

# Context-Free Grammars

$L = \{x \mid x \in \{0,1\}^*, \text{two middle symbols are always equal}\}$

3 NT

2 Ts

12 Rules

$$S \rightarrow A \mid B$$

$$A \rightarrow 0A0 \mid 1A0 \mid 0A1 \mid 1A1 \mid 00$$

$$B \rightarrow 0B0 \mid 1B0 \mid 0B1 \mid 1B1 \mid 11$$

# Context-Free Grammars

0010100

$L = \{x \mid x \in \{0,1\}^*, \text{ First, middle and last Symbols are always equal and } |x| \geq 3\}$

3 NTs

2 Ts

12 Rules

$S \rightarrow 0A0 \mid 1B1$

$A \rightarrow 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \mid 0$

$B \rightarrow 0B0 \mid 0B1 \mid 1B0 \mid 1B1 \mid 1$

S  
0A0

00A00

001A100

0010100

# Context-Free Grammars

$$L = \{ a^m b^m c^n d^n \mid m, n \geq 0 \}$$

$$L = L_1 \cdot L_2$$

$$L_1 = \{ a^m b^m \mid m \geq 0 \}$$

$$L_2 = \{ c^n d^n \mid n \geq 0 \}$$

$$S \rightarrow S_1 S_2 \quad \checkmark$$

$$S_1 \rightarrow a S_1 b \mid \epsilon \quad \checkmark$$

$$S_2 \rightarrow c S_2 d \mid \epsilon \quad \checkmark$$

3 NTs  
 4 Ts  
 5 Rules

$aabb$

$S$   
 $S_1 S_2$

$aS_1 b \cdot S_2$

$aas_1bb s_2$

$aabb \cdot S_2$

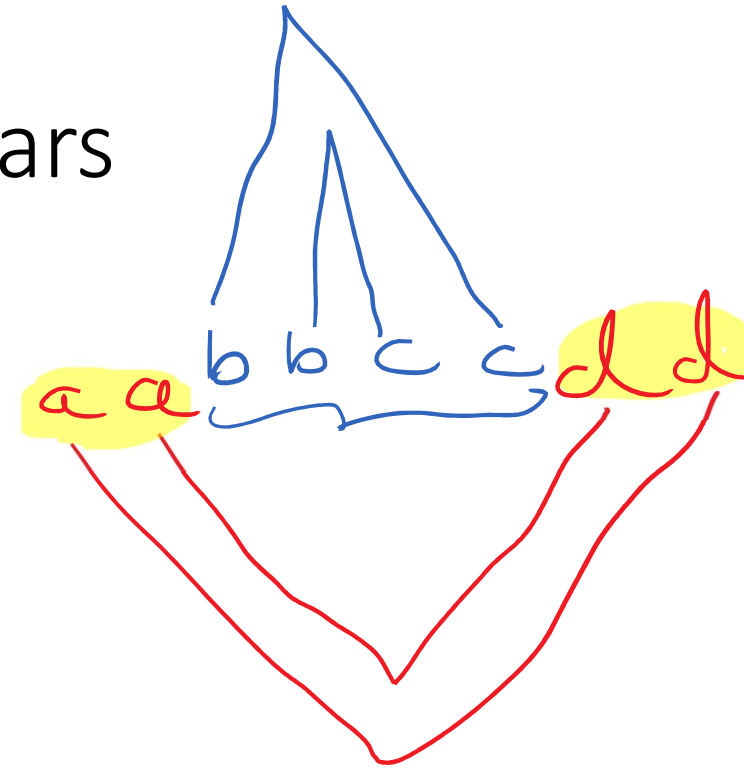
$aabb$

# Context-Free Grammars

$$L = \{ a^m b^n c^n d^m \mid m, n \geq 0 \}$$

$$S \rightarrow aSd \mid A$$

$$A \rightarrow bAc \mid \wedge$$



$abcdd$   
 $abbbccdd$   
 $abbbccdd$

S  
a S d  
a a S d d d  
a a A d d d

# Context-Free Grammars

$$L = \{a^i b^j \mid 0 \leq j \leq 2i\}$$

$$i=0 \quad 0 \leq j \leq 0$$

$$\underline{\underline{i=1}} \quad 0 \leq j \leq 2$$

$$j=0, 1, 2$$

$$i=2 \quad 0 \leq j \leq 4$$

$$j=0, 1, 2, 3, 4$$

✓  $\Lambda$

$a$

✓  $ab$

✓  $abb$

✓  $aa$

✓  $aab$

✓  $aabb$

✓  $aabbb$

✓  $aabbbb$

$$S \rightarrow \Lambda$$

$$S \rightarrow aSb$$

$$S \rightarrow aSbb$$

$$S \rightarrow \underline{\underline{aS}} \text{ or } \underline{\underline{Sa}}$$

$$S \rightarrow \Lambda \mid aSb \mid aSbb \mid aS$$

$$\Lambda: \underline{\underline{S \rightarrow \Lambda}}$$

ab:

aabb

$$S \rightarrow aSb \rightarrow aaSbb \rightarrow aabb$$

$$S \rightarrow aSb \rightarrow ab$$

abb

aaabbb

$$\rightarrow S \rightarrow aSbb \rightarrow aaSbbb \rightarrow aaabbbb$$

aab:

$$S \rightarrow aS \rightarrow aaSb \rightarrow aab$$

aabb

S

aSb

aaaSbbb

aabb

# Context-Free Grammars

$$L = \{ a^i b^j c^k \mid j = i + k, j, k \geq 1 \}$$

$$\frac{a^i b^{i+k} c^k}{a^i b^i b^k c^k}$$

$$L = \frac{\frac{a^i b^i}{L_1} \quad \frac{b^k c^k}{L_2}}$$

$$\begin{aligned} L_1: S_1 &\rightarrow a S_1 b \mid ab \\ L_2: S_2 &\rightarrow b S_2 c \mid bc \\ L: S &\rightarrow S_1 S_2 \end{aligned}$$

3 NTs, 3 Ts, 5 Rules

# Context-Free Grammars

abbbc

$$j = i + k$$

$S$   
 $S_1 \cdot S_2$

$ab S_2$

$ab \underline{b S_2} c$

$ab \underline{bb} cc$

Qabbbc

$S$   
 $S_1 \cdot S_2$

$a S_1 b \cdot S_2$

$aabb \cdot S_2$

Qabbbc



# Context-Free Grammars

$$L = \{ a^i b^j c^k \mid i = j + k, j, k \geq 0 \}$$

$$a^{j+k} b^j c^k = \underbrace{a^k a^j}_{\text{TT}} b^j c^k$$

$S \rightarrow aSc$	$A$
$A \rightarrow aAb$	$\Lambda$

# Context-Free Grammars

$aabc$

$S$

$aSc$

$aAc$

$aAbc$

$abc$

$aabc$

$S$

$aSc$

$aAc$

$aAbc$

$aabc$

$abc$

## Context-Free Grammars

$$L = \{a^i b^j c^k \mid k = i + j, \quad i, j \geq 0\}$$

$$S \rightarrow aSc \mid A$$

$$A \rightarrow bAc \mid \Lambda$$

$$\begin{array}{c} a^i b^j c^{i+j} \\ a^i b^j c^j c^i \\ \underbrace{\hspace{10em}} \end{array}$$