Image Restoration

IMAGE RESTORATION

The main objective of restoration is to improve the quality of a digital image which has been degraded due to Various phenomena like:

- Motion
- Improper focusing of Camera during image acquisition.
- Atmospheric turbulence
- Noise

Preview

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - A prior knowledge of the degradation phenomenon is considered
  - Modeling the degradation and apply the inverse process to recover the original image
- Spatial domain approach
- Frequency domain approach

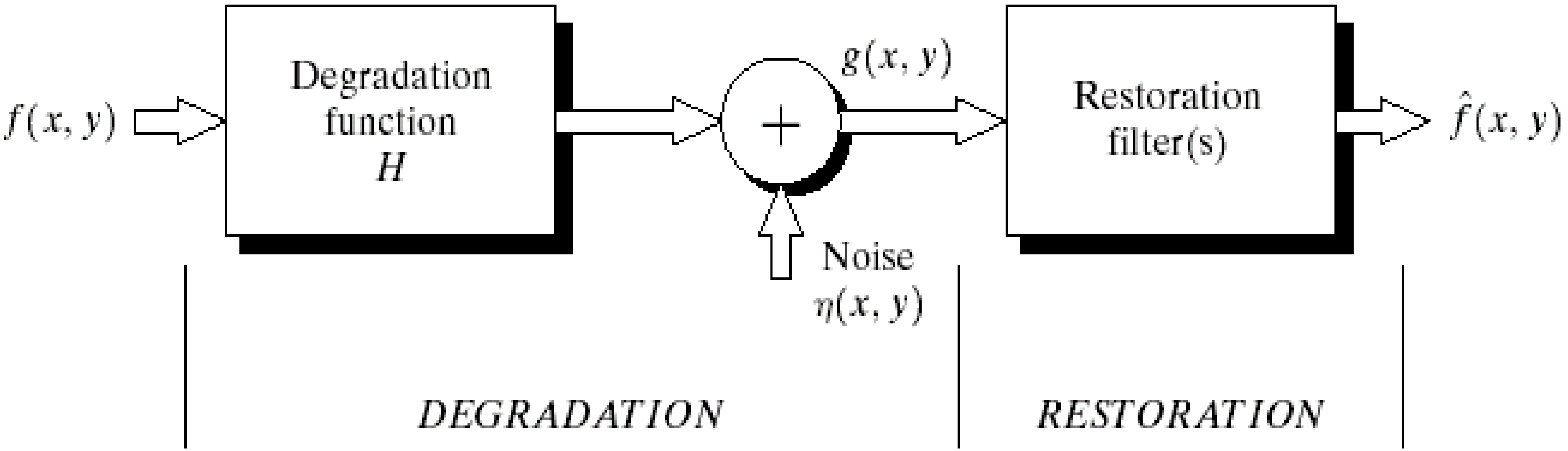
Image restoration vs. image enhancement

	Image restoration	Image enhancement
1.	is an objective process	is a subjective process
2.	formulates a criterion of goodness that will yield an optimal estimate of the desired result	involves heuristic procedures designed to manipulate an image in order to satisfy the human visual system
3.	Techniques include noise removal and deblurring (removal of image blur)	Techniques include contrast stretching

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of <u>noise only</u> – spatial filtering
- <u>Periodic noise</u> reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

A model of the image degradation/restoration process



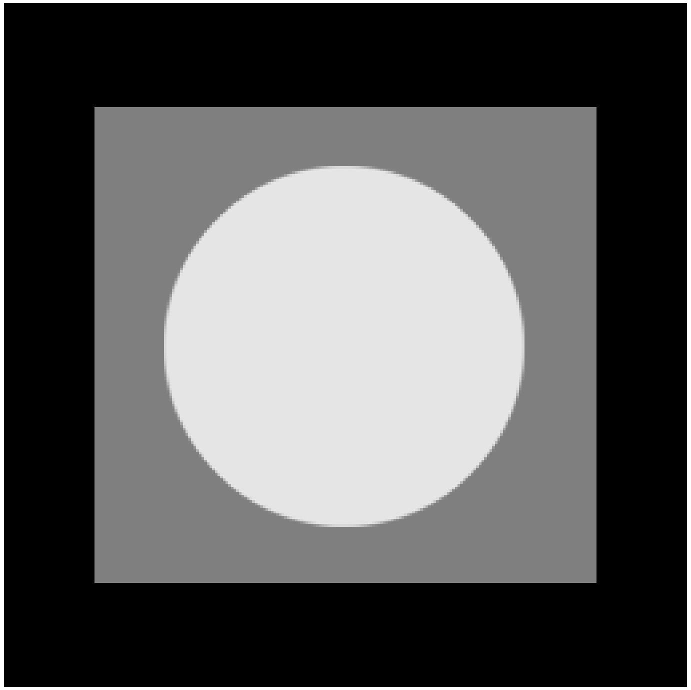
$$
\begin{cases}
g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\
G(u,v)=F(u,v)H(u,v)+N(u,v)
\end{cases}
$$

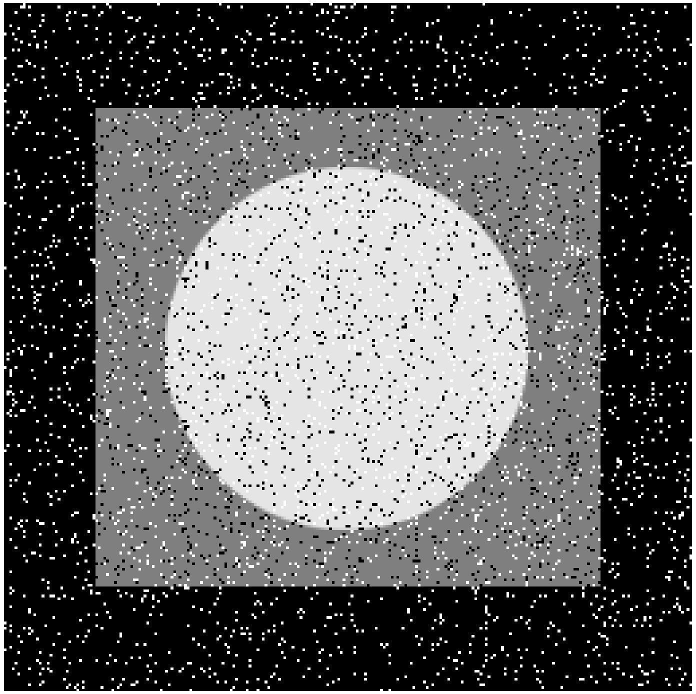
Noise models

- Source of noise
  - Image acquisition (digitization)

  Ex: light levels, sensor temperature, etc.
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - Ex. white noise (a constant Fourier spectrum)

Salt & Pepper Noise





Salt & Pepper Noise

- a = imread(‘C:\lake2.bmp’);
- a = double(a);
- a = mat2gray(a);
- imhist(a);
- b = imnoise(a,’Salt & Pepper’);
- figure,imshow (b);
- figure, imhist(b)

Noise probability density functions

- Noises are taken as random variables
- Random variables
  - Probability density function (PDF)

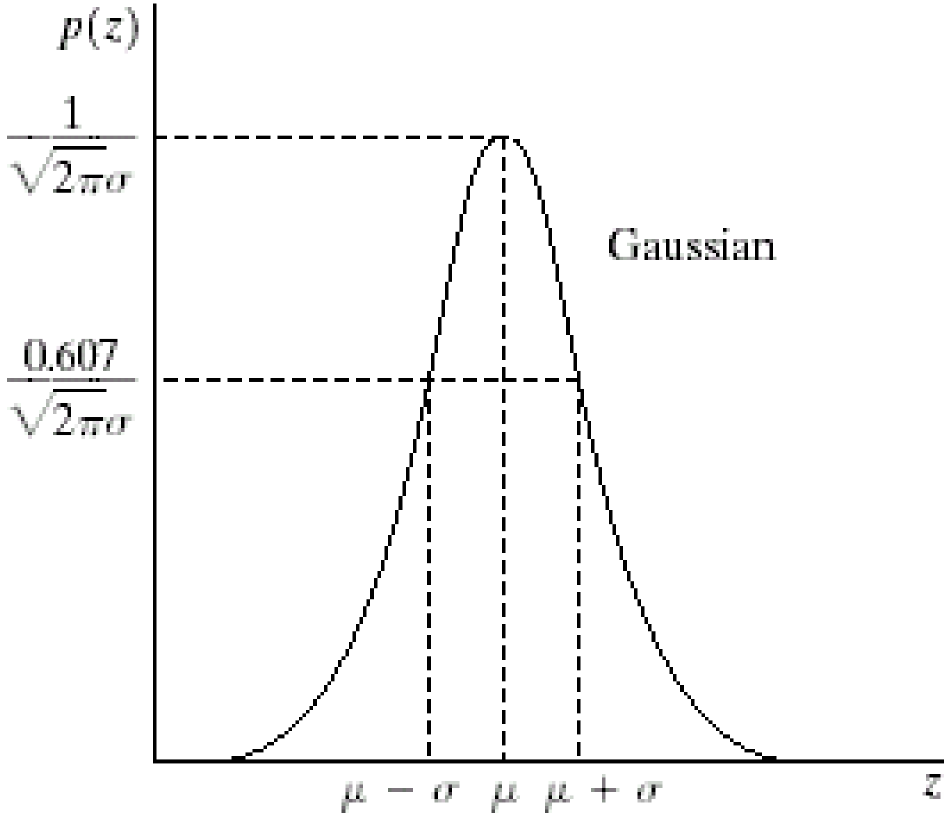
Gaussian Noise

- Noise (image) can be classified according the distribution of the values of pixels (of the noise image) or its (normalized) histogram
- Gaussian noise is characterized by two parameters, μ (mean) and σ^2 (variance), by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

- 70% values of z fall in the range $[(\mu-\sigma),(\mu+\sigma)]$
- 95% values of z fall in the range $[(\mu-2\sigma),(\mu+2\sigma)]$

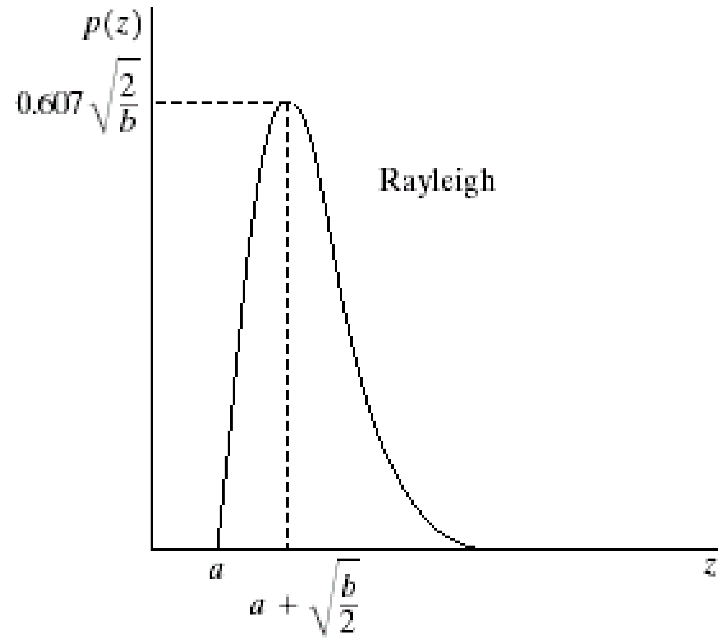
Gaussian Noise



Other Noise Models

- **Rayleigh** noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$



- The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b/4} \text{ and } \sigma^2 = \frac{b(4-\pi)}{4}$$

- a and b can be obtained through mean and variance

Other Noise Models

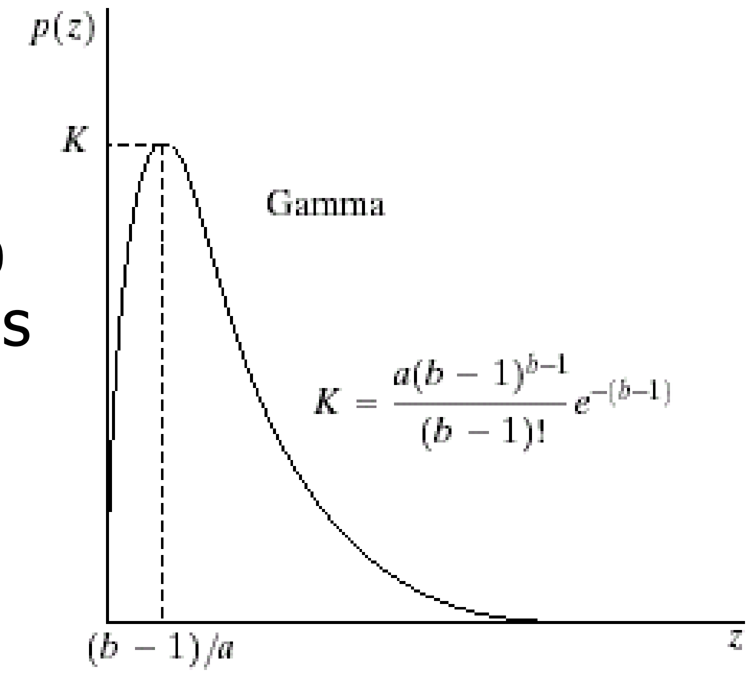
- **Erlang (Gamma)** noise

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$

  - The mean and variance of this density are given by

$$\mu = b/a \ \text{ and } \sigma^2 = \frac{b}{a^2}$$

  - a and b can be obtained through mean and variance

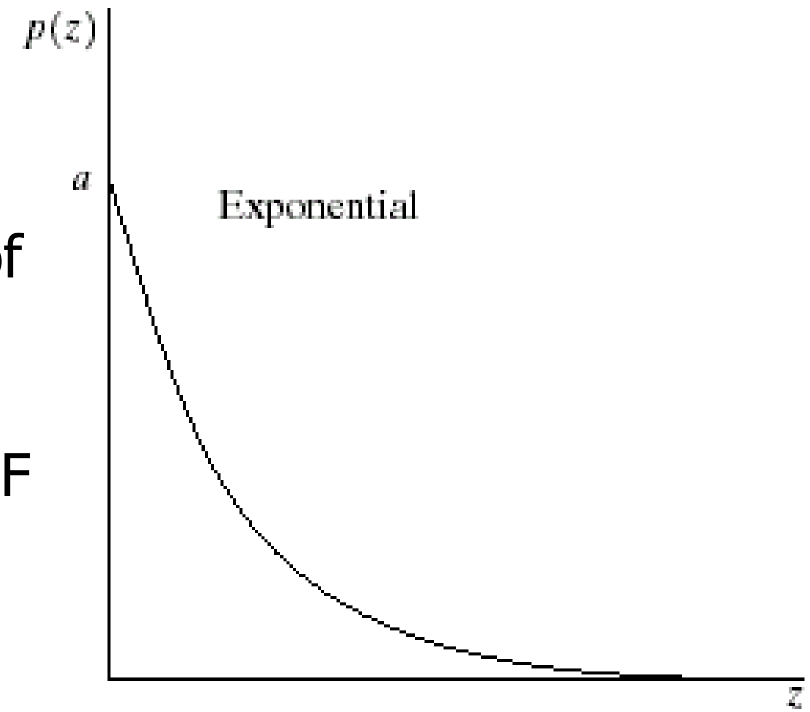


Other Noise Models

- **Exponential** noise
  - $$p(z) = \begin{cases} ae^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases}$$
  - The mean and variance of this density are given by

$$\mu = 1/a \ \text{ and } \sigma^2 = \frac{1}{a^2}$$

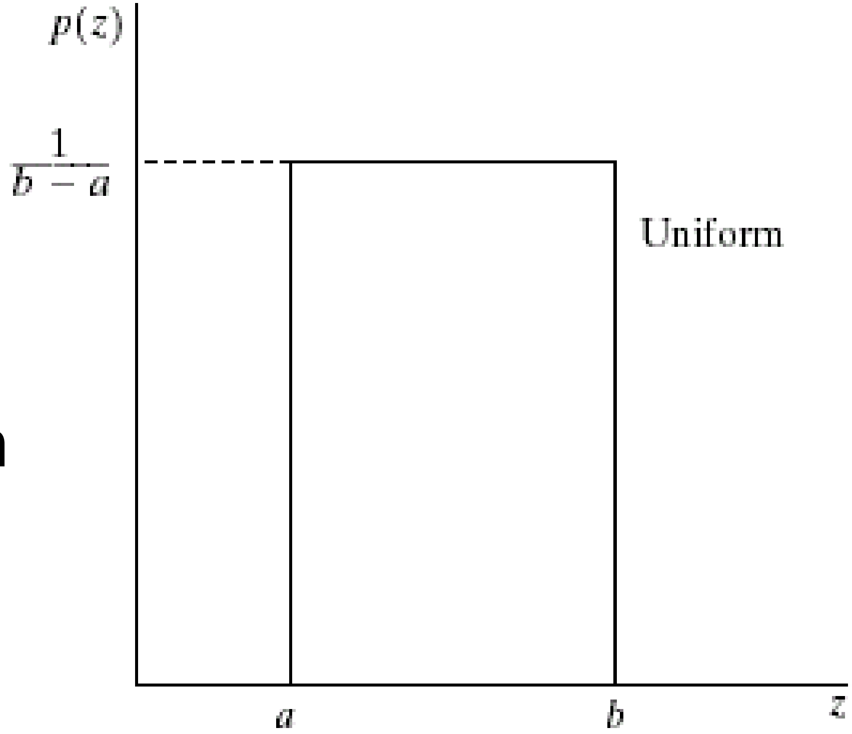
  - Special case pf Erlang PDF with b=1



Other Noise Models

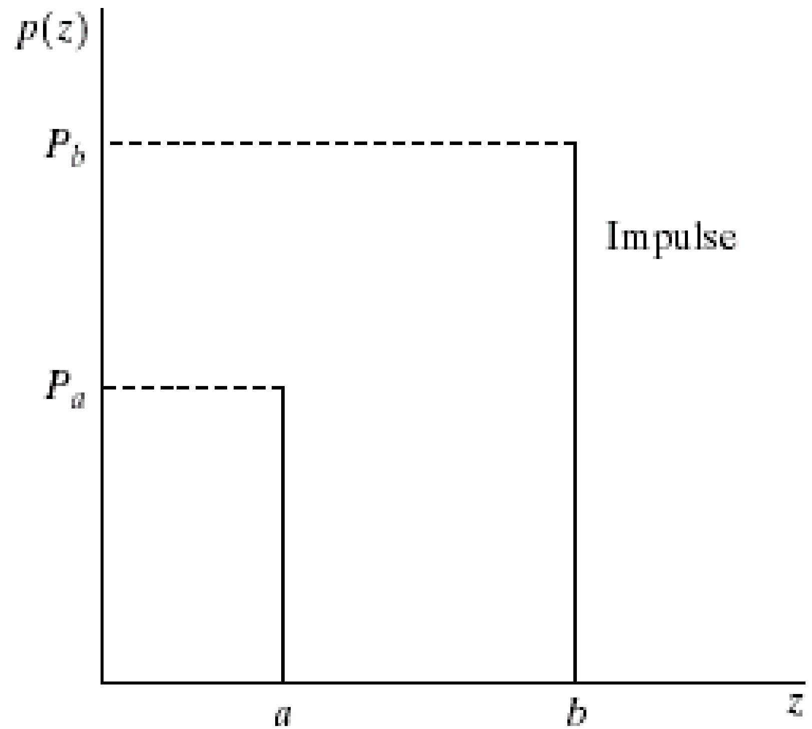
- **Uniform** noise
  - $$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } \; a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$
  - The mean and variance of this density are given by

$$\mu = (a+b)/2 \;\text{ and }\; \sigma^2 = \frac{(b-a)^2}{12}$$



Other Noise Models

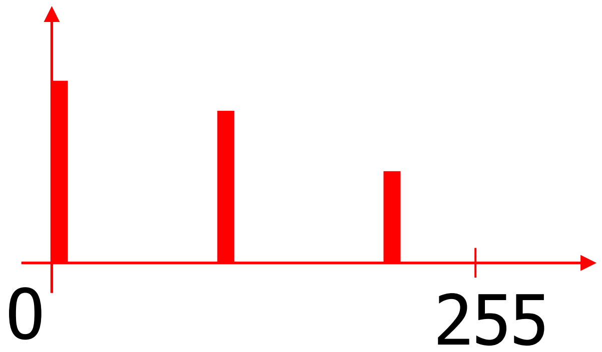
- **Impulse** (salt-and-pepper) noise
  - $$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$
  - If either Pa or Pb is zero, the impulse noise is called unipolar
  - a and b usually are extreme values because impulse corruption is usually large compared with the strength of the image signal
  - It is the only type of noise that can be distinguished from others visually

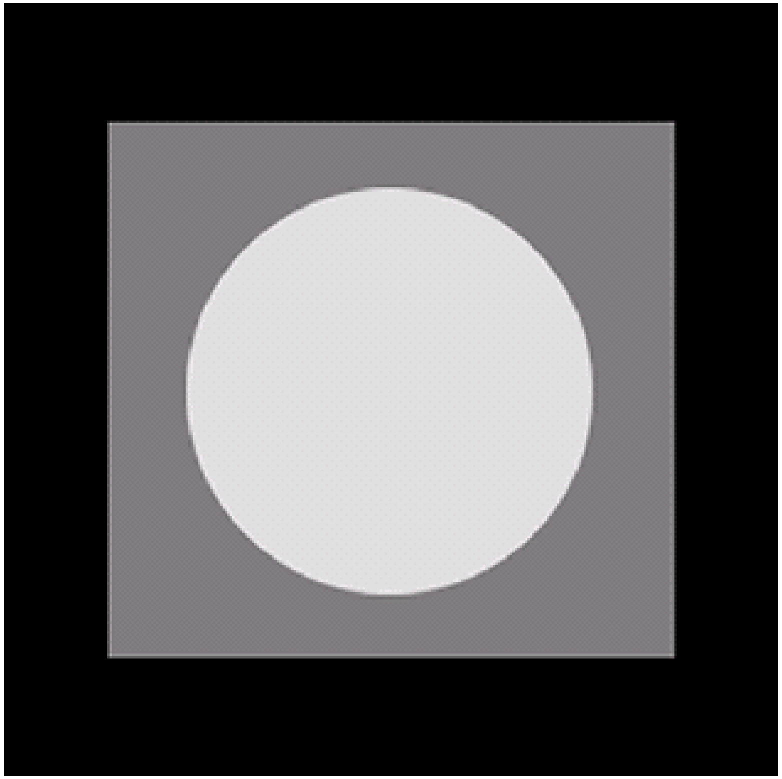


Test for noise behavior

- Test pattern

Its histogram:





Effect of Adding Noise to Sample Image

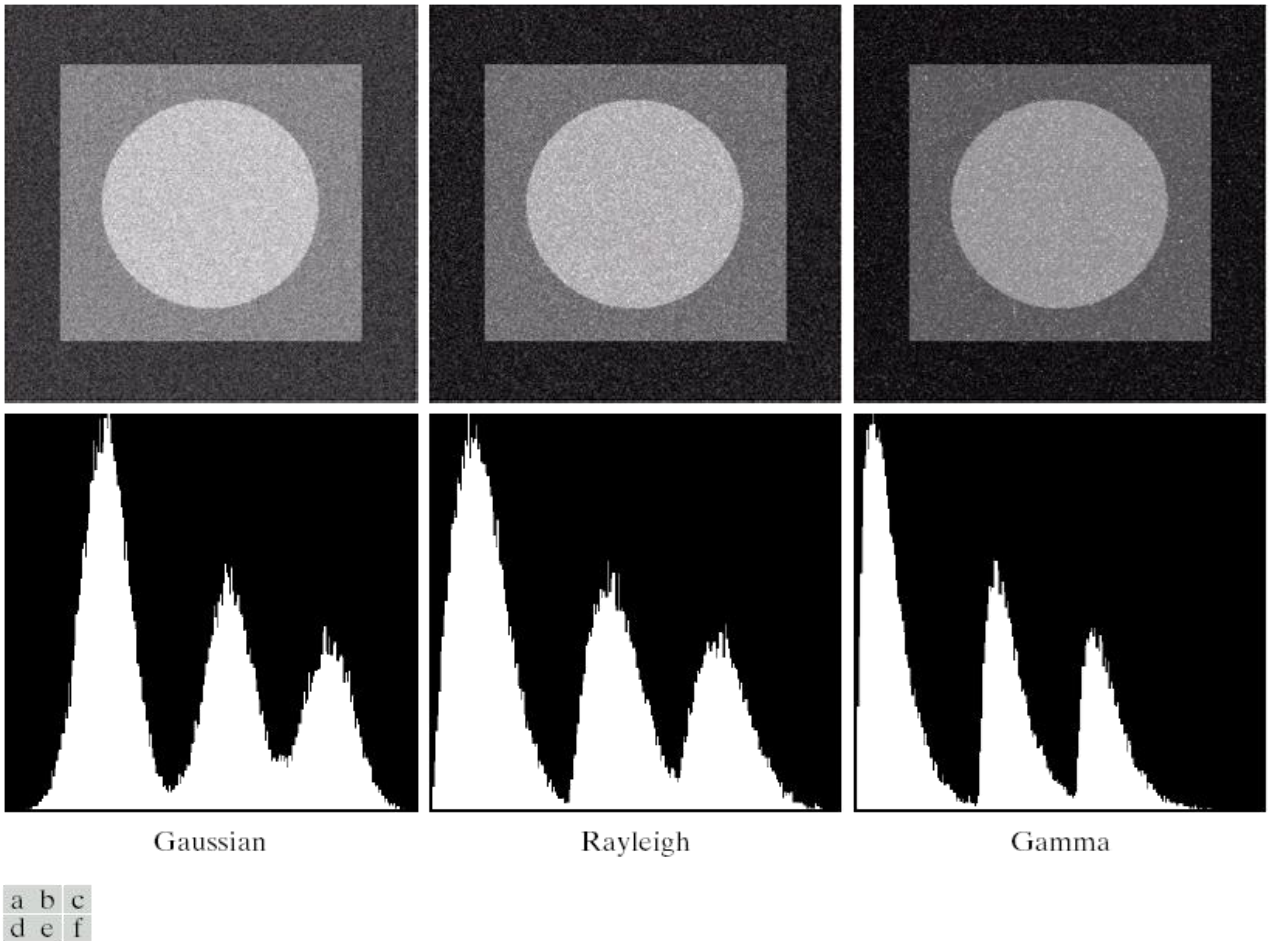


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Effect of Adding Noise to Sample Image

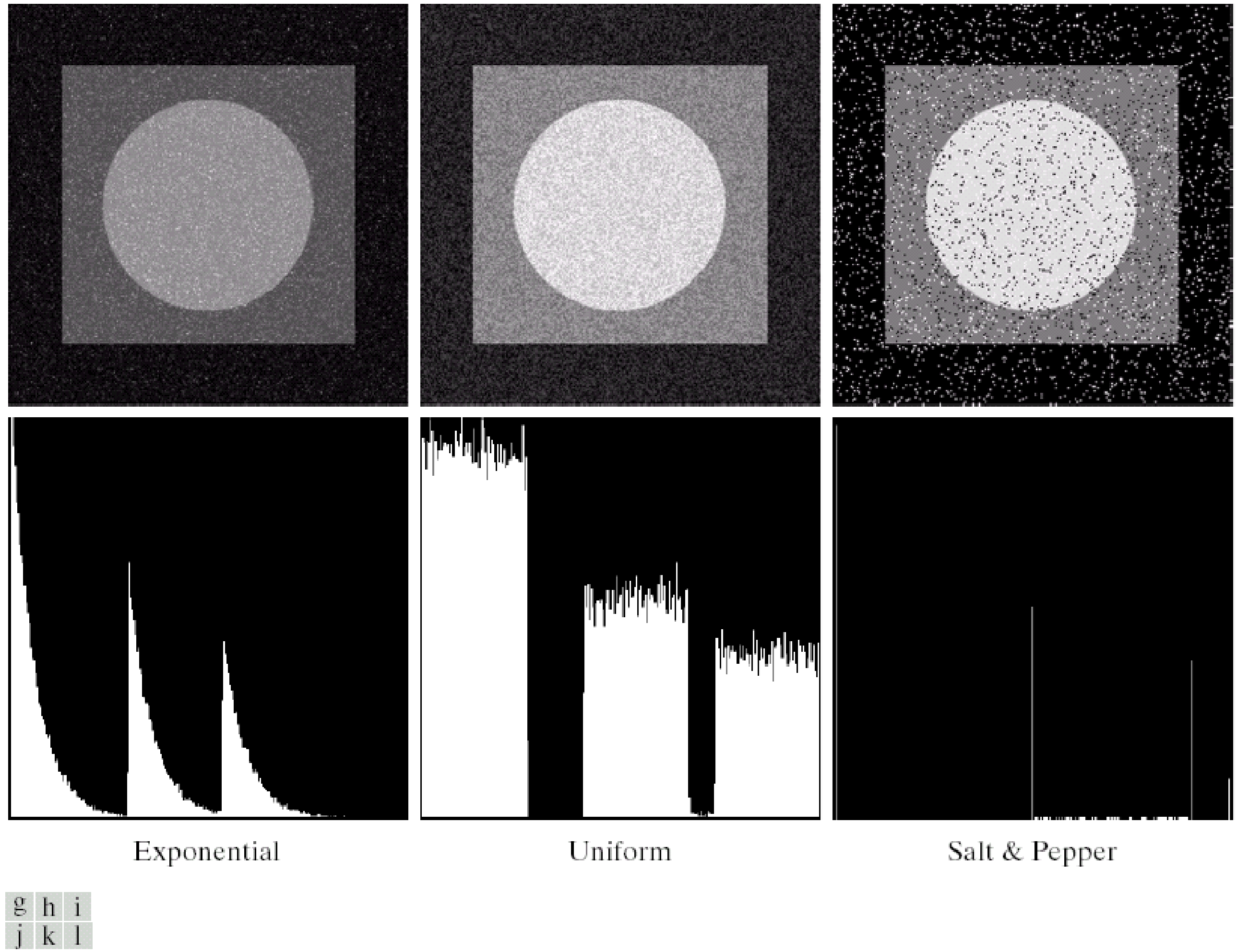
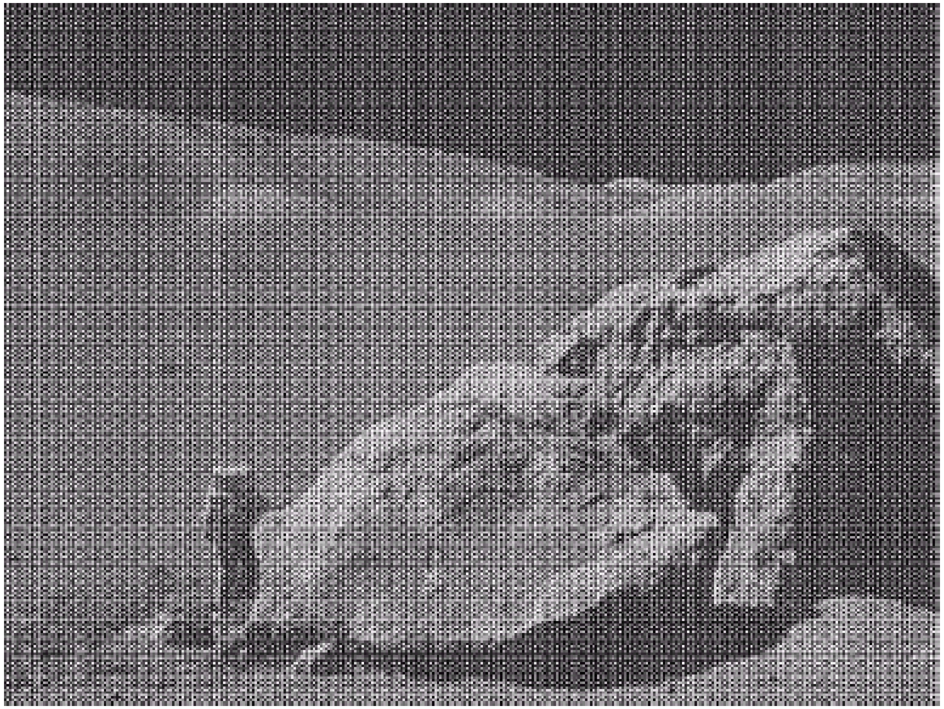
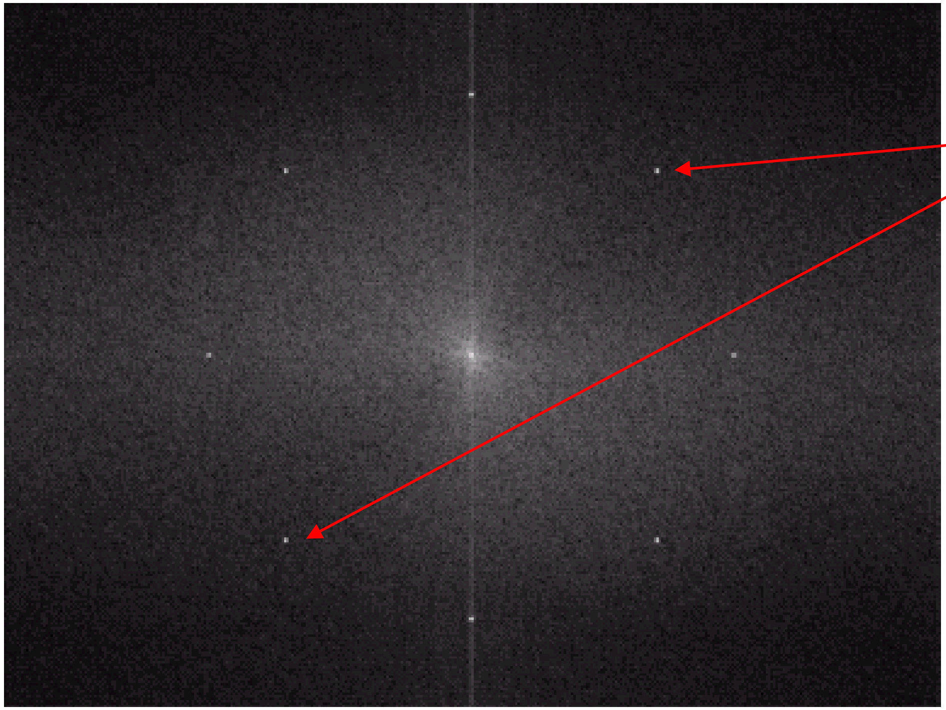


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Periodic noise

- Arise from electrical or electromechanical interference during image acquisition
- Spatial dependence
- Observed in the frequency domain



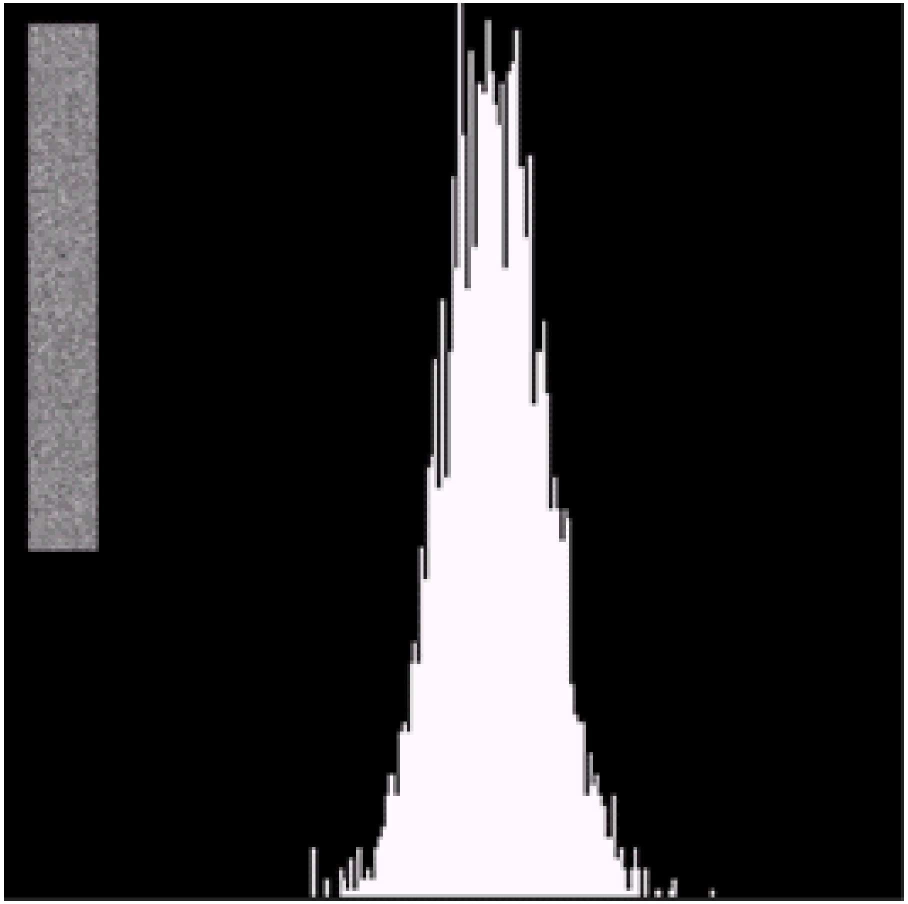


Sinusoidal noise:
Complex conjugate
pair in frequency
domain

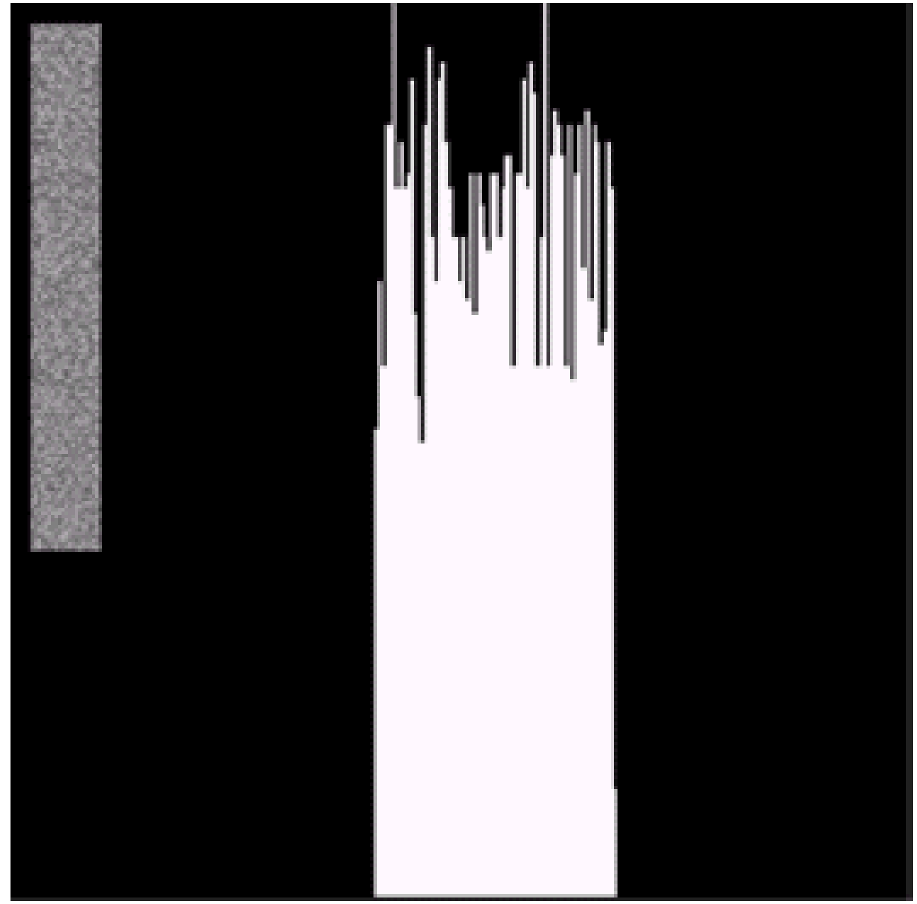
Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of "flat" environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

Observe the histogram



Gaussian



uniform

Measure the mean and variance

- Histogram is an estimate of PDF

$$\left\{\begin{aligned} \mu &= \sum_{z_i \in S} z_i\, p(z_i) \\ \sigma^2 &= \sum_{z_i \in S} (z_i - \mu)^2\, p(z_i) \end{aligned}\right.$$

$\Leftrightarrow$ $\left\{\begin{array}{l} \text{Gaussian: } \mu, \sigma \\ \text{Uniform: } a, b \end{array}\right.$

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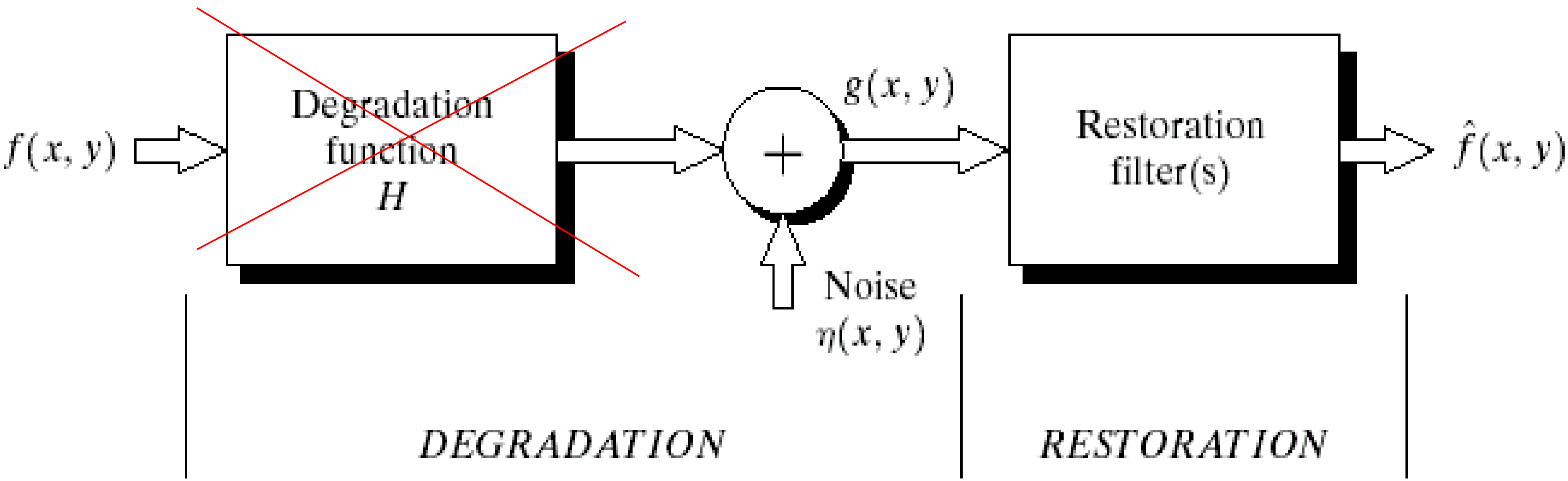
Additive noise only



$$
\begin{cases}
g(x,y)=f(x,y)+\eta\mathbf{(x,y)} \\
G(u,v)=F(u,v)+N(u,v)
\end{cases}
$$

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Mean filters

- Arithmetic mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

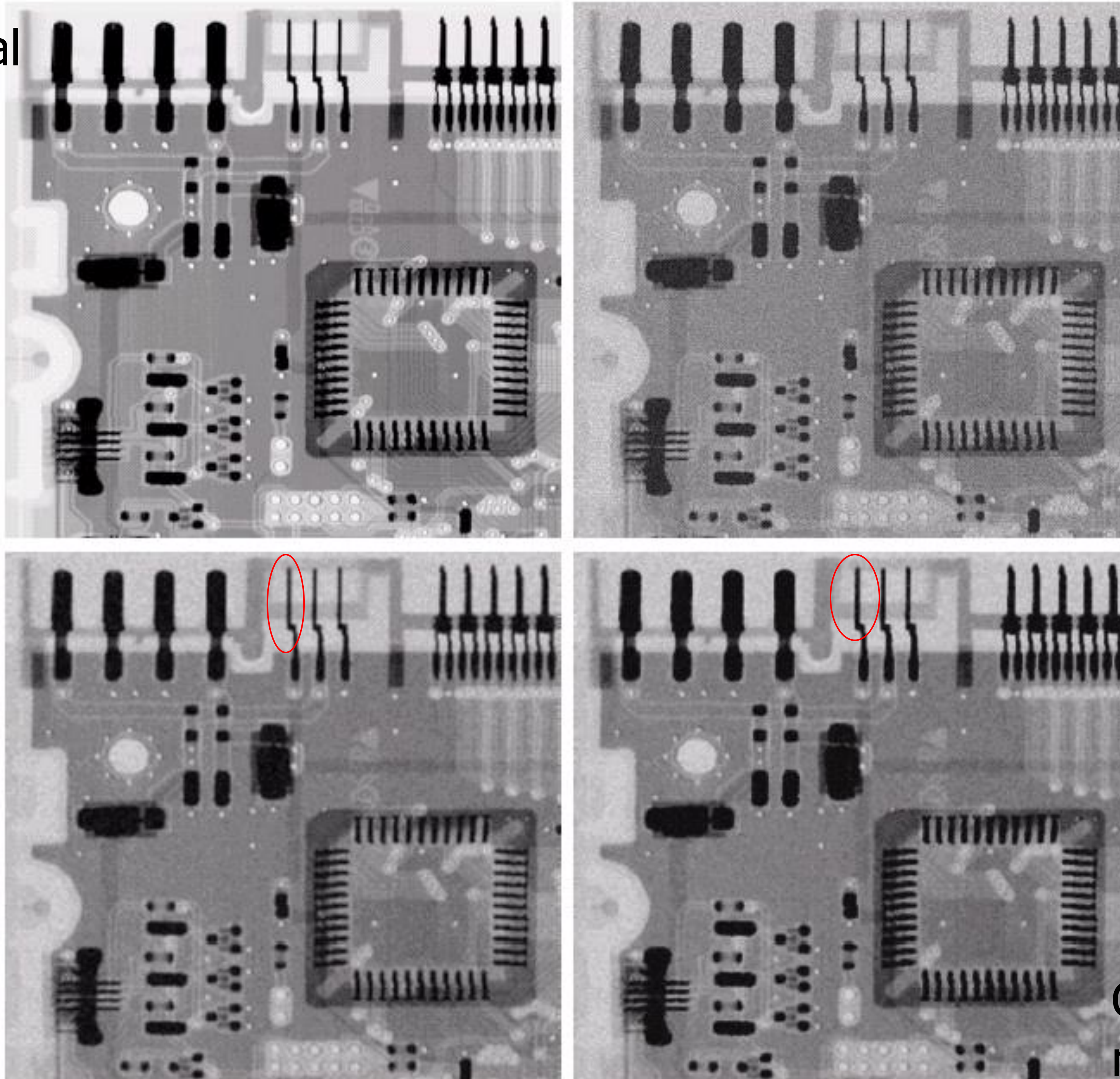
Window centered at (x,y)

- Geometric mean

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{1/mn}$$

original

Noisy Gaussian
$\mu=0$
$\sigma=20$



Arith. mean

Geometric mean

Mean filters (cont.)

- Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Contra-harmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

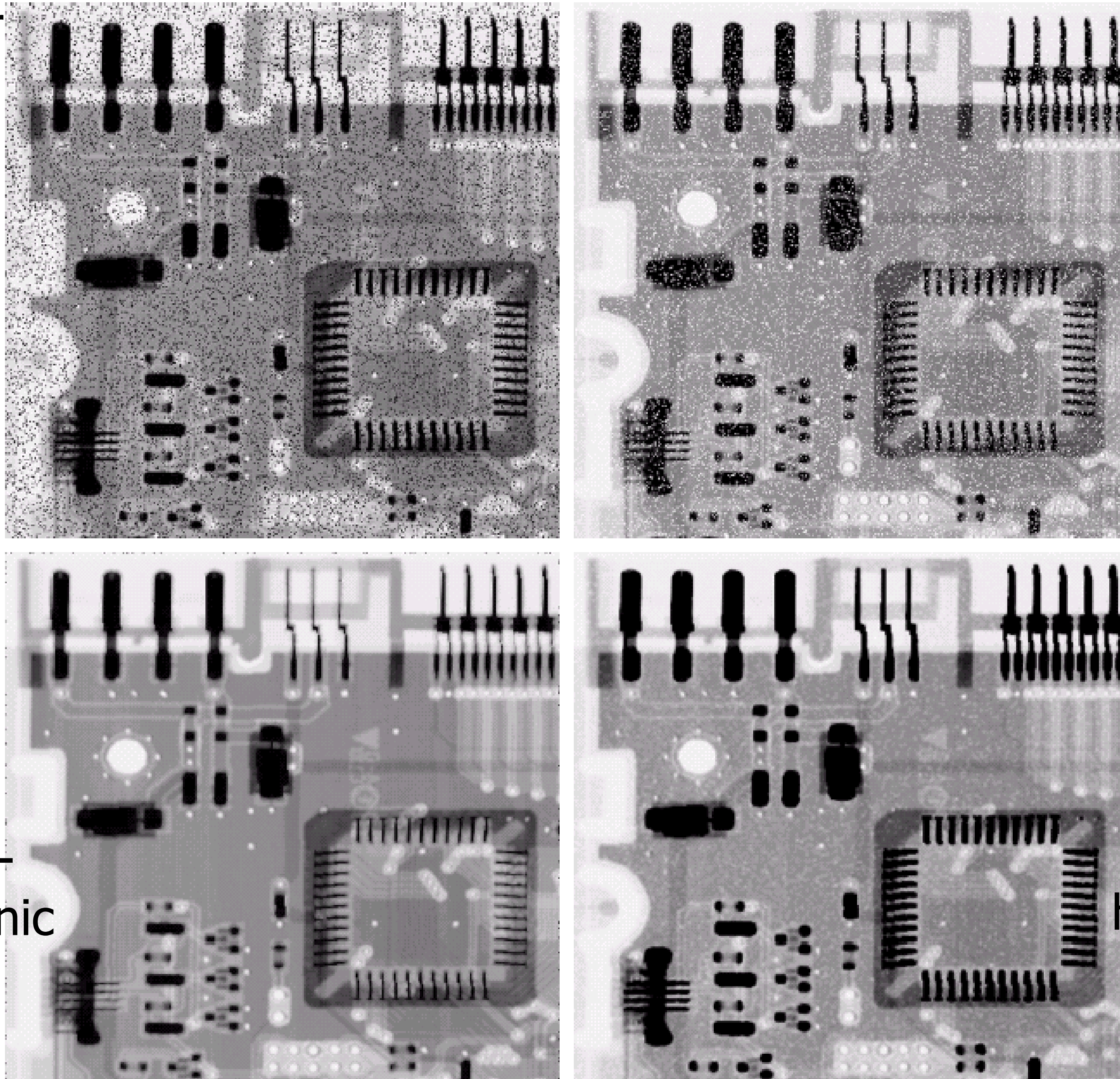
Q=-1, harmonic

Q=0, airth. mean

Q=+, ?

Pepper Noise
黑點

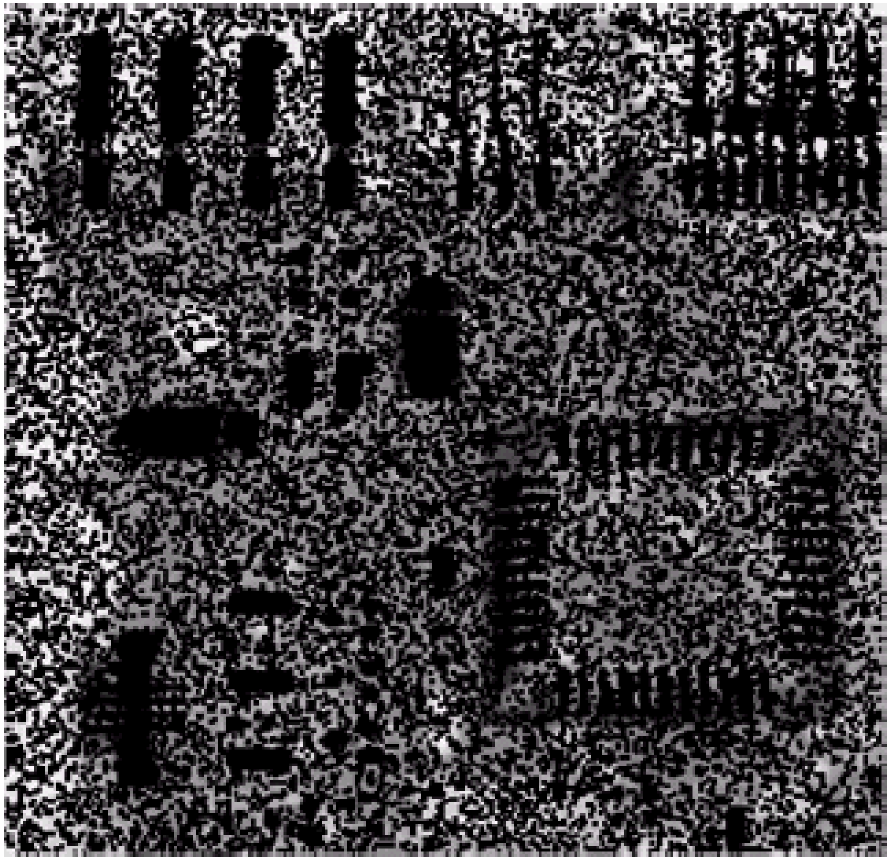
Salt Noise
白點



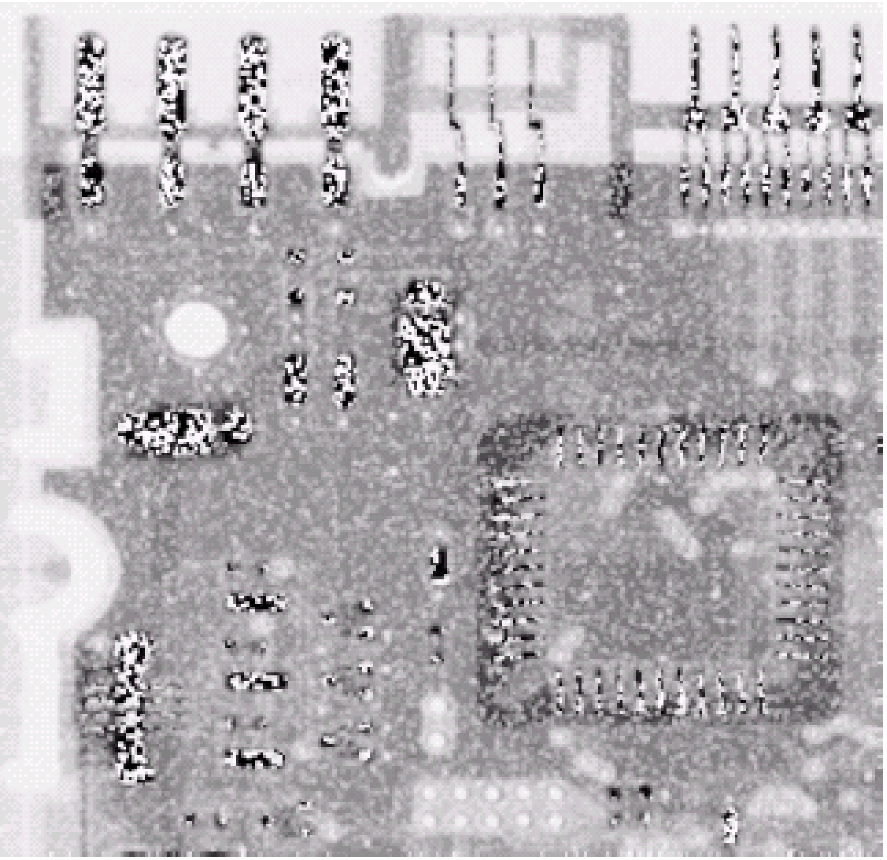
Contra-harmonic Q=1.5

Contra-harmonic Q=-1.5

Wrong sign in contra-harmonic filtering



Q=-1.5



Q=1.5

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

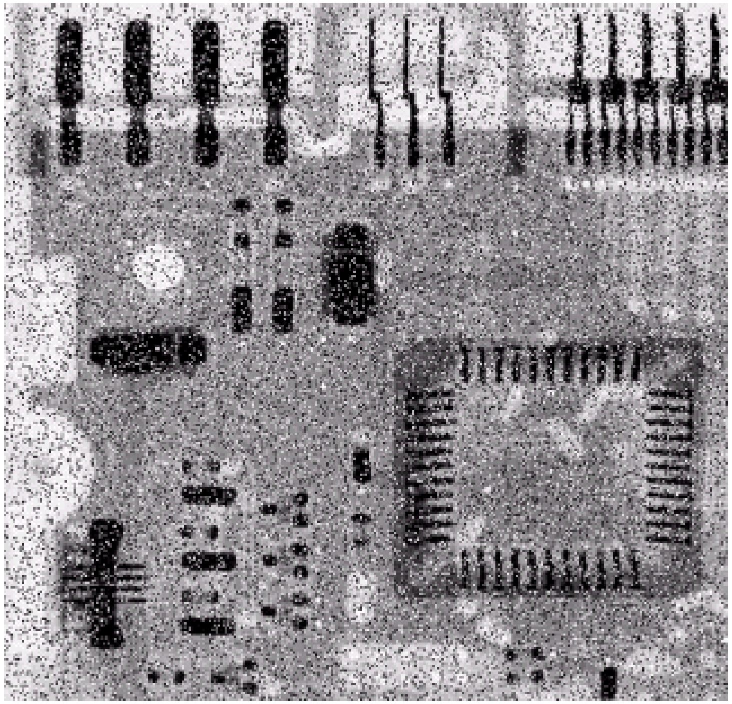
$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

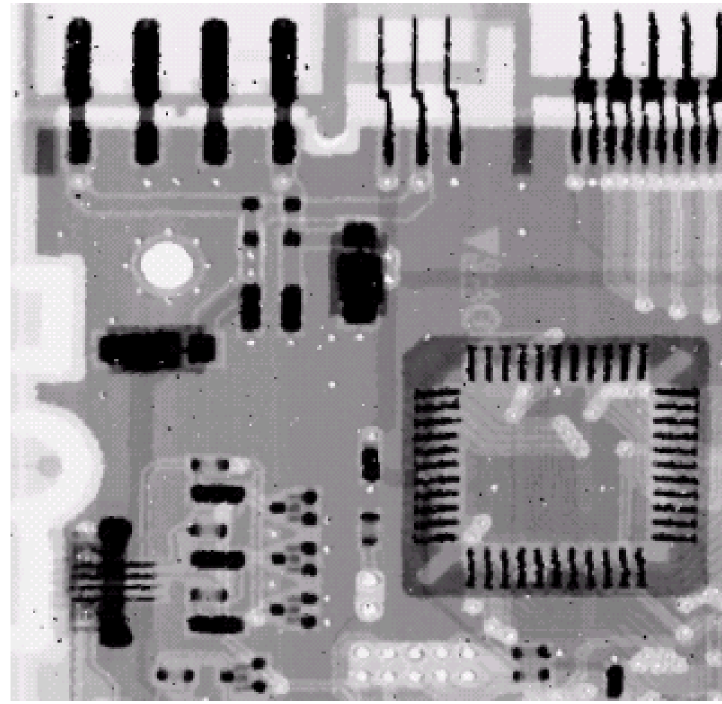
- Max/min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

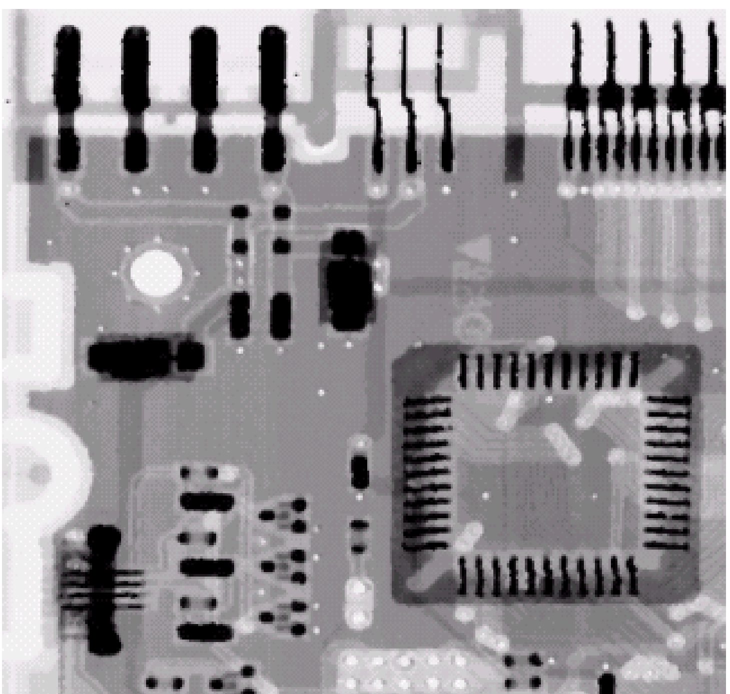
bipolar
Noise
$P_a = 0.1$
$P_b = 0.1$

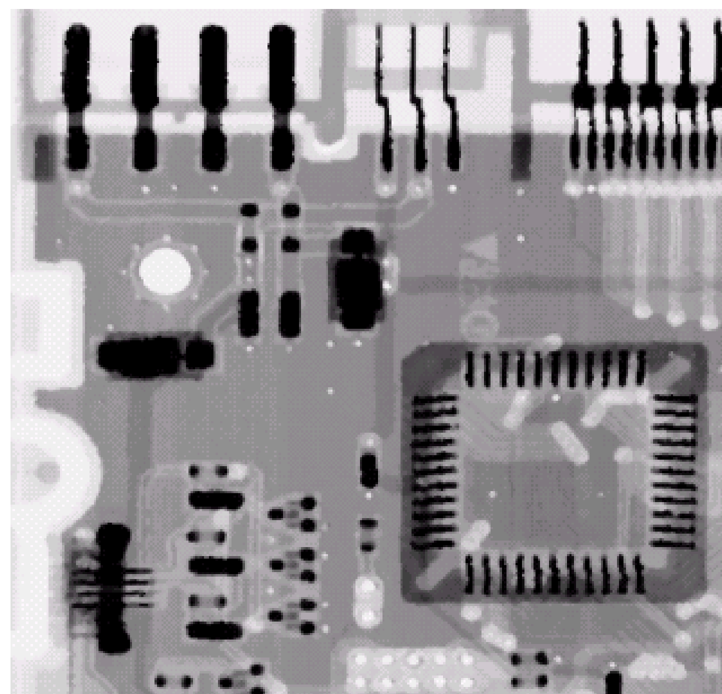




3x3
Median
Filter
Pass 1

3x3
Median
Filter
Pass 2

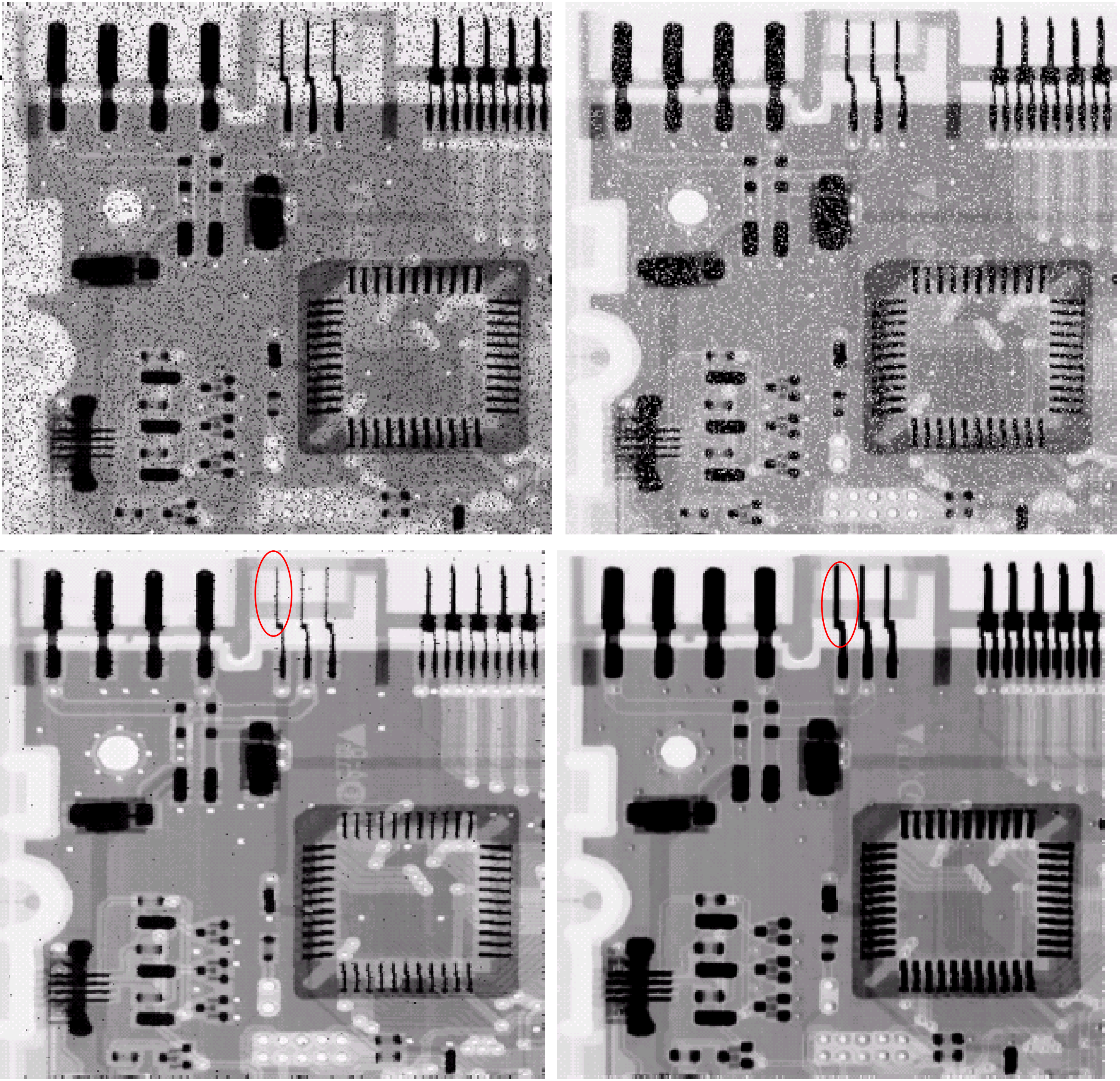




3x3
Median
Filter
Pass 3

Pepper noise

Salt noise



Max filter

Min filter

Order-statistics filters (cont.)

- Midpoint filter

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Middle (mn-d) pixels

Uniform noise

$\mu=0$

$\sigma^2=800$

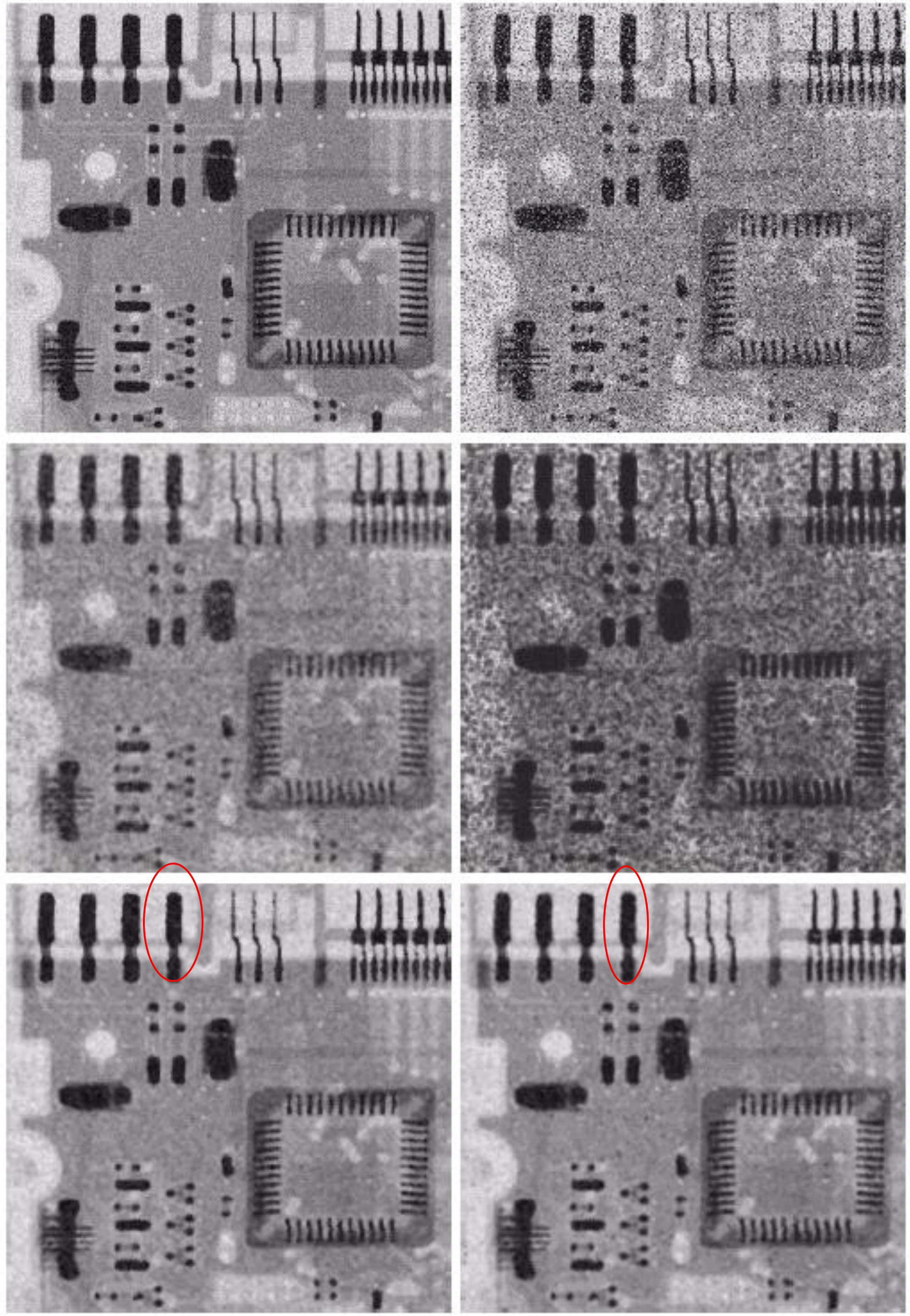
Left +
Bipolar Noise
$P_a = 0.1$
$P_b = 0.1$

5x5
Arith. Mean
filter

5x5
Geometric
mean

5x5
Median
filter

5x5
Alpha-trim.
Filter
d=5



Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - g(x,y): noisy image pixel value
  - σ^2_η: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L : local variance

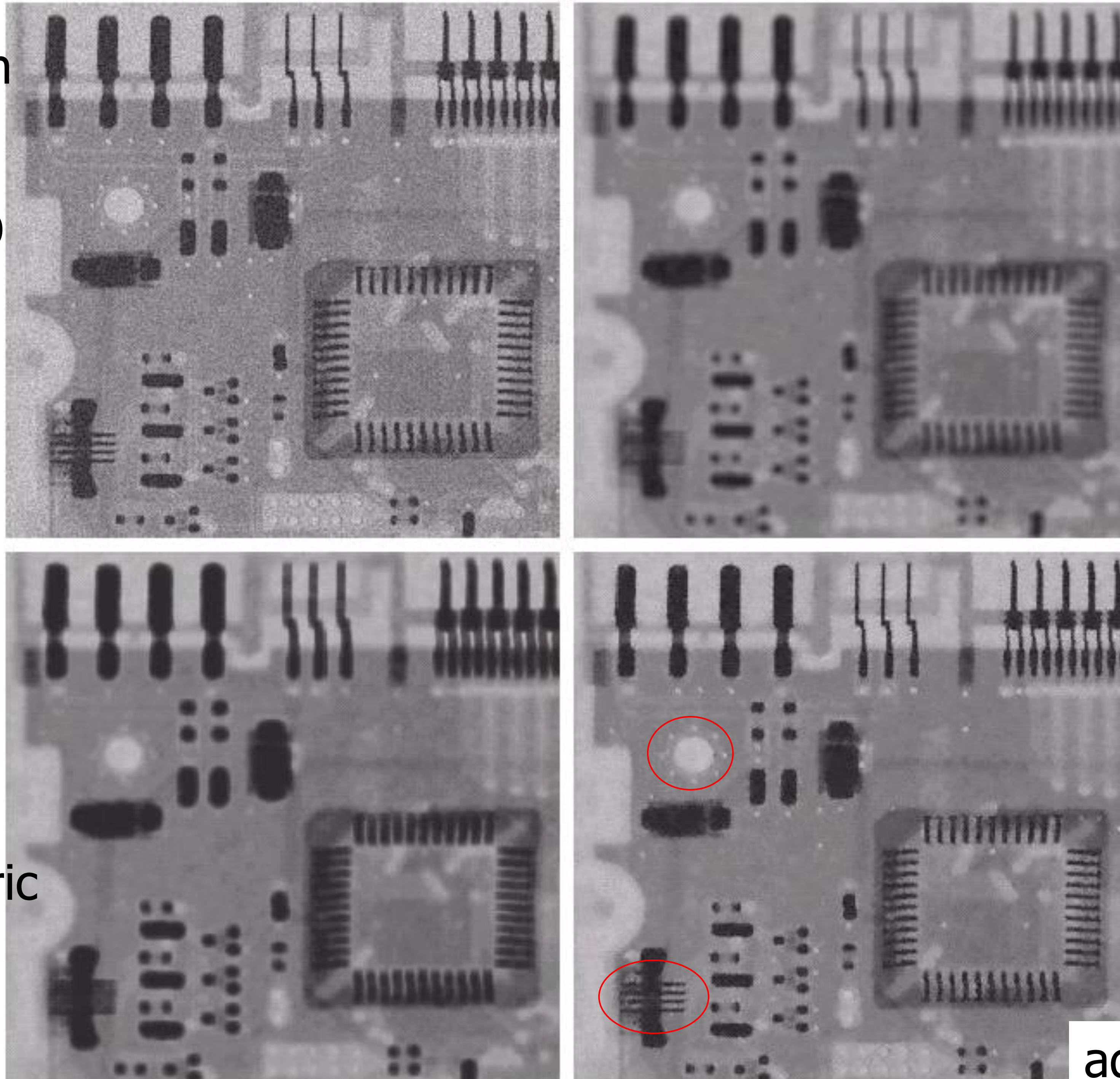
Adaptive local noise reduction filter (cont.)

- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Gaussian noise
$\mu=0$
$\sigma^2=1000$

Arith. mean 7x7



Geometric mean 7x7

adaptive

Adaptive Filters: Adaptive Median Filters

- Median filter is effective for removing salt-and-pepper noise
  - The density of the impulse noise can not be too large
- Adaptive median filter
  - Notation
    - Z_{min}: minimum gray value in S_{xy}
    - Z_{max}: maximum gray value in S_{xy}
    - Z_{med}: median of gray levels in S_{xy}
    - Z_{xy}: gray value of the image at (x,y)
    - S_{max}: maximum allowed size of S_{xy}

Adaptive Median Filter (De-Noising)

- Two levels of operations

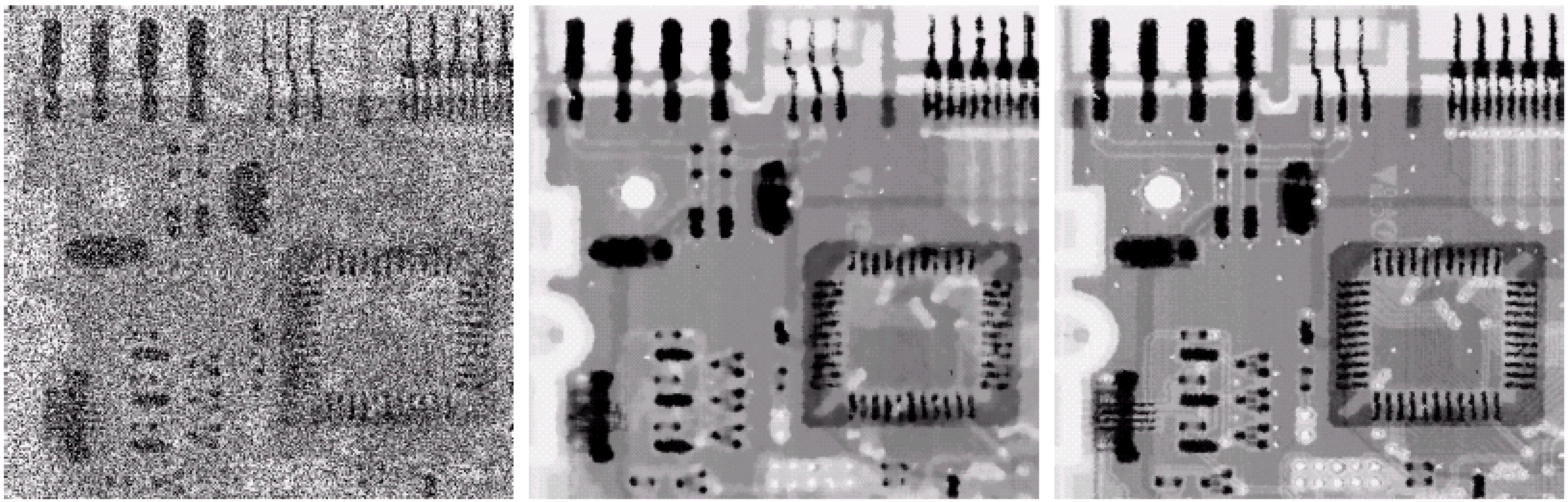
  Level A:

  - $A1 = Z_{med} - Z_{min}$
  - $A2 = Z_{med} - Z_{max}$
  - If $A1 > 0$ AND $A2 < 0$, Go to level B
    else increase the window size by 2

  - If window size $<= S_{max}$ repeat level A
    else output Z_{xy}

  Level B:

  - $B1 = Z_{xy} - Z_{min}$
  - $B2 = Z_{xy} - Z_{max}$
  - If $B1 > 0$ AND $B2 < 0$, output Z_{xy}
    else output Z_{med}



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering

Periodic noise reduction

- Pure sine wave
  - Appear as a pair of impulse (conjugate) in the frequency domain

$$\begin{cases} f(x,y) = A\sin(u_0 x + v_0 y) \\ F(u,v) = -j\dfrac{A}{2}\left[\delta\left(u - \dfrac{u_0}{2\pi}, v - \dfrac{v_0}{2\pi}\right) - \delta\left(u + \dfrac{u_0}{2\pi}, v + \dfrac{v_0}{2\pi}\right)\right] \end{cases}$$

Periodic Noise Reduction by Frequency Domain Filtering

- **Lowpass and highpass filters for image enhancement have been studied**
- **Bandreject, bandpass, and notch filters as tools for periodic noise reduction or removal are to be studied in this section.**

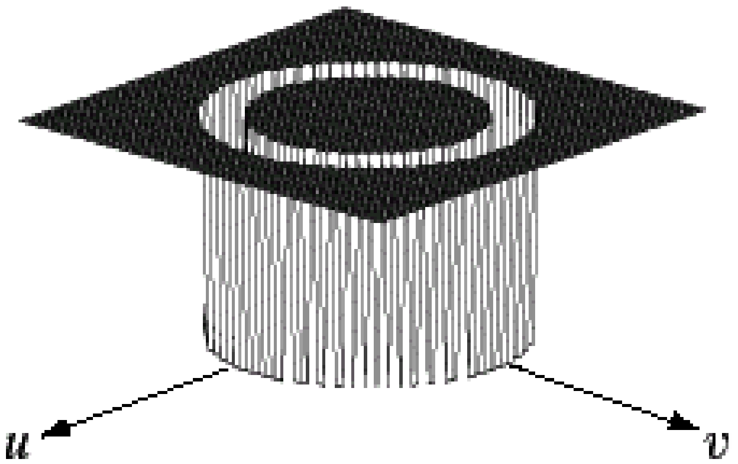
Bandreject Filters

- Bandreject filters remove or attenuate a band of frequencies about the origin of the Fourier transform.
- Similar to those LPFs and HPFs studied, we can construct ideal, Butterworth, and Gaussian bandreject filters

Bandreject Filters

- **Ideal** bandreject filter

$$H(u,v)=\begin{cases}1 & if\ D(u,v)<D_0-\dfrac{W}{2}\\ 0 & if\ D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 1 & if\ D(u,v)>D_0+\dfrac{W}{2}\end{cases}$$

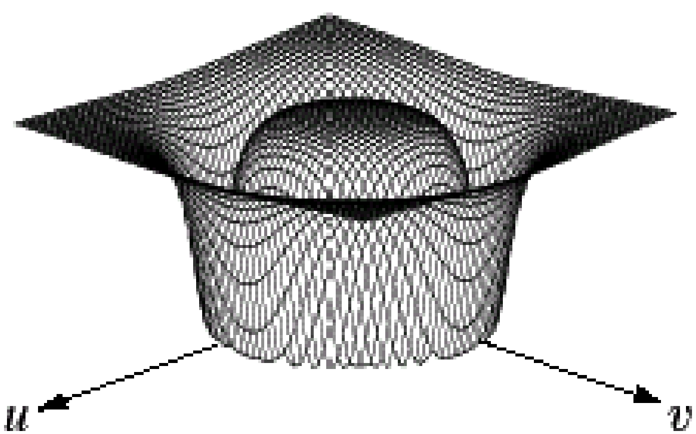


Bandreject Filters

- **Butterworth** bandreject filter

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

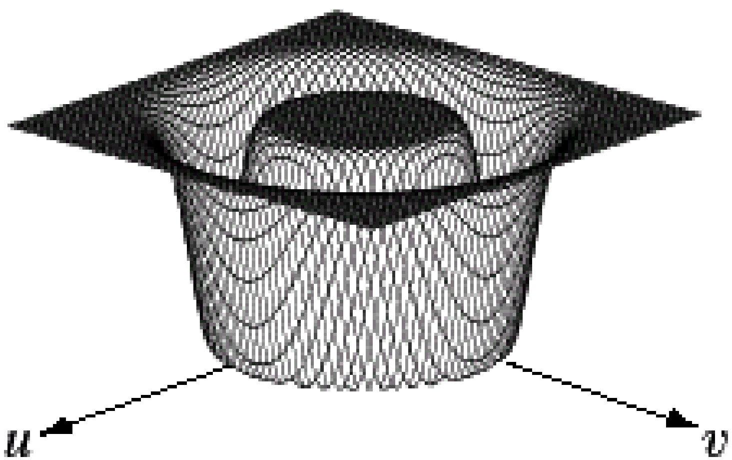
$n=1$



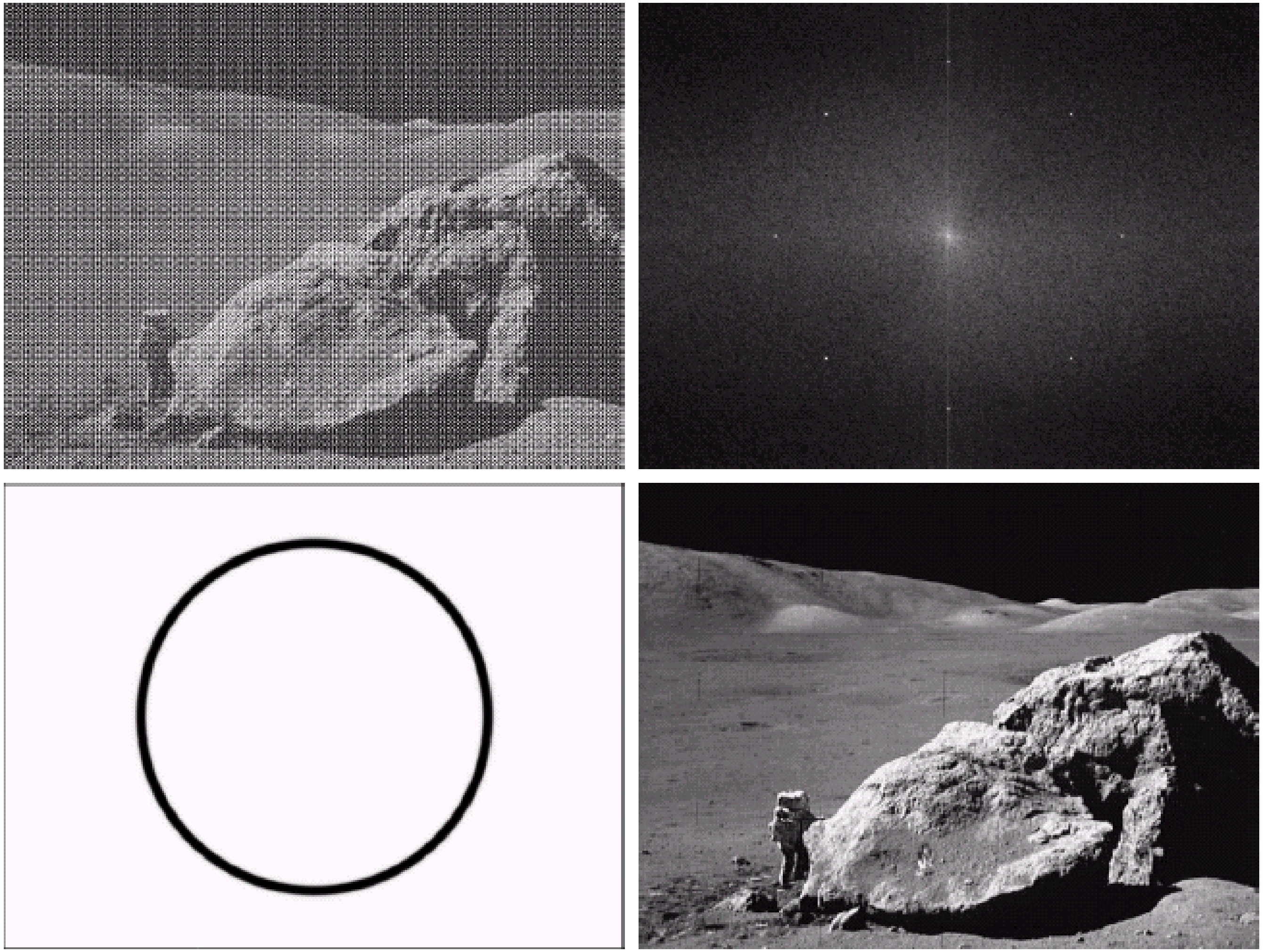
Bandreject Filters

- **Gaussian** bandreject filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)W}\right]}$$



Bandreject Filters



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Bandbass Filters

Bandpass filter performs the opposite of a bandpass filter

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

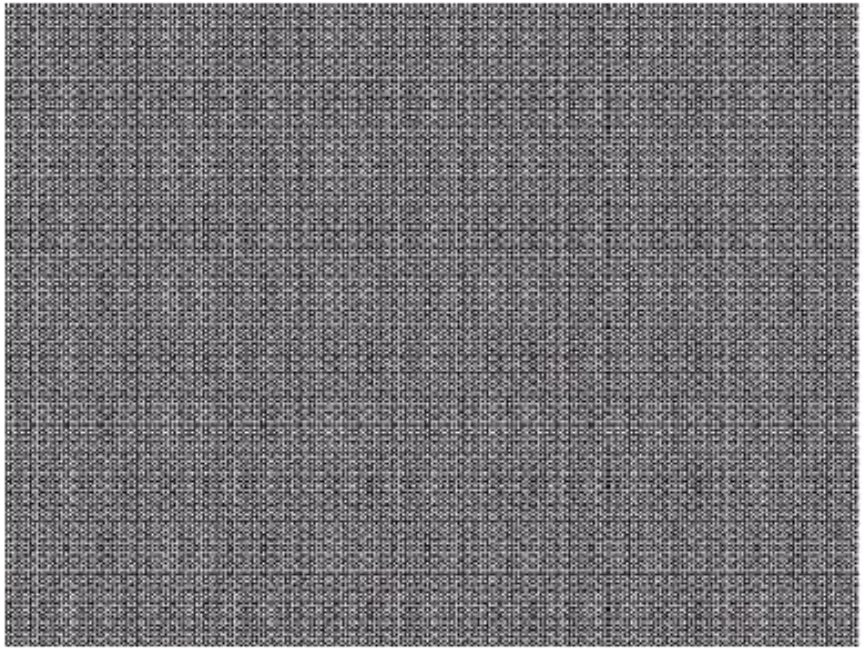


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

Notch Filters

- Notch filter rejects frequencies in predefined neighborhoods about a center frequency.
- It appears in symmetric pairs about the origin because the Fourier transform of a real valued image is symmetric.

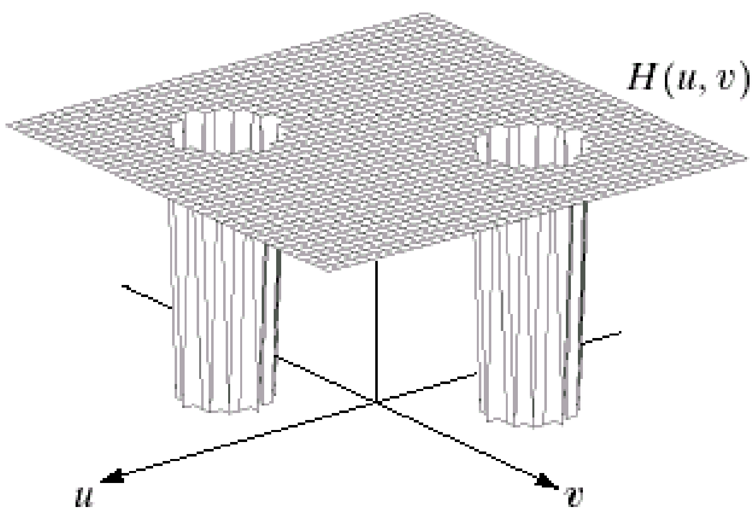
Notch Filters

- **Ideal** notch filter

$$H(u,v)=\begin{cases}0 & if\ D_1(u,v)\le D_0\ or\ D_2(u,v)\le D_0\\ 1 & otherwise\end{cases}$$

$$D_1(u,v)=\left[\left(u-M/2-u_0\right)^2+\left(v-N/2-v_0\right)^2\right]^{1/2}$$

$$D_2(u,v)=\left[\left(u-M/2+u_0\right)^2+\left(v-N/2+v_0\right)^2\right]^{1/2}$$

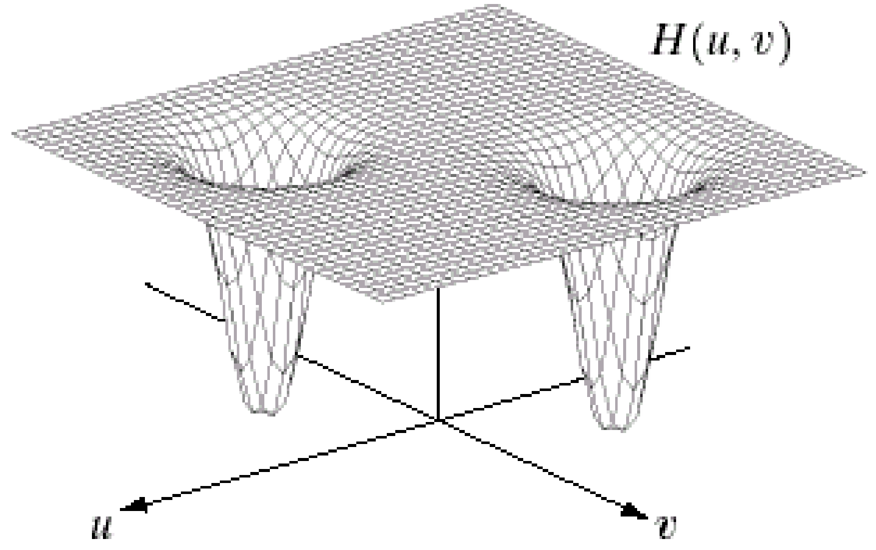


Notch Filters

- **Butterworth** notch filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v) D_2(u,v)} \right]^{2n}}$$

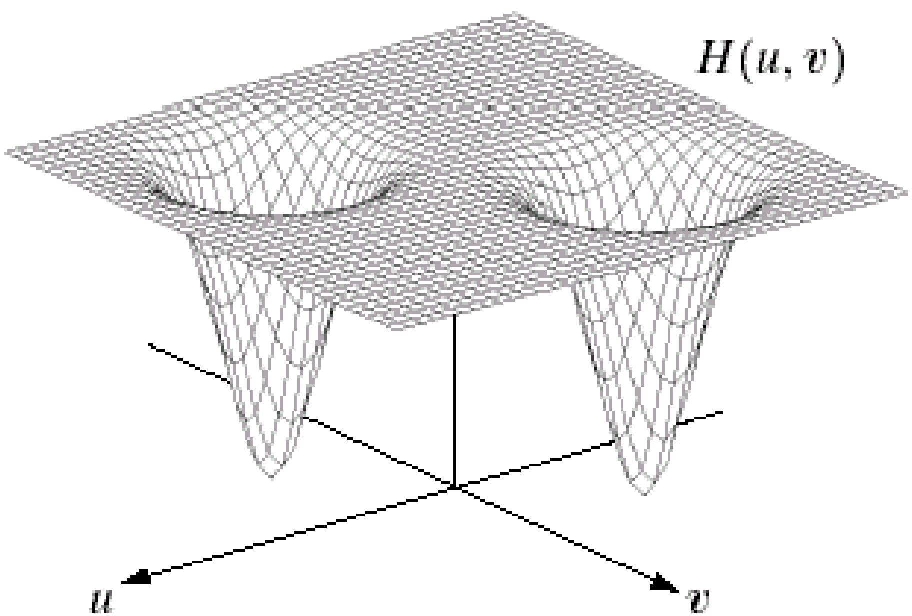
n=2



Notch Filters

- **Gaussian** notch filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]}$$



Notch Filters

Notch filters that pass, rather than suppress:

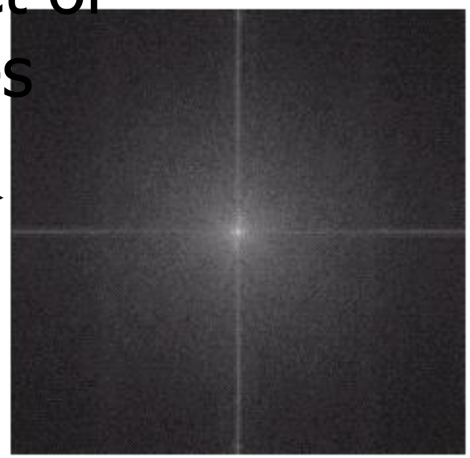
$$H_{np}(u,v) = 1 - H_{nr}(u,v)$$

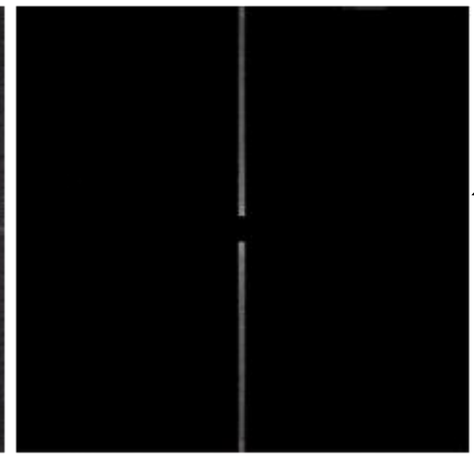
- *NR* filters become highpass filters if $u_0 = v_0 = 0$
- *NP* filters become lowpass filters if $u_0 = v_0 = 0$

Notch Filters

You can see the effect of scan lines

Spectrum of image





Notch pass filter

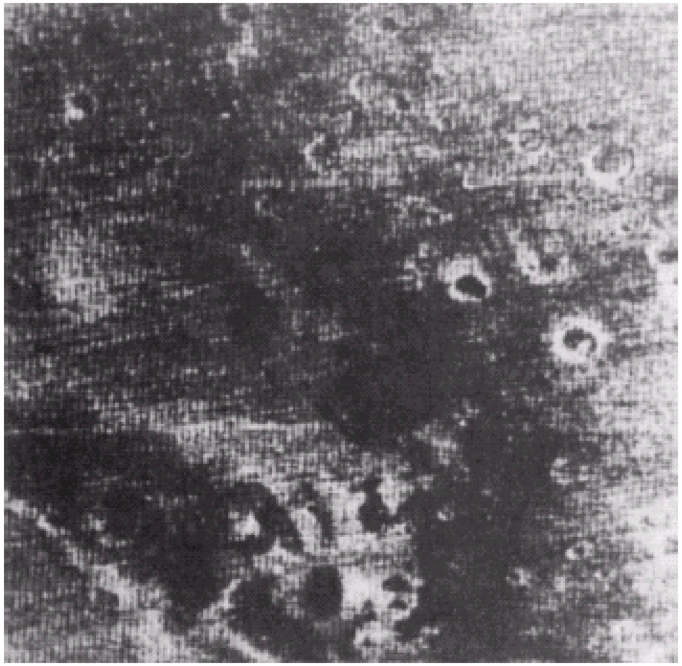
IFT of NP filtered image

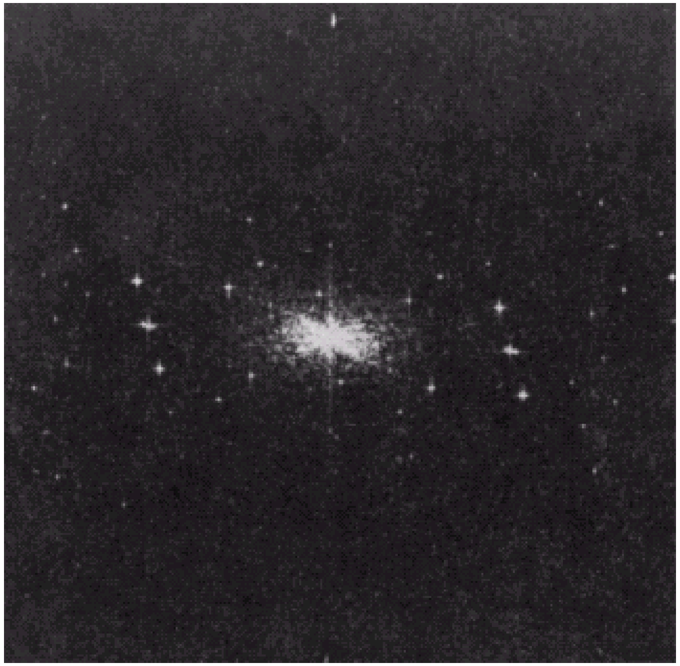
Result of NR filter

Optimum Notch Filtering

a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*.
(b) Fourier spectrum showing periodic interference.
(Courtesy of NASA.)





Optimum Notch Filtering

- In the ideal case, the original image can be restored if the noise can be estimated completely.
  - That is: $f(x,y) = g(x,y) - \eta(x,y)$

- However, the noise can be only partially estimated. This means the restored image is not exact.
  - Which means $\hat{f}(x,y) = g(x,y) - \hat{\eta}(x,y)$

$$\hat{\eta}(x,y) = IFT\left\{H(u,v)G(u,v)\right\}$$

Optimum Notch Filtering

- In this section, we try to improve the restored image by introducing a modulation function
  - $$\hat{f}(x,y) = g(x,y) - w(x,y)\hat{\eta}(x,y)$$
  - Here the modulation function is a constant within a neighborhood of size (2a+1) by (2b+1) about a point (x,y)
  - We optimize its performance by minimizing the local variance of the restored image at the position (x,y)

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x,y) \right]^2$$

$$\bar{\hat{f}}(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s, y+t)$$

Optimum Notch Filtering

Points on or near Edge of the image can be treated by considering partial neighborhoods

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{[g(x+s,y+t) - w(x+s,y+t)\hat{\eta}(x+s,y+t)] - [\bar{g}(x,y) - \overline{w(x,y)\hat{\eta}(x,y}]\}^2$$

Assumption: $w(x+s,y+t) = w(x,y) \quad for \quad -a \le s \le a \ and \ -b \le t \le b$

$$\Rightarrow \overline{w(x,y)\hat{\eta}(x,y)} = w(x,y)\overline{\hat{\eta}}(x,y)$$

Optimum Notch Filtering

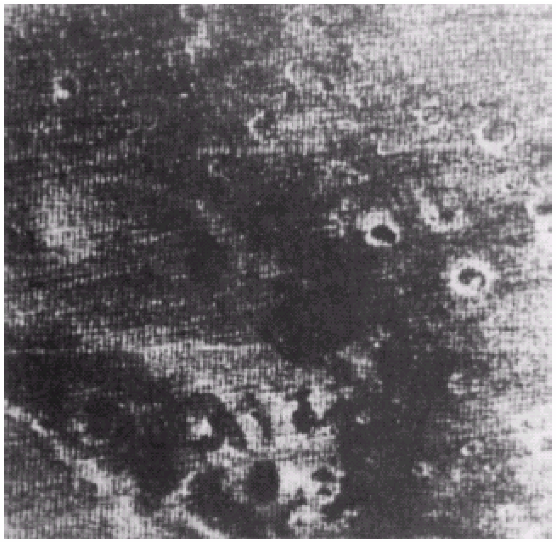
$$\sigma^2(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\{[g(x+s,y+t) - w(x+s,y+t)\hat{\eta}(x+s,y+t)] - [\bar{g}(x,y)-w(x,y)\overline{\hat{\eta}}(x,y)\}^2$$

To minimize $\sigma^2(x,y)$

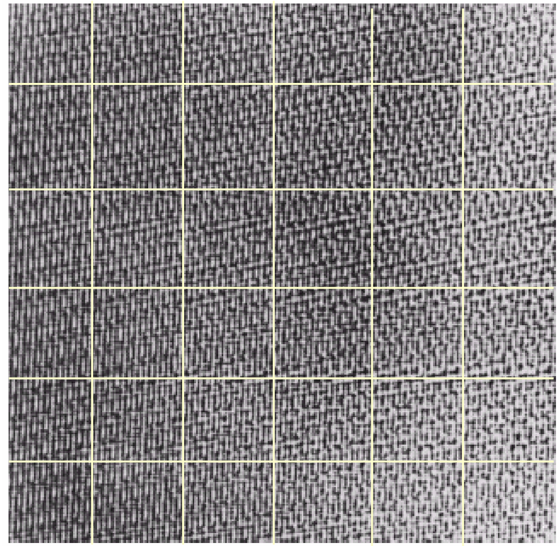
$$\frac{\partial\sigma^2(x,y)}{\partial w(x,y)}=0$$

$$\Rightarrow w(x,y)=\frac{\overline{g(x,y)\hat{\eta}(x,y)}-\bar{g}(x,y)\overline{\hat{\eta}}(x,y)}{\overline{\hat{\eta}^2}(x,y)-\overline{\hat{\eta}}^2(x,y)}$$

Optimum Notch Filtering



$g(x, y)$



$\hat{\eta}(x, y)$

$$\hat{f}(x, y) = g(x, y) - w(x, y)\hat{\eta}(x, y)$$

Optimum Notch Filtering

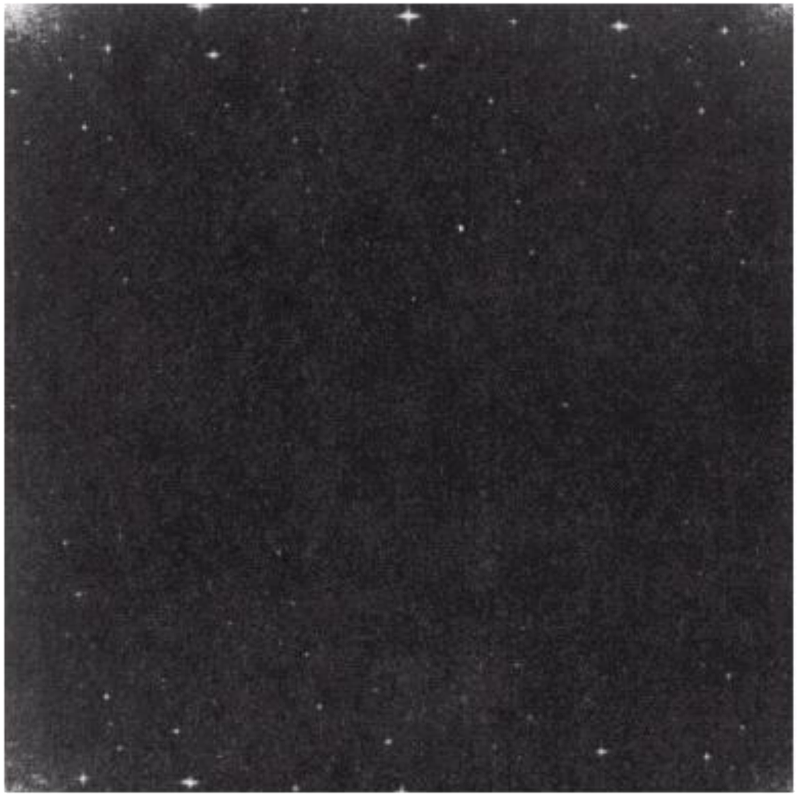
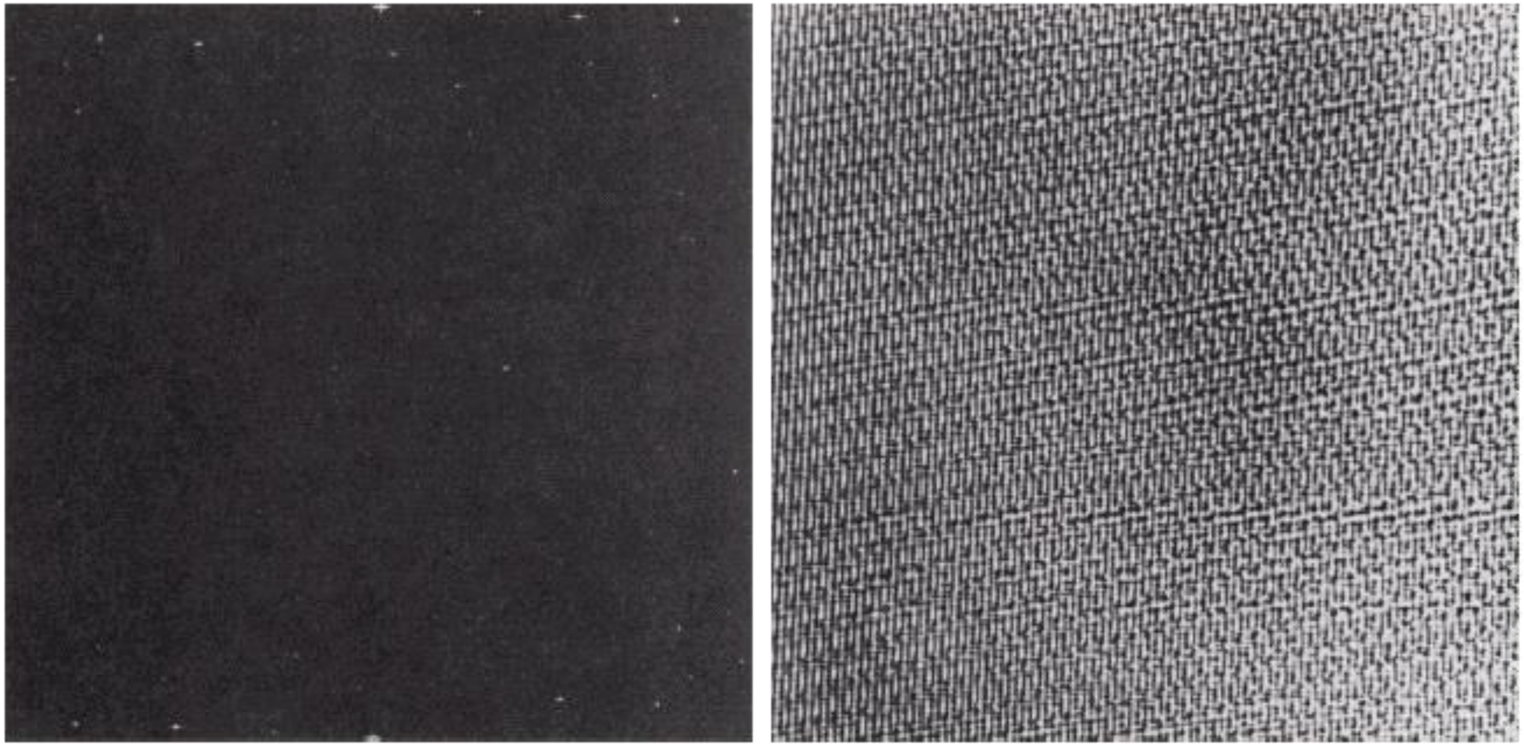


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

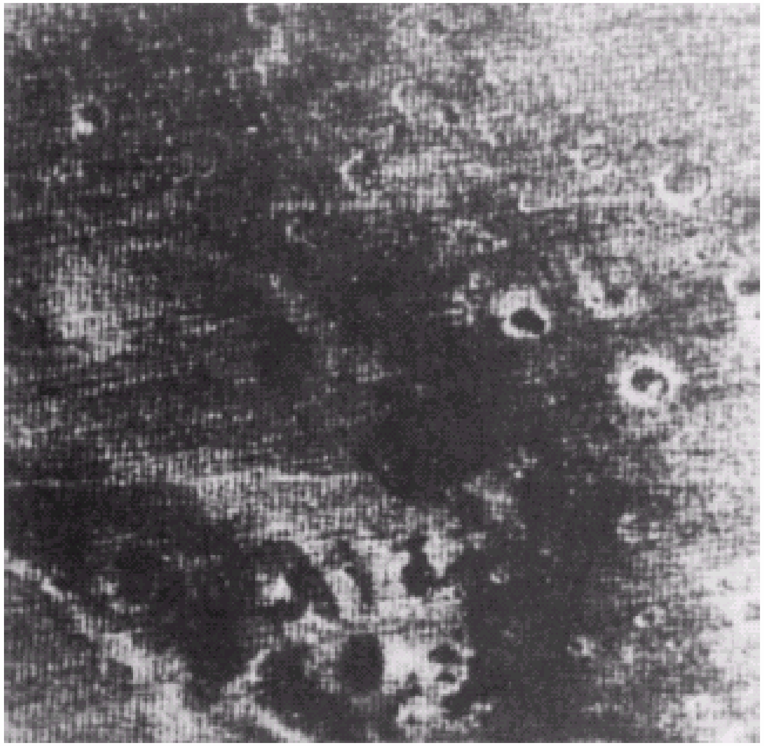
Optimum Notch Filtering



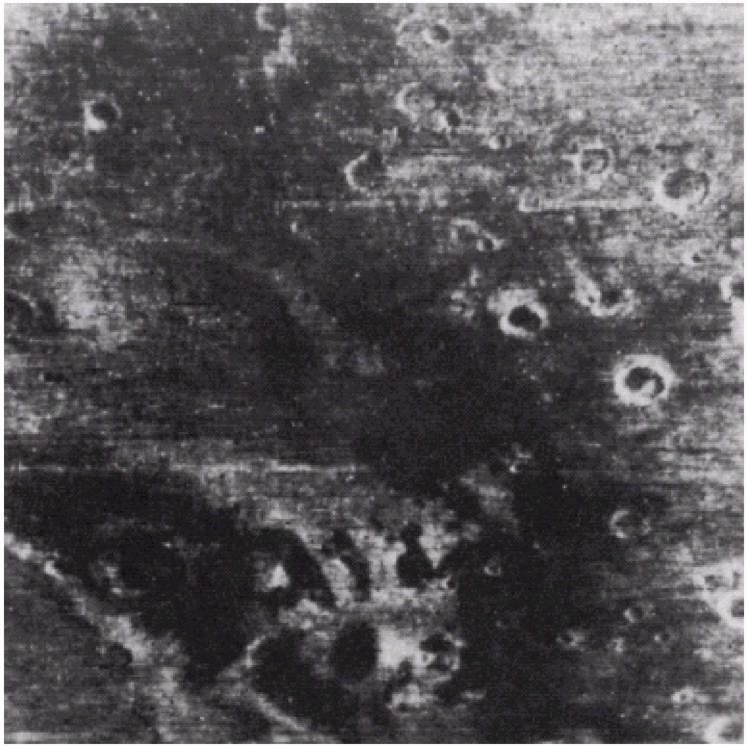
a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

Optimum Notch Filtering



$g(x, y)$



$\hat{f}(x, y)$

Image size:
512x512
a=b=15