GRAPH REPRESENTATION

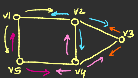
Matrix Representation:

① Adjacency Matrix

Let a_{ij} denote the number of edges (v_i, v_j) then $A = [a_{ij}]_{m \times m}$ is called adjacency matrix of G if

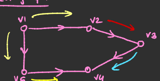
$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \text{ has an edge} \\ 0 & \text{otherwise} \end{cases}$$

Eg.



$$A = \begin{array}{c|ccccc} & v1 & v2 & v3 & v4 & v5 \\ \hline v1 & 0 & 1 & 0 & 0 & 1 \\ v2 & 1 & 0 & 1 & 1 & 0 \\ v3 & 0 & 1 & 0 & 1 & 0 \\ v4 & 0 & 1 & 1 & 0 & 1 \\ v5 & 1 & 0 & 0 & 1 & 0 \end{array}$$

Eg. Directed graph.



$$A = \begin{array}{c|ccccc} & v1 & v2 & v3 & v4 & v5 \\ \hline v1 & 0 & 1 & 0 & 0 & 1 \\ v2 & 0 & 0 & 1 & 0 & 0 \\ v3 & 0 & 0 & 0 & 1 & 0 \\ v4 & 0 & 0 & 0 & 0 & 0 \\ v5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

NOTE: for o═o or o≡o

2 edges, 3 edges

we write accordingly into the matrix

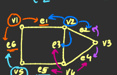
② Incidence Matrix

Let G be a graph with m vertices $v_1, v_2, v_3 \ldots v_m$ & n edges $e_1, e_2, e_3 \ldots e_n$

Let matrix $M = [m_{ij}]_{m \times n}$ defined by

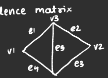
$$m \begin{cases} 1 & \text{if vertex } v_i \text{ is incident on } e_j \\ 0 & \text{if vertex } v_i \text{ is NOT incident on } e_j \\ 2 & \text{if vertex } v_i \text{ is self loop.} \end{cases}$$

Eg.



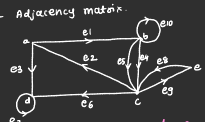
$$A = \begin{array}{c|cccccc} & e1 & e2 & e3 & e4 & e5 & e6 \\ \hline v1 & 1 & 0 & 0 & 0 & 0 & 1 \\ v2 & 1 & 1 & 1 & 0 & 0 & 0 \\ v3 & 0 & 1 & 0 & 1 & 0 & 0 \\ v4 & 0 & 0 & 1 & 1 & 1 & 0 \\ v5 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

Q.1 Incidence matrix



$$\rightarrow A = \begin{array}{c|ccccc} & e1 & e2 & e3 & e4 & e5 \\ \hline v1 & 1 & 0 & 0 & 1 & 0 \\ v2 & 0 & 1 & 1 & 0 & 0 \\ v3 & 1 & 1 & 0 & 0 & 1 \\ v4 & 0 & 0 & 1 & 1 & 1 \end{array}$$

Q.2 Adjacency matrix.

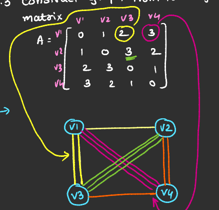


$$\rightarrow A = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 0 & 1 & 0 \\ b & 0 & 1 & 2 & 0 & 0 \\ c & 1 & 0 & 0 & 1 & 1 \\ d & 0 & 0 & 0 & 1 & 0 \\ e & 0 & 0 & 1 & 0 & 0 \end{array}$$

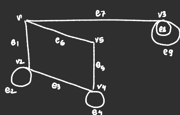
Q.3 Construct graph from full adjacency matrix

$$A = \begin{array}{c|cccc} & v1 & v2 & v3 & v4 \\ \hline v1 & 0 & 1 & 2 & 3 \\ v2 & 1 & 0 & 3 & 2 \\ v3 & 2 & 3 & 0 & 1 \\ v4 & 3 & 2 & 1 & 0 \end{array}$$

→



Q.4 Adjacency & incidence of.



$$A: \begin{array}{c|ccccc} & v1 & v2 & v3 & v4 & v5 \\ \hline v1 & 0 & 1 & 1 & 0 & 1 \\ v2 & 1 & 1 & 0 & 1 & 0 \\ v3 & 1 & 0 & 2 & 0 & 0 \\ v4 & 0 & 1 & 0 & 1 & 1 \\ v5 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$M = \begin{array}{c|ccccccccc} & e1 & e2 & e3 & e4 & e5 & e6 & e7 & e8 & e9 \\ \hline v1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ v2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ v3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 \\ v4 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ v5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array}$$

we count self loop as 2 in incidence matrix.

ISOMORPHISM & HOMEOMORPHISM

ISOMORPHISM

Two graphs G_1 and G_2 are said to be isomorphic if there is one to one correspondence between the edge set E_1 and E_2 in such a way that if e_1 is an edge with end vertices u_1 and v_1 (in G_1) then corresponding edge e_2 in G_2 has its end point vertices u_2 and v_2 (in G_2) which corresponds to u_1 and v_1

Another definition:

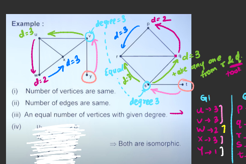
Two graphs are isomorphic if,

① No. of vertices are same

② No of edges are same

③ An equal number of vertices with given degree

④ Vertex correspondence & edge correspondence valid



Checking one one correspondence...

y — t } (∵ same degree & unique)

w — p

v — ? as there are 3 vertices of degree 3 ! in G2

so,

v — s

u — q

w — p

x — r

Vertex correspondence ✓

Edge correspondence,

if these 2 connected [y — t, w — p] ← then these should also be connected for

[v — s, u — q]

[w — p, x — r]

[: connected

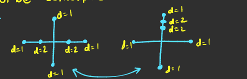
[: not connected

Homeomorphic Graph

Two graphs G and G' are said to be homeomorphic if they can be obtained from the same graph

NOTE: G & G' are homeomorphic, they need not be isomorphic.

Eg.



① no. of vertices = 6

② no of edges = 4 } Homeomorphic ✓

③ d=1 ×4 & d=2 ×2 both places.

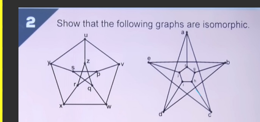
Both are Homeomorphic coz both obtained from

Conditions

→ 1) No. of vertices & edges should be same

2) Sequence of degree of vertices = same

3) Mapping of vertices not same.



1) No of vertices = 10
2) edges = 15
3) all have deg = 3

1) No of vert. = 10
2) edges = 15
3) all have deg = 3

4) mapping (vertices correspond.)

u — a
y — d
x — b
w — e
v — c
p — h

s — i
q — j
z — f
r — g

(p वरून s ला गेलो)
(h वरून कुठे जाऊ g वर की i वर?)
Ans: → s येतो p & y च्या मध्ये
we know p — h & y — d
तर toh vertice h & d च्या मध्ये पाहिजे, जो की "i" आहे !

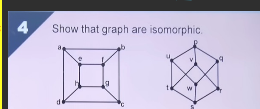
Edges corresp.

all are corresponding correctly.

Eg. u & y connected — a & d connected.

⋮

Hence Proved, they are Isomorphic graphs.



vert = 8
edges = 12
d = 3

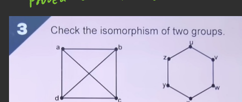
vert = 8
edges = 12
d = 3

start with

a — p
b — q
c — w
d — u
h — t (a & h च्या मध्ये म्हणजे)
e — v (p & t च्या मध्ये)
f — r
g — s

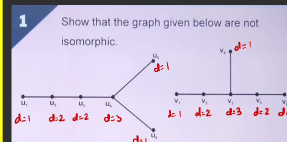
all connected, so,

Proved. Isomorphic graphs.



no of vert = 4 ≠ no of vert = 6

NOT Isomorphic



vert = 6 ⟷ same ⟷ vert = 6

edge = 5 ⟷ same ⟷ edge = 5

[d=3] ×1, [d=2] ×2, [d=1] ×3 ⟷ same ⟷ [d=3] ×1, [d=2] ×2, [d=1] ×3

Vertices correspondence.

Let,

u1 — v1 (d=1, d=1)

u2 — v2

u3 — v3

u2 वरून u3 ला गेलो u3 ची d=2 होती

पण v2 वरून v3 ला जातांना v3 ची d=3 आहे ! # ERROR

If we take

u1 — v6

u2 — v5

u3 — v3 ← same error

If we take

u1 — v4

u2 — v3 ← Error!

d=2, d=3

⇒ Hence, not Isomorphic