

Calculate Model parameters

ID	1	2	3	4	5	6	7	8	9	10	11	12
X_1	0	1	2	0	1	2	0	1	2	0	1	2
X_2	0	1	0	0	1	1	0	0	0	1	1	1
Y	0	0	1	1	2	2	0	0	1	1	2	2

Parameters :

Prior : $P(Y) :$ $P(Y=0)$, $P(Y=1)$, $P(Y=2)$

$\frac{4}{12}$ $\frac{4}{12}$ $\frac{4}{12}$

Y	0	1	2
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Likelihood:

$P(X_1/Y) :$ $P(X_1=0/Y=0)$, $P(X_1=1/Y=0)$, $P(X_1=2/Y=0)$

$P(X_1=0/Y=1)$, $P(X_1=1/Y=1)$, $P(X_1=2/Y=1)$

$P(X_1=0/Y=2)$, $P(X_1=1/Y=2)$, $P(X_1=2/Y=2)$

$X_1 \backslash Y$	0	1	2
0	$\frac{2}{4}$	$\frac{2}{4}$	0
1	$\frac{2}{4}$	0	$\frac{2}{4}$
2	0	$\frac{2}{4}$	$\frac{2}{4}$

$P(X_2/Y) :$ $P(X_2=0/Y=0)$ $P(X_2=1/Y=0)$ $P(X_2=2/Y=0)$

$P(X_2=0/Y=1)$ $P(X_2=1/Y=1)$ $P(X_2=2/Y=1)$

$P(X_2=0/Y=2)$ $P(X_2=1/Y=2)$ $P(X_2=2/Y=2)$

$X_2 \backslash Y$	0	1	2
0	$\frac{3}{4}$	$\frac{3}{4}$	0
1	$\frac{1}{4}$	$\frac{1}{4}$	1

Inference Question

$P(Y=1/X_1=1, X_2=1) ?$

$= \frac{P(X_1=1, X_2=1, Y=1)}{P(X_1=1, X_2=1)}$ { Applied Bayes

$$\begin{aligned}
 &= \frac{P(x_1=1, x_2=1/y=1) \cdot P(y=1)}{P(x_1=1, x_2=1)} \quad \left. \begin{array}{l} \text{Bayes} \\ \text{Rule} \end{array} \right\} \\
 &= \frac{P(x_1=1/y=1) \cdot P(x_2=1/y=1) \cdot P(y=1)}{P(x_1=1, x_2=1)} \quad \left. \begin{array}{l} x_1 \& \\ x_2 \\ \text{are} \\ \text{IID} \end{array} \right\} \\
 &\quad \textcircled{A}, \textcircled{B}, \textcircled{C} \leftarrow \text{From CPT} \\
 &\quad \quad \quad \text{(model parameters)} \\
 &= \frac{(0) \cdot (1/4) \cdot (1/3) + 1}{\textcircled{D} + (1/3 + 1)}
 \end{aligned}$$

$$\textcircled{D} \ P(x_1=1, x_2=1)$$

Marginalize γ

$$\begin{aligned}
 &P(x_1=1/y=0) \cdot P(x_2=1/y=0) \cdot P(y=0) + P(x_1=1/y=1) \cdot P(x_2=1/y=1) \cdot P(y=1) \\
 &= P(x_1=1, x_2=1, y=0) + P(x_1=1, x_2=1, y=1) \\
 &\quad + P(x_1=1, x_2=1, y=2) \\
 &= 0 + 1/24 + 1/6
 \end{aligned}$$

$$= 5/24$$

Denominator is same for
Probability for all outcomes of γ

$$P(\gamma=1/x_1=1, x_2=1) = 0/(5/24) = \frac{0 \times 24}{5} = \underline{0}$$

$$P(\gamma=0/x_1=1, x_2=1) = (1/24)/(5/24) = 1/5$$

Numerator $P(x_1=1, x_2=1, y=0) = 1/24$

$$P(\gamma=2/x_1=1, x_2=1) = (4/24)/(5/24) = 4/5 \checkmark$$

= 4/24

① Why people often avoid calculating denominator?

MLE: Maximum Likelihood Estimation

New knowledge

MAP

Maximum A Posterior

Observation

$P(B)$

Knowledge

"class prior can influence the decision"

Multinomial

Binary / Nominal

What if data is continuous?

Assume
Normal
Distribution

We have calculated for binary or nominal attributes only

- e.g. temp, pressure

How to deal with 'zero' probability?

- Apply smoothing technique