# Context-Free Languages

$$\{a^n b^n\} \qquad \{ww^R\}$$

Regular Languages

# Context-Free Languages

## Context-Free Grammars

## Pushdown Automata

stack

automaton

# Context-Free Grammars

# Example

A context-free grammar $\quad G: \quad S \rightarrow aSb$

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$$S \rightarrow \lambda$$

A derivation:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

A context-free grammar $G$ :  $S \rightarrow aSb$

$S \rightarrow \lambda$

Another derivation:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$L(G) = \{a^n b^n : n \geq 0\}$$

Take a= '('      and      b = ')'

$$((((\ ))))$$

# Example

A context-free grammar $G$:

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda$$

A derivation:

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$$

A context-free grammar $G$:
$$S \rightarrow aSa$$
$$S \rightarrow bSb$$
$$S \rightarrow \lambda$$

Another derivation:

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abaSaba \Rightarrow abaaba$$

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda$$

$$L(G) = \{ww^R : \quad w \in \{a,b\}*\}$$

# Example

A context-free grammar $G$: 

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \rightarrow \lambda$$

A derivation:

$$S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow ab$$

A context-free grammar $G$:  $S \rightarrow aSb$

$$S \rightarrow SS$$

$$S \rightarrow \lambda$$

A derivation:

$$S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow abaSb \Rightarrow abab$$

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \rightarrow \lambda$$

$$L(G) = \{w \ : n_a(w) = n_b(w),$$
$$\text{and } n_a(v) \geq n_b(v)$$
$$\text{in any prefix } v\}$$

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# Definition: Context-Free Grammars

Grammar $G = (V, T, S, P)$

Variables        Terminal        Start
                 symbols         variables

Productions of the form:

$$A \rightarrow x$$

$x$ is string of variables and terminals

# Definition: Context-Free Languages

- A language $L$ is context-free

- if and only if

- there is a grammar $G$ with $L = L(G)$