Morphological Processing

Morphological Processing

- Analyze images
- Define structure
- Define regions and boundaries
- Based on set theory
- Unified theoretical approach

Morphology

The word morphology refers to the scientific branch that deals the forms and structures of animals/plants.

Morphology in image processing is a tool for extracting image components that are useful in the representation and description of region shape, such as **boundaries** and **skeletons**.

Furthermore, the morphological operations can be used for **filtering, thinning and pruning.**

The language of the Morphology comes from the set theory, where image objects can be represented by sets. For example an image object containing black pixels can be considered a set of black pixels in 2D space of Z^2

Morphological Operations

- Dilation
  - Fill in gaps
- Erosion
  - Delete unneeded detail (noise)
- Opening
  - Smooth inner contours
- Closing
  - Smooth outer contours
- Hit or Miss Transformation
  - Detect shapes

Set Theory Summary

- *a* =(x, y) is an element of A
- elements of images
  - pixel coordinates
- subset
- union
- intersection
- complement
- difference

$$a \in A$$

$$A \subseteq B$$

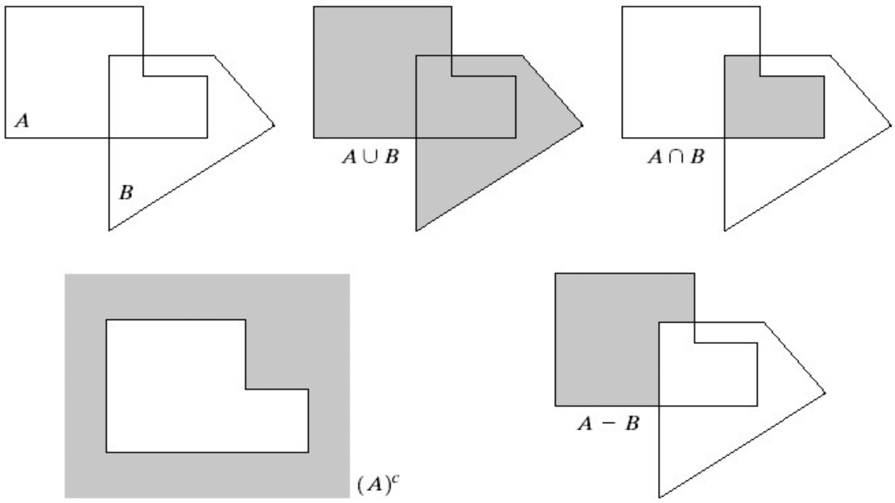
$$A \bigcup B$$

$$A \bigcap B$$

$$A^C = \{w \mid w \notin A\}$$

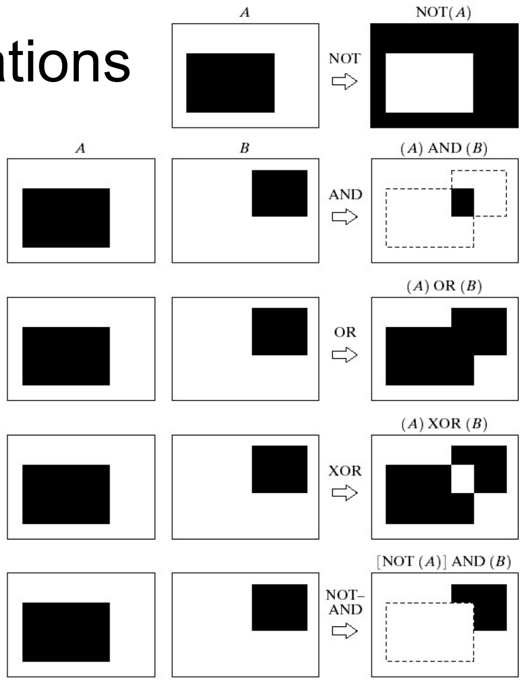
$$A - B = A \bigcap B^C$$

Set Theory



Logic Operations

- Black = 1

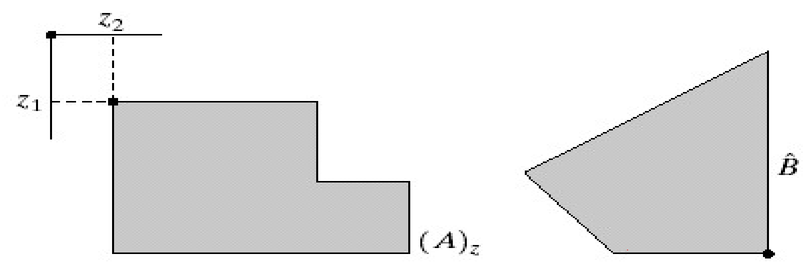


Morphological Operations

- Reflection
- Translation

$$\hat{B} = \{w \mid w = -b \;\; for \; b \in B\}$$

$$(A)_Z = \{c \mid c = a + z \;\; for \; a \in A\}$$



Dilation and Erosion

Dilation and erosion are the two fundamental operations used in morphological image processing. Almost all morphological algorithms depend on these two operations:

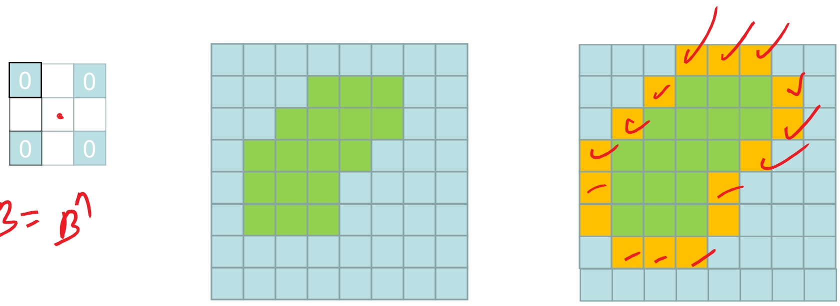
Dilation: Given A and B sets in Z2 , the dilation of A by B, is defined by:

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

The dilation of A and B is a set of all displacements, z , such that B and A overlap by ***at least*** one element.

Set B is referred to as the **structuring element** and used in dilation as well as in other morphological operations. **Dilation expands/dilutes a given image.**

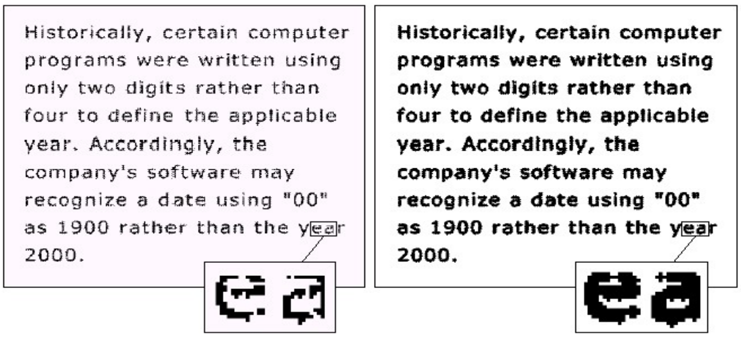
Dilation Operation



$B = \hat{B}$

A

Dilation Operation



0	1	0
1	1	1
0	1	0

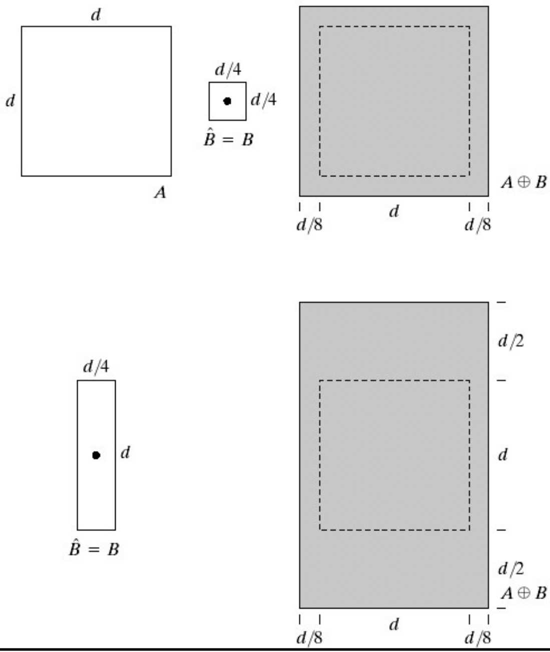
a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Dilation

a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.



Erosion

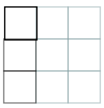
Erosion: Given A and B sets in Z^2 , the erosion of A by structuring element B, is defined by:

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

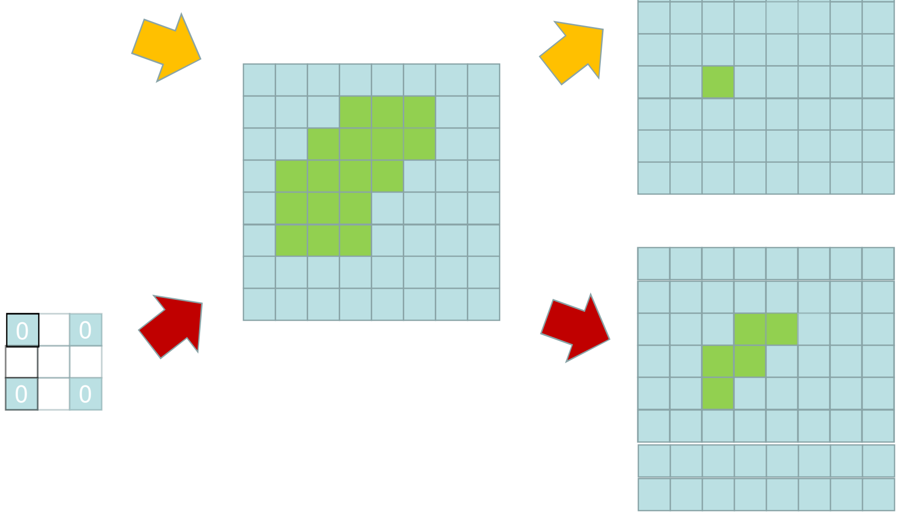
The erosion of A by structuring element B is the **set of all points z, such that B, translated by z, is contained in A.**

Note that in erosion the structuring element B erodes the input image A at its boundaries. Erosion **shrinks** a given image.

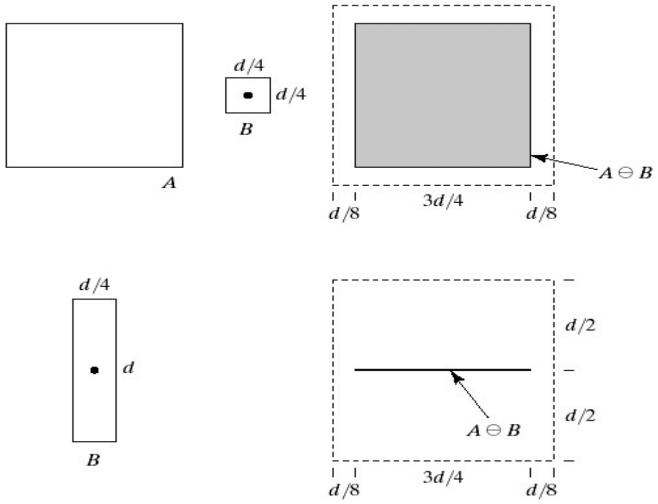
Erosion Operation



Structuring Element

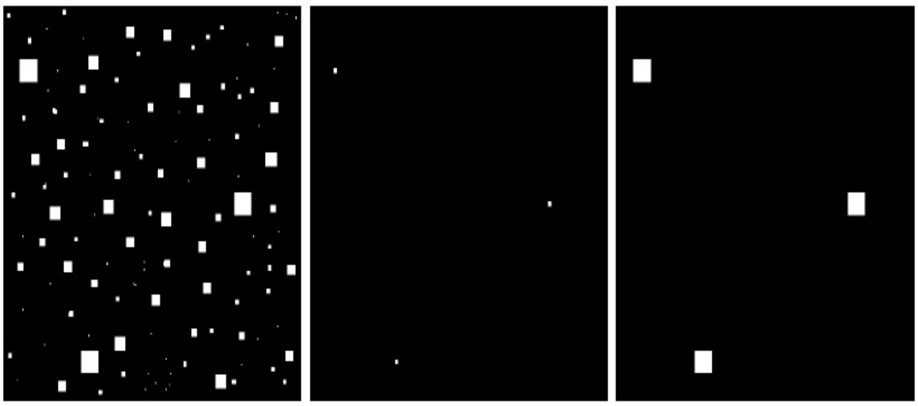


Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening Operation

Opening: The process of erosion followed by dilation is called opening.

It has the effect of eliminating small and thin objects, breaking the objects at thin points and smoothing the boundaries/contours of the objects.

Given set A and the structuring element B. Opening of A by structuring element B is defined by:

$$A \circ B = (A \ominus B) \oplus B$$

The opening of A by the structuring element B is obtained by taking the union of all translates of B that fit into A.

$$A \circ B = \bigcup \{B_z \mid (B_z) \subseteq A\}$$

Opening Operation

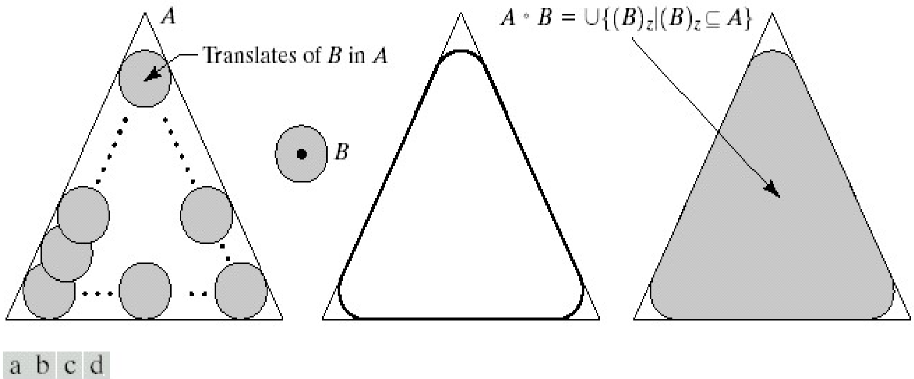
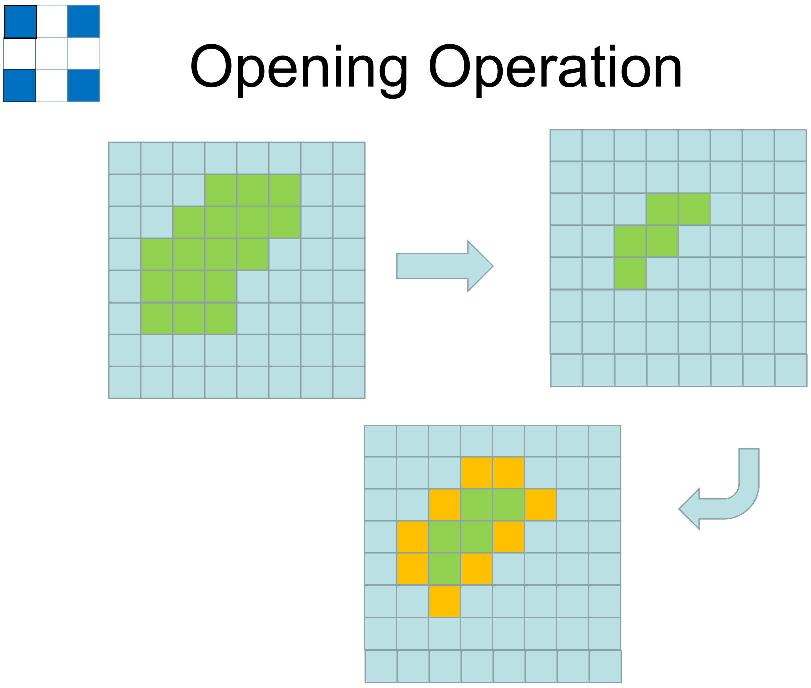
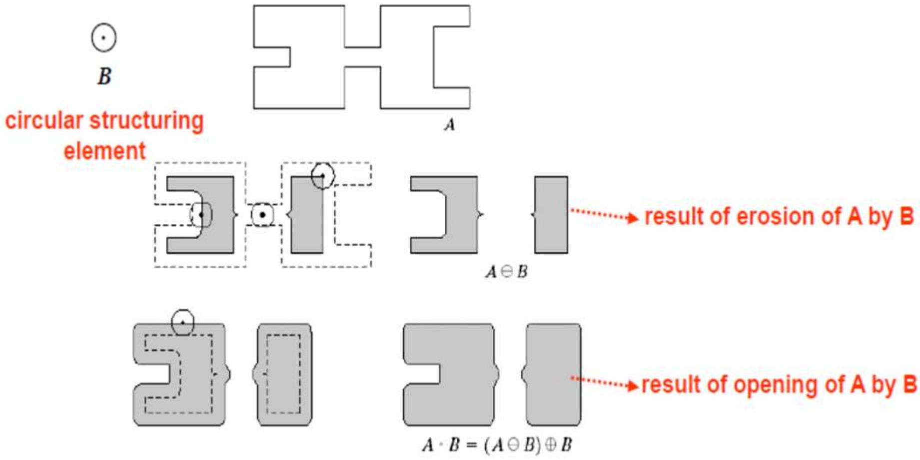


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening Operation



Opening Operation



Closing Operation

Closing: The process of dilation followed by erosion is called closing.

It has the effect of filling small and thin holes, connecting nearby objects and smoothing the boundaries/contours of the objects.

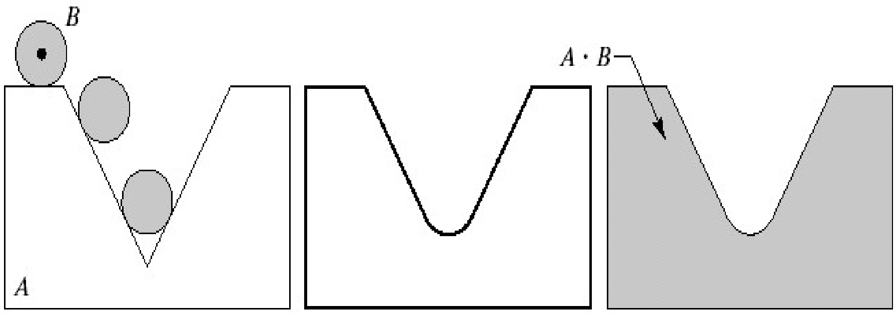
Given set A and the structuring element B. Closing of A by structuring element B is defined by:

$$A \bullet B = (A \oplus B) \ominus B$$

The closing has a similar geometric interpretation except that we roll B on the outside of the boundary.

$$A \bullet B = \bigcup \left\{ (B_z) \middle| (B_z) \cap A \neq \varnothing \right\}$$

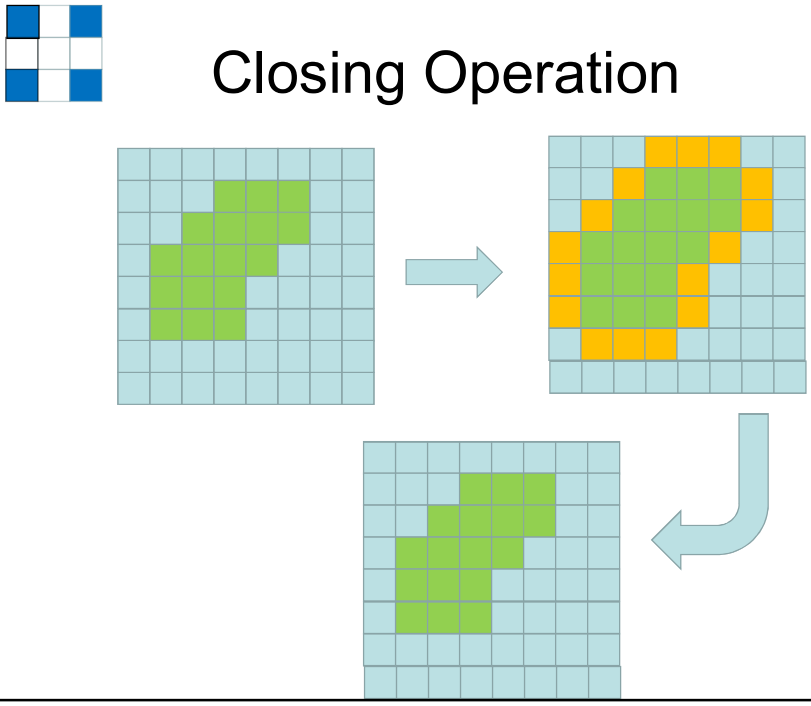
Closing Operation



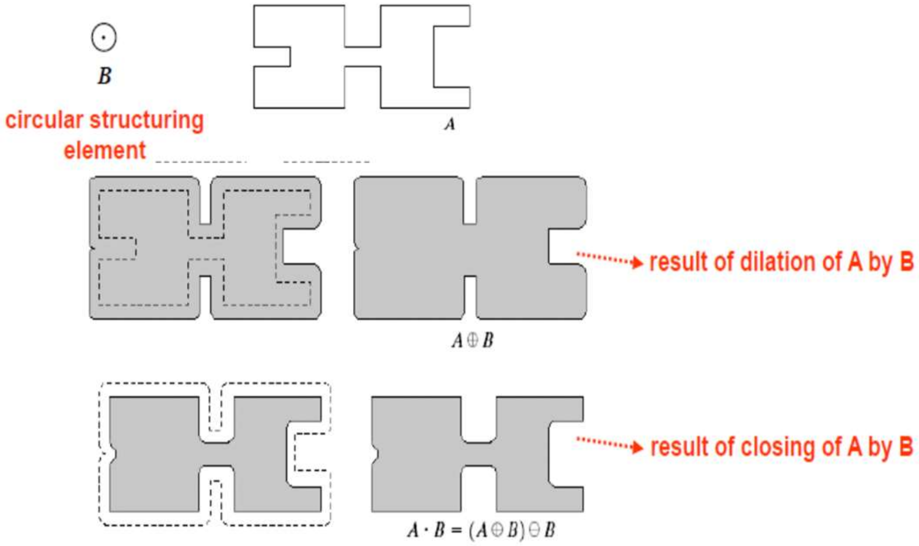
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

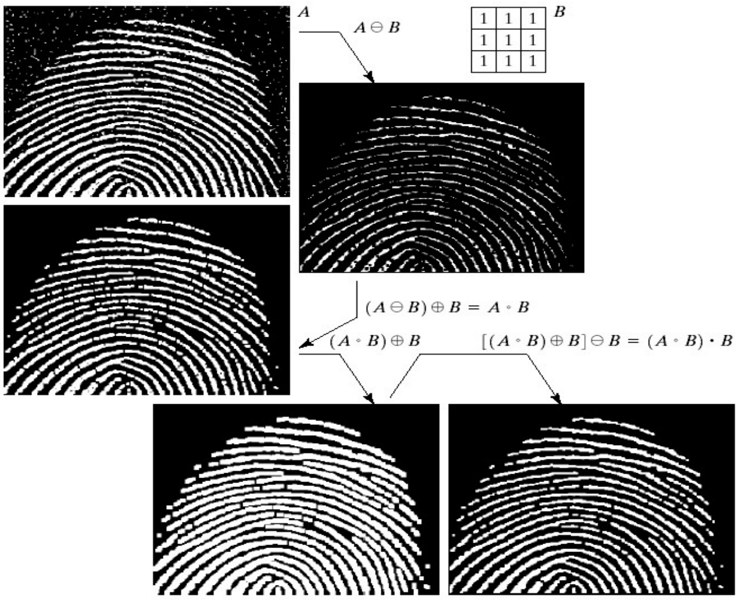
Closing Operation



Closing Operation



Mixed Example



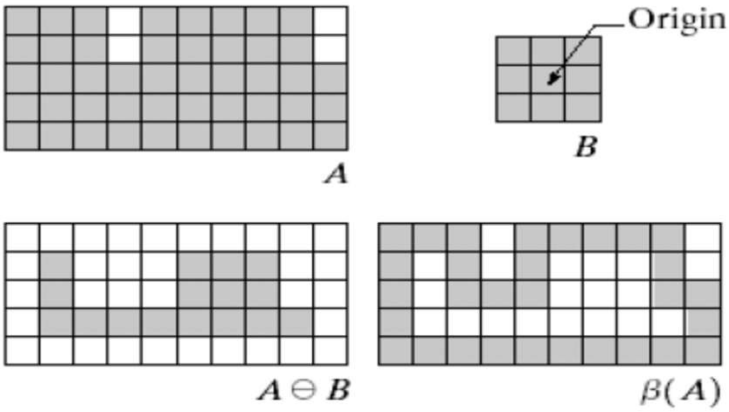
a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Opening and Closing

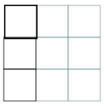
- Opening
  - – smooth contours
  - – break narrow isthmus
  - – eliminate narrow protrusions
- Closing
  - – smooth contours
  - – fuse breaks
  - – eliminate holes
  - – fill in small gaps

Boundary Extraction

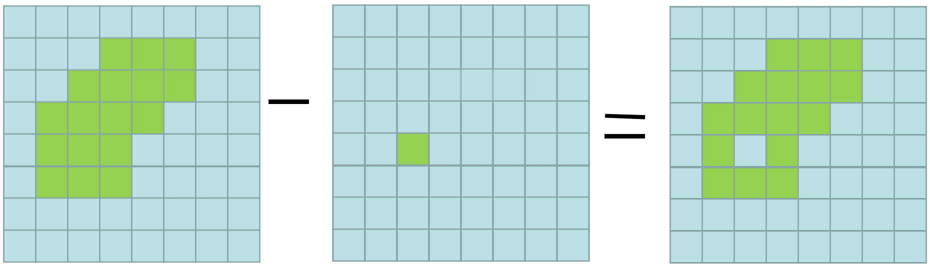


Boundary Extraction

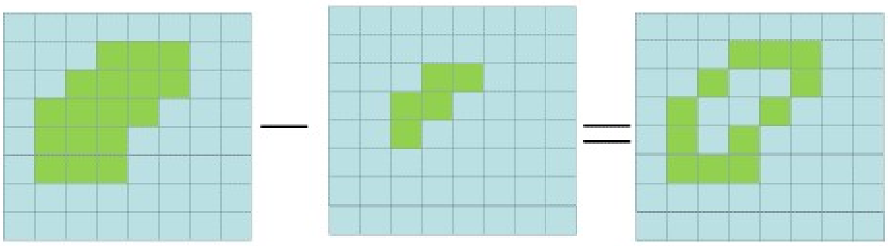
Note that thicker boundaries can be obtained by increasing the size of structuring element

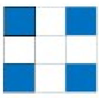


Boundary Extraction



Boundary Extraction





- Erode A by B
- Subtract from original

Region Filling

Region filling can be performed by using the following definition.

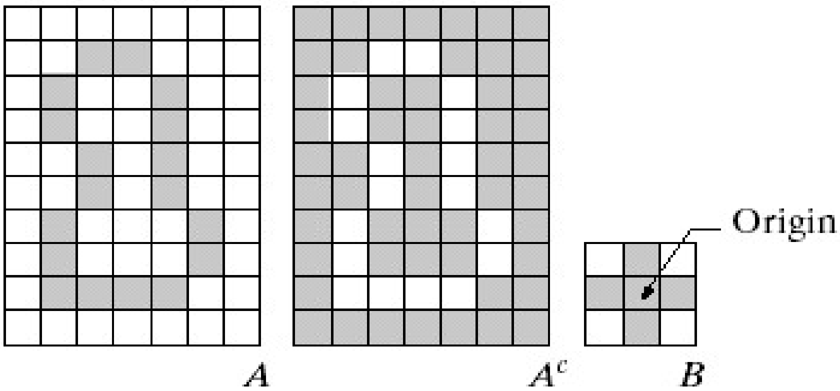
Given a symmetric structuring element B, one of the non-boundary pixels (Xk) is consecutively diluted and its intersection with the complement of A is taken as follows:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad \begin{aligned} k &= 1,2,3,\ldots \\ \text{terminates when } X_k &= X_{k-1} \\ X_0 &= 1 \ (\text{inner pixel}) \end{aligned}$$

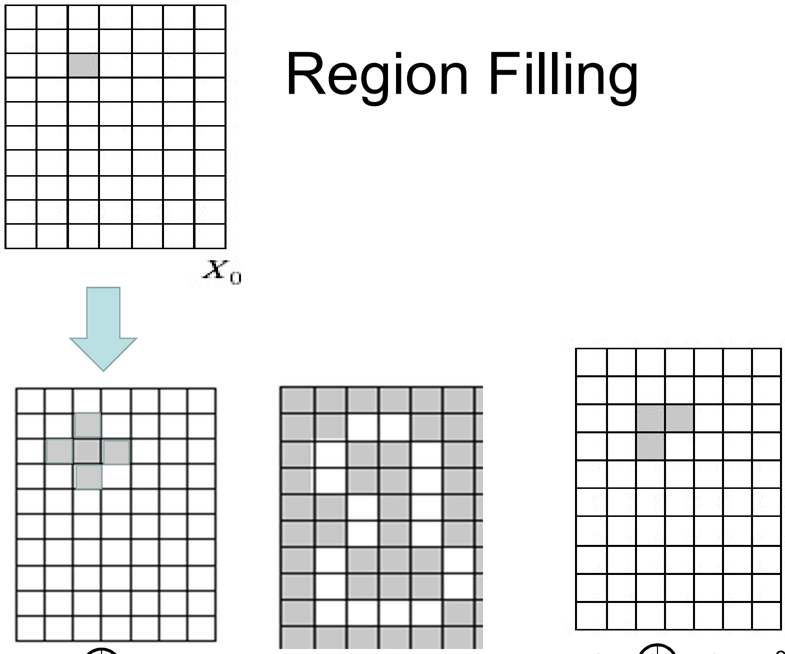
Following consecutive dilations and their intersection with the complement of A, finally resulting set is the filled inner boundary region and its union with A gives the filled region F(A)

$$F(A) = X_k \cup A$$

Region Filling

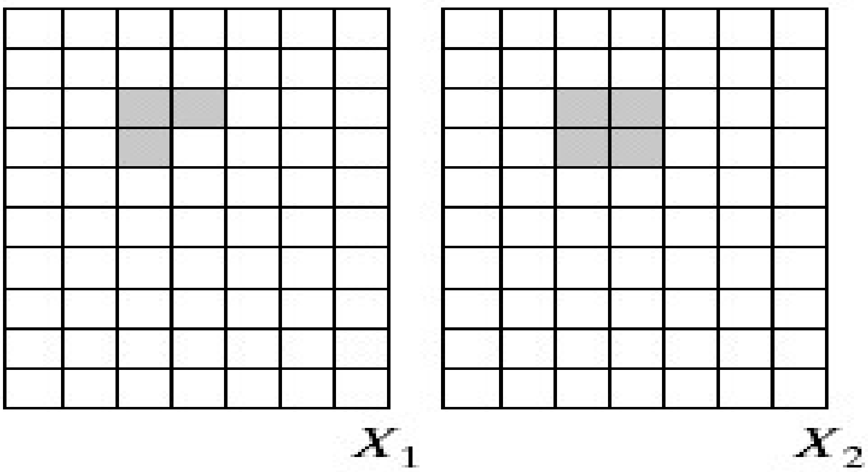


Region Filling

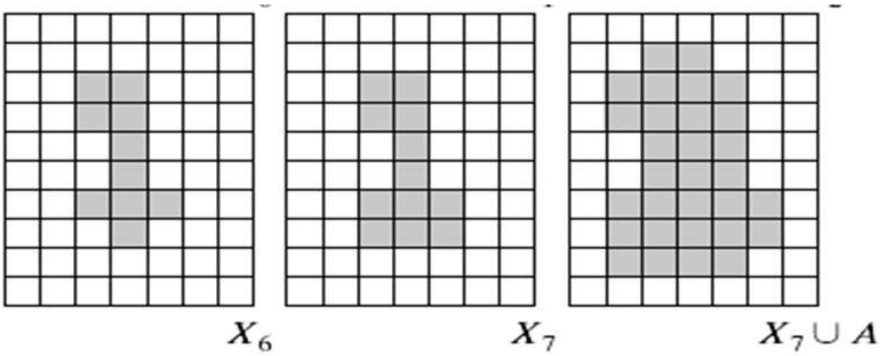


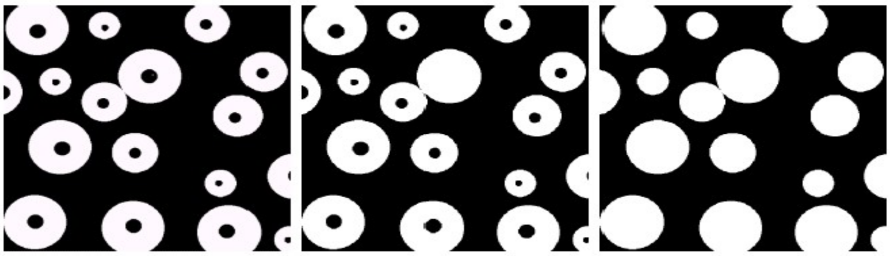
$X_o \oplus B$ $X1 = (X_o \oplus B) \cap A^c$

Region Filling



Region Filling





a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

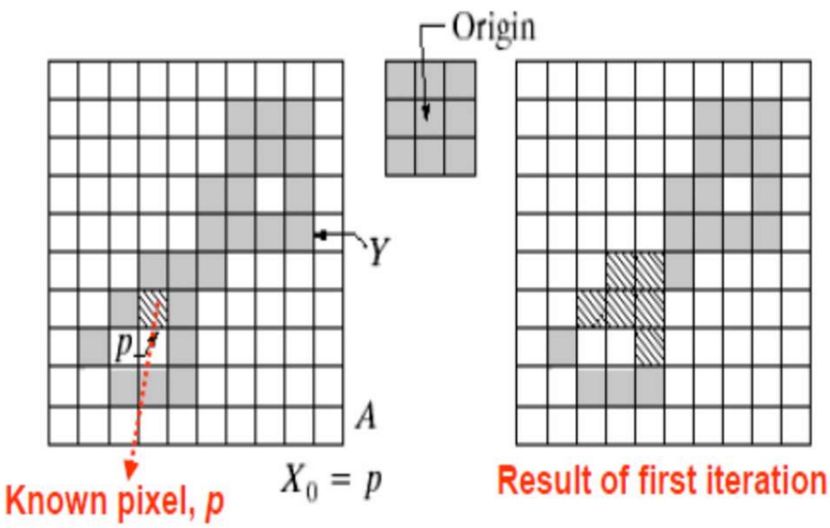
Finding the **starting** points is often done **manually**

Connected Components

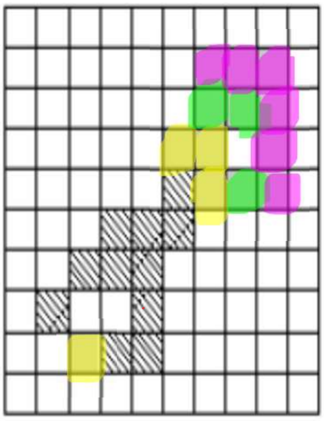
$$X_k = (X_{k-1} \oplus B) \bigcap A \qquad k = 1,2,3,\ldots \quad \text{terminates when } X_k = X_{k-1}$$

- X_0=1 corresponds to one of the pixels on the component Y. Note that one of the pixel locations on the component must be known.
- Consecutive dilations and their intersection with A, yields all elements of component Y.

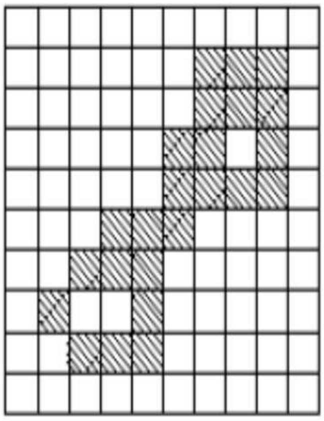
Connected Components



Connected Components



Result of second iteration

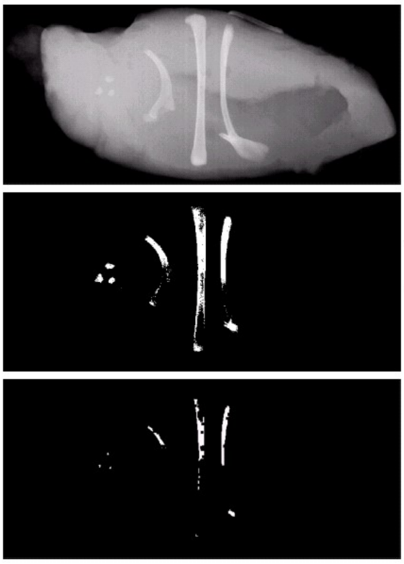


Result of last iteration

Connected components

- a
  b
  c d

  FIGURE 9.18
  (a) X-ray image of chicken filet with bone
- fragments.
  (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's.
  (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



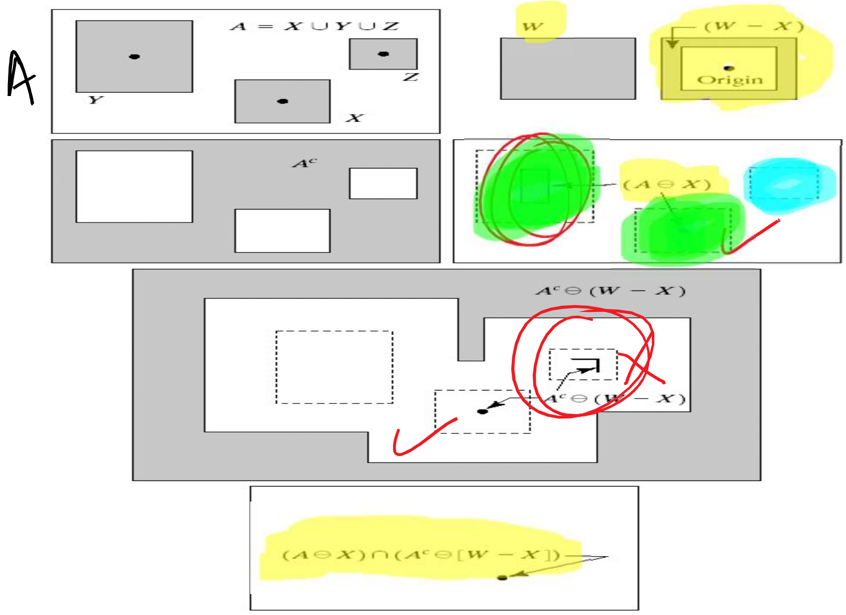
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Hit or Miss Transformation

Used to extract pixels with specific neighbourhood configurations from an image

- Set A has subsets X, Y, Z
- W is a window enclosing X
- W-X is the local background of X
- Erode A by X
- Erode A^c by W-X

Hit or Miss Transform



Hit or Miss Transform

Hit-or-Miss transform is given as

$$A \circledast B = (A \ominus X) \cap \left[A^c \ominus (W - X) \right]$$

where A = *Set in which we want to find the location of object* X

B = *Set composed of* X *and its background* W

$B = (\underline{X, W-X})$

Hit or Miss Transform

We can also write Hit-or-Miss transform as

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

where B_1 = *Object* *and* B_2 = *background*

Hit or Miss Transform

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

A

0	0	0
0	1	0
0	1	0

B_1

0	1	0
0	0	0
0	0	0

B_2

Hit or Miss Transform

STEP 1: Erode A by B_1

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

A

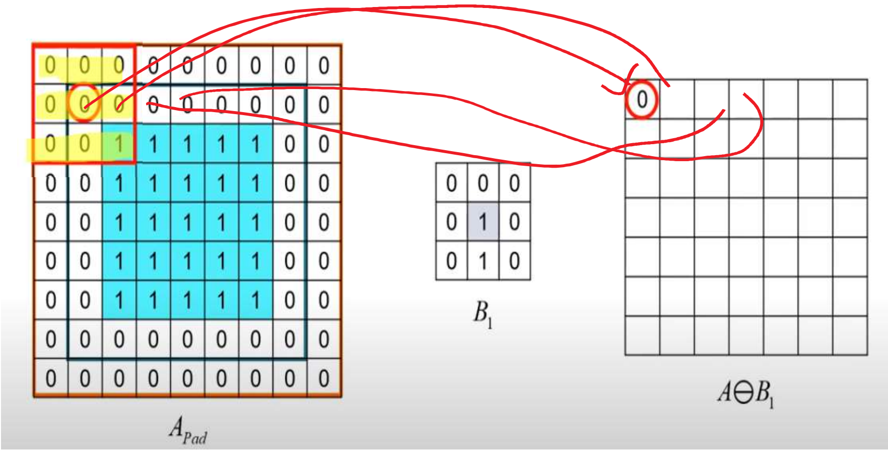
0	0	0
0	1	0
0	1	0

B_1

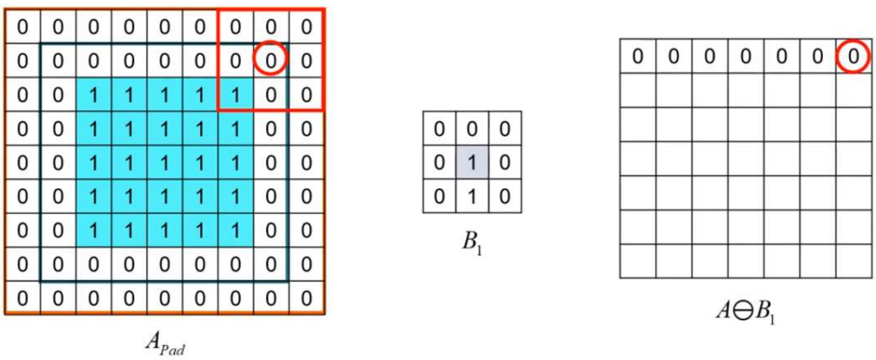
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

A_{Pad}

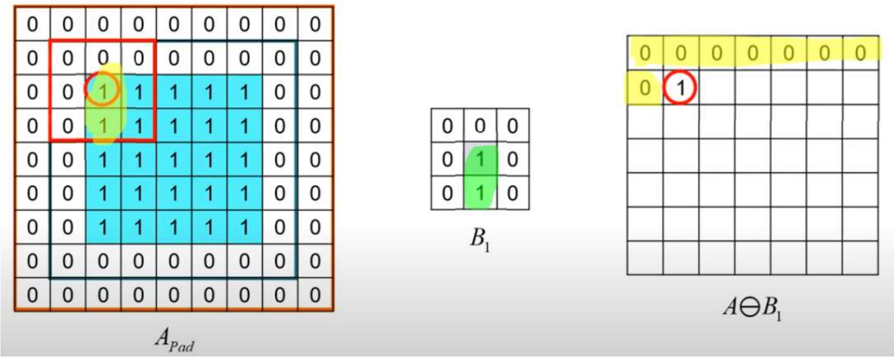
Hit or Miss Transform



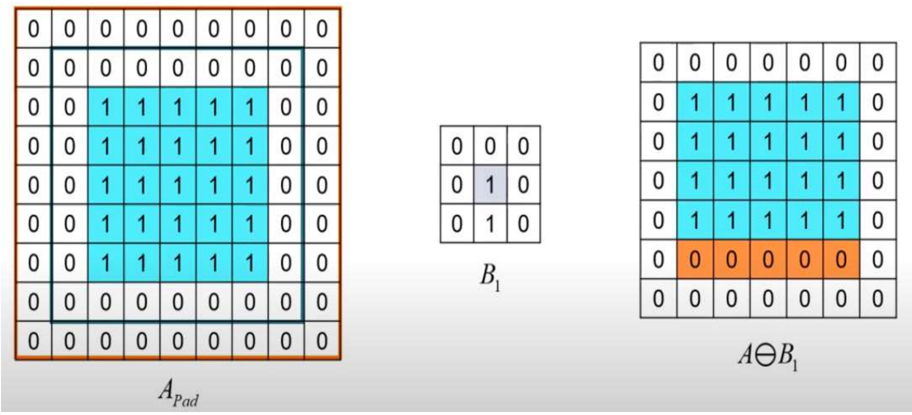
Hit or Miss Transform



Hit or Miss Transform



Hit or Miss Transform



Hit or Miss Transform

We can also write Hit - or - Miss transform as

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

where $B_1 = Object$ *and* $B_2 = background$

Hit or Miss Transform

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

A^c

0	1	0
0	0	0
0	0	0

B_2

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

A^c_{Pad}

Hit or Miss Transform

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

$A^c{}_{Pad}$

0	1	0
0	0	0
0	0	0

B_2

0						

$A^c \ominus B_2$

Hit or Miss Transform

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

$A^c{}_{Pad}$

0	1	0
0	0	0
0	0	0

B_2

0	0	0	0	0	0	0
1	1	1	1	1	1	1

$A^c \ominus B_2$

Hit or Miss Transform

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

$A^c{}_{Pad}$

0	1	0
0	0	0
0	0	0

B_2

0	0	0	0	0	0	0
1						

$A^c \ominus B_2$

Hit or Miss Transform

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	1	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0

$A^c{}_{Pad}$

0	1	0
0	0	0
0	0	0

B_2

0	0	0	0	0	0	0
1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1

$A^c \ominus B_2$

Hit or Miss Transform

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$A \ominus B_1$

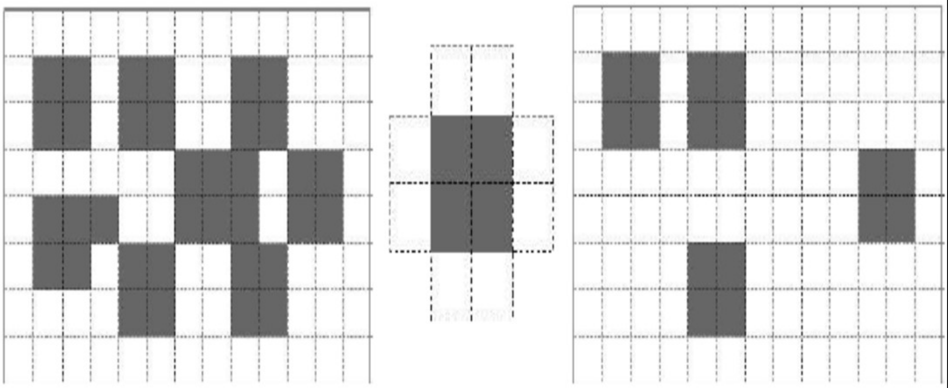
0	0	0	0	0	0	0
1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1

$A^c \ominus B_2$

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

Hit or Miss Transform



Hit or Miss Transform

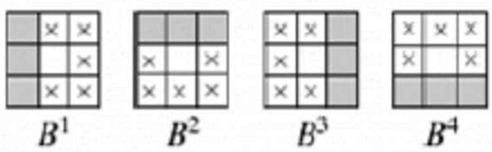
- A background is necessary to detect disjoint sets
- When we only aim to detect certain patterns within a set, a background is not required, and simple erosion is sufficient

Convex Hull

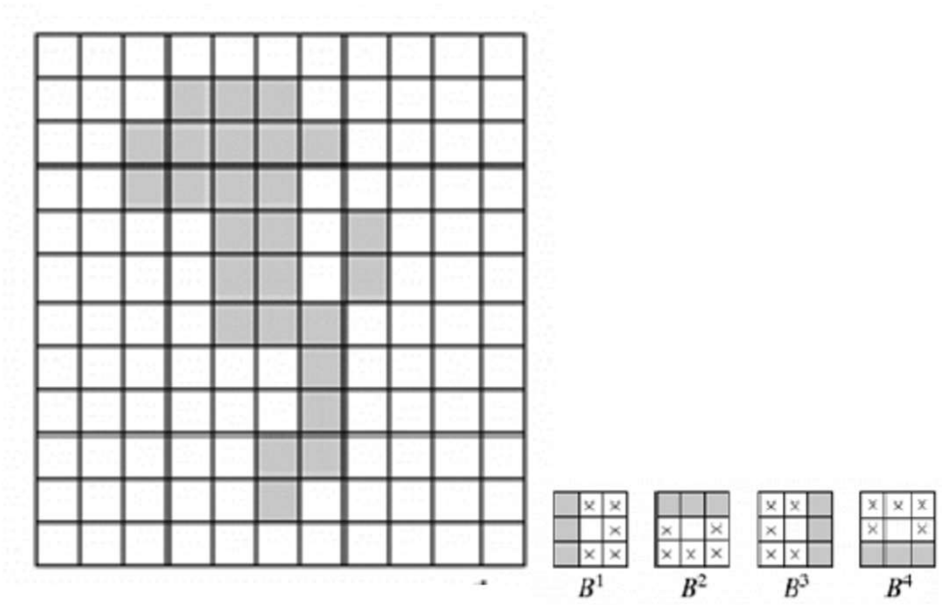
$$X_k^i = (X_{k-1} \circledast B^i) \cup A, \; i = 1,2,3,4, \; k = 1,2,\ldots, \; X_0^i = A$$

Now let $D^i = X_{\text{conv}}^i$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

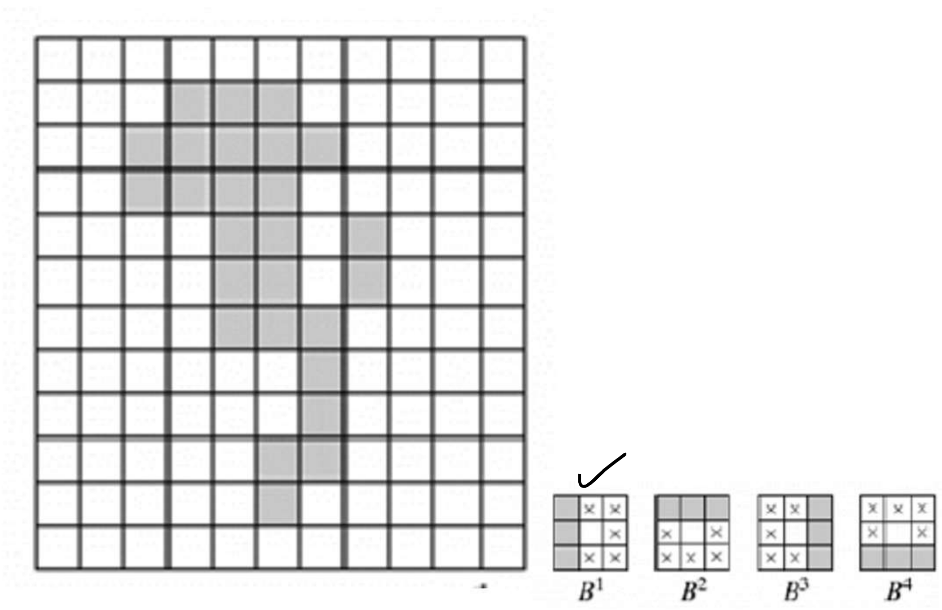
$$C(A) = \cup_{i=1}^{4} D^i$$



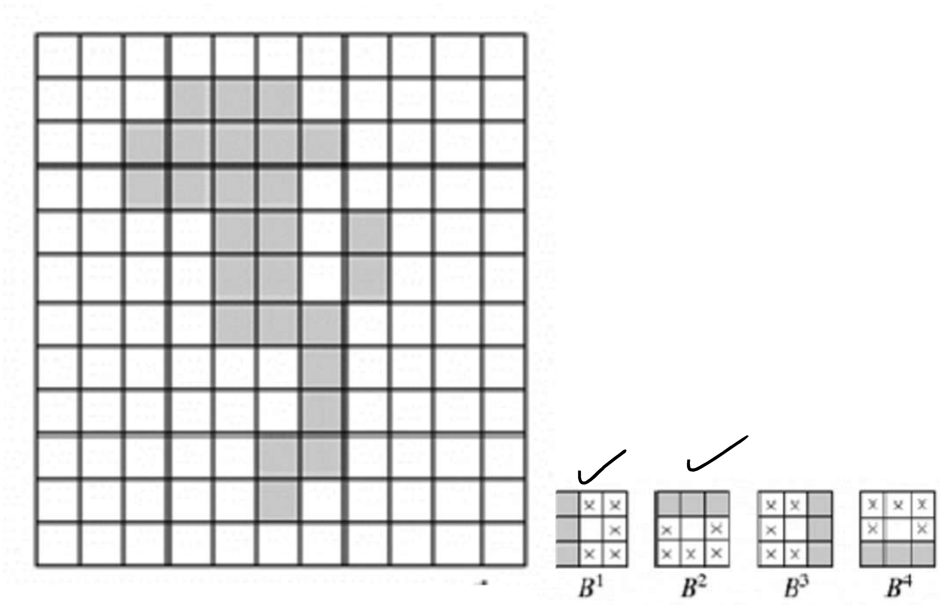
Convex Hull



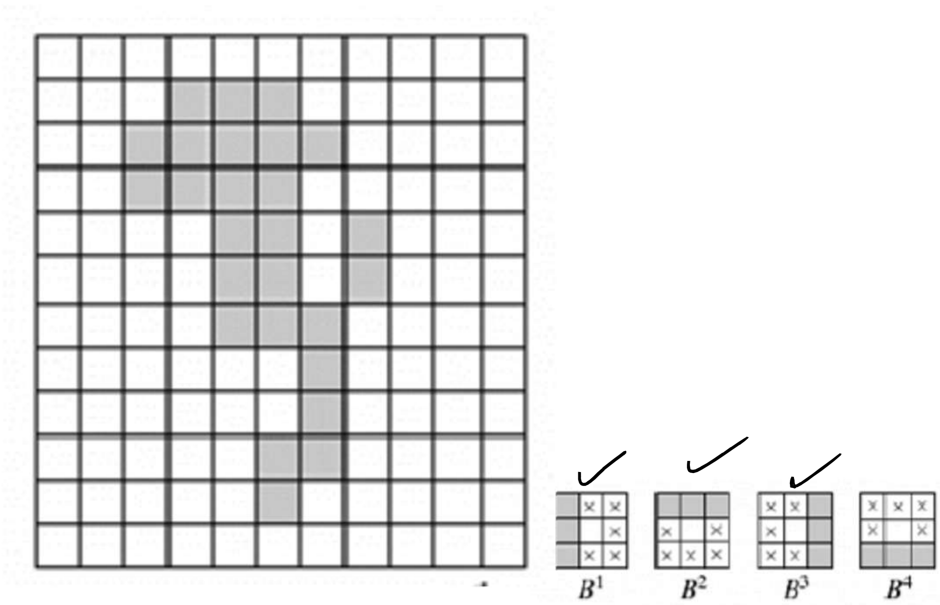
Convex Hull



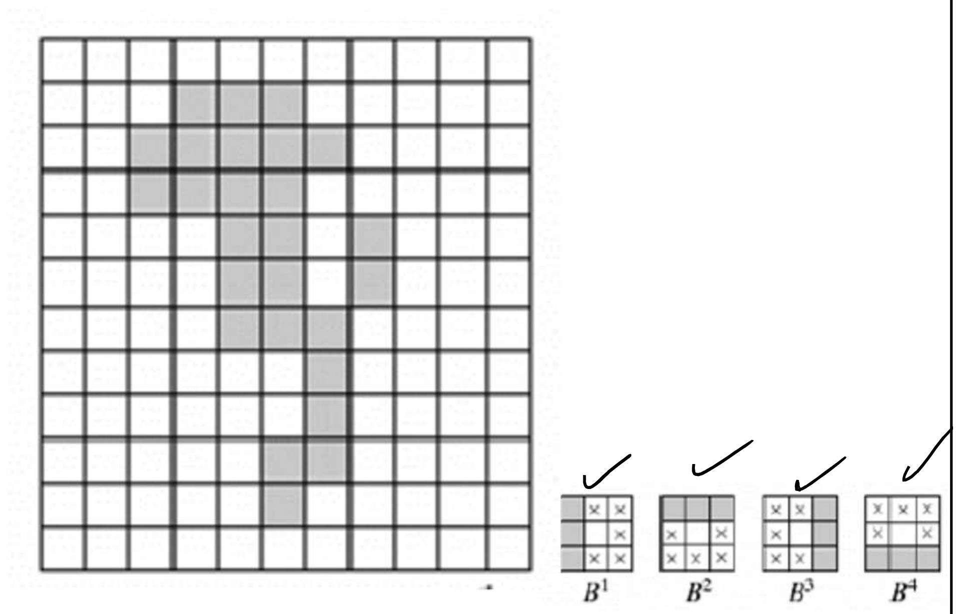
Convex Hull



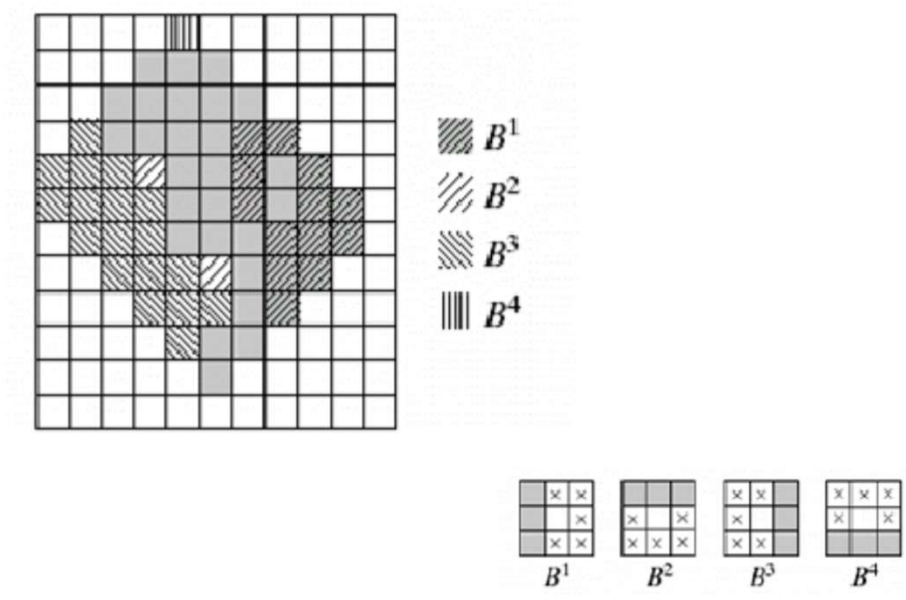
Convex Hull



Convex Hull



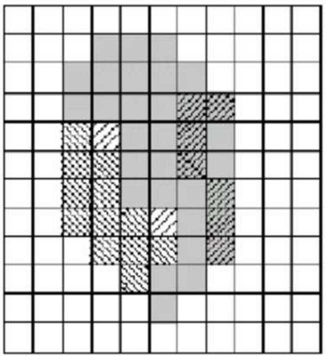
Convex Hull



Convex Hull

Shortcoming of above algorithm: convex hull can grow beyond the minimum dimensions required to guarantee convexity

Possible solution: Limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points



Skeletons

- Compact or minimal representation of objects in an image while retaining homotopy of the image
- As stated earlier, the skeletons of objects in an image can be found by successive thinning until stability
- The thinning cannot be executed in parallel since this may cause the homotopy of the image to change
- Example:

Skeletons

- The skeleton of an object is often defined as the medial axis of that object.
  - Pixels are then defined to be skeleton pixels if they have more than one "closest neighbours".
- Some skeleton algorithms are based on this definition and are computed through the distance transform
- Other algorithms produce skeletons that are smaller than the defined medial axis (such as minimal skeletons)

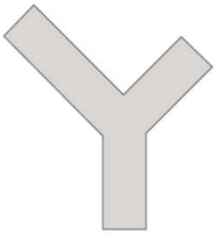
Skeletons

A skeleton, $S(A)$ of a set A has the following properties

a. if z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk.

b. The disk $(D)_z$ touches the boundary of A at two or more different places.

Skeletons



a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

Skeletons

The skeleton of A can be expressed in terms of erosion and openings.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $K = \max\{k \mid A \ominus kB \neq \phi\}$;

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and

$$(A \ominus kB) = ((..((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k successive erosions of A.

Skeletons

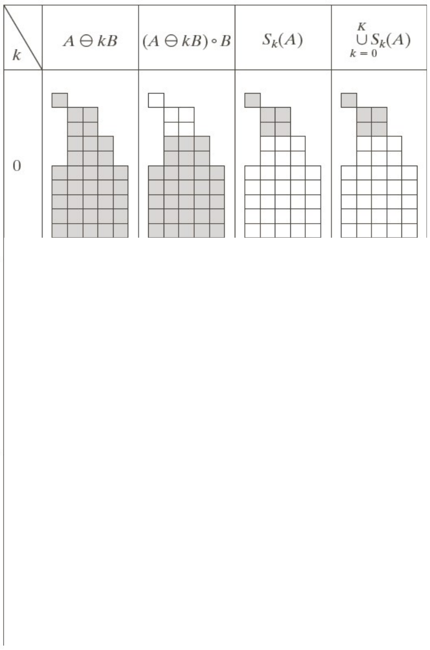
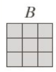


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



Skeletons

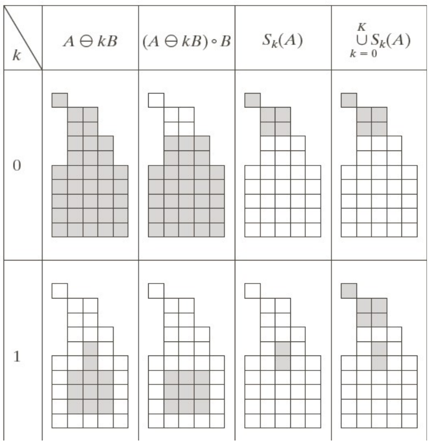
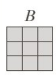


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



Skeletons

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((...((S_k(A) \oplus B) \oplus B)... \oplus B)$$

Skeletons

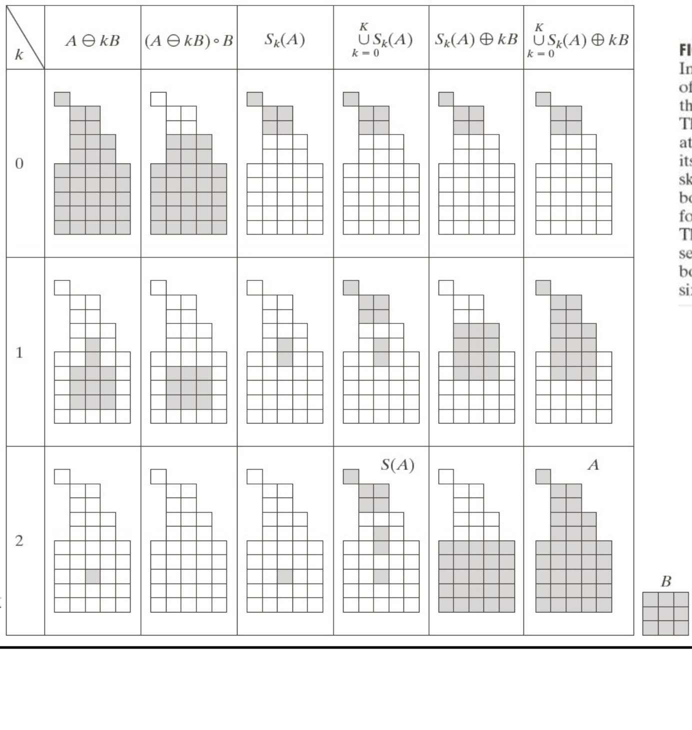


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Thinning

- The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

Thinning

$$A \otimes B = A - (A \circledast B)$$

hit-or-miss transform/template matching

• *Note that we are only interested in pattern matching of B in A, so no background operation is required of the hit-miss-transform.*

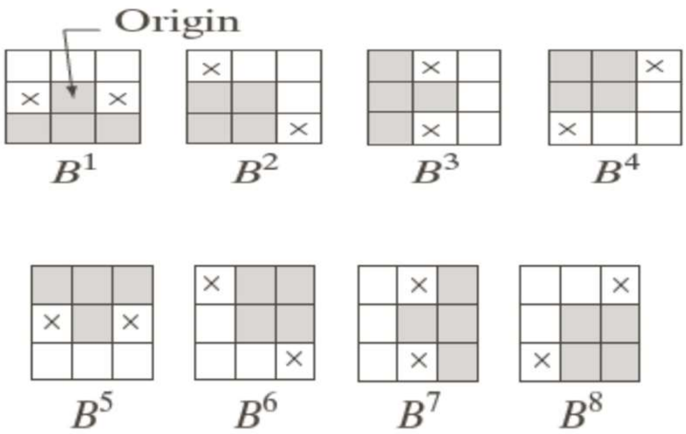
$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

•*The structuring element B consists of a sequence of structuring elements, where B^i is the rotated version of B^{i-1}. Each structuring elements helps thinning in one direction. If there are 4 structuring elements thinning is performed from 4 directions separated by 90°. If 8 structuring elements are used the thinning is performed in 8 directions separated by 45°.*

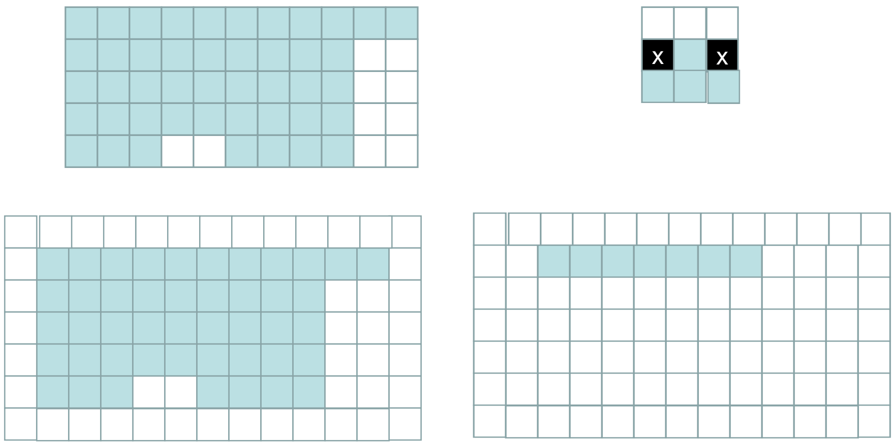
•*The process is to thin A by one pass with B^1, then the result with one pass of B^2, and continue until A is thinned with one pass of B^n.*

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

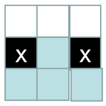
Thinning

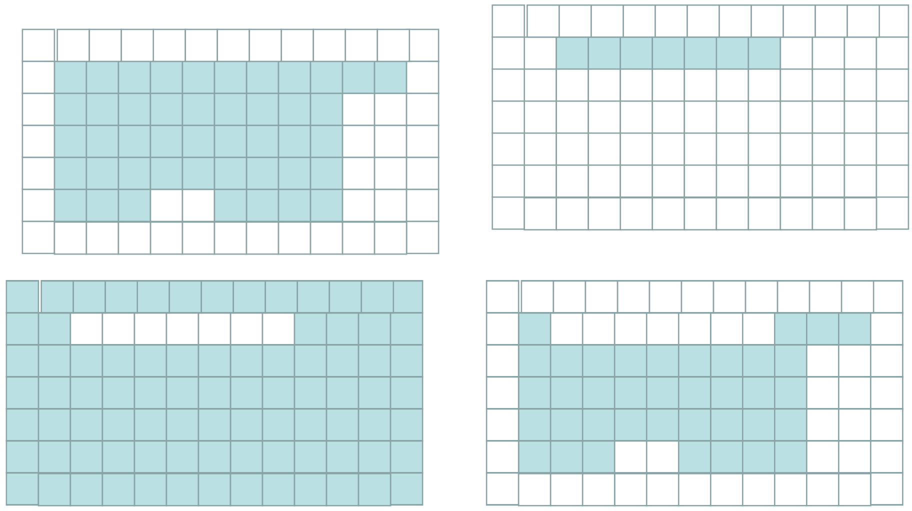


Thinning

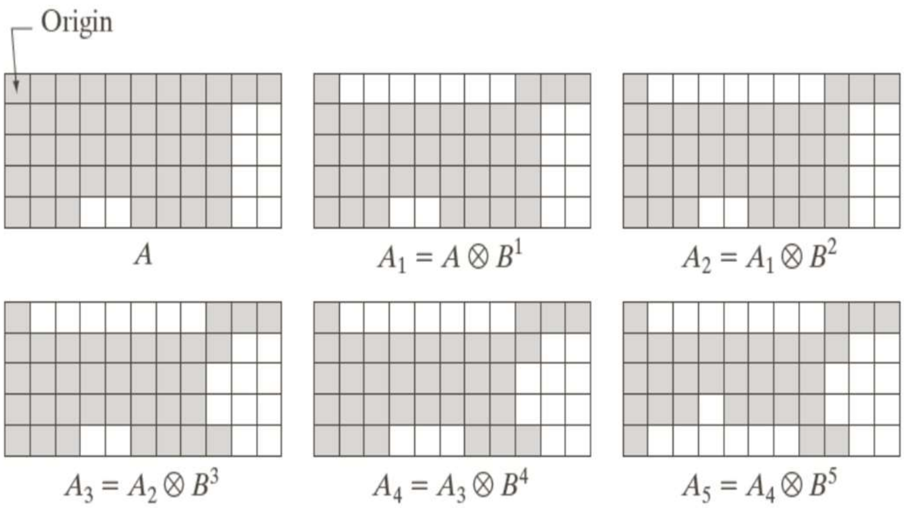


Thinning

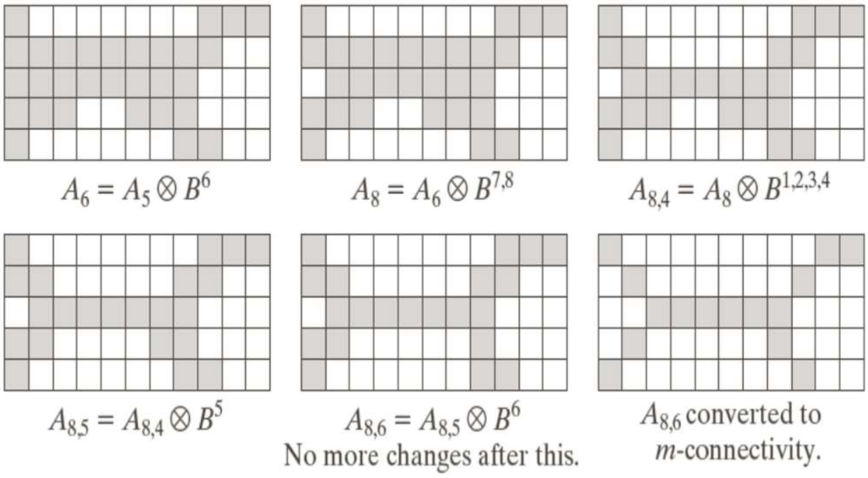




Thinning



Thinning



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Thickening

The thickening is defined by the expression

$$A \odot B = A \cup \left(A \circledast B \right)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

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Thickening

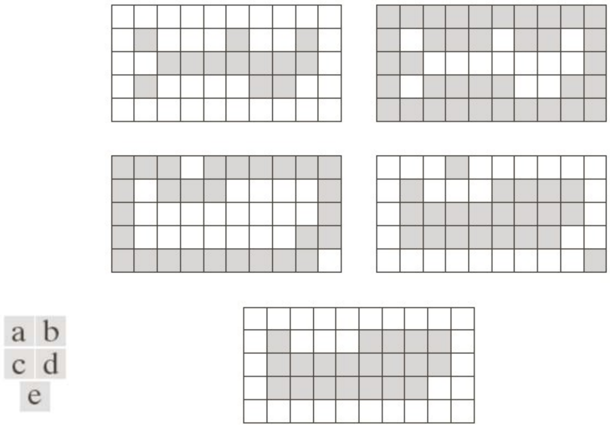

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.