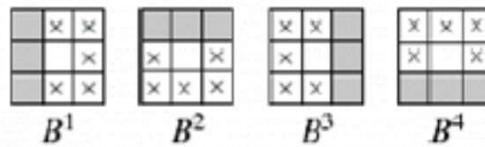


Convex Hull

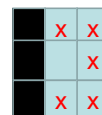
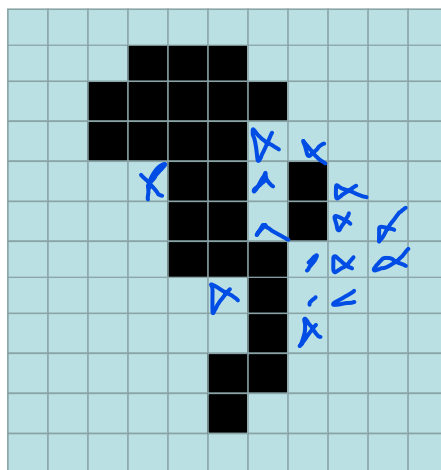
$$X_k^i = (X_{k-1} \circledast B^i) \cup A, \quad i = 1, 2, 3, 4, \quad k = 1, 2, \dots, \quad X_0^i = A$$

Now let $D^i = X_{\text{conv}}^i$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

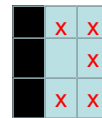
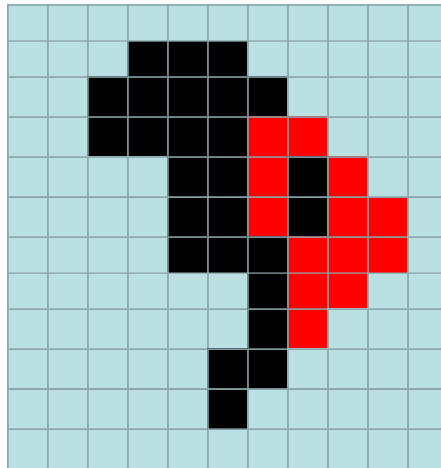
$$C(A) = \bigcup_{i=1}^4 D^i$$



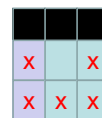
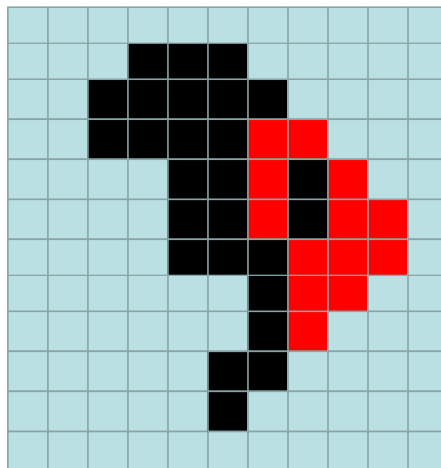
Convex Hull



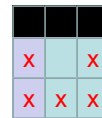
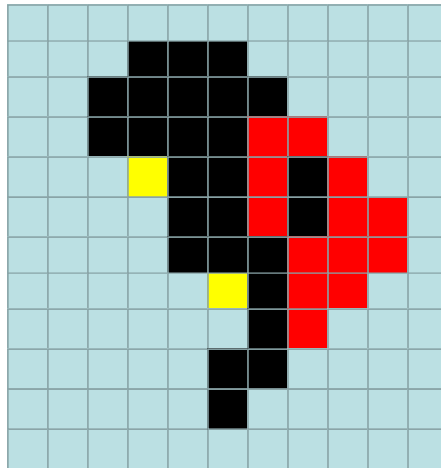
Convex Hull



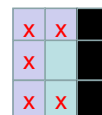
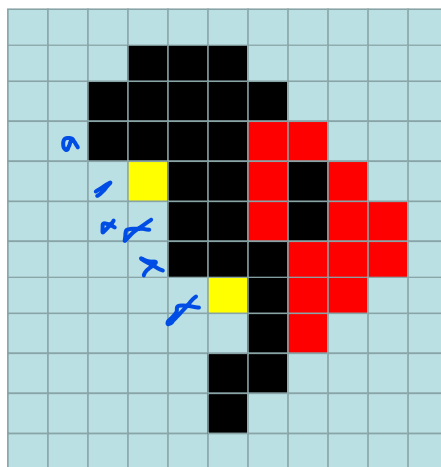
Convex Hull



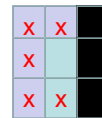
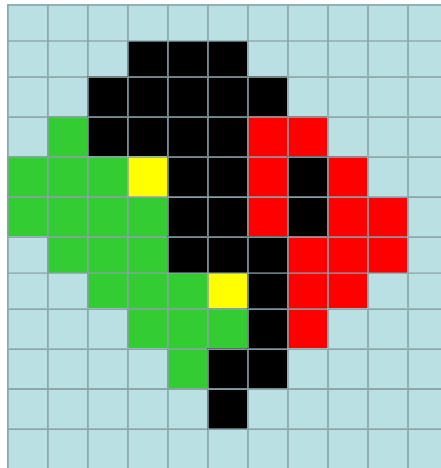
Convex Hull



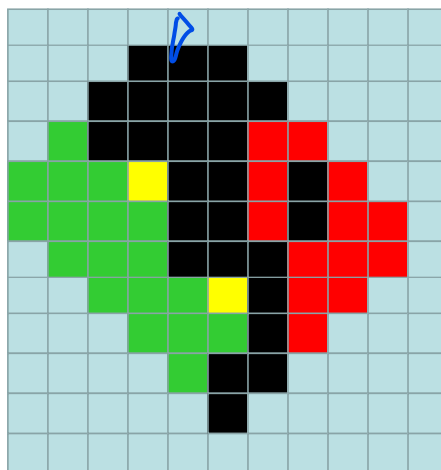
Convex Hull



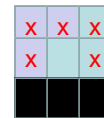
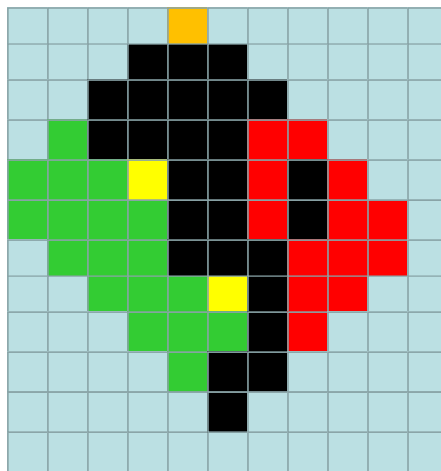
Convex Hull



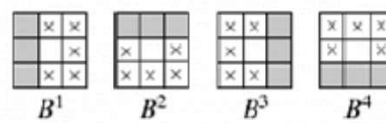
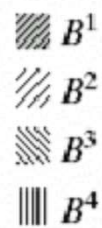
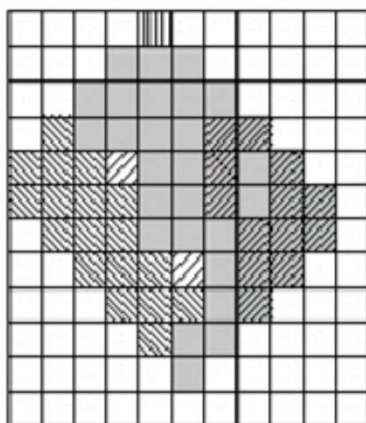
Convex Hull



Convex Hull



Convex Hull

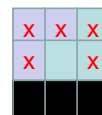
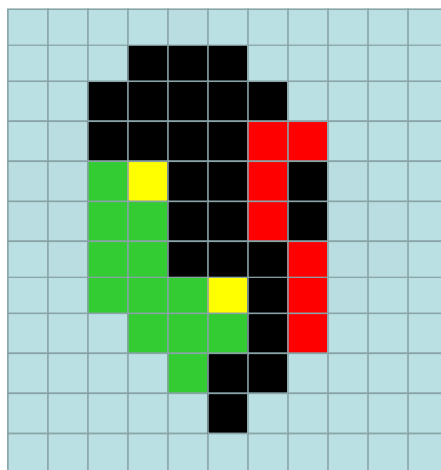


Convex Hull

Shortcoming of above algorithm: convex hull can grow beyond the minimum dimensions required to guarantee convexity

Possible solution: Limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points

Convex Hull



Thinning

- The thinning of a set A by a structuring element B, defined

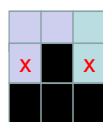
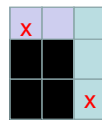
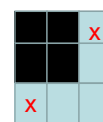
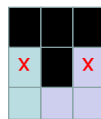
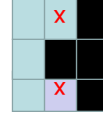
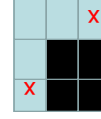
$$A \otimes B = A - (A * B)$$

$$= A \cap (A * B)^c$$

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Thinning

**B¹****B²****B³****B⁴****B⁵****B⁶****B⁷****B⁸**

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Thinning

$$A \otimes B = A - (A \circledast B)$$

hit-or-miss transform/template matching

- Note that we are only interested in pattern matching of B in A , so no background operation is required of the hit-miss-transform.

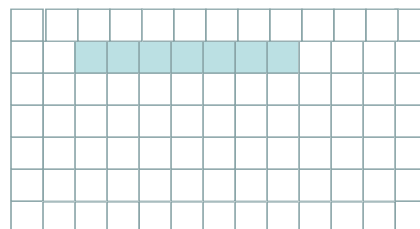
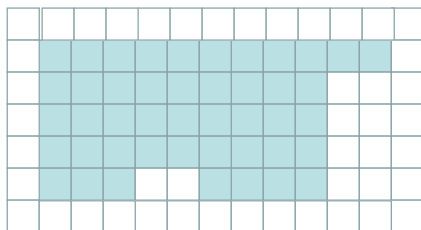
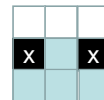
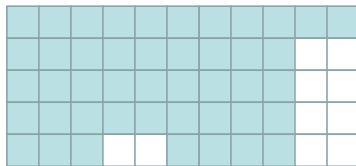
$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

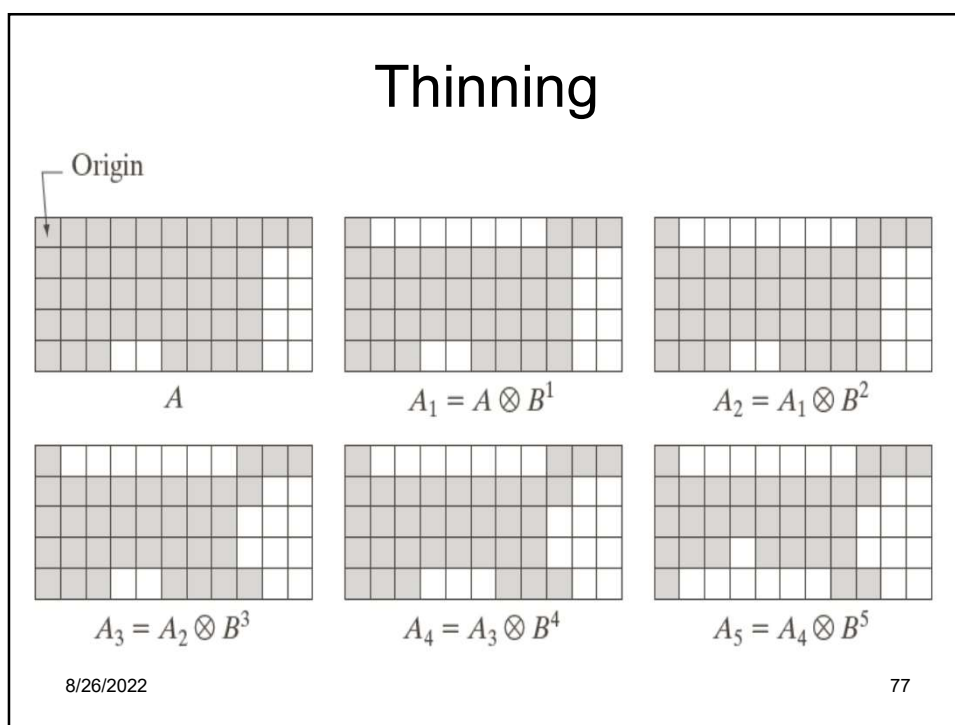
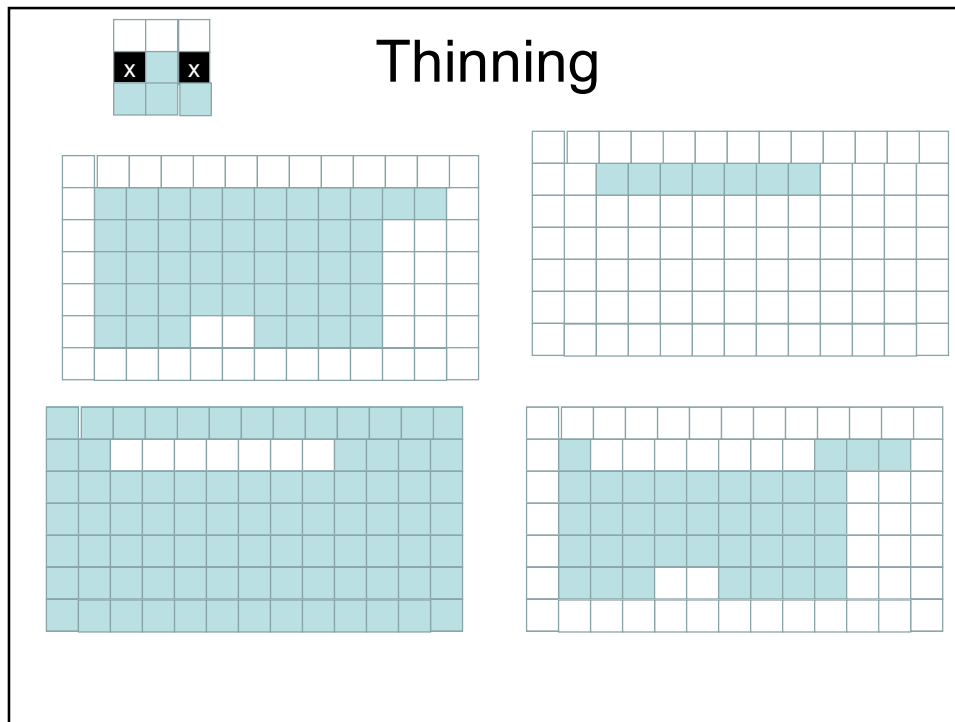
- The structuring element B consists of a sequence of structuring elements, where B^i is the rotated version of B^{i-1} . Each structuring elements helps thinning in one direction. If there are 4 structuring elements thinning is performed from 4 directions separated by 90° . If 8 structuring elements are used the thinning is performed in 8 directions separated by 45° .

- The process is to thin A by one pass with B^1 , then the result with one pass of B^2 , and continue until A is thinned with one pass of B^n .

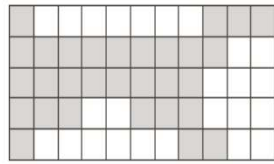
$$A \otimes \{B\} = (((\dots((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$$

Thinning

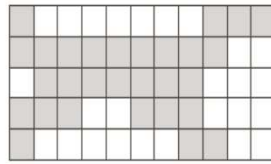




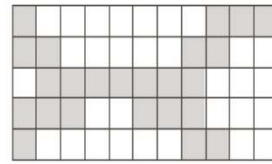
Thinning



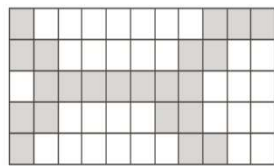
$$A_6 = A_5 \otimes B^6$$



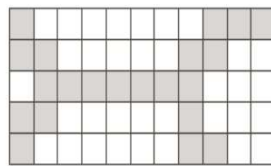
$$A_8 = A_6 \otimes B^{7,8}$$



$$A_{8,4} = A_8 \otimes B^{1,2,3,4}$$

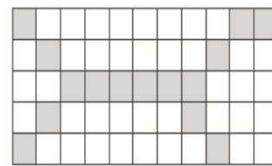


$$A_{8,5} = A_{8,4} \otimes B^5$$



$$A_{8,6} = A_{8,5} \otimes B^6$$

No more changes after this.



$$A_{8,6} \text{ converted to } m\text{-connectivity.}$$

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Thickening

The thickening is defined by the expression

$$A \sqcup B = A \cup (A \circledast B)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \sqcup \{B\} = (((...((A \sqcup B^1) \sqcup B^2)...) \sqcup B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

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Thickening

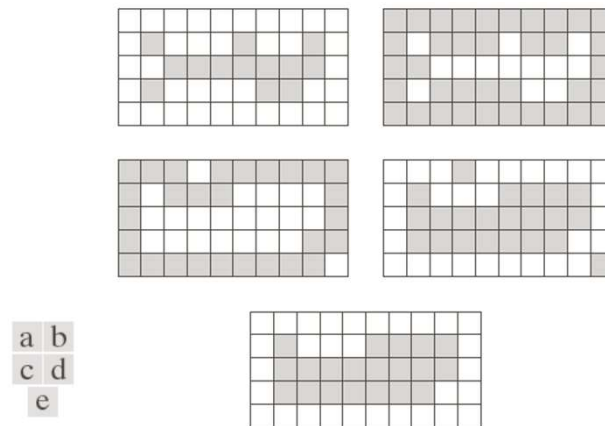


FIGURE 9.22 (a) Set A . (b) Complement of A . (c) Result of thinning the complement of A . (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.