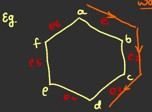
WALK , TRAIL, PATH

① **Walk**

A walk is a finite alternating sequence of vertices and edges, beginning and ending with same or diff. vertices.

Eg.



walk: $ae_1be_2ce_3d$

Starting from a — Ending at d

Length of walk

The number of edges is called as length of the walk

Closed and open walk

If its origin and terminus vertex $(V_0 = V_n)$ is equal then is called Closed walk, otherwise open walk — if diff vertices $(V_0 \neq V_n)$

TRAIL

Any walk having all different edges is called as train — or no repeated edges.

CIRCUIT

A closed trail is called as a circuit.

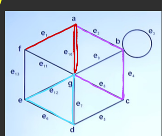
PATH

A walk is called path if all vertices are not repeated

CYCLE

A closed path is called as cycle

Eg.



$ae_1be_2ce_8g$ → Path ✓, Circuit ✗, Trail ✓, Cycle ✗

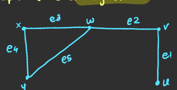
$ge_7de_6ee_{12}g$ → Path ✓, Cycle ✓, Trail ✓, Circuit ✓

$fe_1ae_{10}ge_9a$ → walk ✓, Path ✗ as a is repeated, Trail ✗ as e_{10} repeated

NOTE: This repetition is allowed #closed loop आहे so allowed आहे.

EULERIAN PATH

A path in a graph is said to be Eulerian Path iff it traverses each & every edge in graph once & only once



then $ue_1ve_2we_3xe_4ye_5w$ is Eulerian.

{NOTE: vertice repetition is accepted here.}

EULERIAN CIRCUIT

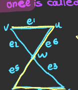
A circuit in a graph is said to be an Eulerian circuit if it traverses each edge in graph once & only once.

(for closed graph)

HAMILTONIAN PATH

A path which contain every vertex of a graph & exactly once is called Hamiltonian graph.

Eg.

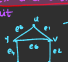


$ve_1ue_6we_5ye_4x$ — Hamiltonian path

HAMILTONIAN CIRCUIT

A circuit that passes through each of the vertices in a group G exactly once except the starting vertex and end vertex is called Hamiltonian circuit

Eg.1



$ue_1ve_2we_3xe_4ye_5u$ — Hamiltonian ckt

Eg.2 Above diag.

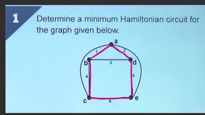
on joining this line vertex w repeats which is not allowed so, NOT Hamiltonian circuit.

Hamiltonian Graph:

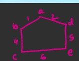
A connected graph containing Hamiltonian circuit is called as Hamiltonian graph.

✓ Hamiltonian graph

✗ Not Hamiltonian graph.

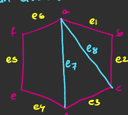


1 Determine a minimum Hamiltonian circuit for the graph given below.



⇒ $a2d5e6c4b1a$ minimum Hamiltonian ckt.

Q.2 Draw a graph with 6 vertices containing a Hamiltonian circuit but not Eulerian circuit.



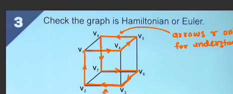
$ae_1be_2ce_3de_4ee_5fe_6a$ — Hamiltonian Ckt, #all vertices not repeated

but become Eulerian ckt also as #all edges not repeated

for Eulerian ckt, added e7 & e8 ←

$ae_1be_2ce_3de_4ee_5fe_6ae_7de_3ce_8a$ — Error: e_3 repeated

so now, its just Hamiltonian circuit & not Eulerian circuit

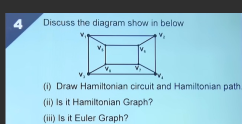


3 Check the graph is Hamiltonian or Euler.

arrows r only for understanding

Hamiltonian circuit in a graph ∴ Hamiltonian graph ✓

Cannot be Eulerian graph as some or other edge misses out / repeats



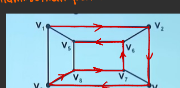
4 Discuss the diagram show in below

(i) Draw Hamiltonian circuit and Hamiltonian path.

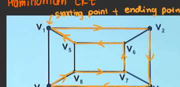
(ii) Is it Hamiltonian Graph?

(iii) Is it Euler Graph?

Hamiltonian path



Hamiltonian ckt — starting point + ending point



Hamiltonian ckt present in graph so ✓ Hamiltonian Graph

Eulerian Graph (trials) for Eulerian ckt.

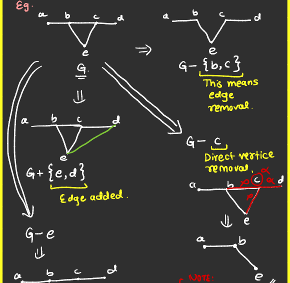
so not Eulerian Graph!

Operations on Graph

Removing vertex from Graph:

When we remove a vertex v and all edges incident/connected to it from Graph $\{G = (V, E)\}$ we create a subgraph, denoted by $G - v$ (minus)

Eg.



NOTE: we removed all edges associated with c also

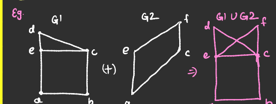
Graph Union

Two or more graphs can be combined in various ways. The new graph that contains all the vertices and edges of these graphs is called as union of graph.

The union of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$

The union of G_1 and G_2 is denoted by $G_1 \cup G_2$

Eg.



Cut Vertex & Cut Set

Connectivity:

A graph is said to be connected iff there is path between every pair of vertex.

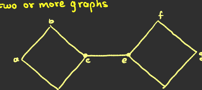
From one vertex to any other vertex, such that there should be atleast some path to traverse

Cut Vertex

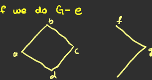
A vertex $V \in G$ is called as Cut-Vertex of G if $G - V$ (G minus V) results in a disconnected graph.

Removing a cut vertex from a graph breaks it into two or more graphs

Eg.

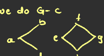


Here 'e' & 'c' are Cut vertices as if we do $G - e$



#disconnected

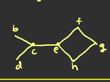
& if we do $G - c$



#disconnected.

if we remove any other vertices except c & e the graph is not gonna break / disconnect

Eg. $G - a$



#connected only!

Applications of Hamilton Circuits

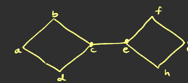
Hamilton paths and circuits can be used to solve practical problems. For example, many applications ask for a path or circuit that visits each road intersection in a city, each place pipelines intersect in a utility grid, or each node in a communications network exactly once. Finding a Hamilton path or circuit in the appropriate graph model can solve such problems. The famous **traveling salesperson problem** or **TSP** (also known in older literature as the **traveling salesman problem**) asks for the shortest route a traveling salesperson should take to visit a set of cities. This problem reduces to finding a Hamilton circuit in a complete graph such that the total weight of its edges is as small as possible. We will return to this question in

Cut-Edge (Bridge)

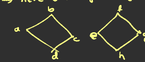
An edge $e \in G$ is called as a cut edge, if $G - e$ results into a disconnected graph.

If removing an edge results into two / more graph then that edge is called as a cut edge.

Eg.



⇒ here Cut edge is $\{(c, e)\}$



NOTE 'G' graph with 'n' vertices

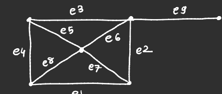
1. Cut edge e exist iff it is not a part of cycle!
2. maximum number of cut edges possible is 'n−1'
3. whenever cut edges exist, cut vertices also exists.
4. whenever cut vertices exist, cut edge may or may not exist.

Cut Set of a Graph

Let Graph be $G = (V, E)$ be a connected graph. A subset E' of E, is called cut-set of G if deletion of all edges given in E' set causes G to disconnect

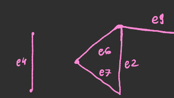
→ If deleting certain number of edges from a graph makes it disconnected, then those deleted edges are called as cut set (set of) of graph

Eg



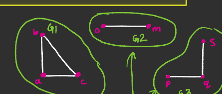
Cut set is $E_1 = \{e1, e3, e5, e8\}$

(#remove all these edges)



disconnected graph!

CONNECTED COMPONENTS.



These can be single graph also.

Suppose in Q" had it been given draw graph with 8 vertices & 6 edges so that

a — b
b — c
a — c
o — m
p — q
q — s

} then we would have drawn this graph 'G'

So, the graph G is the union of 3 disjoint connected subgraphs G1, G2 & G3

These subgraphs are called as Connected components of G.

CONNECTED COMPONENTS A **connected component** of a graph G is a connected subgraph of G that is not a proper subgraph of another connected subgraph of G. That is, a connected component of a graph G is a maximal connected subgraph of G. A graph G that is not connected has two or more connected components that are disjoint and have G as their union.

APPLICATIONS

APPLICATIONS OF PLANAR GRAPHS Planarity of graphs plays an important role in the design of electronic circuits. We can model a circuit with a graph by representing components of the circuit by vertices and connections between them by edges. We can print a circuit on a single board with no connections crossing if the graph representing the circuit is planar. When this graph is not planar, we must turn to more expensive options. For example, we can partition the vertices in the graph representing the circuit into planar subgraphs. We then construct the circuit using multiple layers. (See the preamble to Exercise 30 to learn about the thickness of a graph.) We can construct the circuit using insulated wires whenever connections cross. In this case, drawing the graph with the fewest possible crossings is important. (See the preamble to Exercise 26 to learn about the crossing number of a graph.)

The planarity of graphs is also useful in the design of road networks. Suppose we want to connect a group of cities by roads. We can model a road network connecting these cities using a simple graph with vertices representing the cities and edges representing the highways connecting them. We can built this road network without using underpasses or overpasses if the resulting graph is planar.