Morphological Processing

Morphological Processing

- Analyze images
- Define structure
- Define regions and boundaries
- Based on set theory
- Unified theoretical approach

Morphology

The word morphology refers to the scientific branch that deals the forms and structures of animals/plants.

Morphology in image processing is a tool for extracting image components that are useful in the representation and description of region shape, such as **boundaries** and **skeletons**.

Furthermore, the morphological operations can be used for **filtering, thinning and pruning.**

The language of the Morphology comes from the set theory, where image objects can be represented by sets. For example an image object containing black pixels can be considered a set of black pixels in 2D space of Z^2

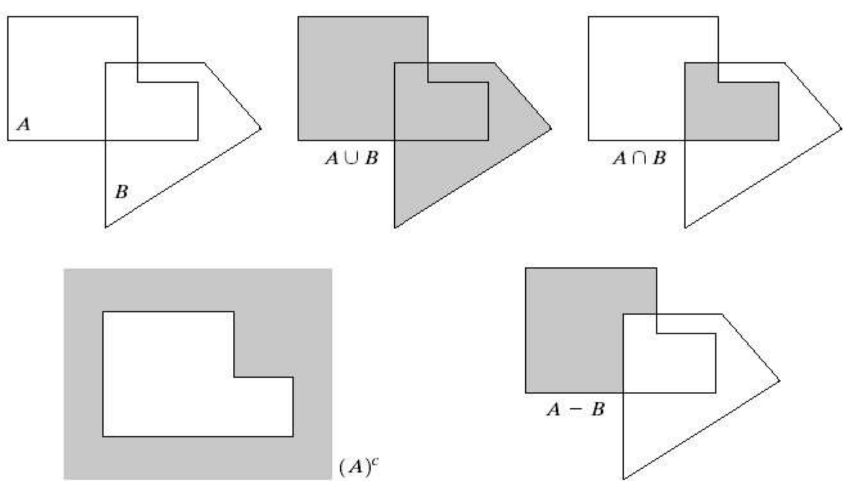
Morphological Operations

- Dilation
  - Fill in gaps
- Erosion
  - Delete unneeded detail (noise)
- Opening
  - Smooth inner contours
- Closing
  - Smooth outer contours
- Hit or Miss Transformation
  - Detect shapes

Set Theory Summary

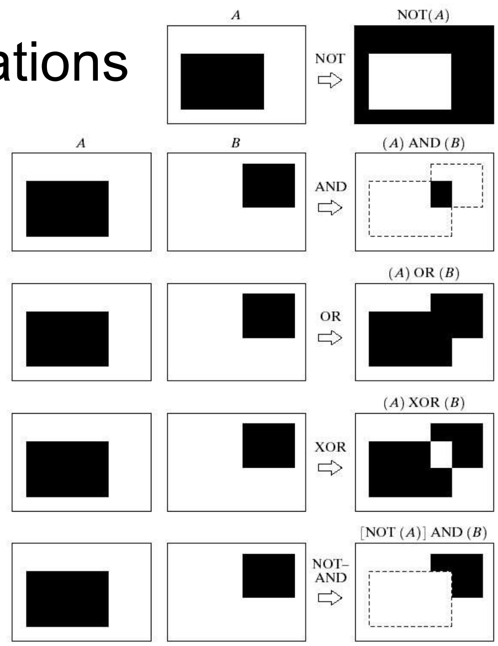
- *a* =(x, y) is an element of A — $a \in A$
- elements of images
  - pixel coordinates
- subset — $A \subseteq B$
- union — $A \bigcup B$
- intersection — $A \bigcap B$
- complement — $A^C = \{w \mid w \notin A\}$
- difference — $A - B = A \bigcap B^C$

Set Theory



Logic Operations

- Black = 1

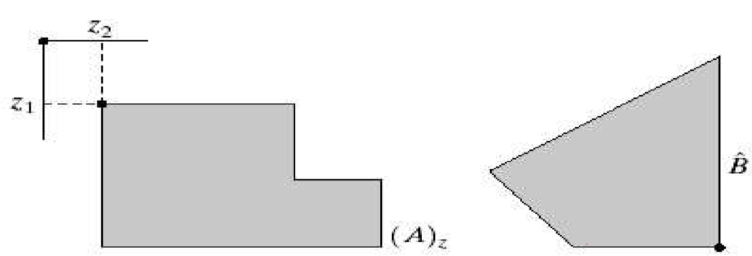


Morphological Operations

- Reflection
- Translation

$$\hat{B} = \{ w \mid w = -b \ \ for\ b \in B \}$$

$$(A)_Z = \{ c \mid c = a + z \ \ for\ a \in A \}$$



Dilation and Erosion

Dilation and erosion are the two fundamental operations used in morphological image processing. Almost all morphological algorithms depend on these two operations:

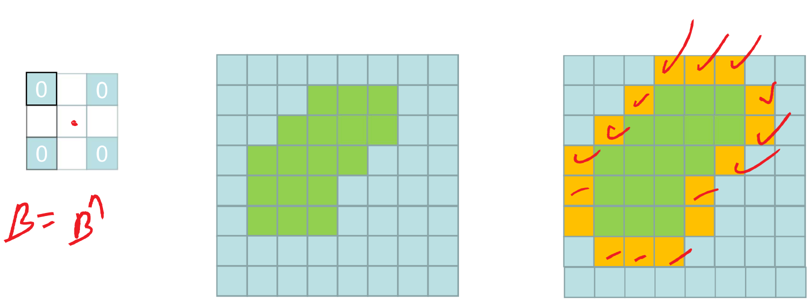
Dilation: Given A and B sets in Z2 , the dilation of A by B, is defined by:

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

The dilation of A and B is a set of all displacements, z , such that B and A overlap by ***at least*** one element.

Set B is referred to as the **structuring element** and used in dilation as well as in other morphological operations. **Dilation expands/dilutes a given image.**

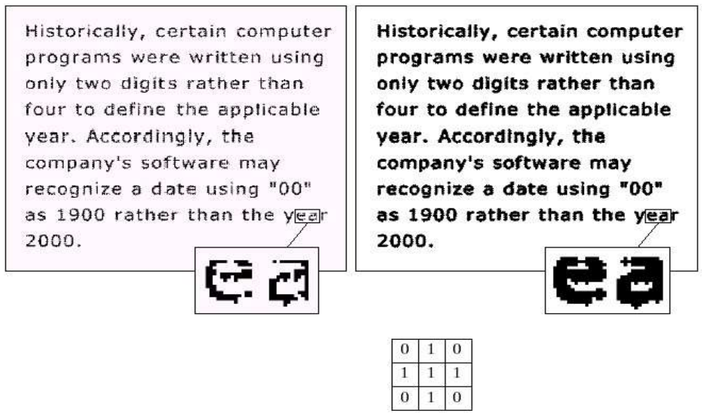
Dilation Operation



$B = \hat{B}$

A

Dilation Operation



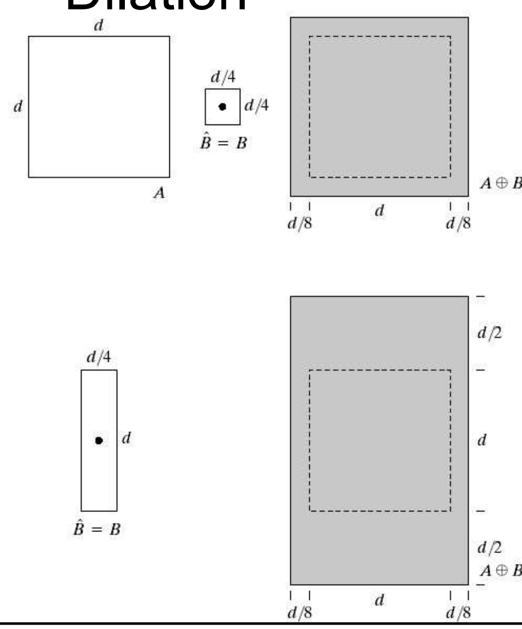
a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Erosion

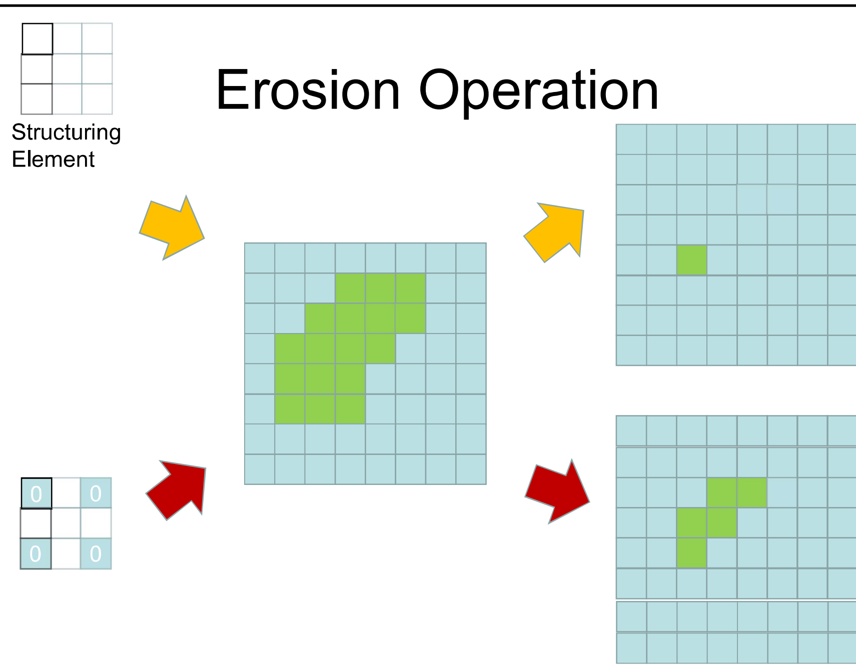
Erosion: Given A and B sets in Z^2 , the erosion of A by structuring element B, is defined by:

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

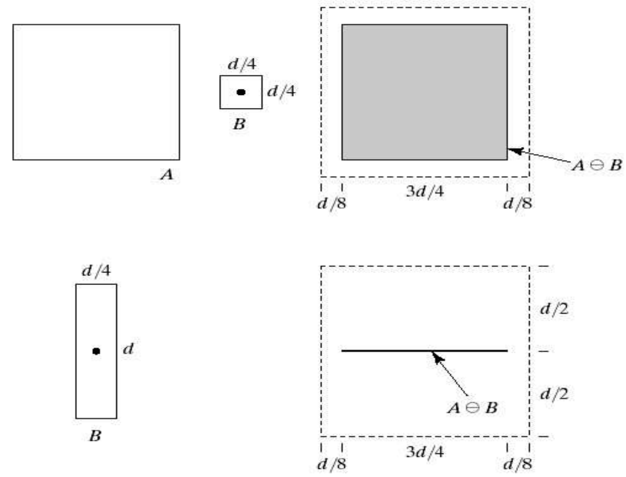
The erosion of A by structuring element B is the **set of all points z, such that B, translated by z, is contained in A.**

Note that in erosion the structuring element B erodes the input image A at its boundaries. Erosion **shrinks** a given image.

Erosion Operation

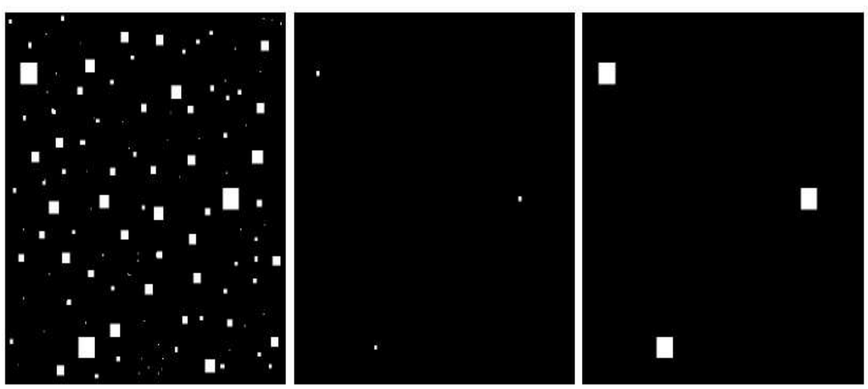


Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening Operation

Opening: The process of erosion followed by dilation is called opening.

It has the effect of eliminating small and thin objects, breaking the objects at thin points and smoothing the boundaries/contours of the objects.

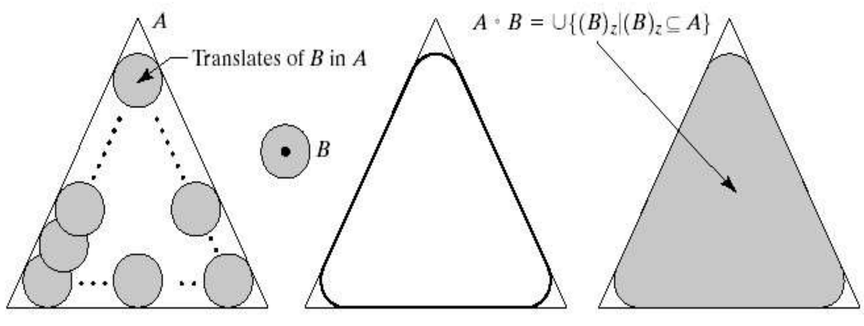
Given set A and the structuring element B. Opening of A by structuring element B is defined by:

$$A \circ B = (A \ominus B) \oplus B$$

The opening of A by the structuring element B is obtained by taking the union of all translates of B that fit into A.

$$A \circ B = \bigcup \{B_z \mid (B_z) \subseteq A\}$$

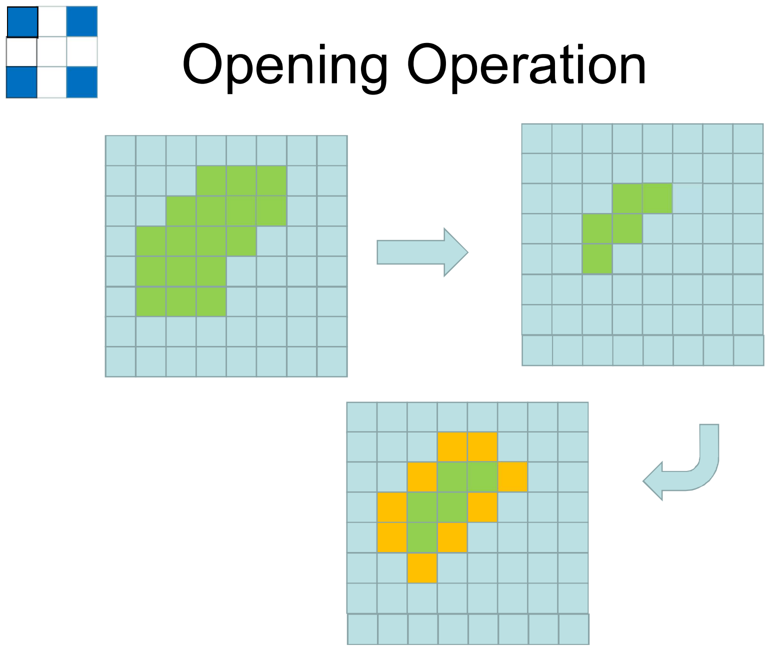
Opening Operation



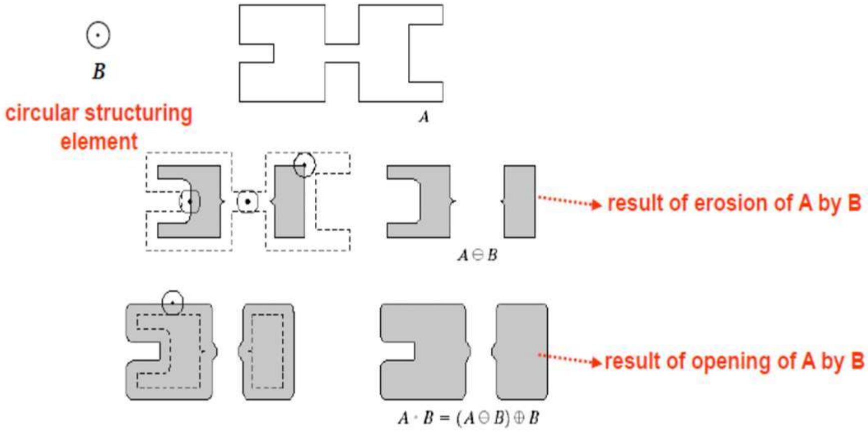
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening Operation



Opening Operation



Closing Operation

Closing: The process of dilation followed by erosion is called closing.

It has the effect of filling small and thin holes, connecting nearby objects and smoothing the boundaries/contours of the objects.

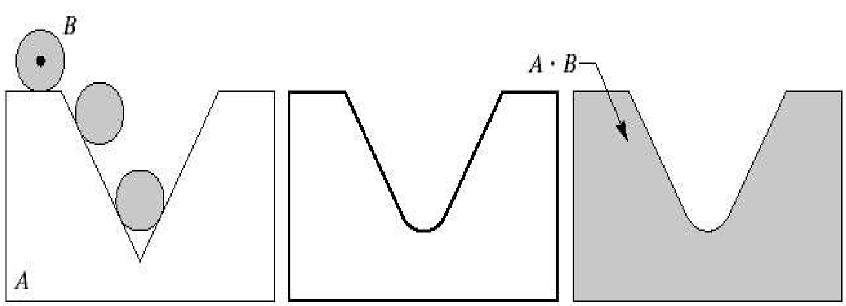
Given set A and the structuring element B. Closing of A by structuring element B is defined by:

$$A \bullet B = (A \oplus B) \ominus B$$

The closing has a similar geometric interpretation except that we roll B on the outside of the boundary.

$$A \bullet B = \bigcup \{ (B_z) \mid (B_z) \cap A \neq \varnothing \}$$

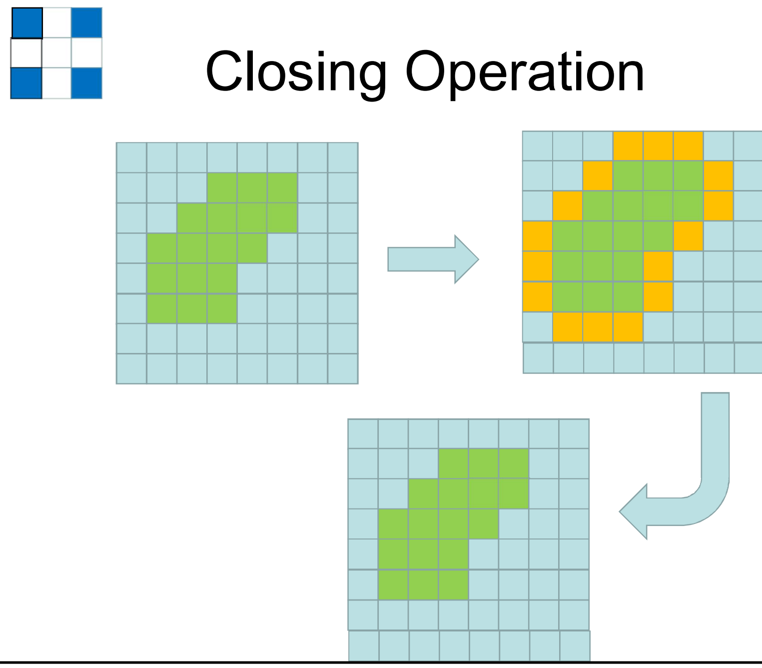
Closing Operation



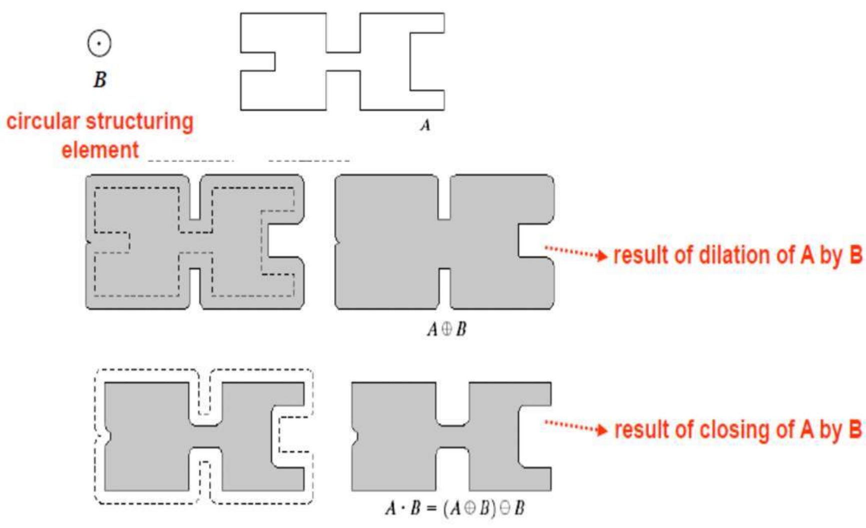
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

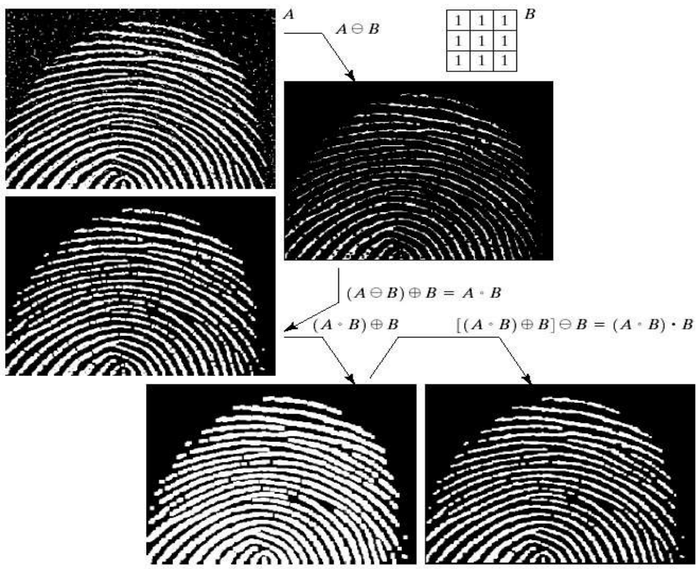
Closing Operation



Closing Operation



Mixed Example



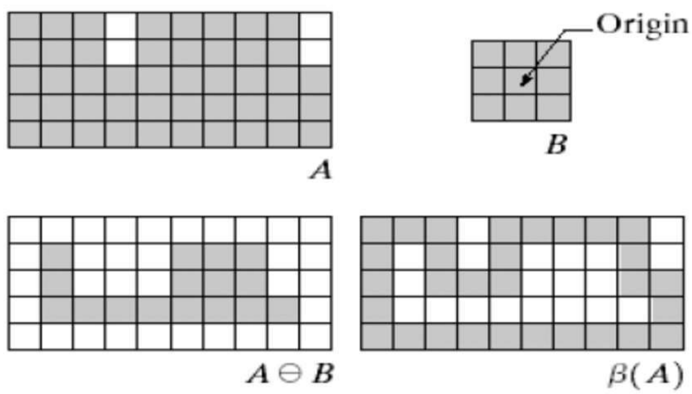
a b c d e f

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Opening and Closing

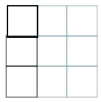
- Opening
  - smooth contours
  - break narrow isthmus
  - eliminate narrow protrusions
- Closing
  - smooth contours
  - fuse breaks
  - eliminate holes
  - fill in small gaps

Boundary Extraction

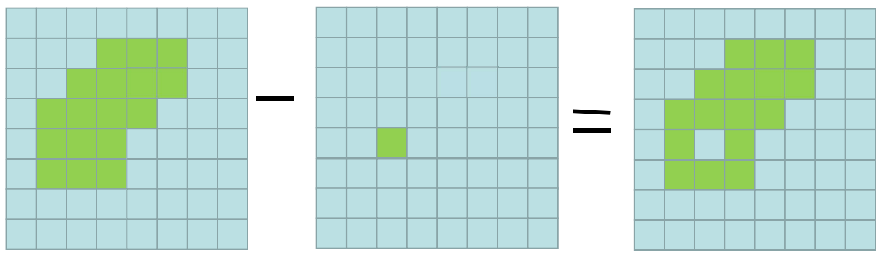


Boundary Extraction

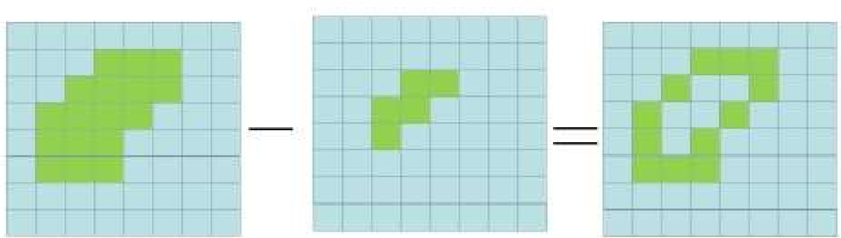
Note that thicker boundaries can be obtained by increasing the size of structuring element

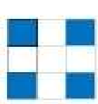


Boundary Extraction



Boundary Extraction





- Erode A by B
- Subtract from original

Region Filling

Region filling can be performed by using the following definition.

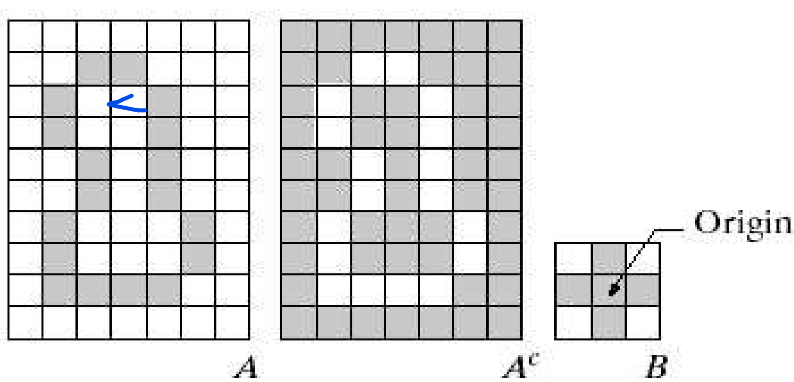
Given a symmetric structuring element B, one of the non-boundary pixels (Xk) is consecutively diluted and its intersection with the complement of A is taken as follows:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad \begin{aligned} k &= 1,2,3,\ldots \\ \textit{terminates when } X_k &= X_{k-1} \\ X_0 &= 1 \ (\textit{inner pixel}) \end{aligned}$$

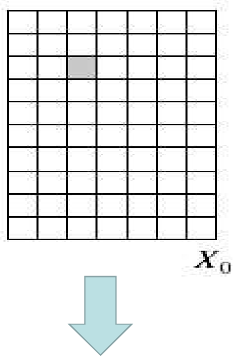
Following consecutive dilations and their intersection with the complement of A, finally resulting set is the filled inner boundary region and its union with A gives the filled region F(A)

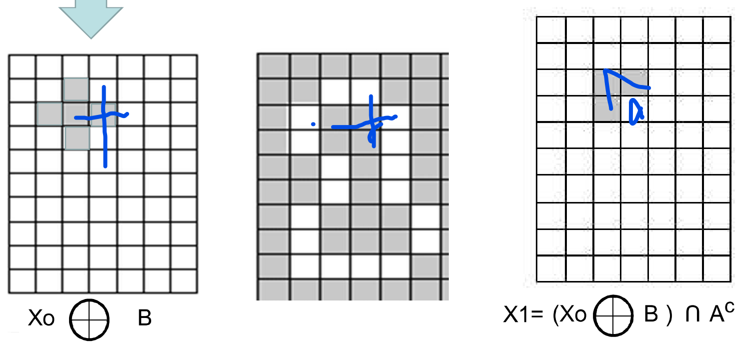
$$F(A) = X_k \cup A$$

Region Filling

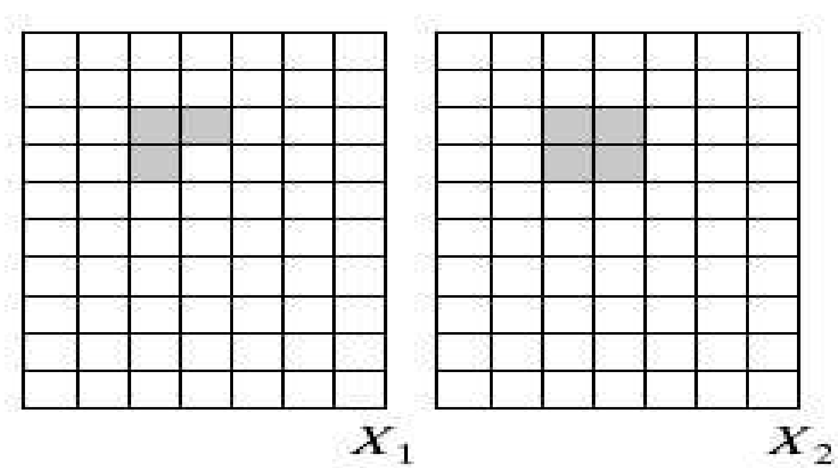


Region Filling

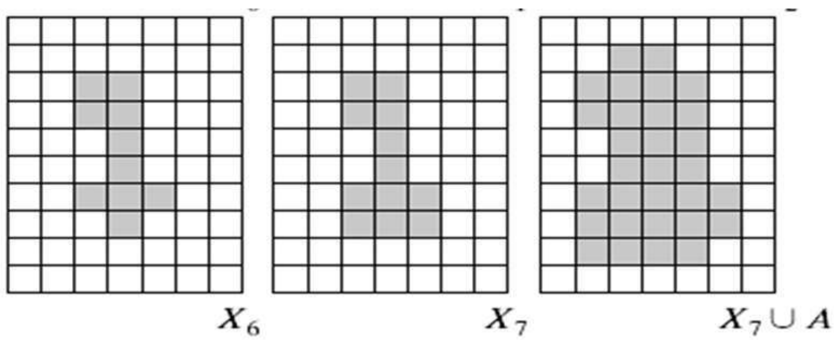


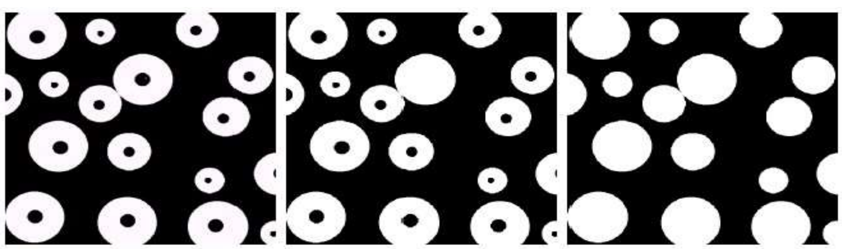


Region Filling



Region Filling





a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Finding the **starting** points is often done **manually**