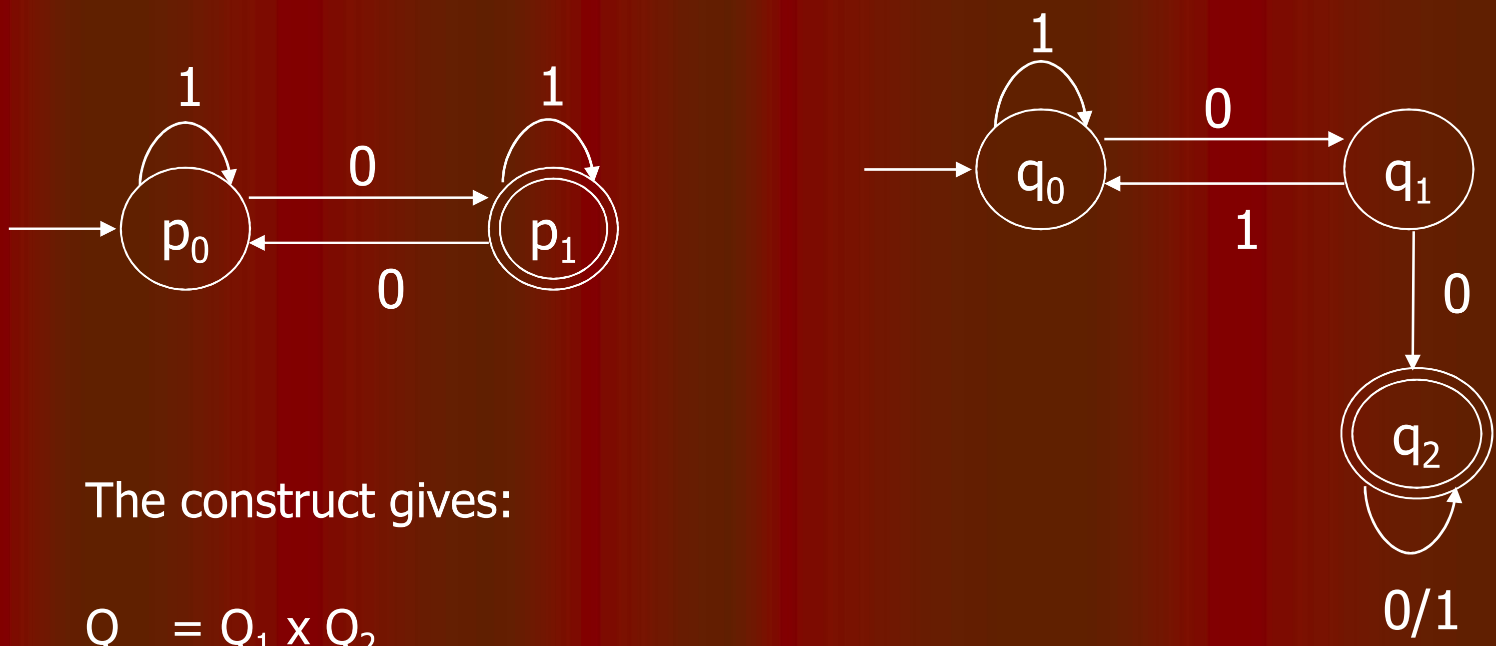
Theory of Automata & Formal Languages

(Theory of Computation)

RLs: Intersection, Union & Complement

RLs are closed Under Intersection



The construct gives:

Q $= Q_1 \times Q_2$
$= \{[p_0, q_0], [p_0, q_1], [p_0, q_2], [p_1, q_0], [p_1, q_1], [p_1, q_2]\}$

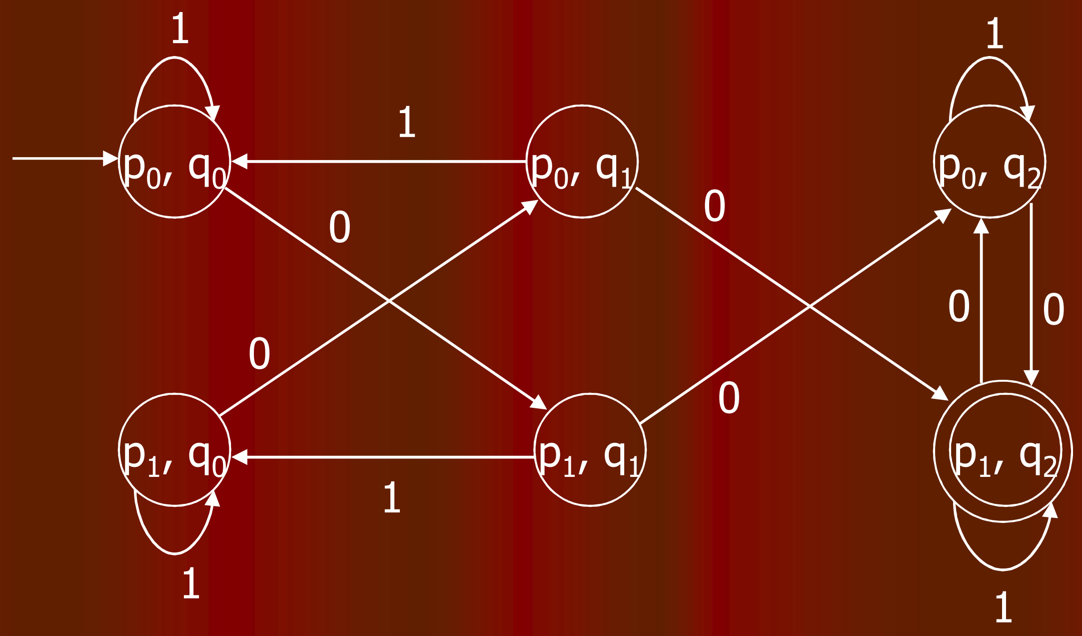
$\Sigma = \{0, 1\}$

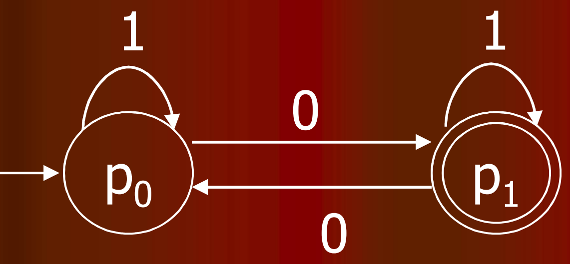
F $= \{[p_1, q_2]\}$

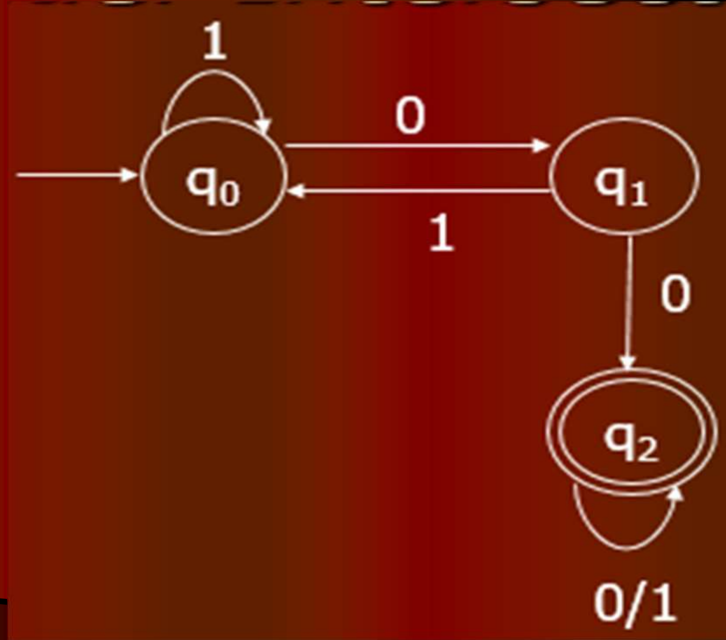
start state $= [p_0, q_0]$

δ – Some examples: $\delta([p_0, q_2], 0) = [\delta_1(p_0, 0), \delta_2(q_2, 0)] = [p_1, q_2]$
$\delta([p_1, q_2], 1) = [\delta_1(p_1, 1), \delta_2(q_2, 1)] = [p_1, q_2]$

- **Final Result:**







- A direct construction of a DFA M such that $L(M) = L_1 \cap L_2$:

- Let:

  $M_1 = (Q_1, \Sigma, \delta_1, p_0, F_1)$, where $Q_1 = \{p_0, p_1, \ldots\}$
  $M_2 = (Q_2, \Sigma, \delta_2, q_0, F_2)$, where $Q_2 = \{q_0, q_1, \ldots\}$

  where $L(M_1) = L_1$ and $L(M_2) = L_2$.

- Construct M where:

  $Q = Q_1 \times Q_2$ Note that M has a state for each
  $= \{[p_0, q_0], [p_0, q_1], [p_0, q_2], \ldots\}$ pair of states in M_1 and M_2

  Σ = as with M_1 and M_2

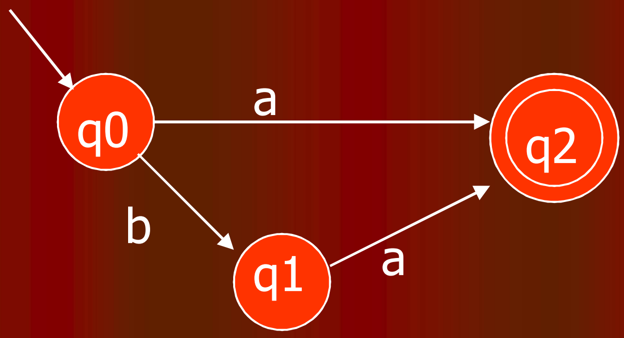
  $F = F_1 \times F_2$

  start state = $[p_0, q_0]$

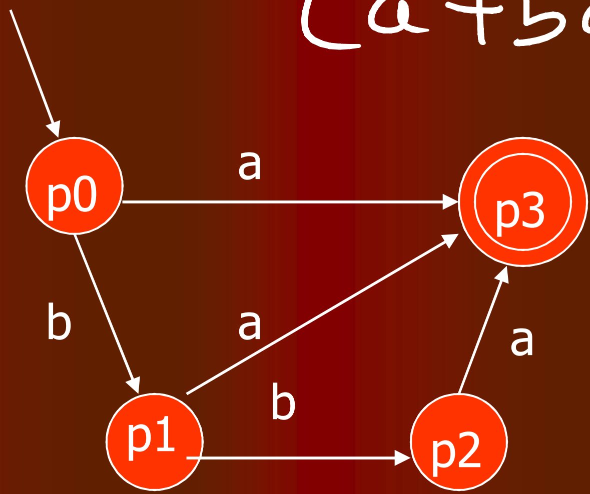
  $\delta([p_i, q_j], a) = [\delta_1(p_i, a), \delta_2(q_j, a)]$ for all $[p_i, q_j]$ in Q and a in Σ

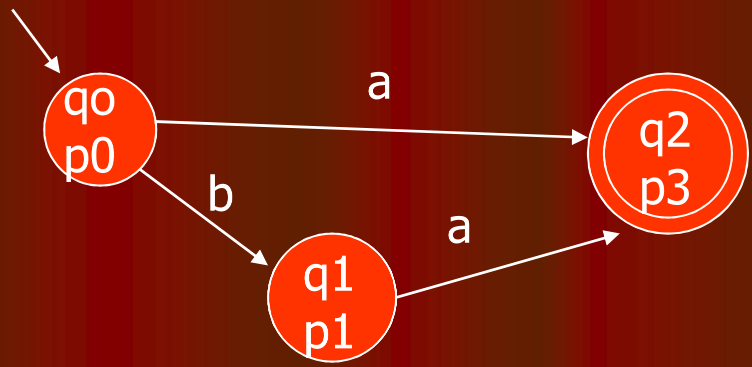
RLs are closed Under Intersection

+ba)



(a+ba+bba)





(a+ba)

Closure Under Intersection

DeMorgan's Law: $\overline{L1 \cap L2} = \overline{L1}$ U $\overline{L2}$

L1 and L2 are regular

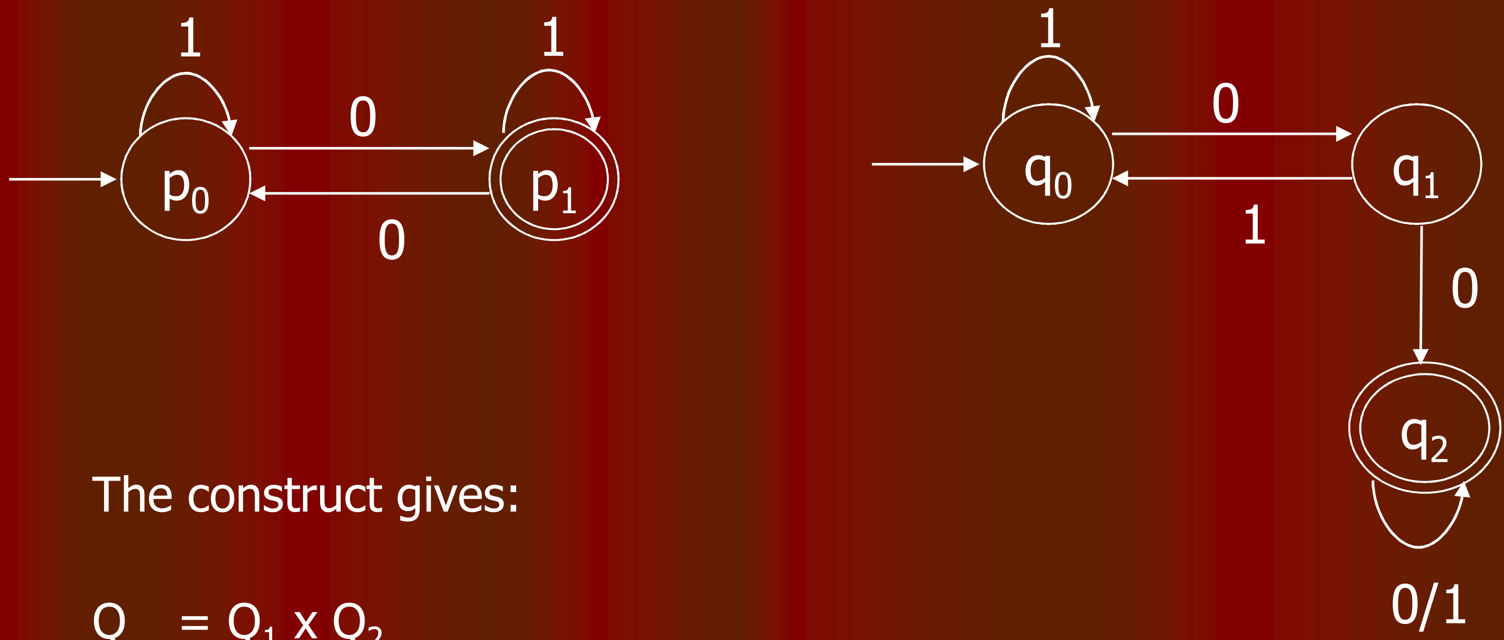
So $\overline{L1}$ and $\overline{L2}$ are regular (Closure under complementation)

So $\overline{L1}$ U $\overline{L2}$ is regular (Closure under union)

So $\overline{\overline{L1} \text{ U } \overline{L2}}$ is regular. (Closure under comp.)

So L1 $\cap$ L2 is regular.

RLs are closed Under Union



The construct gives:

$Q = Q_1 \times Q_2$
$= \{[p_0, q_0], [p_0, q_1], [p_0, q_2], [p_1, q_0], [p_1, q_1], [p_1, q_2]\}$

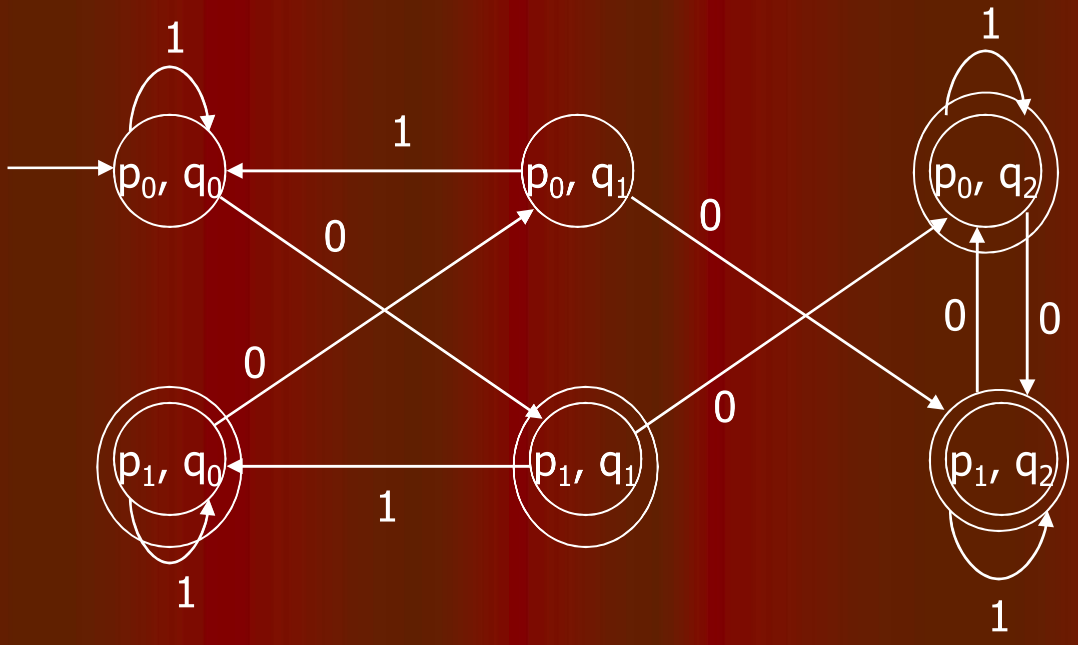
$\Sigma = \{0, 1\}$

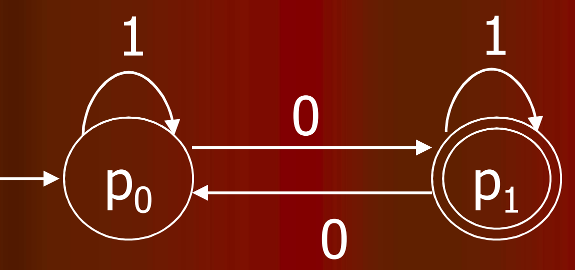
$F = \{[p_1, q_2], [p_1, q_0], [p_1, q_1], [p_0, q_2]\}$

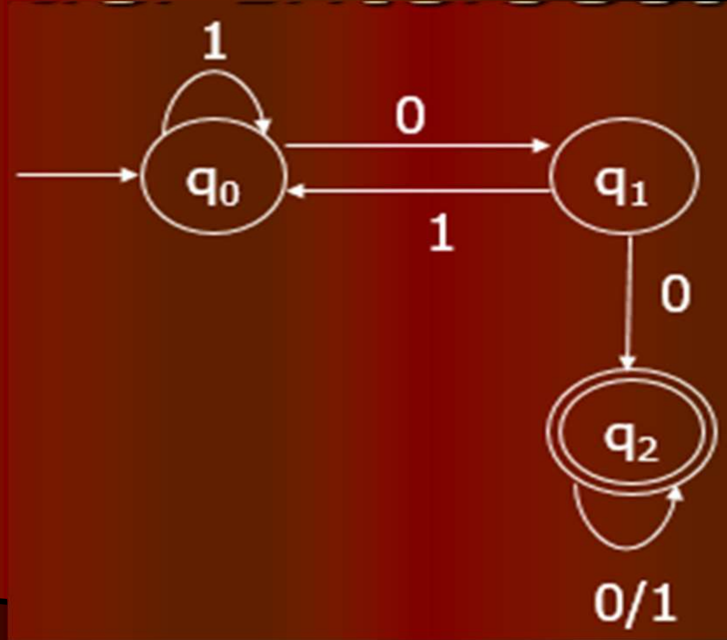
start state = $[p_0, q_0]$

δ – Some examples: $\delta([p_0, q_2], 0) = [\delta_1(p_0, 0), \delta_2(q_2, 0)] = [p_1, q_2]$
$\delta([p_1, q_2], 1) = [\delta_1(p_1, 1), \delta_2(q_2, 1)] = [p_1, q_2]$

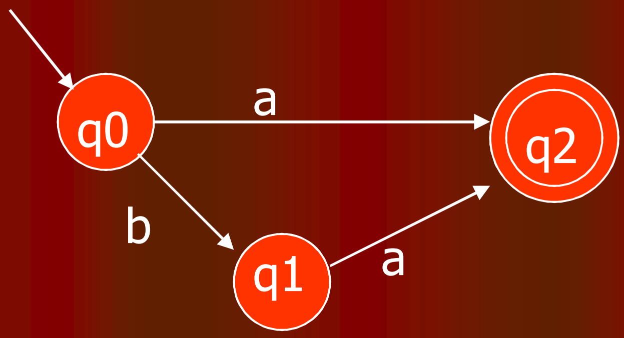
- Final Result:

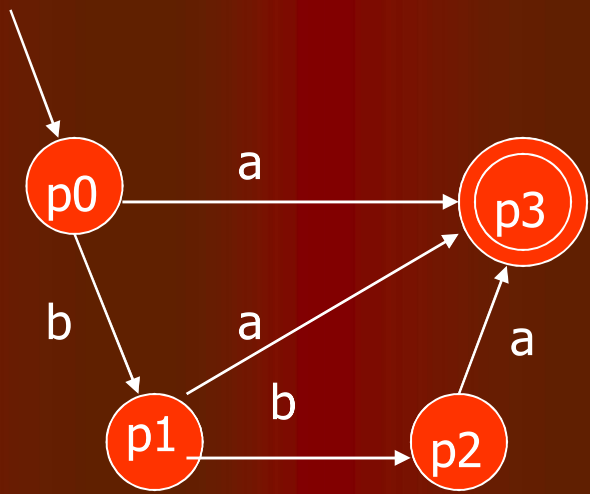






RLs are closed Under Union

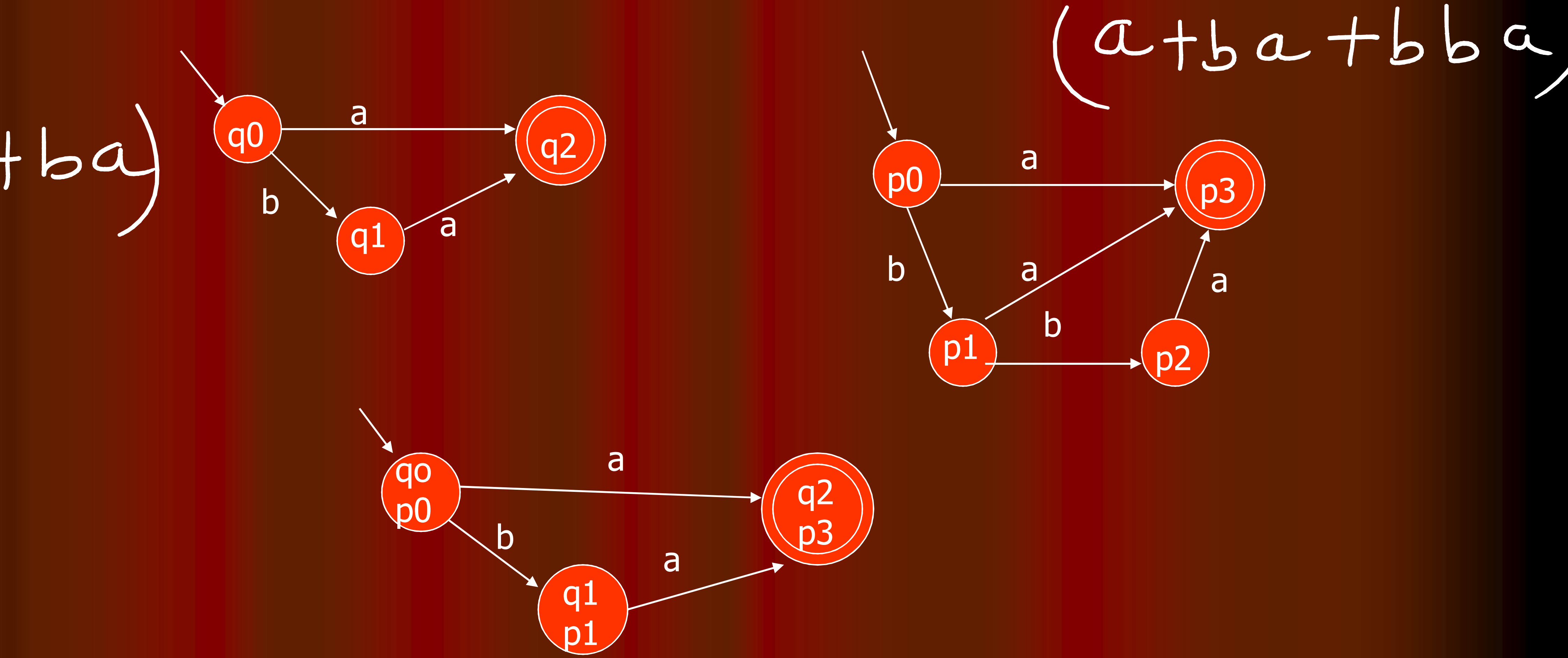




$(a+ba)$

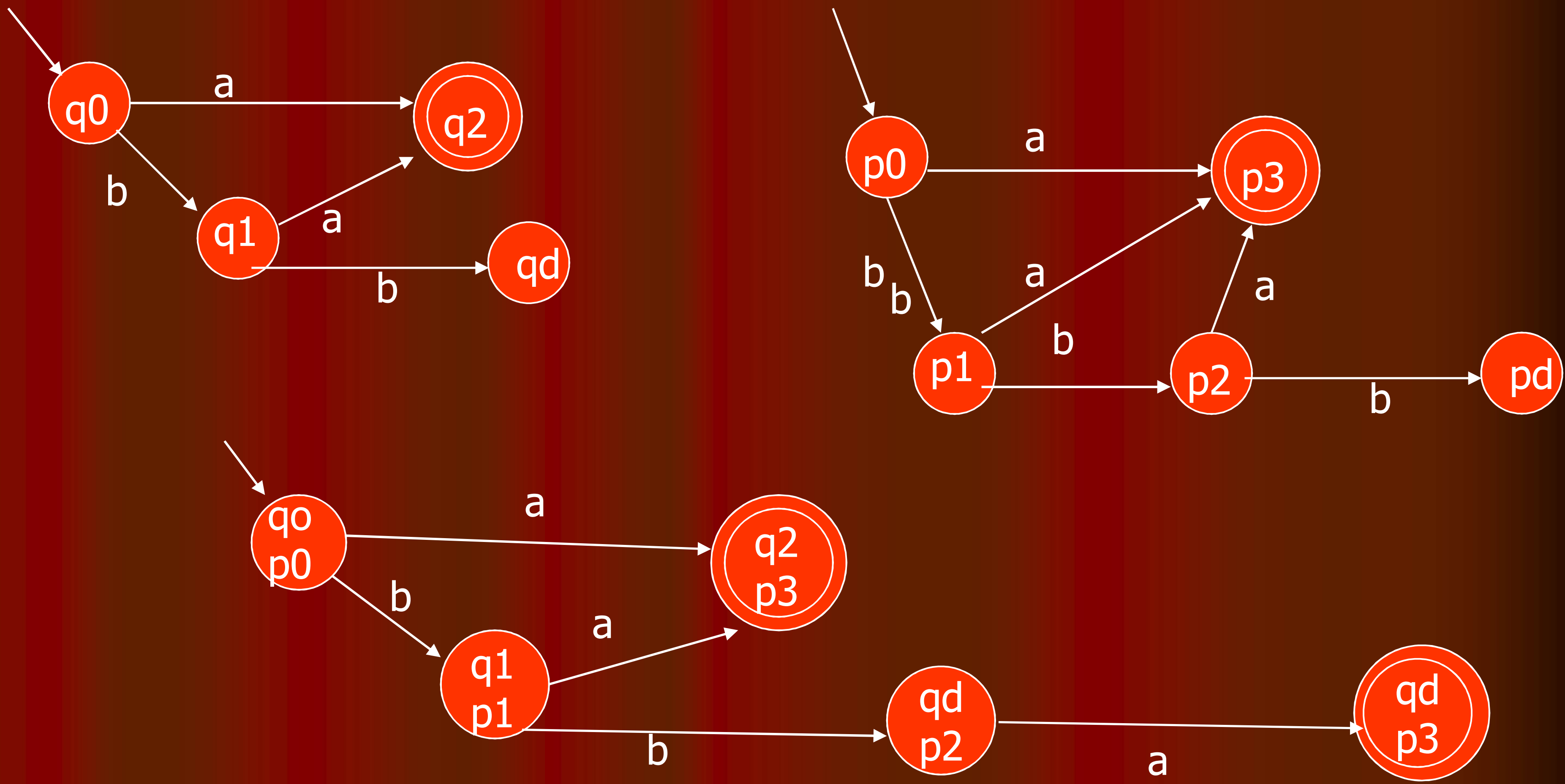
$(a+ba+bba)$

RLs are closed Under Union



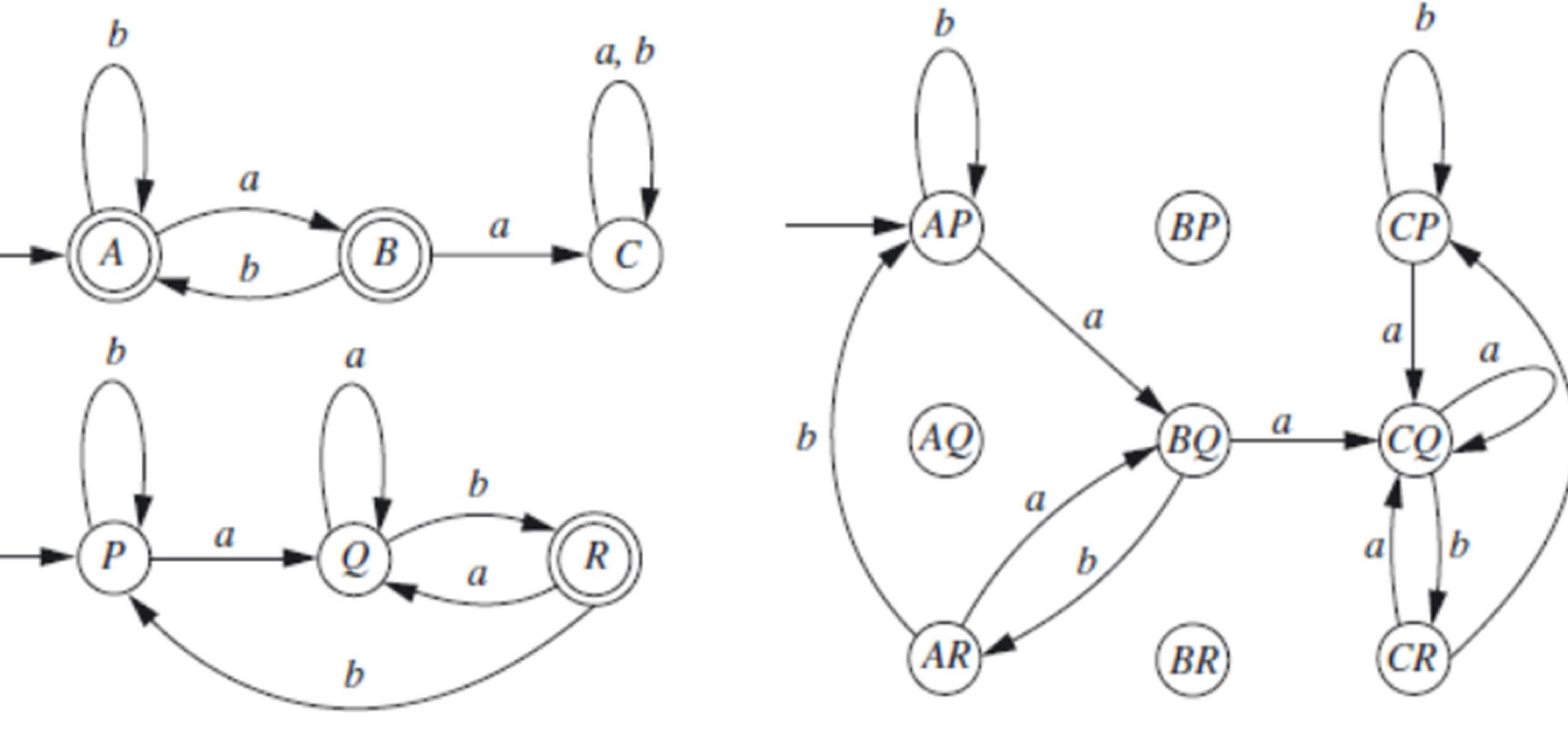
Is the above union operation Correct ?

RLs are closed Under Union



Above union operation is Correct

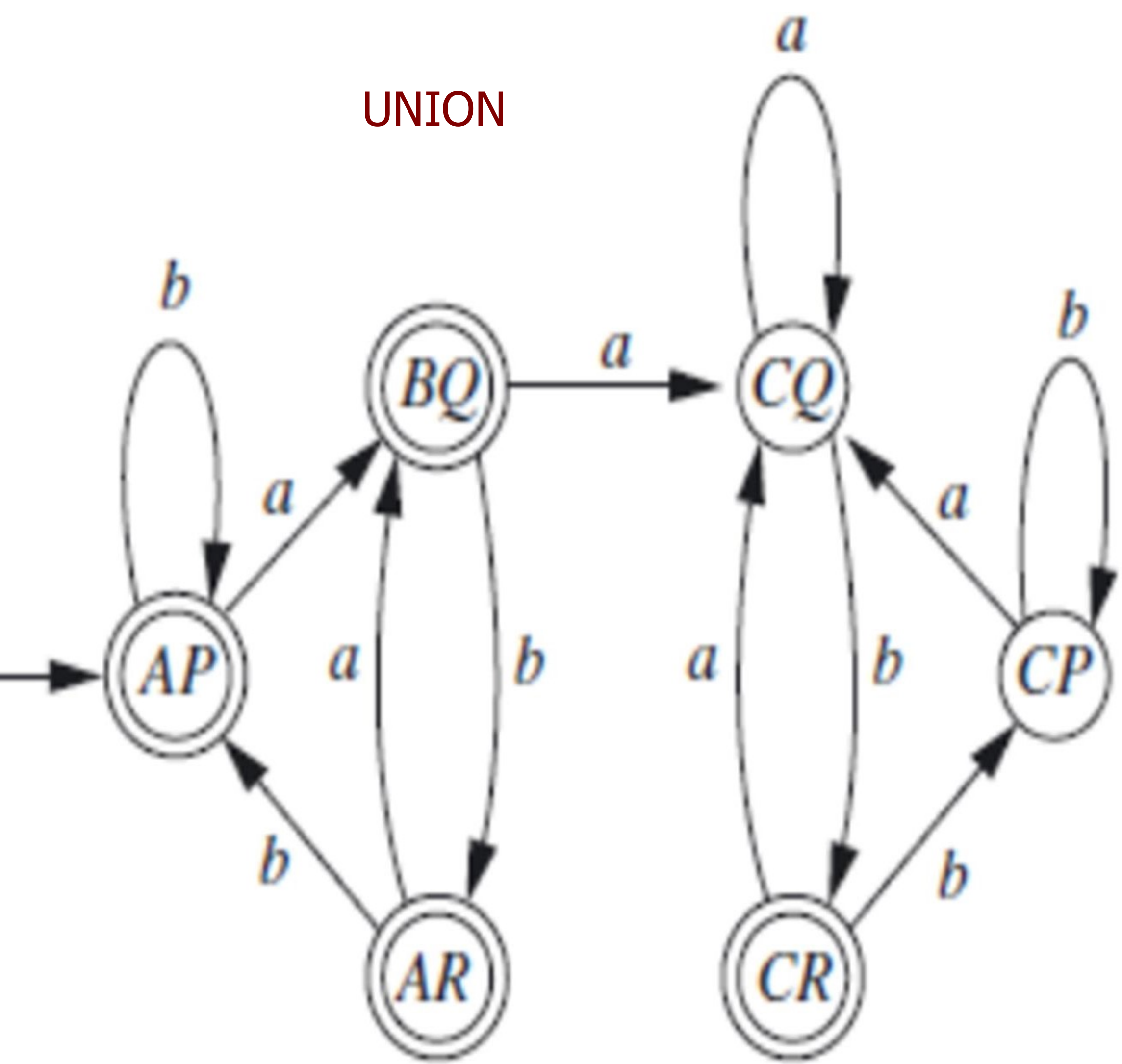
EXAMPLE -2



(a) (b)

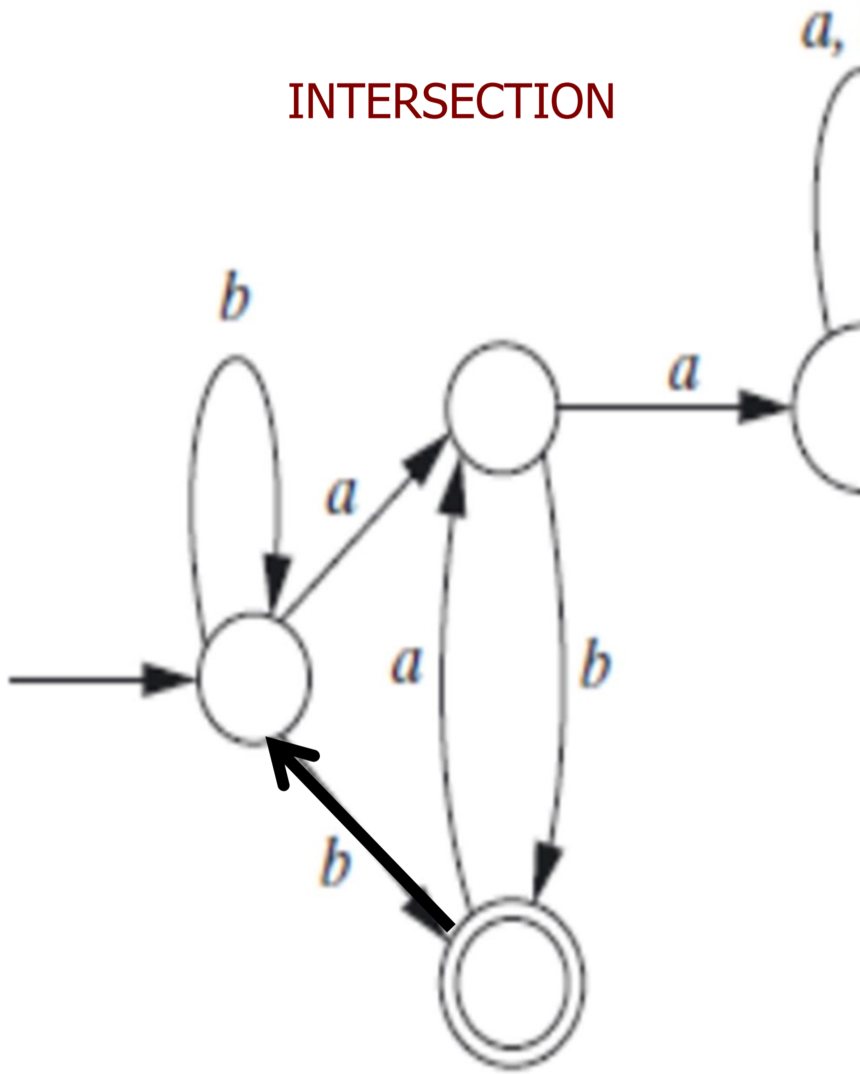
EXAMPLE -2

UNION



(c)

INTERSECTION



(d)

Closure Under Union

DeMorgan's Law: $L1 \cup L2 = \overline{\overline{L1} \cap \overline{L2}}$

L1 and L2 are regular

So $\overline{L1}$ and $\overline{L2}$ are regular (Closure under complementation)

So $\overline{L1} \cap \overline{L2}$ is regular (Closure under intersection)

So $\overline{\overline{L1} \cap \overline{L2}}$ is regular. (Closure under comp.)

So $L1 \cup L2$ is regular.