Week-3

Lecture-1

Max. $10x_1 + 9x_2$

S.t.

$$3x_1 + 3x_2 \leq 21$$
$$4x_1 + 3x_2 \leq 24$$
$$x_1, x_2 \geq 0$$

Max. $10x_1 + 9x_2 + 0x_3 + 0x_4$

S.t.

$$3x_1 + 3x_2 + x_3 = 21$$
$$4x_1 + 3x_2 + x_4 = 24$$
$$x_1, x_2, x_3, x_4 \geq 0$$

$$x_3 = 21 - 3x_1 - 3x_2$$
$$x_4 = 24 - 4x_1 - 3x_2$$
$$Z = 0 + 10x_1 + 9x_2$$

x_1, x_2 are NBV and x_3, x_4 are BV

$\therefore x_3 = 21$

$x_4 = 24$

$Z = 0$

Z can be increased by either increasing x_1 or x_2

But Coefficient of x_1 is 10

$\therefore$ We will increase x_1 when we start increasing x_1, x_3 and x_4 will start reducing.

So $x_3 = 21 - 3x_1 - 3x_2$

$x_4 = 24 - 4x_1 - 3x_2$

When $x_1 = 7$ $\quad 21 - 3x_1 = 0$

but $\quad 24 - 4x_1 = 24 - 28 = -4 < 0$

When $x_1 = 6$ $\quad 21 - 3x_1 = 21 - 18 = 3$

$24 - 4x_1 = 24 - 24 = 0$

$\therefore$ We can increase x_1 upto 6 only.

$x_4 = 24 - 4x_1 - 3x_2$ | x_4 will become B.V.

$\therefore 4x_1 = 24 - 3x_2 - x_4$

$\therefore x_1 = 6 - \frac{3}{4}x_2 - \frac{1}{4}x_4$

Put value of x_1 in $x_3 = 21 - 3x_1 - 3x_2$, we get

$$x_3 = 21 - 3\left(6 - \frac{3}{4}x_2 - \frac{1}{4}x_4\right) - 3x_2$$

$$= 21 - 18 + \frac{9}{4}x_2 + \frac{3}{4}x_4 - 3x_2$$

$$= 3 - \frac{3}{4}x_2 + \frac{3}{4}x_4$$

Put $x_4 = 6 - \frac{3}{4}x_2 - \frac{1}{4}x_4$ in Z also We will get

$$Z = 10x_4 + 9x_2$$

$$= 10\left(6 - \frac{3}{4}x_2 - \frac{1}{4}x_4\right) + 9x_2$$

$$= 60 - \frac{15}{2}x_2 - \frac{5}{2}x_4 + 9x_2$$

$$= 60 + \frac{3}{2}x_2 - \frac{5}{2}x_4$$

$\therefore$ $x_4 = 6$ $\quad$ x_2, x_4 NBV

$x_3 = 3$ $\quad$ x_4, x_3 BV

$Z = 60$

→ We can increase Z by increasing x_2 only. ($\because$ increase in x_4, will decrease Z) and we can't make $x_4 < 0$ because of non-negativity Constraint

$$6 - \frac{3}{4}x_2 = 0 \implies x_2 = 8$$

$$3 - \frac{3}{4}x_2 = 0 \implies x_2 = 4$$

$$\text{Min } x_2 = 4$$

$\therefore$ We can increase x_2 upto 4 only.

$\therefore$ x_2 will replace x_3 in

$$x_3 = 3 - \frac{3}{4}x_2 + \frac{3}{4}x_4$$

$$\therefore \frac{3}{4}x_2 = 3 - x_3 + \frac{3}{4}x_4$$

$$\therefore x_2 = 4 - \frac{4}{3}x_3 + x_4$$

Put Value of x_2 in

$$x_1 = 6 - \frac{3}{4}x_2 - \frac{1}{4}x_4$$

$$= 6 - \frac{3}{4}\left(4 - \frac{4}{3}x_3 + x_4\right) - \frac{1}{4}x_4$$

$$= 6 - 3 + x_3 - \frac{3}{4}x_4 - \frac{1}{4}x_4$$

$$\therefore x_4 = 3 + x_3 - x_4$$

$$Z = 60 + \frac{3}{2}x_2 - \frac{5}{2}x_4$$

$$= 60 + \frac{3}{2}\left(4 - \frac{4}{3}x_3 + x_4\right) - \frac{5}{2}x_4$$

$$= 60 + 6 - 2x_3 + \frac{3}{2}x_4 - \frac{5}{2}x_4$$

$$= 66 - 2x_3 - x_4$$

Here x_3, x_4 are N.B.V.

x_1, x_2 are B.V.

$\therefore Z = 66$

$x_2 = 4$

$x_4 = 3$

Verify

$Z = 10x_4 + 9x_2$

$= 10.3 + 9.4$

$= 30 + 36$

$= 66$

This gives only

1) ~~feasible~~ soln

2) progressively ~~better~~ solns

3) Terminates when you reach soln

⇓

why?

Now $Z = 66 - 2x_3 - x_4$

We can't improve Z by increasing x_3 or x_4.

∴ We reached soln

∴ Stop.