

# **Sugeno Fuzzy Inference**

# Sugeno Fuzzy Inference

- Difference between Mamdani and Sugeno is that Sugeno output membership functions are either linear or constant
- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.
- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else

# Sugeno Fuzzy Inference

- Sugeno fuzzy model ( also known as **TSK fuzzy model**) was proposed by Takagi, Sugeno and Kang
- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent: instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the Sugeno-style fuzzy rule is

IF         $x$  is  $A$     AND     $y$  is  $B$     THEN     $z$  is  $f(x, y)$

where:

- $x$ ,  $y$  and  $z$  are linguistic variables;
- $A$  and  $B$  are fuzzy sets on universe of discourses  $X$  and  $Y$ , respectively;
- $f(x, y)$  is a mathematical function.

# Sugeno Fuzzy Inference

- $z = f(x, y)$  is a crisp function in the consequent.
- Usually  $f(x, y)$  is a polynomial of the input variables  $x$  and  $y$ . but it can be any function.
- When  $f(x, y)$  is a first-order polynomial, the resulting fuzzy inference system is called a first-order Sugeno fuzzy model, for example  $z = ax + by + c$ .
- When  $f$  is a constant, we then have a zero-order Sugeno fuzzy model, ( $z = ax + by + c$  where  $(a = b = 0)$ ) which can be viewed either as a special case of the Mamdani Fuzzy inference system in which each rule's consequent is specified by a fuzzy singleton.

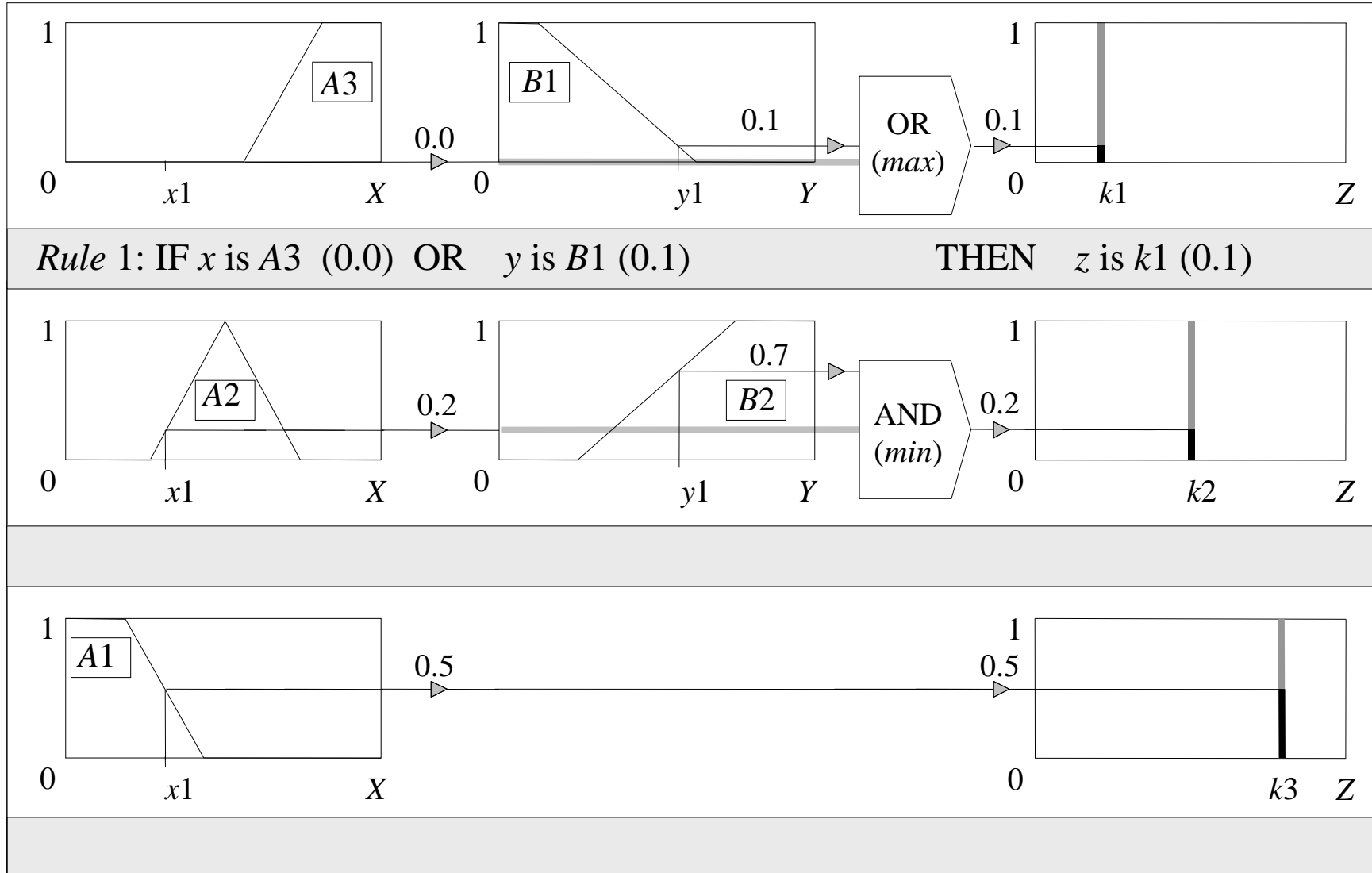
# Sugeno Fuzzy Inference

- The most commonly used zero-order Sugeno fuzzy model applies fuzzy rules in the following form:

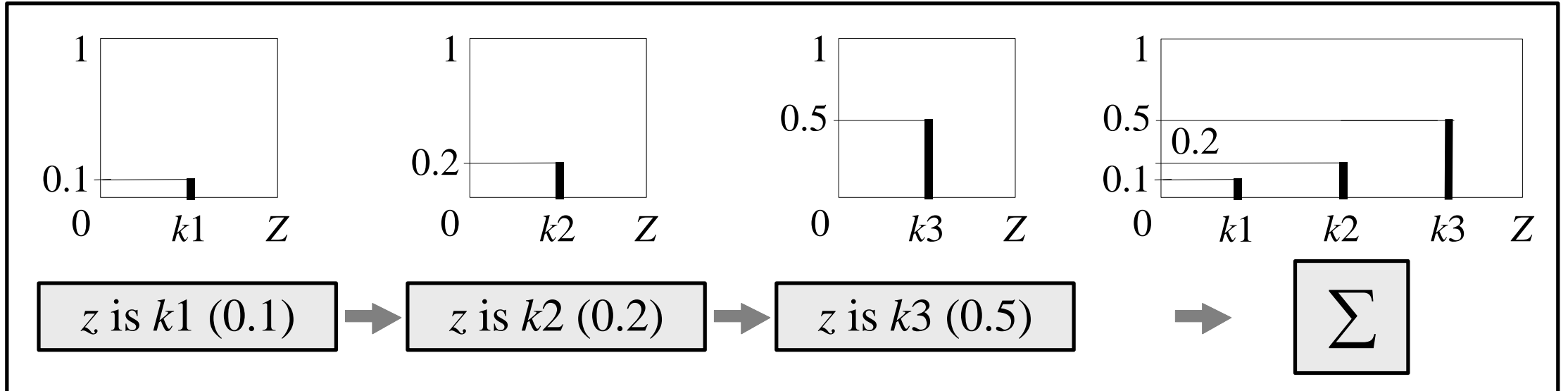
IF  $x$  is  $A$  AND  $y$  is  $B$  THEN  $z$  is  $k$

- where  $k$  is a constant.
- In this case, the output of each fuzzy rule is constant and all consequent membership functions are represented by singleton spikes.

# Sugeno rule evaluation



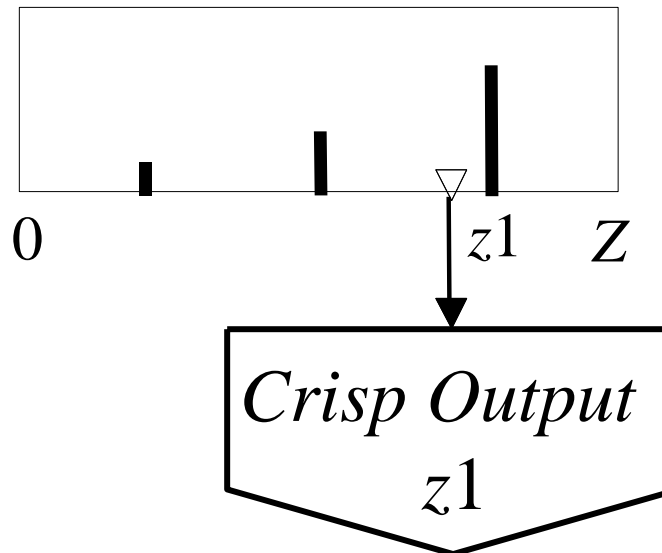
# Sugeno Aggregation



# Sugeno Defuzzification

Weighted Average (WA)

$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$





# How to make a decision on which method to apply – Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.