Fuzzy membership functions

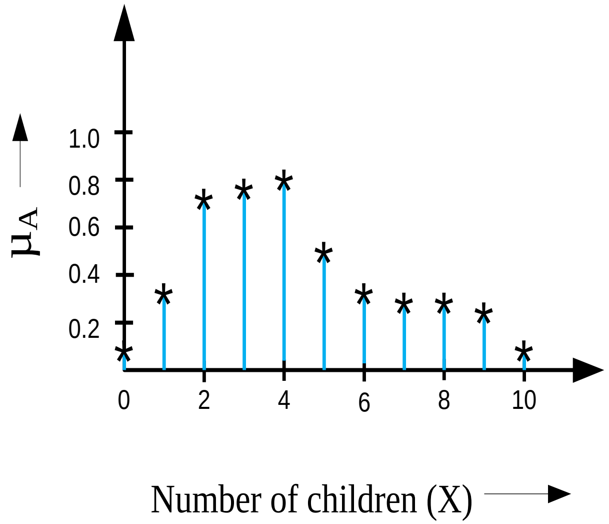
Fuzzy membership functions

A fuzzy set is completely characterized by its membership function (sometimes abbreviated as MF and denoted as μ). So, it would be important to learn how a membership function can be expressed (mathematically or otherwise).

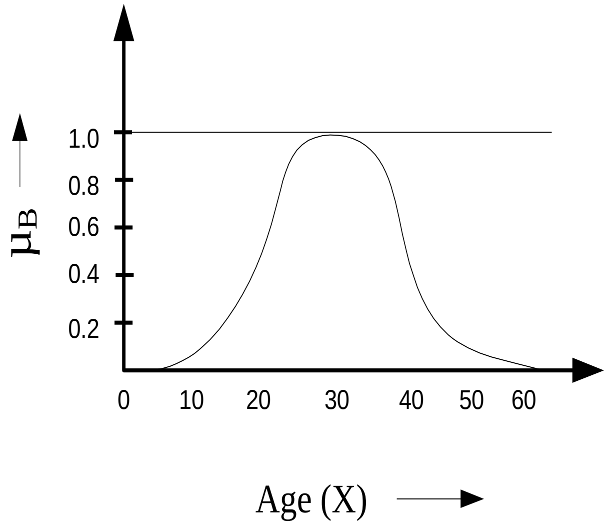
Note: A membership function can be on

a) a discrete universe of discourse and

b) a continuous universe of discourse.



A = Fuzzy set of "Happy family"

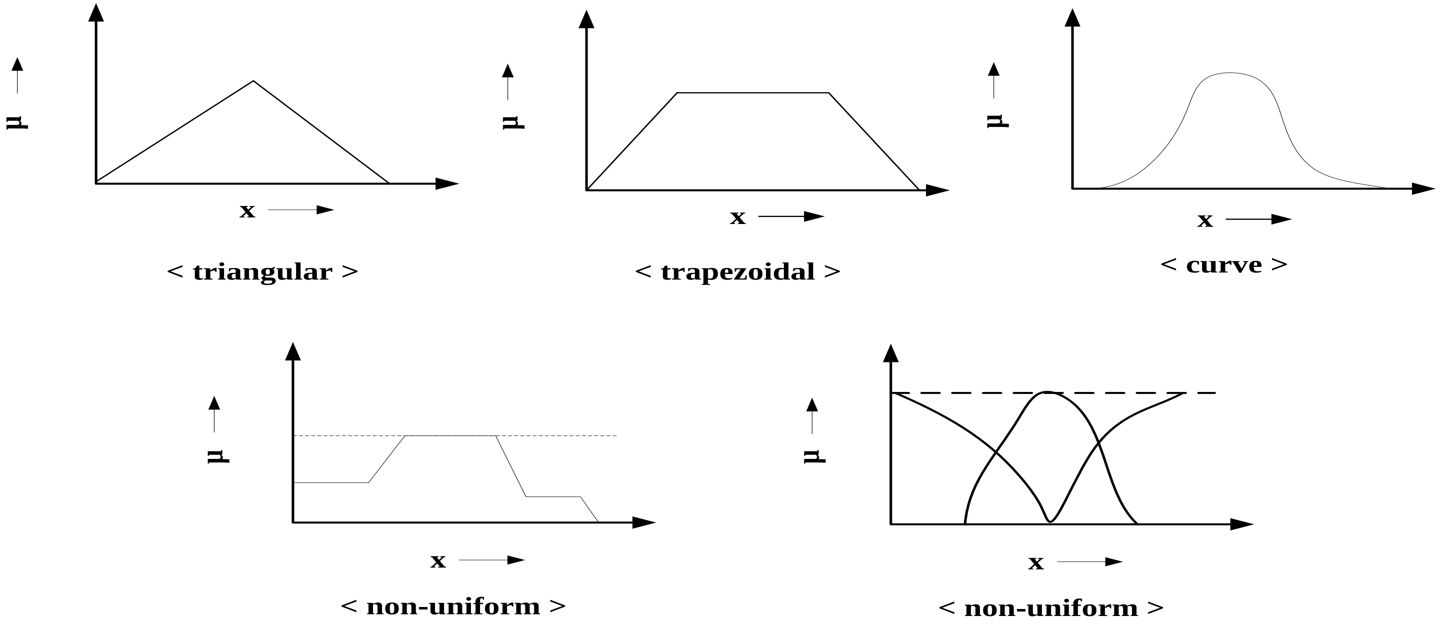


B = "Young age"

Fuzzy membership functions

So, membership function on a discrete universe of course is trivial. However, a membership function on a continuous universe of discourse needs a special attention.

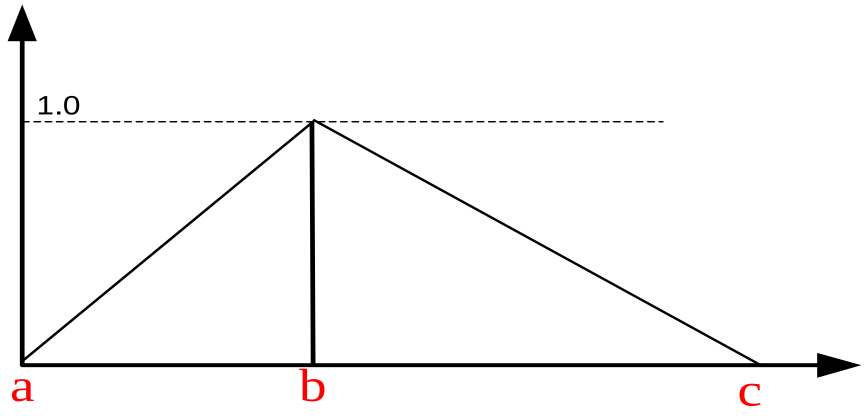
Following figures shows typical examples of membership functions.



Fuzzy MFs : Formulation and parameterization

In the following, we try to parameterize the different MFs on a continuous universe of discourse.

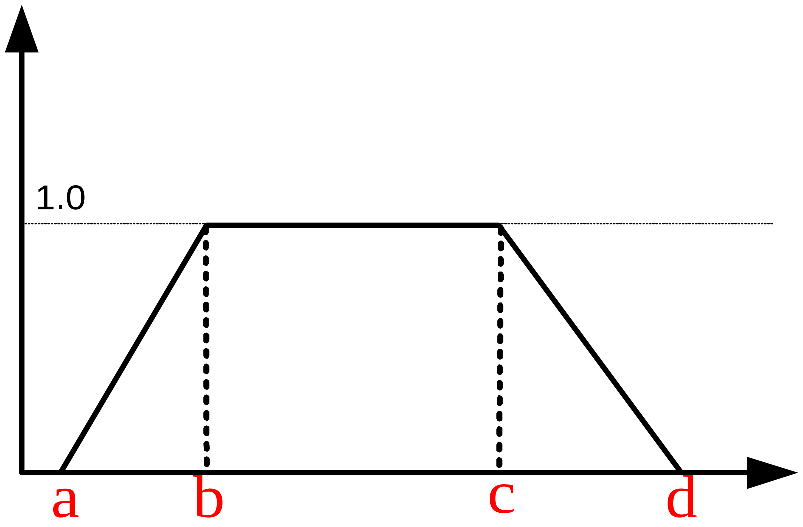
Triangular MFs : A triangular MF is specified by three parameters $\{a, b, c\}$ and can be formulated as follows.



$$\text{triangle}\,(x; a, b, c) = \begin{Bmatrix} 0 & if\ x \leq a \\ \dfrac{x-a}{a-b} & if\ a \leq x \leq b \\ \dfrac{c-x}{c-b} & if\ b \leq x \leq c \\ 0 & if\ c \leq x \end{Bmatrix}$$

Fuzzy MFs: Trapezoidal

A trapezoidal MF is specified by four parameters $\{a, b, c, d\}$ and can be defined as follows:

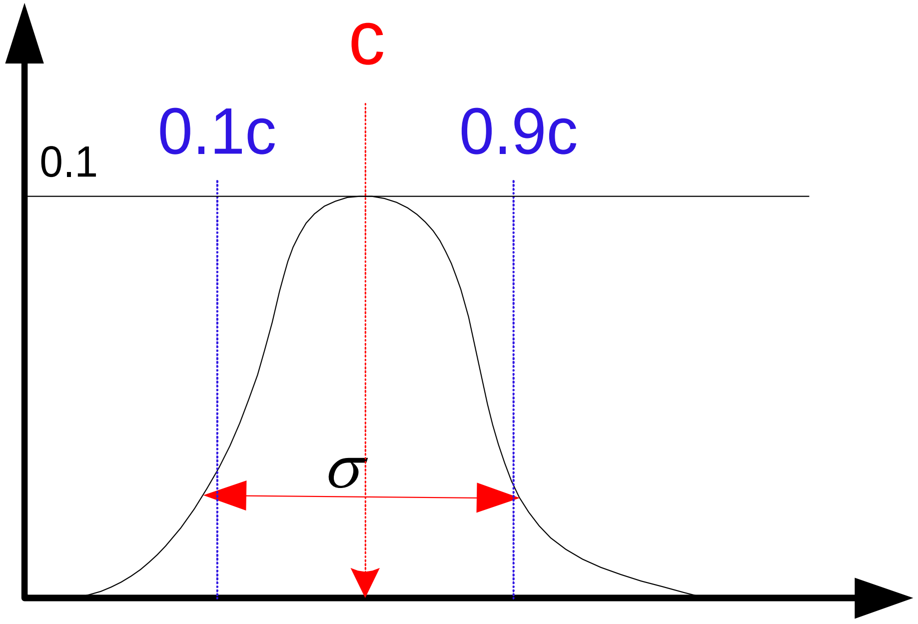


$$\text{trapozoid}\,(x; a, b, c, d) = \begin{Bmatrix} 0 & if\ x \leq a \\ \dfrac{x-a}{b-a} & if\ a \leq x \leq b \\ 1 & if\ b \leq x \leq c \\ \dfrac{d-x}{d-c} & if\ c \leq x \leq d \\ 0 & if\ d \leq x \end{Bmatrix}$$

Fuzzy MFs: Gaussian

A **Gaussian MF** is specified by two parameters $\{c, \sigma\}$ and can be defined as below:

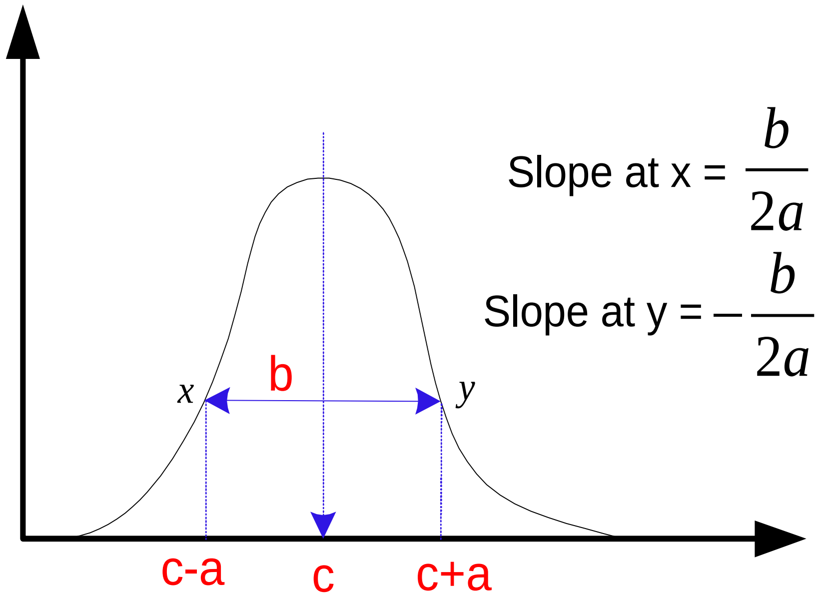
$$gaussian\ (x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$



Fuzzy MFs: Generalized bell

It is also called Cauchy MF. A generalized bell MF is specified by three parameters $\{a, b, c\}$ and is defined as:

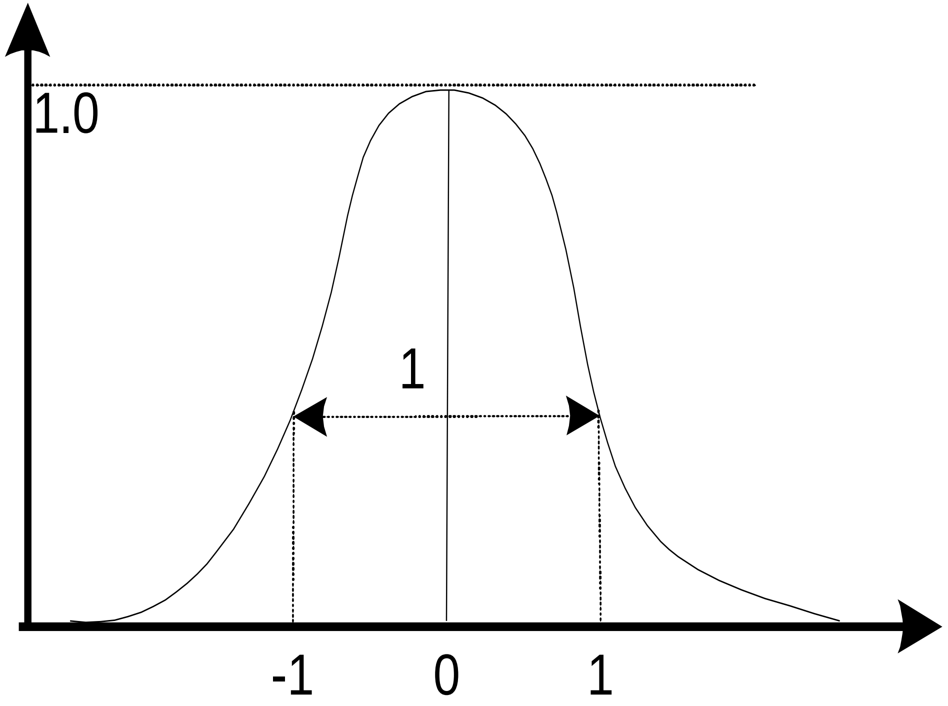
$$bell(x;\ a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



Example: Generalized bell MFs

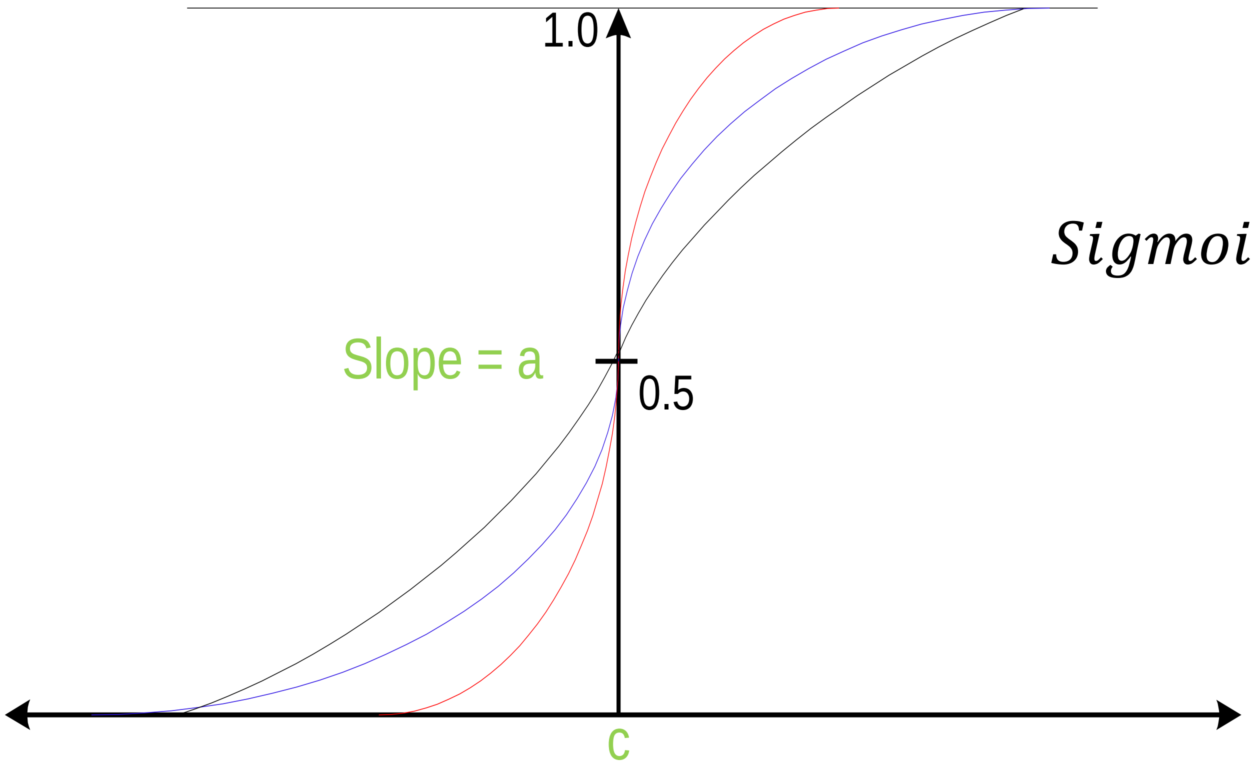
Example: $\mu(x) = \frac{1}{1+|x|^2}$;

a = b = 1 and c = 0;



Fuzzy MFs: Sigmoidal MFs

Parameters: $\{a, c\}$; where c = crossover point and a = slope at c;



$$Sigmoid(x;\ a, c) = \frac{1}{1 + e^{-\left[\frac{x-c}{a}\right]}}$$

Generation of MFs

Given a membership function of a fuzzy set representing a linguistic hedge, we can derive many more MFs representing several other linguistic hedges using the concept of Concentration and Dilation.

1. **Concentration:** $A^k = [\mu_A(x)]^k; k > 1$
2. **Dilation:** $A^k = [\mu_A(x)]^k; k < 1$

Example : Age = { Young, Middle-aged, Old }

Thus, corresponding to Young, we have : Not young, Very young, Not very young and so on.

Similarly, with Old we can have : Not old, Very old, Very very old, Extremely old, etc.

Thus, $\mu_{Extremely\ old}(x) = (((\mu_{Old}(x))^2)^2)^2$ and so on

Or, $\mu_{More\ or\ less\ old}(x) = A^{0.5} = (\mu_{Old}(x))^{0.5}$

Linguistic variables and values

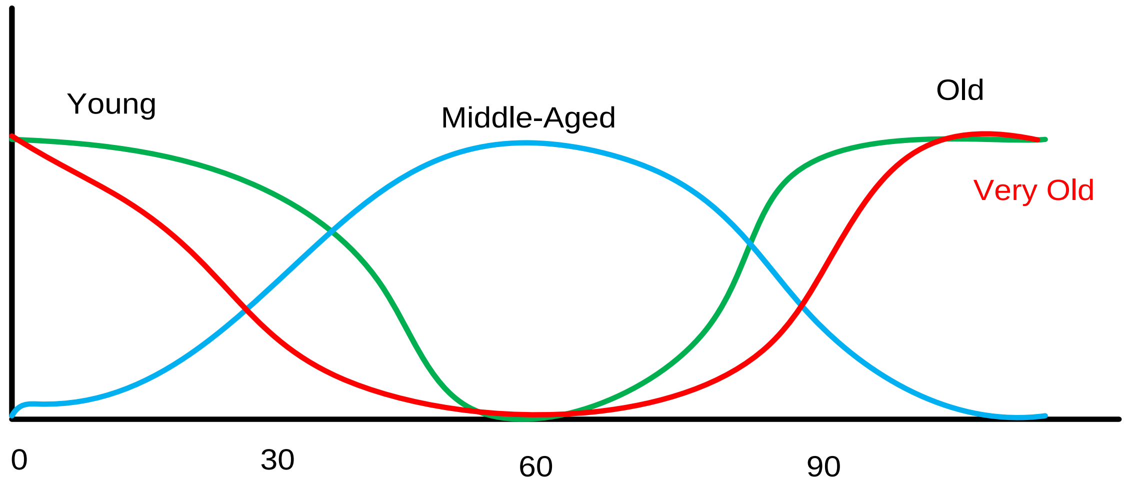
$\mu_{young}(x)$ =bell(x,20,2,0) $= \frac{1}{1+(\frac{x}{20})^4}$

$\mu_{old}(x)$ =bell(x,30,3,100) $= \frac{1}{1+(\frac{x-100}{30})^6}$

$\mu_{middle-aged}(x)$ =bell(x,30,60,50)

Not young=$\overline{\mu_{young}(x)} = 1 - \mu_{young}(x)$

Young but not too young $= \mu_{young}(x) \cap \overline{\mu_{young}(x)}$



* Fuzzy membership function - Example 1

Consider the following example

$X = \{5, 15, 20, 25, 35, 45, 55, 65, 75, 85, 90\}$

Fuzzy sets = infant, young, adult, senior

Age	infant	young	adult	senior
5	0	0	0	0
15	0	0.2	0	0
20	0	0.8	0.9	0
25	0	1	1	0
35	0	0.6	1	0
45	0	0.5	1	0
55	0	0.1	1	0.5
65	0	0	1	1
75	0	0	1	1
85	0	0	1	1
90	0	0	1	1

* Fuzzy membership function - Example 2

- Let the value of temperature in °C

$T = \{0, 5, 10, 15, 20, 25, 30, 35, 40\}$

Then the term HOT can be defined by fuzzy set as follows.

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HOT = {(0,0), (5,0.1), (10,0.3), (15,0.5),
(20,0.6), (25,0.7), (30,0.8), (35,0.9)
(40,1.0)}

This fuzzy set reflects the point of view that 0°c is not hot at all, 5,10, 15°c are somewhat hot.

And 40°c is indeed hot

Another person could have defined the set differently.