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16BCE0789

ALA - C2

① Find the LDU factorization of the following matrix.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \quad \& \quad b = \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$$

Also solve the given system of equations.

Ans - $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 4 & -3 & 2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 - R_1 \\ R_3 \end{matrix}$ $\quad A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 1 & -2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 - 4R_1 \end{matrix}$

$U = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & +2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_2 - 2R_3 \end{matrix}$ $\quad \bar{U} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1 \\ R_2/2 \\ R_3/2 \end{matrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$ $\quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & [illegible] & -2 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3/(-2) \end{matrix}$

$\therefore E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ $\quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & [illegible] \end{bmatrix}$

$$\therefore L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 1/2 & 1 \end{bmatrix}$$

$$A = LDU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \text{(Ans)}$$

$\boxed{Ly = b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$$

$\boxed{\therefore y_1 = 5}$

$y_1 + y_2 = -1$

$\Rightarrow 5 + y_2 = -1$

$\Rightarrow \boxed{y_2 = -6}$

$4y_1 + \frac{y_2}{2} + y_3 = 3$

$\Rightarrow 4 \times 5 + (-6)/2 + y_3 = 3 \Rightarrow 20 - 3 + y_3 = 3$

$\Rightarrow \boxed{y_3 = -14}$

$$y = Ux \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \\ -14 \end{bmatrix}$$

$\therefore 2x_3 = -14 \Rightarrow \boxed{x_3 = -7}$

$2x_2 - 2x_3 = -6 \Rightarrow x_2 - x_3 = -3$

$\Rightarrow x_2 - (-7) = -3 \Rightarrow x_2 = -3 - 7 = -10 \Rightarrow \boxed{x_2 = -10}$

$x_1 - x_2 + x_3 = 5 \Rightarrow x_1 - (-10) + (-7) = 5$

$\Rightarrow x_1 + 10 - 7 = 5 \Rightarrow \boxed{x_1 = 2}$

$(x_1, x_2, x_3) = (2, -10, -7)$ (Ans)

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(2) Find the LDU factorization $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix}$

Ans – $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix}$ $A = \begin{array}{c} \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 4.5 & -10.5 & -6 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 - 2.5(R_1) \end{array}$

$U = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & (-19.5) & -1.5 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 - 4.5(R_2) \end{array}$ $U = \begin{bmatrix} 1 & -0.5 & 1.5 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 0.076 \end{bmatrix} \begin{array}{l} R_1/2 \\ R_2 \\ R_3/19.5 \end{array}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2.5 & 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4.5 & 1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2.5 & 0 & 1 \end{bmatrix}$

$E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4.5 & 1 \end{bmatrix}$ $L = E_1^{-1} E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4.5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2.5 & 4.5 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -19.5 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2.5 & 4.5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -19.5 \end{bmatrix} \begin{bmatrix} 1 & -0.5 & 1.5 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 0.076 \end{bmatrix}$

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$$w + x + y = 3$$
$$-3w - 17x + y + 2z = 1$$
$$4w - 17x + 8y - 5z = 1$$
$$-5x - 2y + z = 1$$

(5) $x + [illegible]y - [illegible]z = b_1$, $[illegible]x - [illegible]y + [illegible]z = b_2$, $[illegible]x + [illegible]y + z = b_3$

Augmented matrix $= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ -3 & -17 & 1 & 2 & 1 \\ 4 & -17 & 8 & -5 & 1 \\ 0 & -5 & -2 & 1 & 1 \end{bmatrix}$

$R_2 \leftrightarrow R_4$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -5 & -2 & 1 & 1 \\ 4 & -17 & 8 & -5 & 1 \\ -3 & -17 & 1 & 2 & 1 \end{bmatrix}$

$R_3 \to 4R_3 - (-8)R_1$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -5 & -2 & 1 & 1 \\ 0 & -17 & 28 & -7 & 7 \\ -3 & -17 & 1 & 2 & 1 \end{bmatrix}$

$R_4 \to 3R_1 + R_4$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -5 & -2 & 1 & 1 \\ 0 & -17 & 28 & -7 & 7 \\ 0 & -14 & 4 & 2 & 10 \end{bmatrix}$

$R_3 \to (-17)R_2 - (-5)R_3$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & -5 & -2 & 1 & 1 \\ 0 & 0 & 174 & -52 & 18 \\ 0 & -14 & 4 & 2 & 10 \end{bmatrix}$

$R_4 \to (-14)R_2 - (-5)R_4$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & (-5) & (-2) & 1 & 1 \\ 0 & 0 & 174 & (-52) & 18 \\ 0 & 0 & 48 & (-4) & 36 \end{bmatrix}$

$R_4 \to R_4/4$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & (-5) & (-2) & 1 & 1 \\ 0 & 0 & 174 & (-52) & 18 \\ 0 & 0 & 12 & -1 & 9 \end{bmatrix}$

$R_3 \to R_3/2$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & (-5) & (-2) & 1 & 1 \\ 0 & 0 & 87 & (-26) & 9 \\ 0 & 0 & 12 & -1 & 9 \end{bmatrix}$

$R_4 \to 12(R_3) - 87(R_4)$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 0 & (-5) & (-2) & 1 & 1 \\ 0 & 0 & 87 & (-26) & 9 \\ 0 & 0 & 0 & (-225) & (-675) \end{bmatrix}$

$\therefore (-225)z = (-675) \Rightarrow \boxed{z = 3}$

$\therefore (87)y + (-26)z = 9 \Rightarrow y(87) - 78 = 9 \Rightarrow \boxed{y = 1}$

$\therefore (-5)x + (-2)y + (1)z = 1 \Rightarrow (-5)x - 2 + 3 = 1 \Rightarrow x =$

$\therefore (1)w + (1)x + (1)y + (0)z = 3 \Rightarrow w + 0 + 1 + 0 = 3 \Rightarrow \boxed{w = 2}$

Answer: $w = 2,\ x = 0,\ y = 1,\ z = 3$.

(2) Augmented matrix $= \begin{bmatrix} 1 & 3 & -2 & b_1 \\ 2 & -1 & 3 & b_2 \\ 4 & 2 & 1 & b_3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 4R_1$

$\Rightarrow \begin{bmatrix} 1 & 3 & -2 & b_1 \\ 0 & -7 & 7 & b_2 - 2b_1 \\ 0 & -10 & 9 & b_3 - 4b_1 \end{bmatrix}$

$R_3 \to (-10)R_2 - (-7)R_3$

$= \begin{bmatrix} 1 & 3 & -2 & b_1 \\ 0 & -7 & 7 & b_2 - 2b_1 \\ 0 & 0 & -7 & (-10)(b_2 - 2b_1) \\ & & & +7(b_3 - 4b_1) \end{bmatrix}$

$\therefore (-7)z = 20b_1 + 10b_2 + 7b_3 - 28b_1$

$\Rightarrow (-7)z = -8b_1 - 10b_2 + 7b_3$

$\Rightarrow z = \dfrac{-8b_1 - 10b_2 + 7b_3}{(-7)} = \dfrac{8b_1 + 10b_2 - 7b_3}{7}$ (Ans)

$\therefore (-7)y + (7)z = b_2 - 2b_1$

$\Rightarrow (-7)y + 8b_1 + 10b_2 - 7b_3 = b_2 - 2b_1$

$\Rightarrow (-7)y = -10b_1 - 9b_2 + 7b_3$

$\Rightarrow y = \dfrac{10b_1 + 9b_2 - 7b_3}{7}$ (Ans)

$\therefore (1)x + (3)y + (-2)z = b_1$

$\Rightarrow x + 3\left(\dfrac{10b_1 + 9b_2 - 7b_3}{7}\right) - 2\left(\dfrac{8b_1 + 10b_2 - 7b_3}{7}\right)$

$\Rightarrow x = \dfrac{7b_1 - 3(10b_1 + 9b_2 - 7b_3) + 2(8b_1 + 10b_2 - 7b_3)}{7}$

$\Rightarrow x = \dfrac{7b_1 - 30b_1 - 27b_2 + 21b_3 + 16b_1 + 20b_2 - 14b_3}{7}$

$\Rightarrow x = \dfrac{-7b_1 - 7b_2 + 7b_3}{7} = -b_1 - b_2 + b_3$ (Ans)

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Q1) Let $V_1 = \langle 1\ 1\ 2\ 4 \rangle$, $V_2 = \langle 2\ -1\ -5\ 2 \rangle$

$V_3 = \langle 1\ -1\ -4\ 0 \rangle$, $V_4 = \langle 2\ 1\ 1\ 6 \rangle$

Verify if the vector forms a basis.

Q2) Is a vector $\langle 3\ -1\ 0\ -1 \rangle \in$ subspace of R^4 spanned by the vectors

$V_1 = \langle 2\ -1\ 3\ 2 \rangle$ $V_2 = \langle -1\ 1\ 1\ -3 \rangle$

$V_3 = \langle 1\ 1\ 9\ -5 \rangle$.

Q3) Let $\alpha = V_1\ V_2 \dots V_n$ be a basis for a vector space V then each vector except $x \in V$ can be uniquely explained as linear combination of $V_1, V_2 \dots V_n$. ~~Let V be a vector space~~.

Q4) Let V be a vector space $V_1\ V_2 \dots V_n$ be a set of n ~~bas~~ vectors in V. If α spans V then prove that every set of vectors with more than n vectors cannot be linearly independent.

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Ans 1)

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & -5 & 2 \\ 1 & -1 & 4 & 0 \\ 2 & 1 & 1 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 9 & 6 \\ 1 & -1 & 4 & 0 \\ 2 & 1 & 1 & 6 \end{bmatrix} (2R_1 - R_2) = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 9 & 6 \\ 0 & 2 & -1 & 4 \\ 2 & 1 & 1 & 6 \end{bmatrix} (R_1 -$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 9 & 6 \\ 0 & 2 & -7 & 4 \\ 0 & 1 & 3 & 2 \end{bmatrix} (2R_1 - R_4) \quad \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & 3 & 2 \end{bmatrix} \begin{matrix} \\ (R_2 \\ (R_3 \\ \\ \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix} (R_2 - R_3) \quad \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} (R_2$$

Since we got the ~~f~~ last row as all zeros. Therefore the condition of linearly independency is not satisfied. Therefore the given vectors do form a span.

Ans) Spanned element: $\langle 3\ -1\ 0\ -1 \rangle$

Vectors given:

$v_1 = \langle 2\ -1\ 3\ -2 \rangle$

$v_2 = \langle -1\ 1\ 1\ -3 \rangle$

$v_3 = \langle 1\ 1\ 9\ -5 \rangle$

$\therefore$ Let the scalar coefficients be :- a_1, a_2, a_3

$\therefore$ In order to satisfy span:-

$\langle 3\ -1\ 0\ -1 \rangle = a_1 \langle 2\ -1\ 3\ -2 \rangle +$

$a_2 \langle -1\ 1\ 1\ 3 \rangle + a_3 \langle 1\ 1\ 9\ -5 \rangle$

$\Rightarrow \langle 3\ -1\ 0\ -1 \rangle = \langle 2a_1\ -a_1\ 3a_1\ -2a_1 \rangle +$

$\langle -a_2\ a_2\ a_2\ -3a_2 \rangle + \langle a_3\ a_3\ 9a_3\ -5a_3 \rangle$

$\Rightarrow \langle 3\ -1\ 0\ -1 \rangle = \langle (2a_1 - a_2 + a_3)$

$(-a_1 + a_2 + a_3) \cdot (3a_1 + a_2 + 9a_3)$

$(-2a_1 - 3a_2 - 5a_3) \rangle$

$\therefore$ The equations are:-

① $\rightarrow 2a_1 - a_2 + a_3 = 3$

② $\rightarrow -a_1 + a_2 + a_3 = -1$

③ $\rightarrow 3a_1 + a_2 + 9a_3 = 0$

④ $\rightarrow -2a_1 - 3a_2 - 5a_3 = -1$

$a_1 + 2a_3 = 2$ ① + ②

$\cancel{5a_1 + 10a_3 = 3}$

$5a_1 + 2a_3 = 4$ ⑤

$3 \times ① + ④$

$5(2 - 2a_3) + 2a_3 = 4$

$10 - 10a_3 + 2a_3 = 4$

$\Rightarrow 6 = 8a_3 \Rightarrow \boxed{a_3 = \frac{3}{4}}$

$a_1 = 2 - 2a_3 = 2 - 2 \times \frac{3}{4} = 2 - \frac{3}{2} = \frac{1}{2}$

$\boxed{a_1 = \frac{1}{2}}$

$2a_1 - a_2 + a_3 = 3$

$\Rightarrow 2\left(\frac{1}{2}\right) - a_2 + \frac{3}{4} = 3$

$\Rightarrow 1 - a_2 + \frac{3}{4} = 3$

$\Rightarrow \frac{7}{4} - a_2 = 3$

$\Rightarrow a_2 = \frac{7}{4} - 3 \Rightarrow a_2 = \frac{7-12}{4} = \frac{-5}{4}$

$\therefore \boxed{a_1 = \frac{1}{2}} \quad \boxed{a_2 = \frac{-5}{4}} \quad \boxed{a_3 = \frac{3}{4}}$

$\therefore \langle 3 \ -1 \ 0 \ -1 \rangle = \frac{1}{2} \langle 2 \ -1 \ 3 \ -2 \rangle +$
$\left(-\frac{5}{4}\right) \langle -1 \ 1 \ 1 \ -3 \rangle + \left(\frac{3}{4}\right) \langle 1 \ 1 \ 9 \ -5 \rangle$

Hence the given vector can be spanned.

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Ans 3)

Given: $\alpha = v_1 v_2 \dots v_n \in V$ is basis

∴ (i) The V is linearly independent

(ii) The v can be spanned.

Let $u \in V$

$$u = c_1 v_1 + c_2 v_2 + c_3 v_3 + \dots + c_n v_n$$

$$u = k(c_1 v_1) + k(c_2 v_2) + k(c_3 + v_3) + \dots + k(c_n v_n)$$

If $x \notin V$

$(x_1, x_2) \notin V \qquad (u_1, 5u_2) \in V$

~~$(x_1, x_2) \; x_1 \; (u_1 - x_1) = (x_2 - k_1)$~~

$u_1 \in V \qquad u_2 \in V$

$(u_1 - u_2) \in V$ is a unique solution.

Ans 4) Let $\alpha \in V$ as it spans V.

$\therefore$ V has properties:-

(i) Linearly independent

(ii) basis is formed.

The Given ~~set~~ with n elements the spanning is possible.

$\therefore$ Let $n = 3$ & $W = \{v_1, v_2, v_3\}$

$$\therefore A = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & x_{23} \\ 0 & 0 & x_{31} \end{bmatrix}$$

Now this is linearly independent and also satisfies the condition of spanning where $n = 3$.

The moment we take $n = 4$ then:-

$$\therefore A = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & x_{23} \\ 0 & 0 & x_{31} \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \text{Non zeroes Absent.}$$

$\therefore$ The vector space contains linearly dependent elements.

$\therefore$ Spanning is not ~~to~~ possible as $c_1 = c_2 = c_3 \neq 0$ or constant.

$\therefore$ Basis cannot be formed.

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Hence we need vectors with more than n vectors if we need to produce linearly independent elements in vector space.