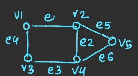
GRAPHS

I: BASICS

GRAPHS: definition

A graph G is mathematical structure consisting of two sets V and E where V is a non-empty set of vertices and E is non-empty set of edges

Eg.



BASIC TERMINOLOGIES

(1) Trivial graph

A graph consisting only one vertex and no edge

Eg. o v1

(2) NULL graph

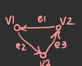
A graph consisting n vertices and NO EDGE is a null graph

Eg. o v1   o v2

(3) Directed Graph

A graph consisting direction of edges is called as directed graph

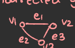
Eg.



(4) Undirected Graph

A graph which is not directed is undirected graph

Eg.



(5) Self loop in Graph

If edge having same vertex as both its end vertices is called as a self loop.
(जिथून सुरू तिथेच समाप्त)

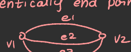
Eg.

(6) Proper edge

An edge which is not self loop is called proper edge.

(7) Multi edge.

Collection of two or more edges having identically end point



e1, e2, e3 are multiedges with same endpoints v1 & v2

(8) Simple Graph

A graph does not contain any self loop and multiedge is Simple graph.

Eg.

(9) Multigraph

A graph not containing any self loop but containing a multiedge (atleast one) is called as multigraph

Eg.

(10) Pseudo graph

A graph containing both multiedge and self loop is called pseudo graph.

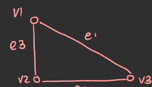
Eg.

(11) Incidence and Adjacency.

Let e be an edge joining two vertices v1 and v2, then e is said as Incidence of v1 & v2
and
v1 and v2 are said to be Adjacency of e

Two vertices are said to be adjacent if there exists an edge joining those vertices.

Eg.

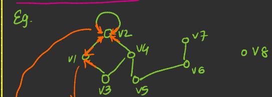


↳ e1 is incidence of v1 & v3
e2 is incidence of v2 & v3
{e1 is not incidence of v1 & v2 or v2 & v3}
↳ v1 & v2 is adjacency of e3

(12) Degree of vertex.

The degree of vertex V in a graph G {written as d(V)} is equal to the number of edges incident on V with self loop countered twice.

Eg.



d(v1) = 2
d(v2) = 4
d(v3) = 2
d(v4) = 3
d(v7) = 1
d(v8) = 0

(13) Isolated Vertex

A vertex having 0 degree is called as an isolated vertex

Eg. above
v8 is isolated vertex

Pendant Vertex

A vertex having 1 degree is called as Pendant vertex

Eg.
v7 is pendant vertex.

(14) Finite and Infinite Graphs.

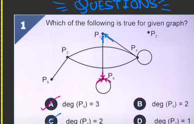
Graph with finite no. of vertices
Graph with infinite no. of vertices.

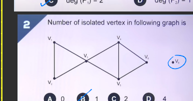
Eg.

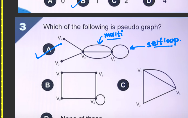
Finite

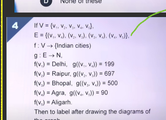
Infinite graph

= QUESTIONS =

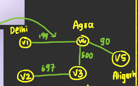








4
If V = {v1, v2, v3, v4, v5},
E = {(v1, v2), (v2, v3), (v3, v4), (v4, v5), (v2, v4)},
f : V → {Indian cities}
g : E → N,
f(v1) = Delhi, g((v1, v2)) = 199
f(v2) = Raipur, g((v2, v3)) = 697
f(v3) = Bhopal, g((v3, v4)) = 500
f(v4) = Agra, g((v4, v5)) = 90
f(v5) = Aligarh.
Then to label after drawing the diagrams of the graph.

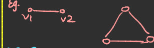


II: TYPES OF GRAPH

(1) Complete Graph

A simple connected graph is said to be complete if each vertex is connected to every other vertex (basically no open chain)

Eg.



(2) Regular Graph.

A graph is said to be regular if every vertex has the same degree. If degree of each vertex is k, then Graph G is called as k-regular graph.

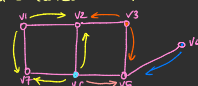
Eg.

2 regular graph
Degree of each vertices = 2
4 regular graph

(3) Bigraph (or Bipartite)

If the vertex set V of graph G can be "partitioned" into two non-empty disjoint subsets X and Y, in such a way that edge of G has one end in X and other end in Y, Then G is called as Bipartite/Bigraph

Eg.



here V = {v1, v2, v3, v4, v5, v6, v7}

X = {v1, v6, v3, v4}

Y = {v2, v7, v5}

CONNECTED GRAPH

An undirected graph is said to be connected if there is a path between every two vertices.

Eg.

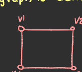
NOTE: If graph is connected, then it CANNOT be BIPARTITE!

(4) Complete Bipartite Graph

If every vertex in X is disjoint

If X and Y condition m & n vertices, then this graph is denoted as K_{mn}

Eg.



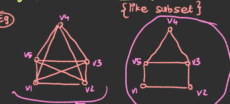
X = {v1, v3}
Y = {v2, v4}

each element in X is connected to every element in Y ⇒ Complete Bipartite

(5) Subgraph

Let G(V, E) be a graph, let V' be a subset of V and let E' be a subset of E whose end point belong to V'. Then G(V', E') is graph called as subgraph of G(V, E)
{like subset}

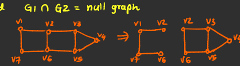
Eg.



DECOMPOSITION OF GRAPH

A graph is said to be decomposed into two subgraphs G_1 and G2 if G1 ∪ G2 = G and G1 ∩ G2 = null graph

Eg.

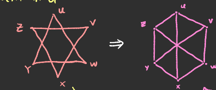


= Union
Intersection
=
common
=
[o v2, o v6] → # NULL GRAPH

COMPLEMENT OF GRAPH

The complement of a graph G is defined as a simple graph with the same vertex set as G and where any two vertices (u & v) are adjacent only when they are not adjacent in G

Eg.

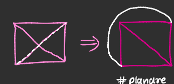


→ Jithe lines astil te kadhun taka
& Jithe lines nastil tithe lines draw kara
⇒ # complement of graph

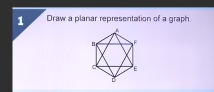
PLANARE GRAPH

A graph which can be drawn in the plane, so that none of its edge do not cross each other is called as planare graph

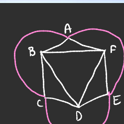
Eg.



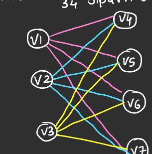
planare graph



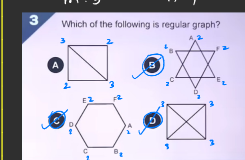
⇒

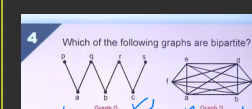


Q.2 Draw K_{34} bipartite graph



m : 3   n : 4





Bipartite ✓

as connected graphs are not Bipartite!

HANDSHAKING THEOREM

The sum of degree of vertices of graph G is equal to twice the total number of edges in G

Proof:

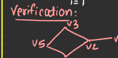
Consider graph G with 'e' edges & 'n' vertices, $V_1, V_2, V_3 \ldots V_n$

Each edge contribute 2 degree in G

∴ sum of degree of all vertices

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

Verification:



deg(v1) = 1
deg(v2) = 3
deg(v3) = 2
deg(v4) = 2
deg(v5) = 2

$\sum_{i=1}^{4} \deg(v_i) = 10$

e: no. of edges = 5

∴ Hence Verified!