 + 6\lambda + 8\} + \{\lambda + 4\} - \{-\lambda - 4\} = 0$$

$$\Rightarrow \{-3\lambda^2 - 18\lambda - 24 - \lambda^3 - 6\lambda^2 - 8\lambda\} + 2\lambda + 8 = 0$$

$$\Rightarrow -\lambda^3 - 9\lambda^2 - 24\lambda - 16 = 0$$

$$\Rightarrow \lambda^3 + 9\lambda^2 + 24\lambda + 16 = 0$$

$$\Rightarrow (\lambda + 1)(\lambda + 4)^2 = 0 \Rightarrow \lambda = -1, -4, -4$$ are the eigen values.

Eigen vectors are given by :-

$$(-3-\lambda)x_1 + (-1)x_2 + (-1)x_3 = 0$$
$$(-1)x_1 + (-3-\lambda)x_2 + x_3 = 0$$
$$(-1)x_1 + x_2 + (-3-\lambda)x_3 = 0$$

when $\lambda = -1$, we get :-

$$-2x_1 - x_2 - x_3 = 0$$
$$-x_1 - 2x_2 + x_3 = 0$$
$$-x_1 + x_2 - 2x_3 = 0$$

From the first two equations

$$\frac{x_1}{(-1)-(2)} = \frac{-x_2}{(-2)-(1)} = \frac{x_3}{4-1}$$

$$\Rightarrow \frac{x_1}{-3} = \frac{x_2}{3} = \frac{x_3}{3}$$

Hence the eigen vector is $X_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

when $\lambda = -4$, we get :-

$$x_1 - x_2 - x_3 = 0$$
$$-x_1 + x_2 + x_3 = 0$$
$$-x_1 + x_2 + x_3 = 0$$

Putting $x_2 = 0$

$\therefore x_1 = x_3$

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore x_1 + 0x_2 + x_3 = 0$

$x_1 - x_2 - x_3 = 0$

$$X_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\frac{X_1}{1} = \frac{-X_2}{-2} = \frac{X_3}{-1}$$

Normalized Model Matrix $N = \begin{bmatrix} -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0/\sqrt{2} & \frac{1}{\sqrt{6}} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$

Now,

$$N^TAN = \begin{bmatrix} -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} -3 & -1 & -1 \\ -1 & -3 & 1 \\ -1 & 1 & -3 \end{bmatrix} \begin{bmatrix} -1/\sqrt{3} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = D$$

So canonical form,

$$y^TDy = (y_1\ y_2\ y_3) \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= -y_1^2 - 4y_2^2 - 4y_3^2 \quad \text{(Ans)}$$

Nature : Negative definite

Index : 0

Signature : $0 - 3 = -3$

③ A periodic function $f(t)$ of period 2 is defined by $f(t) = \begin{cases} 3t, & 0 < t < 1 \\ 3, & 1 < t < 2 \end{cases}$. Determine a Fourier Series expansion for the function.

Ans: According Fourier series :-

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{l} + b_n \sin\frac{n\pi t}{l}.$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(t)\,dt = \frac{2}{2}\int_0^2 f(t)\,dt = \int_1^2 3\,dt + \int_0^1 3t\,dt$$

$$= 3[t]_1^2 + 3\left[\frac{t^2}{2}\right]_0^1 = 3[2-1] + 3\left[\frac{1}{2} - 0\right]$$

$$= 3 + \frac{3}{2} = \frac{9}{2}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(t)\cos\frac{n\pi t}{l}\,dt = \int_0^2 f(t)\cos n\pi t\,dt$$

$$= \int_1^2 3\cos n\pi t\,dt + \int_0^1 3t\cos n\pi t\,dt$$

$$= 3\left[\frac{\sin n\pi t}{n\pi}\right]_1^2 + 3\left[t\frac{\sin n\pi t}{n\pi} - \frac{\cos n\pi t}{(-n\pi)(n\pi)}\right]_0^1$$

$$= 0 + 3\left[0 + \frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right]$$

$$= 3\left[\frac{(-1)^n - 1}{n^2\pi^2}\right] \quad \therefore a_n = \begin{cases} \frac{-6}{n^2\pi^2}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(t)\sin n\pi t\,dt = \frac{2}{2}\int_0^2 f(t)\sin n\pi t\,dt$$

$$= \int_1^2 3\sin n\pi t\,dt + \int_0^1 3t\sin n\pi t\,dt$$

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$$= 3\left[\frac{\cos n\pi t}{(-n\pi)}\right]_1^2 + 3\left[t\,\frac{\cos n\pi t}{(-n\pi)} + \frac{\sin n\pi t}{n^2\pi^2}\right]_0^1$$

$$= 3\left[\frac{1}{(-n\pi)} - \frac{(-1)^n}{(-n\pi)}\right] + 3\left[\left(\frac{(-1)^n}{(-n\pi)} + 0\right) - (0+0)\right]$$

$$= \frac{-3}{n\pi} - \frac{6(-1)^n}{n\pi} \qquad \therefore b_n = \begin{cases} \frac{3}{n\pi} & ; n \to \text{odd} \\ \frac{-9}{n\pi} & ; n \to \text{even} \end{cases}$$

$$\therefore f(t) = \frac{9}{4} + \sum_{n=1,3,5}^{\infty} \frac{-6}{n^2\pi^2}\cos n x + \sum_{n=1,3,5\ldots}^{\infty} \frac{3}{n\pi}\sin n x$$

$$~~(Ans)~~ + \sum_{n=2,4,6\ldots}^{\infty}\left(\frac{-9}{n\pi}\right)\sin n x \quad \text{(Ans)}.$$

(4) Find the Fourier series to represent $|\cos x|$ in the interval $(-\pi, \pi)$.

Ans: As $f(-x) = |\cos(-x)| = |\cos x| = f(x)$,

$|\cos x|$ is an even function. $\therefore b_n = 0$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} |\cos x|\,dx = \frac{2}{\pi}\int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,dx$$

[as $\cos x$ is negative when $\pi/2 < x < \pi$]

$$= \frac{2}{\pi}\left\{ \left|\sin x\right|_0^{\pi/2} - \left|\sin x\right|_{\pi/2}^{\pi} \right\}$$

$$= \frac{2}{\pi}\left[(1-0) - (0-1)\right] = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} |\cos x| \cos nx\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos nx\,dx + \int_{\pi/2}^{\pi} (-\cos x)(\cos nx)\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2} [\cos(n+1)x + \cos(n-1)x]\,dx - \int_{\pi/2}^{\pi} [\cos(n+1)x + \cos(n-1)x]\,dx\right]$$

$$= \frac{1}{\pi}\left[\left|\frac{\sin(n+1)x}{(n+1)} + \frac{\sin(n-1)x}{(n-1)}\right|_0^{\pi/2} - \left|\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right|_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{\sin(n+1)\pi/2}{(n+1)} + \frac{\sin(n-1)\pi/2}{(n-1)}\right) + \left(\frac{\sin(n+1)\pi/2}{(n+1)} + \frac{\sin(n-1)\pi/2}{(n-1)}\right)\right]$$

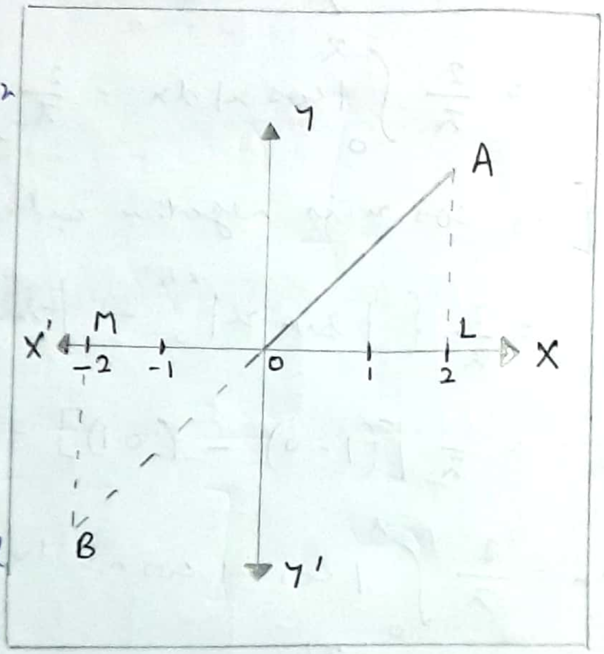
$$= \frac{2}{\pi}\left[\frac{\cos n\pi/2}{n+1} - \frac{\cos n\pi/2}{n-1}\right] = -\frac{4\cos n\pi/2}{\pi(n^2-1)} \quad (n \neq 1)$$

In particular $a_1 = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos^2 x\, dx - \int_{\pi/2}^{\pi} \cos^2 x \, dx\right] =$

Hence $|\cos x| = \frac{2}{\pi} + \frac{4}{\pi}\left\{\frac{1}{3}\cos 2x - \frac{1}{15}\cos 4x + \right.$

(Ans)

(5) Express $f(x) = x$ as a half range sine series in $0 < x < 2$.



Ans: The graph of $f(x) = x$ in $0 < x < 2$ is the line OA. Let us extend the function $f(x)$ in the interval $-2 < x < 0$ (shown by the line BO) so that the new function is symmetrical about the origin and, therefore, represents an odd function in $(-2, 2)$.

Hence the Fourier series for $f(x)$ over the full period $(-2, 2)$ will contain only sine terms given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}$$

where $b_n = \frac{2}{2}\int_0^2 f(x) \sin \frac{n\pi x}{2}\, dx = \int_0^2 x \sin \frac{n\pi x}{2} \, dx$

$$= \left| \frac{-2x}{n\pi} \cos \frac{n\pi x}{2} + \frac{4}{n^2\pi^2} \sin \frac{n\pi x}{2} \right|_0^2$$

$$= -\frac{4(-1)^n}{n\pi}$$

Hence the Fourier sin series for $f(x)$ over the half-range $(0,2)$ is

$$f(x) = \frac{4}{\pi}\left(\sin\frac{\pi x}{2} - \frac{1}{2}\sin\frac{2\pi x}{3} + \frac{1}{3}\sin\frac{3\pi x}{2} - \frac{1}{4}\sin\frac{4\pi x}{2} + \cdots\right) \text{ (Ans).}$$

⑥ Obtain a half range cosine series for

$$f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l. \end{cases}$$ Hence deduce

the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty$.

Ans: Let the half-range cosine series be

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

Then, $$a_0 = \frac{2}{l}\left\{\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\right\}dx$$

$$a_0 = \frac{2k}{l}\left\{\left|\frac{x^2}{2}\right|_0^{l/2} - \left|\frac{(l-x)^2}{2}\right|_{l/2}^{l}\right\}$$

$$= \frac{2k}{l}\cdot\frac{1}{2}\left\{\frac{l^2}{4} - \left(0 - \frac{l^2}{4}\right)\right\} = \frac{kl}{2}$$

$$a_n = \frac{2}{l}\left\{\int_0^{l/2} kx\cos\frac{n\pi x}{l}\,dx + \int_{l/2}^{l} k(l-x)\cos\frac{n\pi x}{l}\,dx\right\}$$

$$= \frac{2k}{l}\left| x\left(\frac{\sin n\pi x/l}{n\pi/l}\right) - 1\left(-\cos\frac{n\pi x/l}{(n\pi/l)^2}\right)\right|_0^{l/2}$$

$$+ \frac{2k}{l}\left|\left\{\frac{(l-x)\sin n\pi x/l}{n\pi/l} - (-1)\left(\frac{-\cos n\pi x/l}{(n\pi/l)^2}\right)\right|_{l/2}^{l}\right.$$

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$$= \frac{2k}{l}\left[\left(\frac{l^2}{2n\pi}\sin\frac{n\pi}{2}\right) + \frac{l^2}{n^2\pi^2}\left(\cos\frac{n\pi}{2} - \cos 0\right)\right]$$ (2

$$+ \frac{2k}{l}\left[\frac{l}{n\pi}\left(-\frac{l}{2}\sin\frac{n\pi}{2}\right) - \frac{l^2}{n^2\pi^2}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)\right]$$

$$= \frac{2k}{l}\cdot\frac{l^2}{n^2\pi^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]$$

$$= \frac{2kl}{n^2\pi^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]$$

Hence the required Fourier series is :-

$$f(x) = \frac{kl}{4} - \frac{8kl}{\pi^2}\left[\frac{1}{2^2}\cos\frac{2\pi x}{l} + \frac{1}{6^2}\cos\frac{6\pi x}{l} + \frac{1}{10^2}\cos\frac{10\pi x}{l} + \cdots\right] \text{ (Ans)}$$

Putting $x = l$, we get :-

$$0 = \frac{kl}{4} - \frac{8kl}{\pi^2}\left(\frac{1}{2^2} + \frac{1}{6^2} + \frac{1}{10^2} + \cdots \infty\right)$$

Thus, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$.

(Ans)