

Compiler Construction

Finite State Machines with Output

- Mealy Machine
- Moore Machine

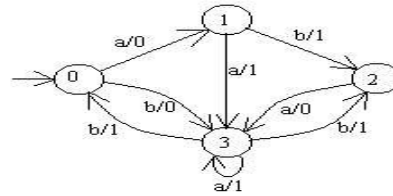
Mealy Machine & Moore Machine

- Finite automata are like computers in that they receive input and process the input by changing states. The only output that we have seen finite automata produce so far is a yes/no at the end of processing.
- We will now look at two models of finite automata that produce more output than a yes/no.

Mealy Machine

Transitions are labelled i/o where

- i is a character in the input alphabet and
- o is a character in the output alphabet.



Input: aaabb
Output: 01110

- Mealy machines are complete in the sense that there is a transition for each character in the input alphabet leaving every state.
- There are no accept states in a Mealy machine because it is not a language recogniser, it is an output producer. Its output will be the same length as its input.

Mealy Machine

- Mealy Machines are exactly as powerful as Moore machines
 - (we can implement any Mealy machine using a Moore machine, and vice versa).
- However, Mealy machines move the output function from the state to the transition. This turns out to be easier to deal with in practice, making Mealy machines more practical.



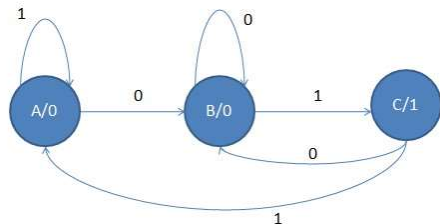
Input: 010110
Output: 101001

The above Mealy machine takes the one's complement of its binary input. In other words, it flips each digit from a 0 to a 1 or from a 1 to a 0.

Mealy Machine & Moore Machine

- Moore Machine $M1=(Q,\Sigma,\lambda,O,\delta,q_0)$
 - ✓ Q : Set of States
 - ✓ Σ : input alphabet
 - ✓ O : Output alphabet
 - ✓ λ : Output function, $Q \rightarrow O$
 - ✓ δ : Transition Function, $Q \times \Sigma \rightarrow Q$
 - ✓ q_0 : Initial State
- Mealy Machine $M1=(Q,\Sigma,O,\lambda',\Delta,q_0)$
 - ✓ Q : Set of States
 - ✓ Σ : input alphabet
 - ✓ O : Output alphabet
 - ✓ Δ : Transition Function, $Q \times \Sigma \rightarrow Q$
 - ✓ λ' : Output function, $Q \times \Sigma \rightarrow O$
 - ✓ q_0 : Initial State

Mealy Machine (Count Substring 01)



$$\begin{aligned}\lambda'(A,0) &= \lambda(\delta(A,0)) \\ &= \lambda(B) \\ &= 0\end{aligned}$$

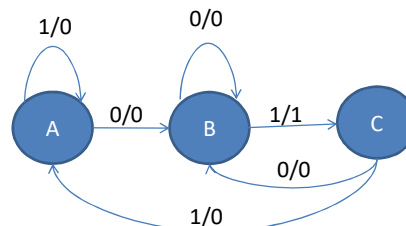
$$\begin{aligned}\lambda'(A,1) &= \lambda(\delta(A,1)) \\ &= \lambda(A) \\ &= 0\end{aligned}$$

$$\begin{aligned}\lambda'(B,0) &= \lambda(\delta(B,0)) \\ &= \lambda(B) \\ &= 0\end{aligned}$$

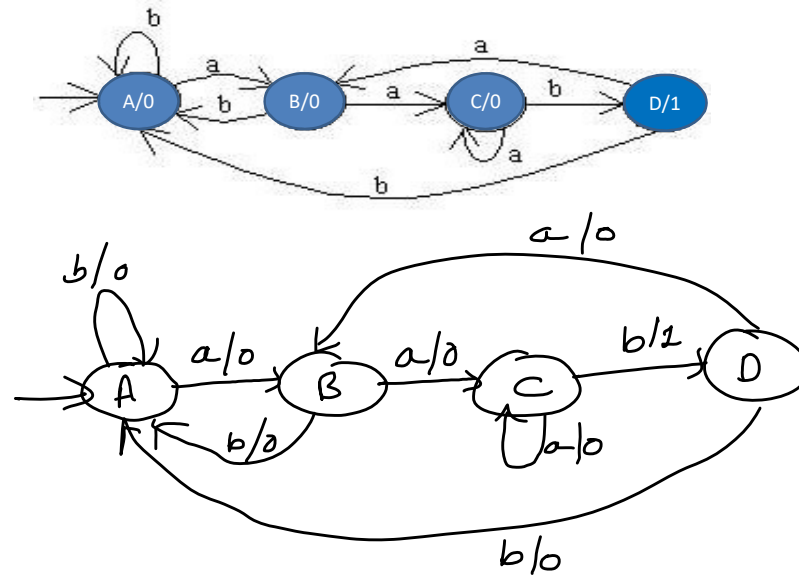
$$\begin{aligned}\lambda'(B,1) &= \lambda(\delta(B,1)) \\ &= \lambda(C) \\ &= 1\end{aligned}$$

$$\begin{aligned}\lambda'(C,0) &= \lambda(\delta(C,0)) \\ &= \lambda(B) \\ &= 0\end{aligned}$$

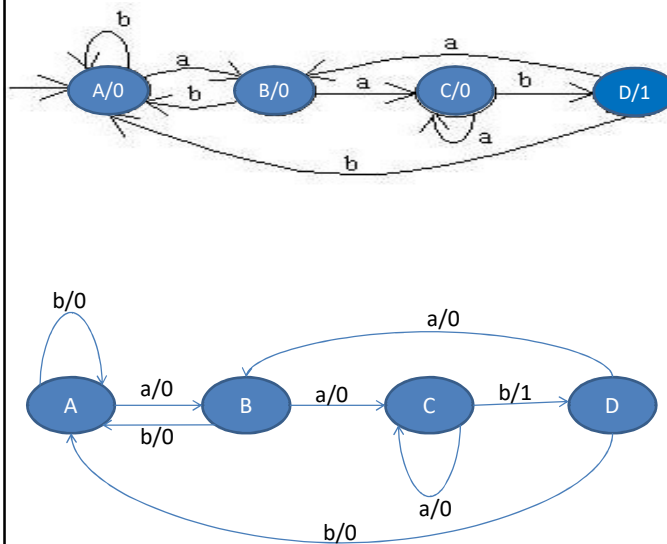
$$\begin{aligned}\lambda'(C,1) &= \lambda(\delta(C,1)) \\ &= \lambda(A) \\ &= 0\end{aligned}$$



Mealy Machine (Count Substrings aab)

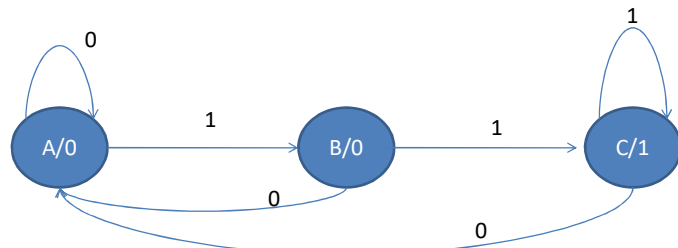


Mealy Machine (Count Substrings aab)



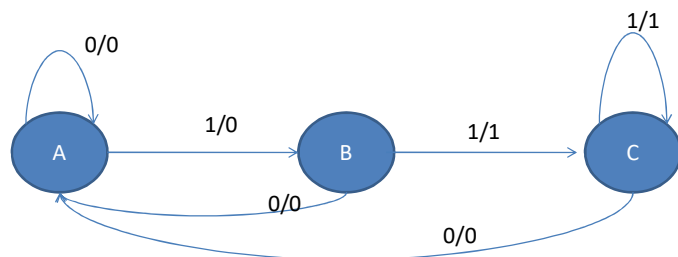
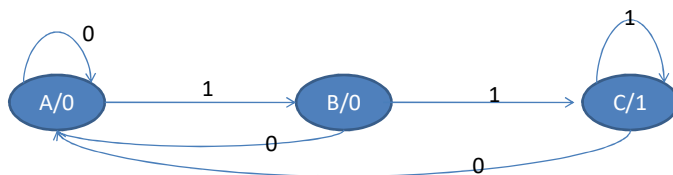
Moore Machine To Mealy Machine

- Replaces the first 1 with 0 from every substring starting with 1



Input	Result
0111001	0011000
0001001110	0000000110
111010110	011000010

Moore Machine To Mealy Machine



Input	Result
0111001	0011000
0001001110	0000000110
111010110	011000010