**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans :- We have a normal distribution with = 45 and = 8. Let X be the amount of time it takes to complete the repair on customers car. To finish in one hour you must have X 50 so the question is to find P ( X > 50 ).

P ( X > 50 ) = 1 - P ( X 50 )

Z = ( X - )/ = ( X – 45 )/ 8

Thus the question can be answered by using the normal table to find

P ( X 50 ) = P ( Z ( 50 – 45 )/8 )

= P ( Z 0.625 )

= 73.24 %

Therefore, probability that the service manager will not meet his demand will be = 100 - 73.24 = 26.76% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans :- We have a normal distribution with = 38 and = 6. Let be the number of employees.

a) Probability of employees greater than age of 44 = P ( X > 44 )

P ( X > 44 ) = 1 – P ( X 44) = 1-0.8413 = 0.1587

Probability that employee will be greater than age of 44 = 100- 84.1345 = 15.86 %

So the probability of number of employees between 38-44 years of age = P ( 38 < X < 44 ) = P( X 44) – P( X 38) = 0.8413 – 0.5 = 0.3413

Therefore the statement that “ More employees at the processing center are older than 44 than between 38 to 44” is FALSE.

b) Probability of employees less than age of 30 = P ( X < 30 )

Z = ( X - )/ = ( 30 - 38 )/6

Thus the question can be answered by using the normal table to find

P ( X 30 ) = P ( Z ( 30-38)/6 )

P ( Z -1.333 ) = 9.12 %

So the number of employees with probability 0.0912 of them being under age 30 = 0.0912 \* 400 = 36.48 or 36 employees

Therefore the statement B of the question is FALSE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans :- As we know that if X N (1 , ), and Y N( , ) are two independent random variables then X + Y N( + , ^2 +2^2 ) , and X – Y N( 1 - 2, 1^2 + 2^2 ).

Similarly if Z = aX + bY , where X and Y are as defined as above, i.e Z is linear combination of X and Y , then Z N( a1 + b2, a^21^2 + b^22^2 ).

Therefore,

2X1 N( 2 , 4^2 ) and

X1 + X2 N( + , ^2 + ^2 ) N ( 2 , 2 ^2 )

2X1 - ( X1 + X2 ) = N ( 4, 6 ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans :- Since we need to find out the value of a and b, which are symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99

The probability of getting value between a and b should be 0.99

So the probability of going wrong, or the probability outside the a and b area is 0.01 ( i.e., 1-0.99

The probability towards left from a = -0.005

The probability towards right from b = +0.005

So since we have the probabilities of a and b , we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z, we can calculate the X values.

Z = ( X -

For probability 0.005 the Z value is -2.57 ( from Z table )

Z \*

Z ( -0.005 ) \* 20 + 100 = - ( -2.57 ) \* 20 + 100 = 151.4

Z ( +0.005 ) \* 20 + 100 = ( -2.57 ) \* 20 + 100 = 48.6

Therefore, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:- Total Profit = profit 1 + profit 2

Mean = profit 1(mean) + profit 2 (mean)

= 5 +7

= 12

Std = sqrt(9+16)

= sqrt(25)

= 5

Mean in Rs = 12\*45 = 540

Std in Rs = 5\*45 = 225

Mean profit is Rs 540 Million

Std deviation is Rs 225 Million

State norms interval ( 0.95,540,255)

Range is Rs (99.0081034, 980.991896) Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:- 5th percentile= From z score = 0.5000 – 0.050 = 0.4500

We get = -1.645

Therefore the 5th percentile of profit = mean + (-1.645)\*std

= 540 - 1.645\*225

= 540 – 370.125

= 169.87

5th percentile of profit = 170 Million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:- Probability of 1st division making loss = stats.norm.cdf (0,5,3) = 0.0479

Probability of 2nd division making loss = stats.norm.cdf (0,7,4) = 0.04005

Therefore, we can say that 1st division can make more loss compared to 2nd division.