Digital Image Processing

Chapter 9: Morphological Image Processing

Mathematic Morphology

- used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Mathematic Morphology

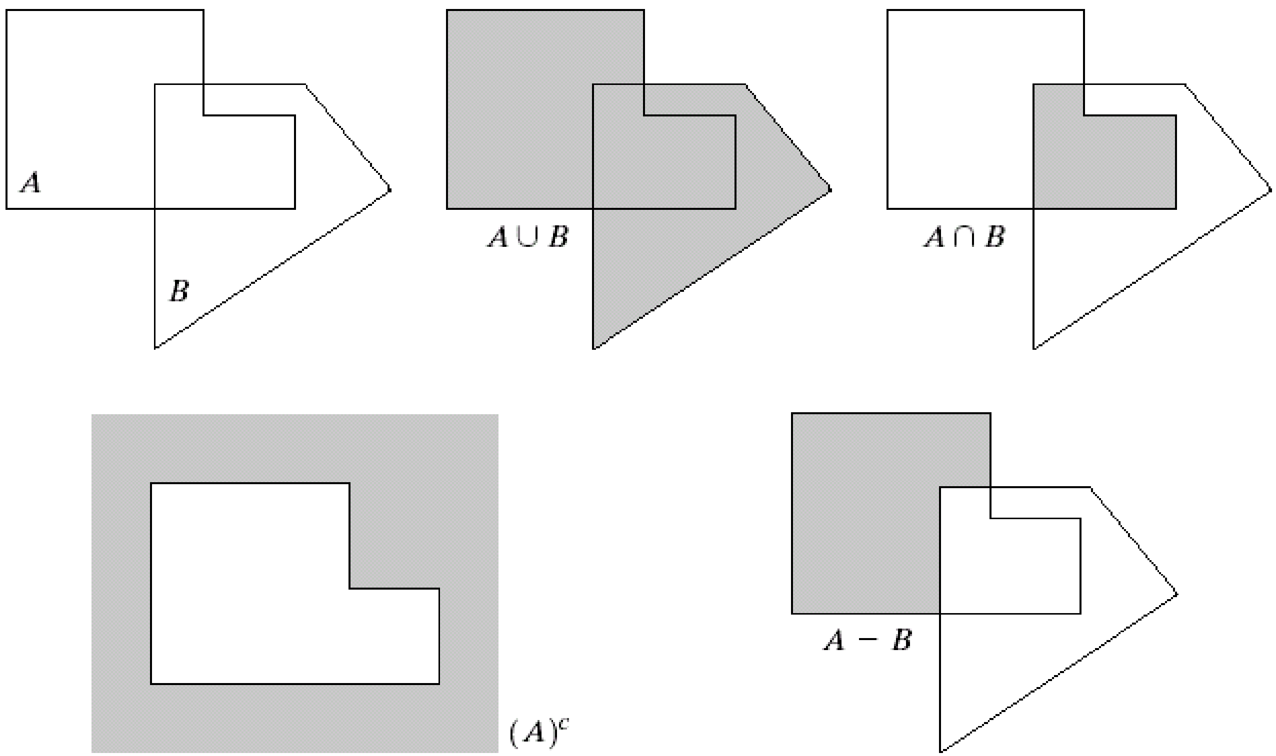
mathematical framework used for:

- pre-processing
  - noise filtering, shape simplification, ...
- enhancing object structure
  - skeletonization, convex hull...
- Segmentation
  - watershed,...
- quantitative description
  - area, perimeter, ...

Z^2 and Z^3

- set in mathematic morphology represent objects in an image
  - binary image (0 = white, 1 = black) : the element of the set is the coordinates (x,y) of pixel belong to the object ⇨ Z^2
- gray-scaled image : the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels ⇨ Z^3

Basic Set Theory



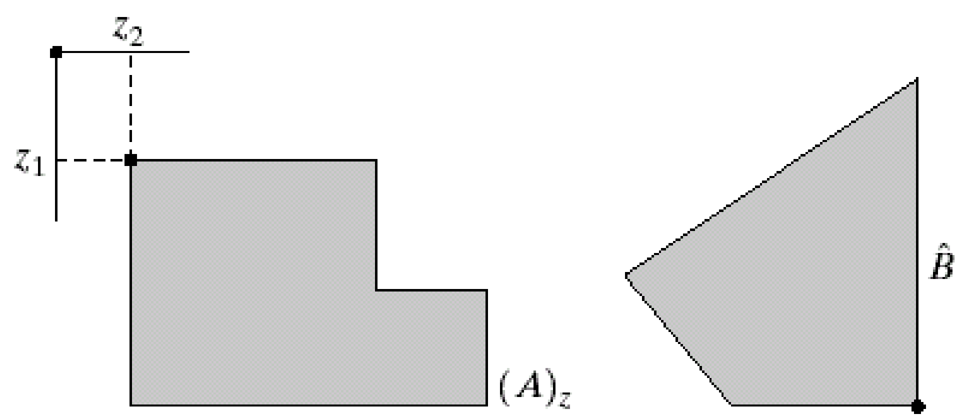
a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflection and Translation

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Logic Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

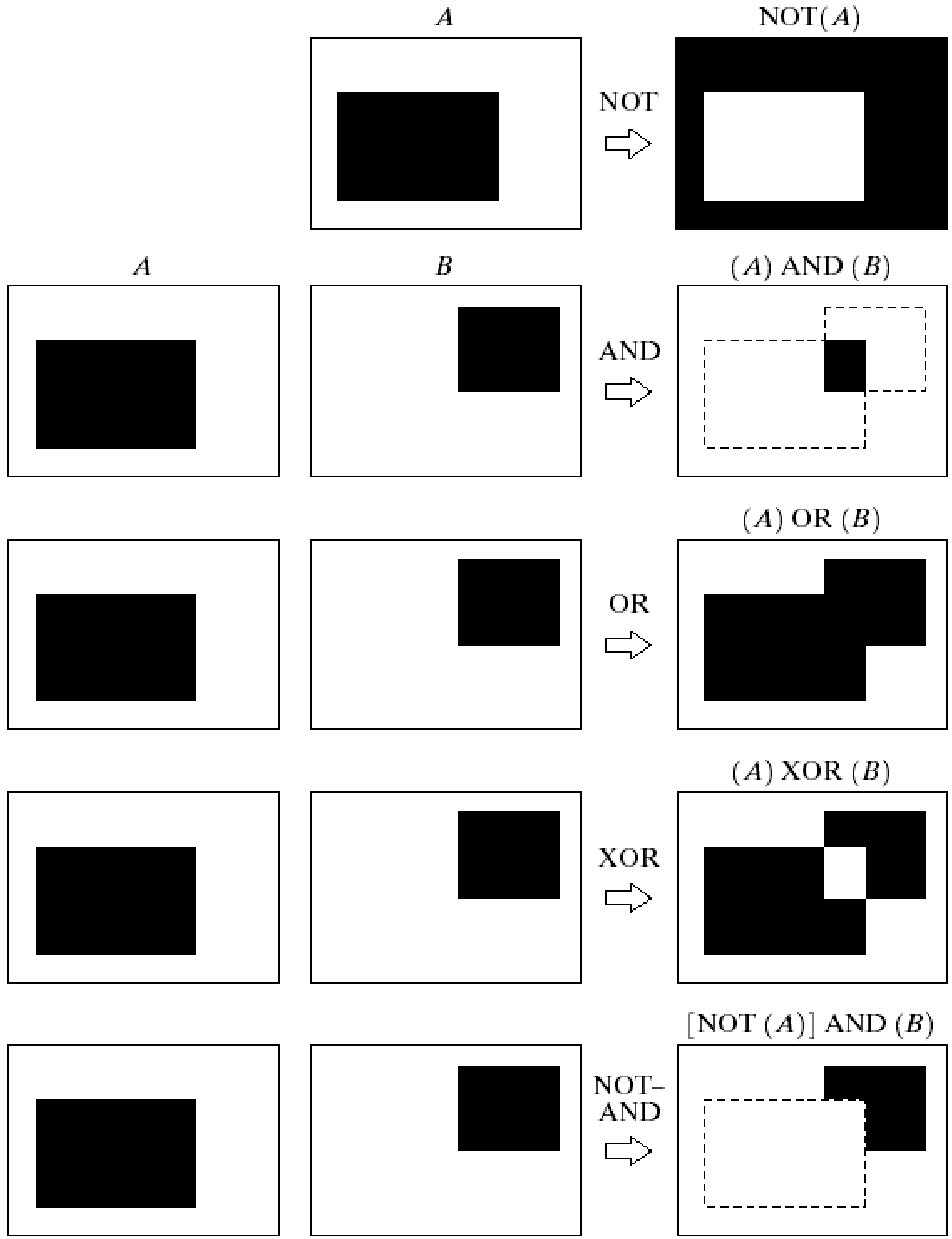
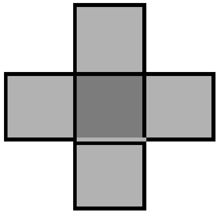
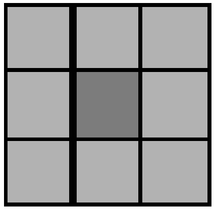


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring element (SE)

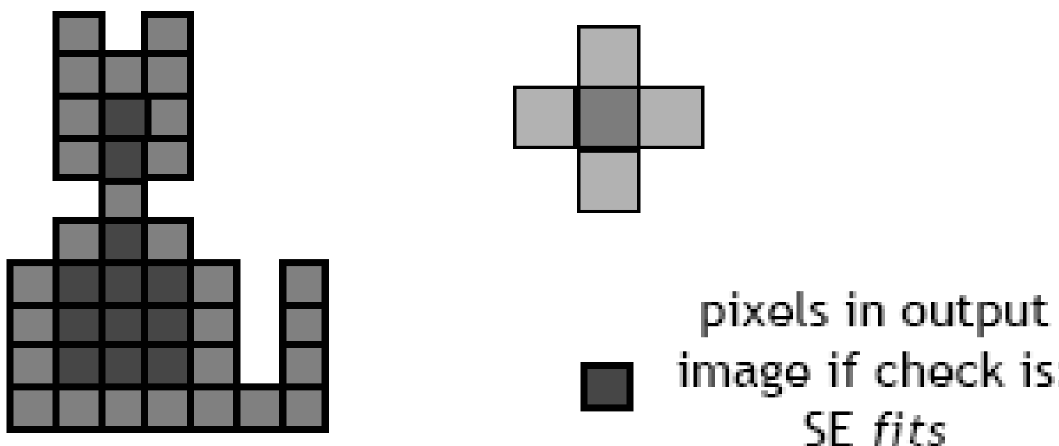
- small set to probe the image under study
- for each SE, define origo
- shape and size must be adapted to geometric properties for the objects





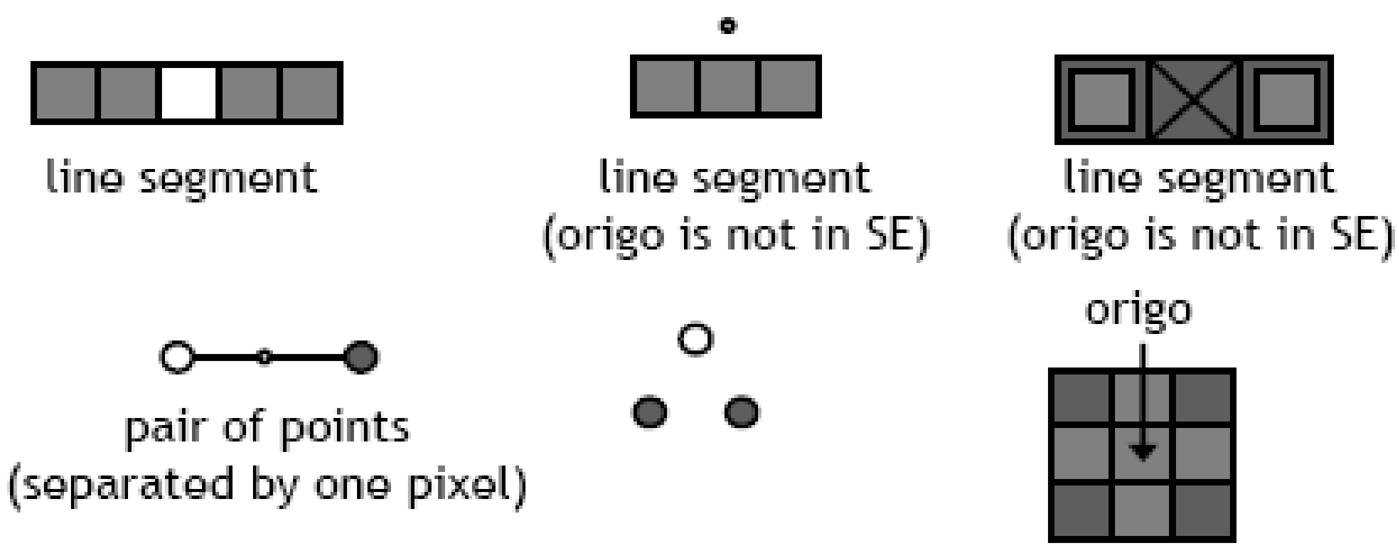
Basic idea

- in parallel for each pixel in binary image:
  - check if SE is ”satisfied”
  - output pixel is set to 0 or 1 depending on used operation



How to describe SE

- many different ways!
- information needed:
  - position of origo for SE
  - positions of elements belonging to SE



Basic morphological operations

- Erosion 

- Dilation 

- combine to
  - Opening → object
  - Closening → background

keep general shape but smooth with respect to

Erosion

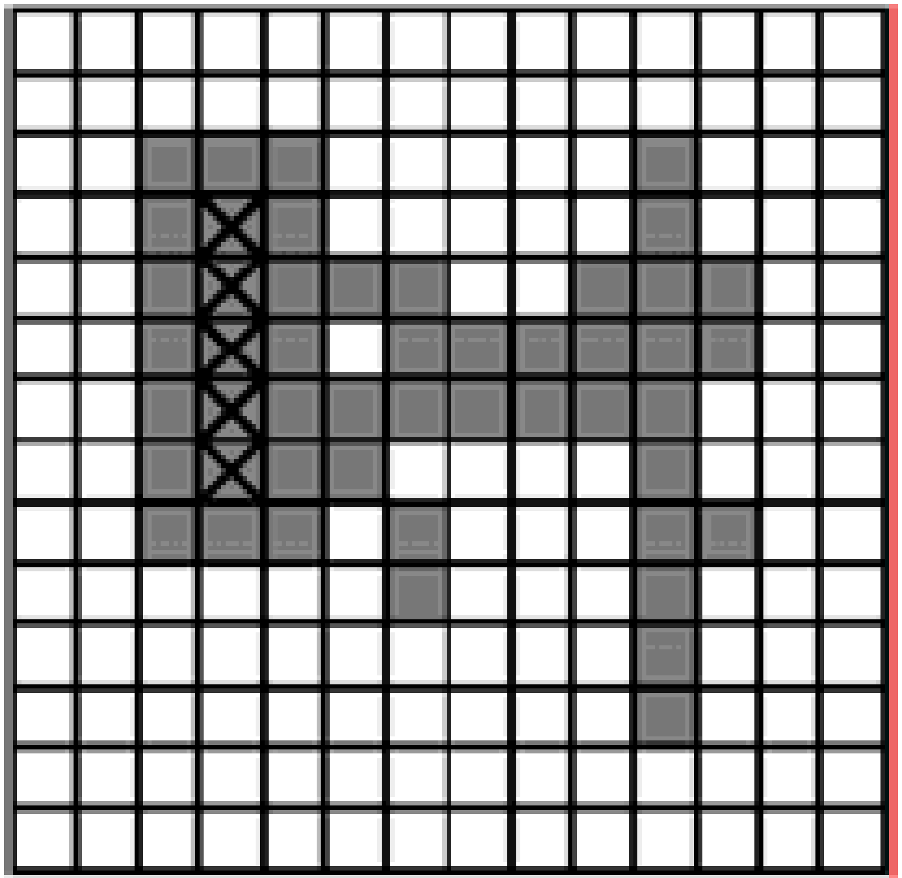
- Does the structuring element **fit the set?**

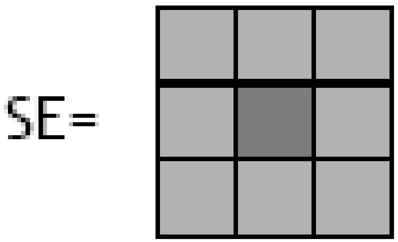
erosion of a set A by structuring element B: all z in A such that B is in A when origin of B=z

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

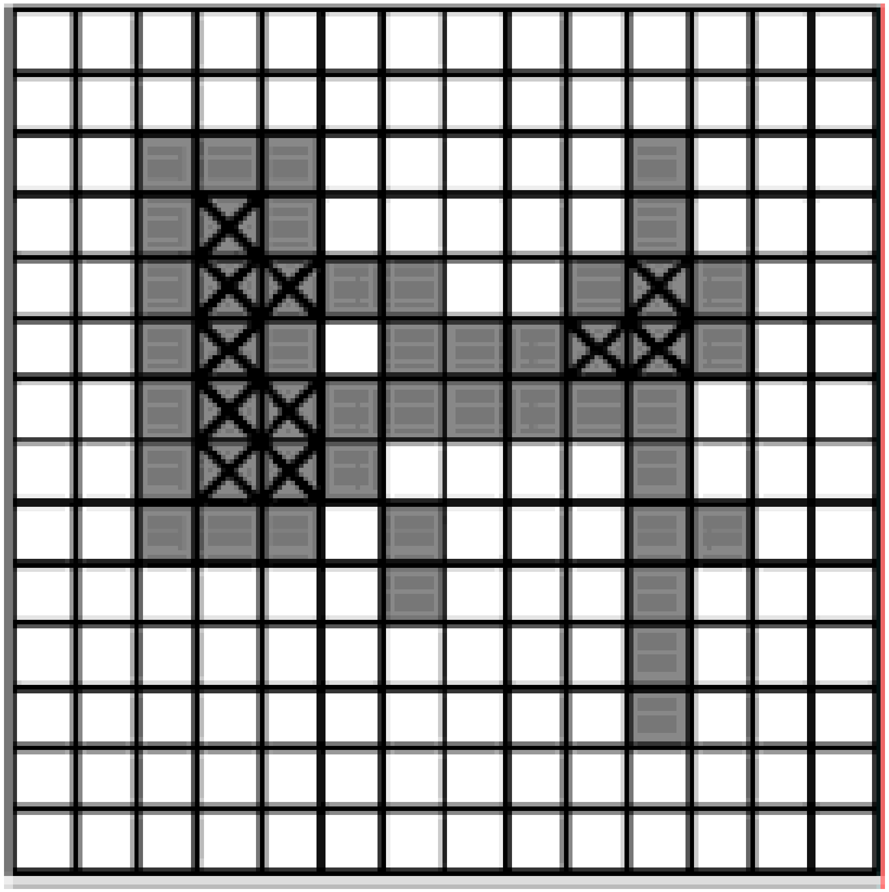
shrink the object

Erosion

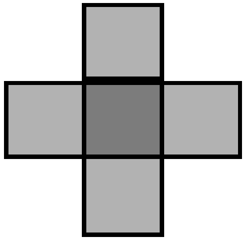


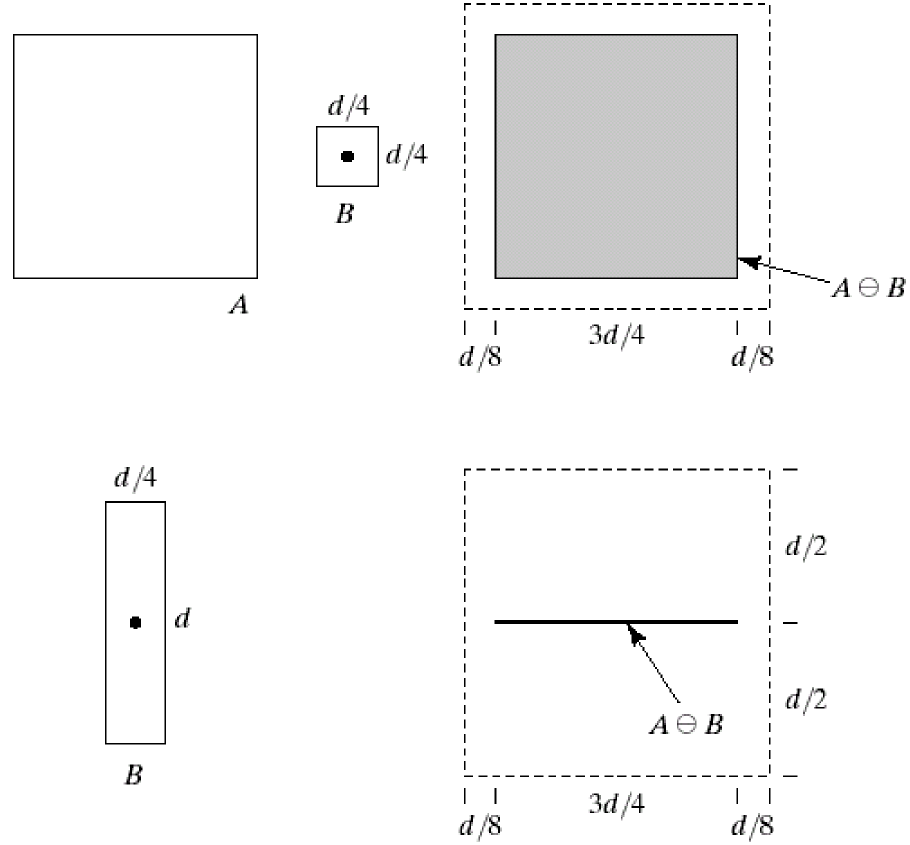


Erosion



SE=





a	b	c
d		e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

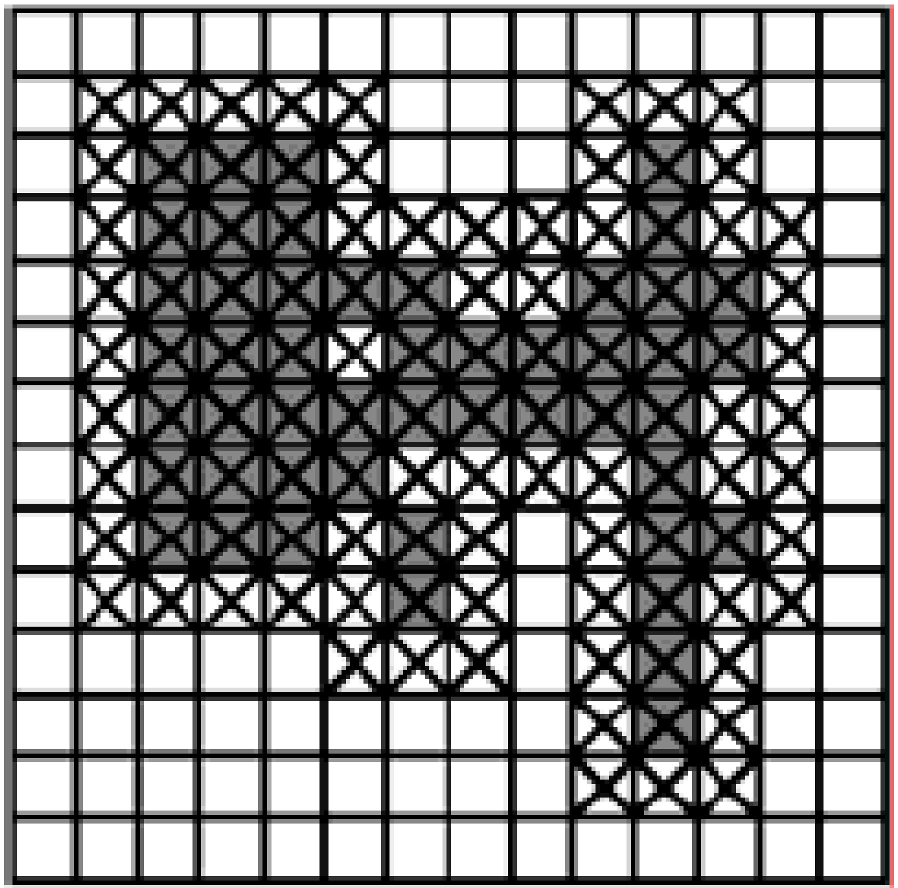
Dilation

- Does the structuring element **hit the set?**
- dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z

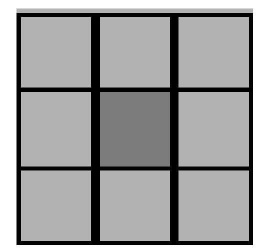
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \Phi\}$$

- **grow the object**

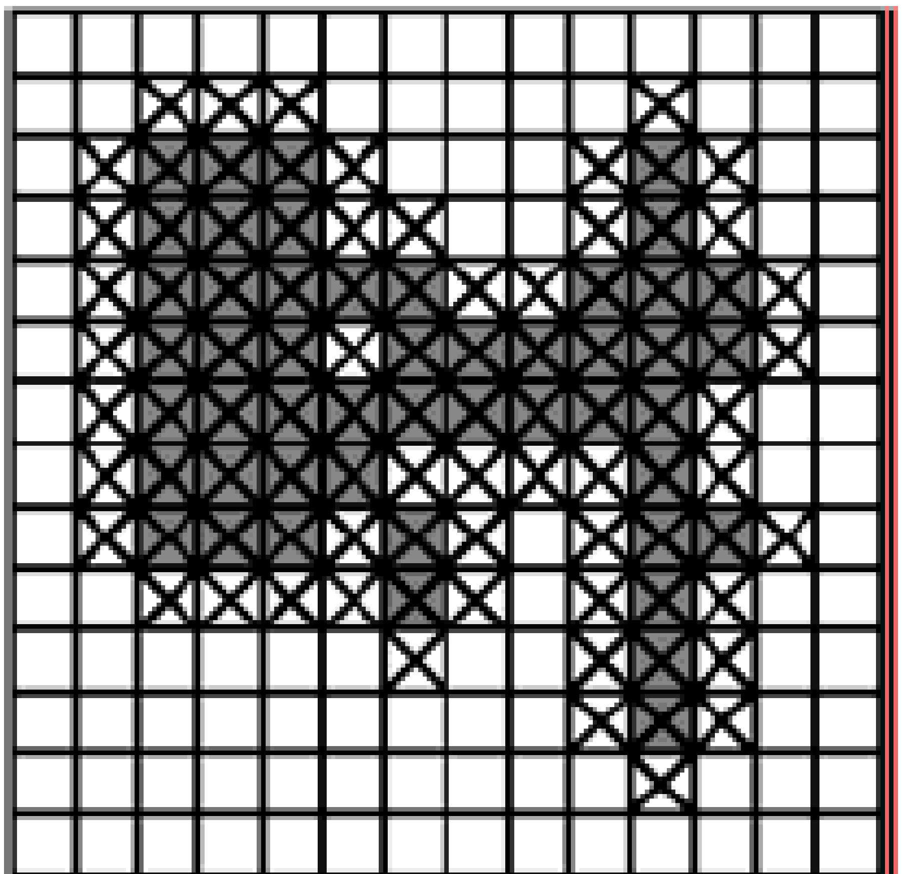
Dilation



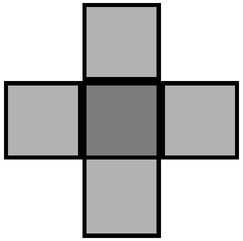
SE=



Dilation

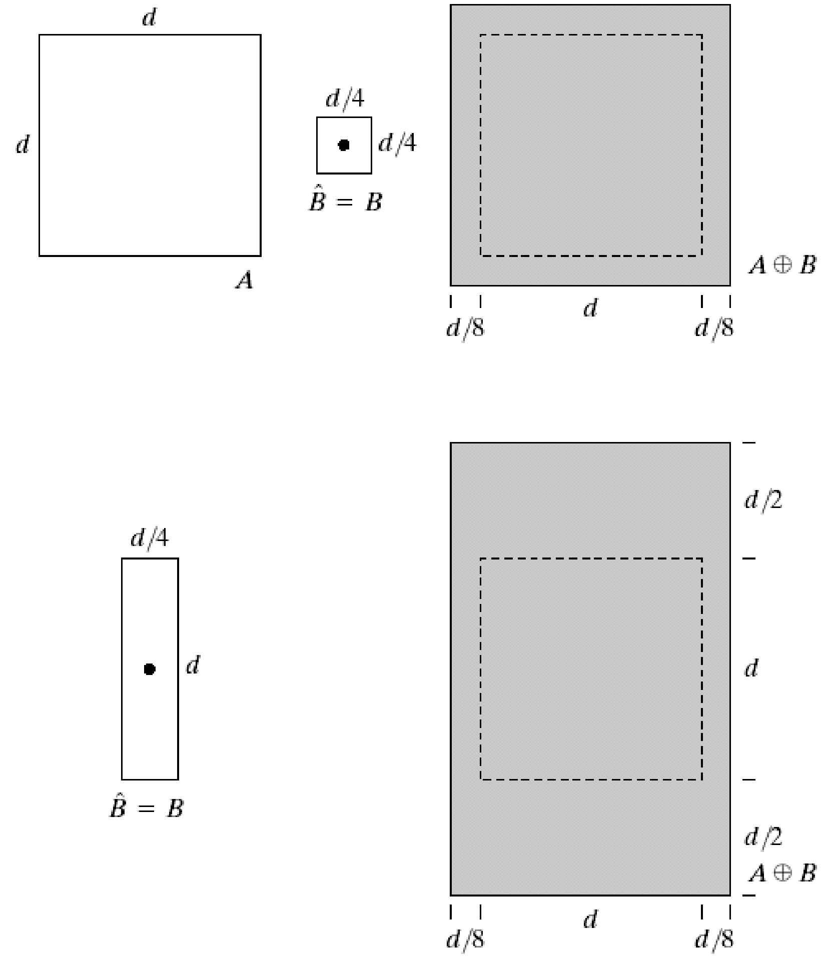


SE=



a b c
d e

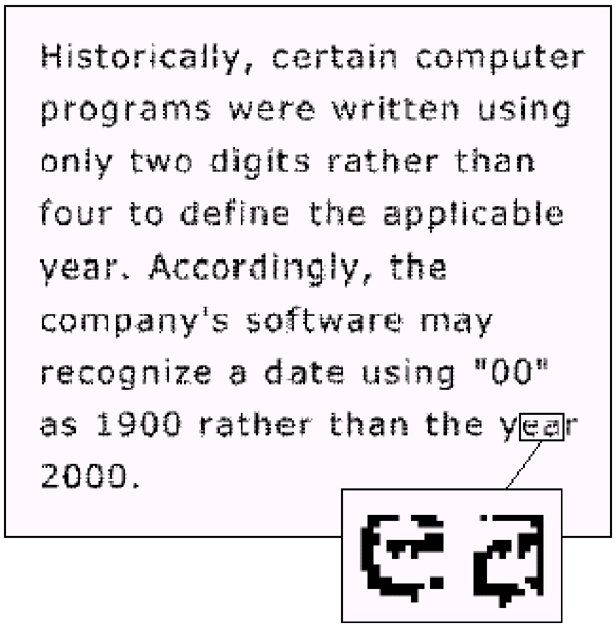
FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



B = structuring element

$$A \oplus B = \{z \,|\, (\hat{B})_z \cap A \neq \Phi\}$$

Dilation : Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

useful

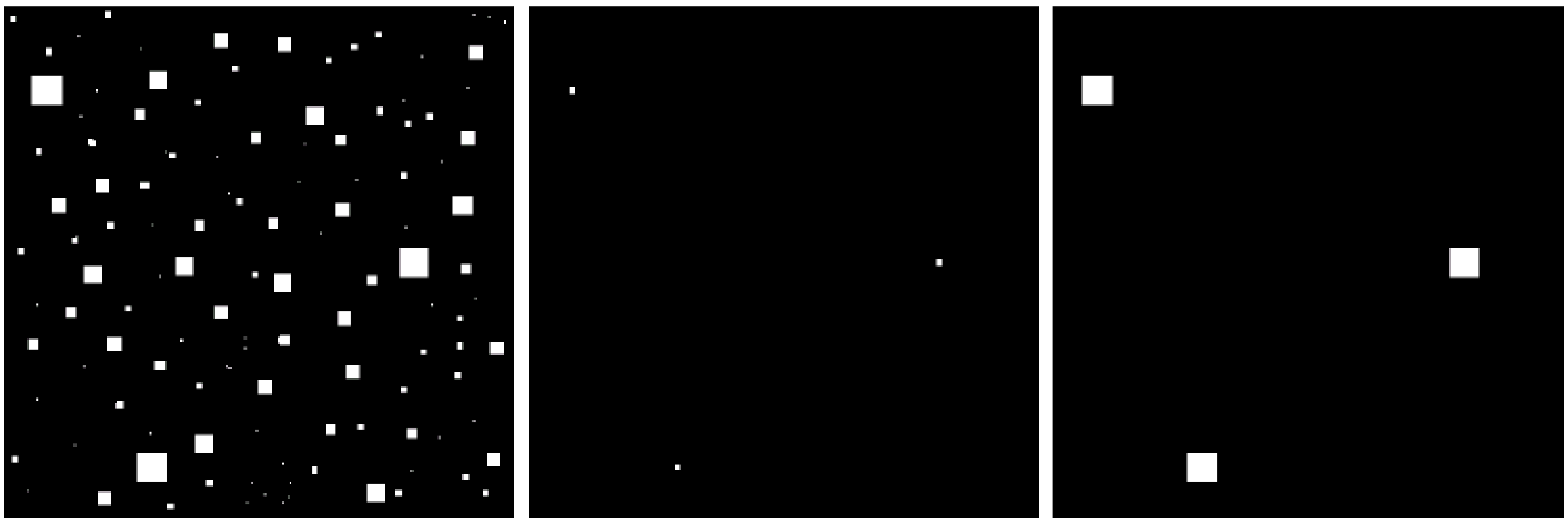
- erosion
  - removal of structures of certain shape and size, given by SE
- Dilation
  - filling of holes of certain shape and size, given by SE

Combining erosion and dilation

- WANTED:
  - remove structures / fill holes
  - without affecting remaining parts

- SOLUTION:
- combine erosion and dilation
- (using same SE)

Erosion : eliminating irrelevant detail



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

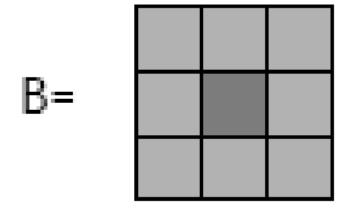
Opening

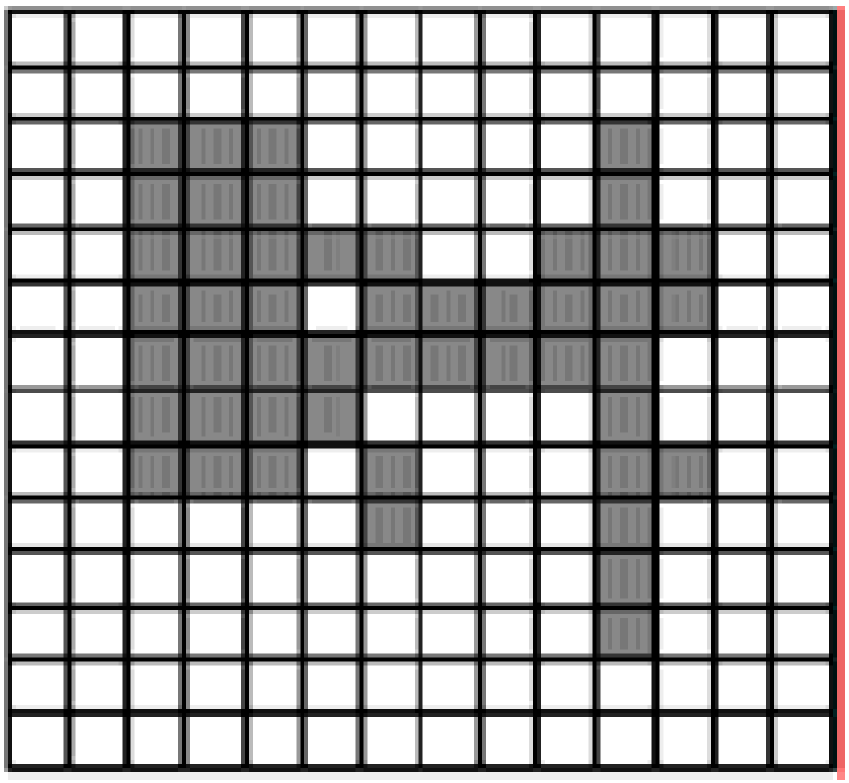
erosion followed by dilation, denoted ∘

$$A \circ B = (A \ominus B) \oplus B$$

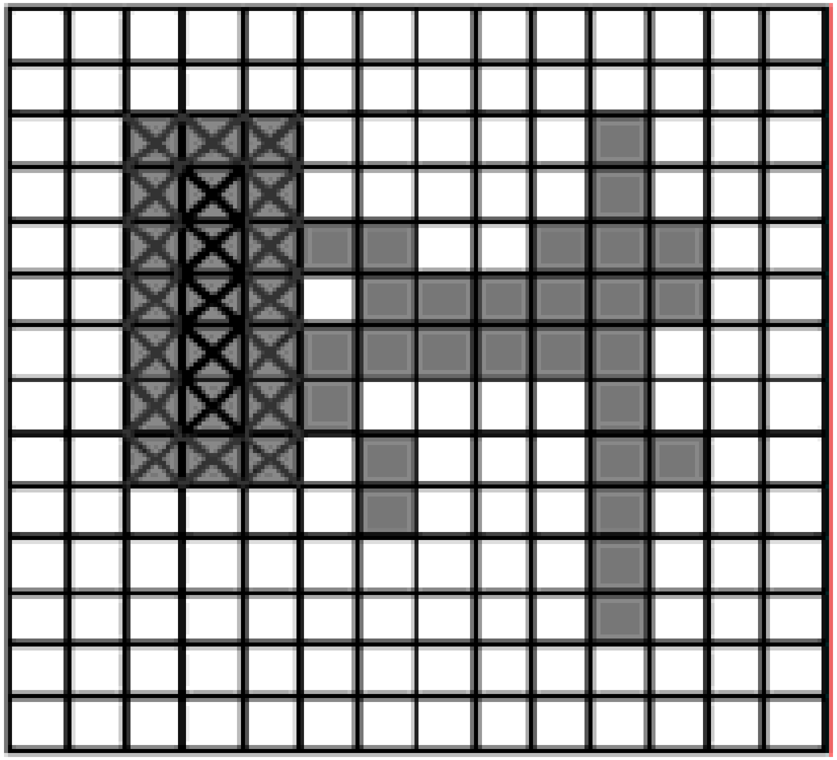
- eliminates protrusions
- breaks necks
- smoothes contour

Opening



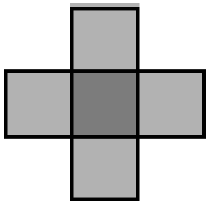


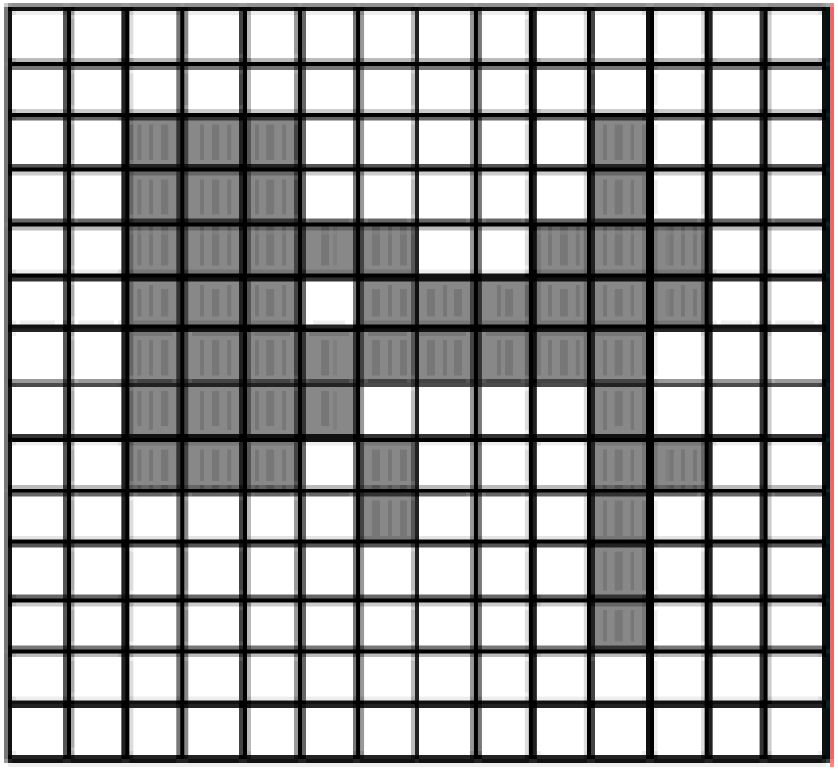
A



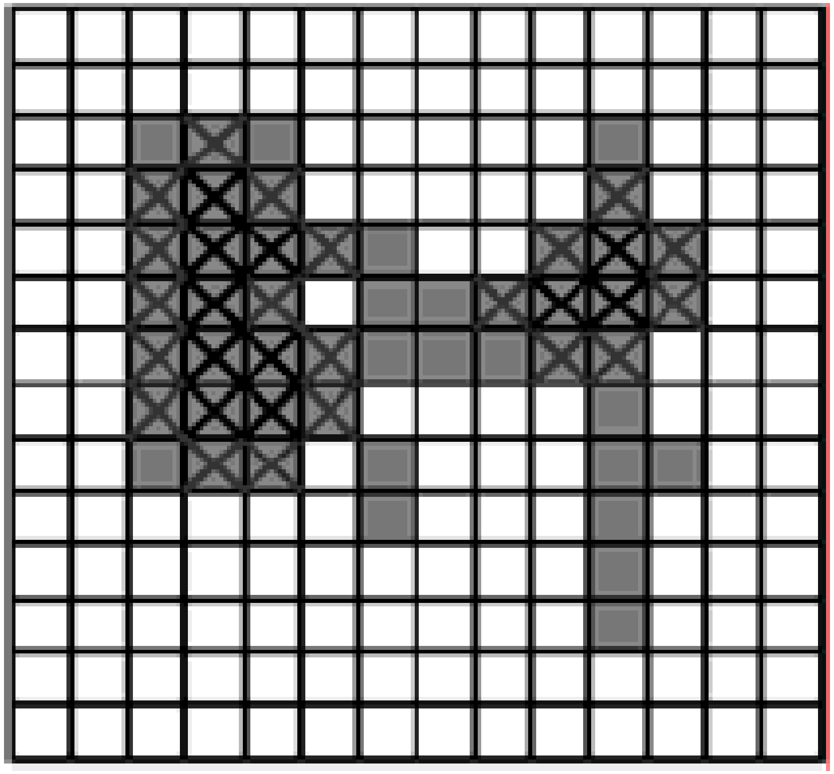
A⊖B A∘B

Opening

B=




A



A⊖B A∘B

Opening

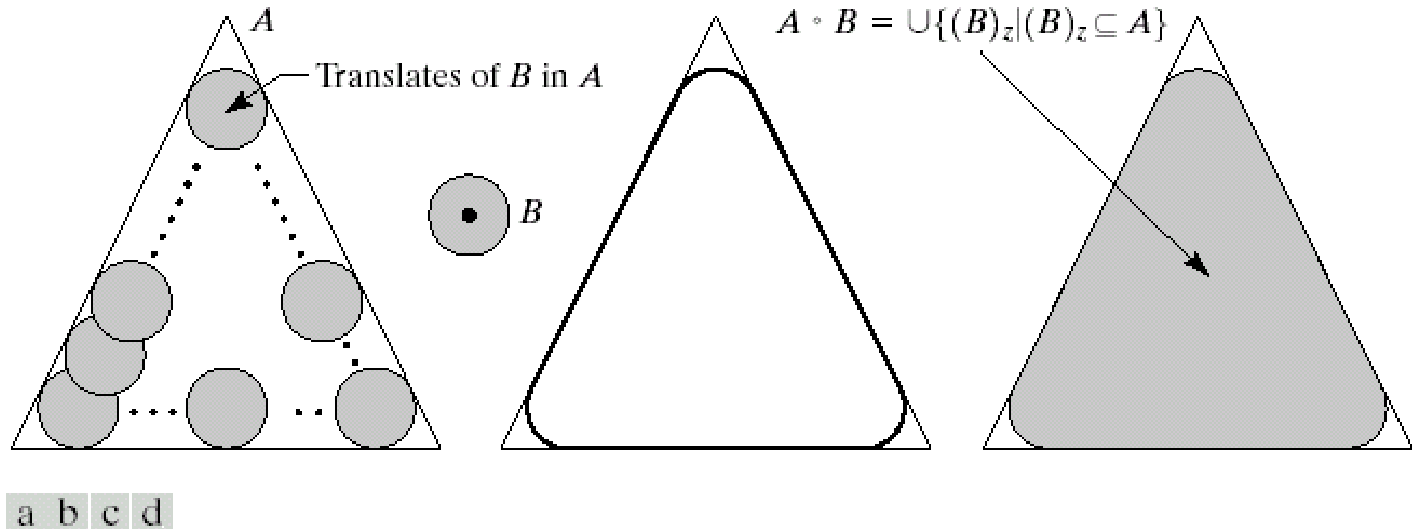


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

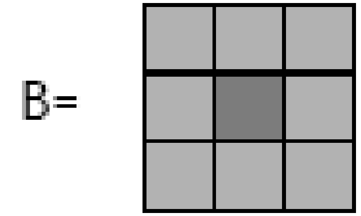
Closing

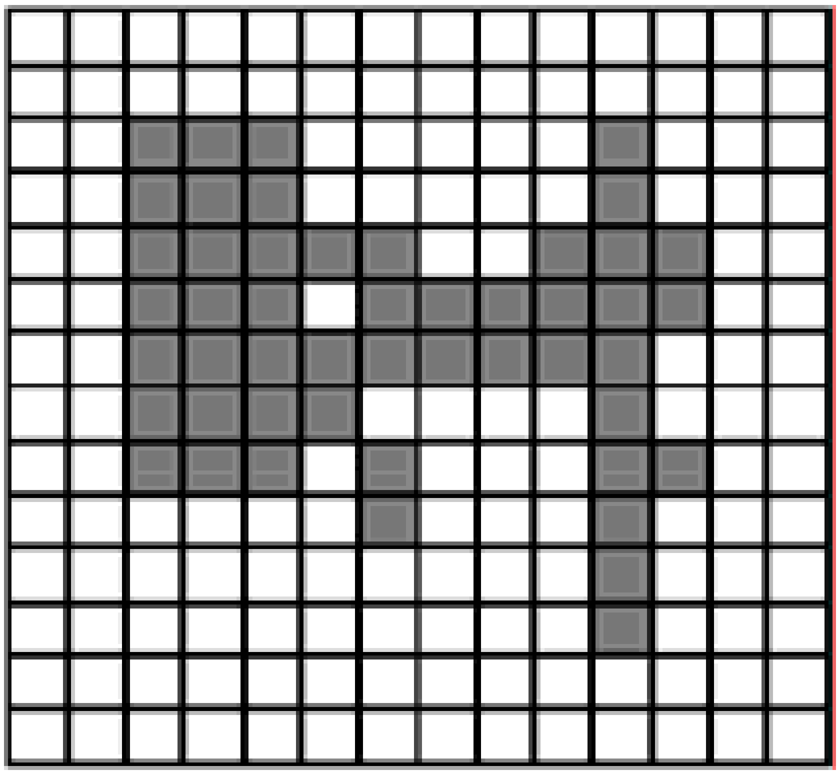
dilation followed by erosion, denoted •

$$A \bullet B = (A \oplus B) \ominus B$$

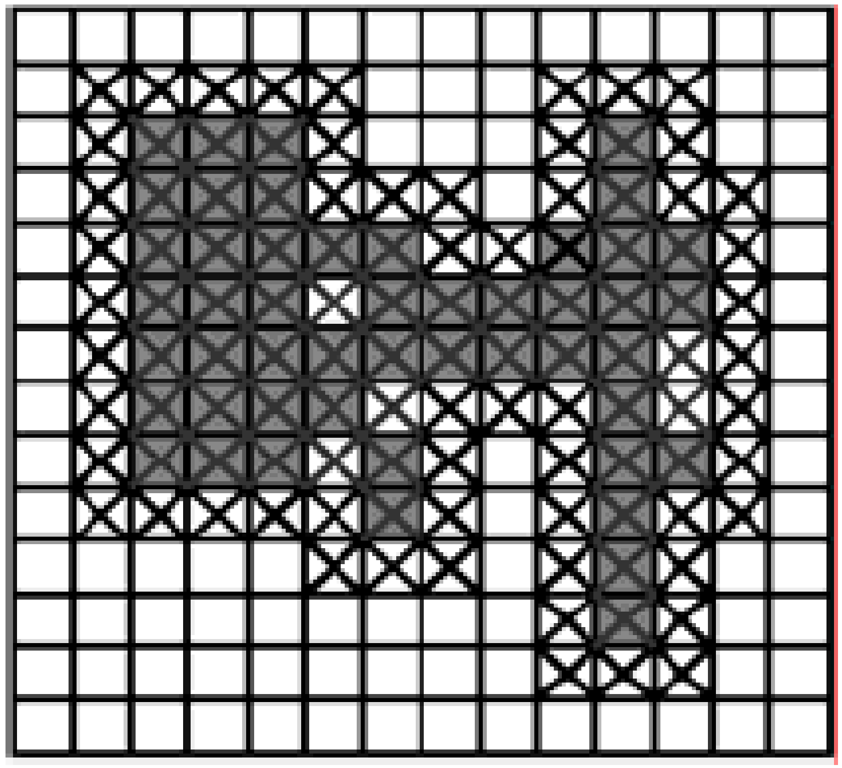
- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

Closing



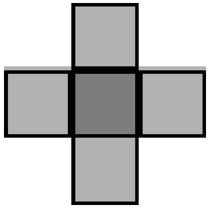


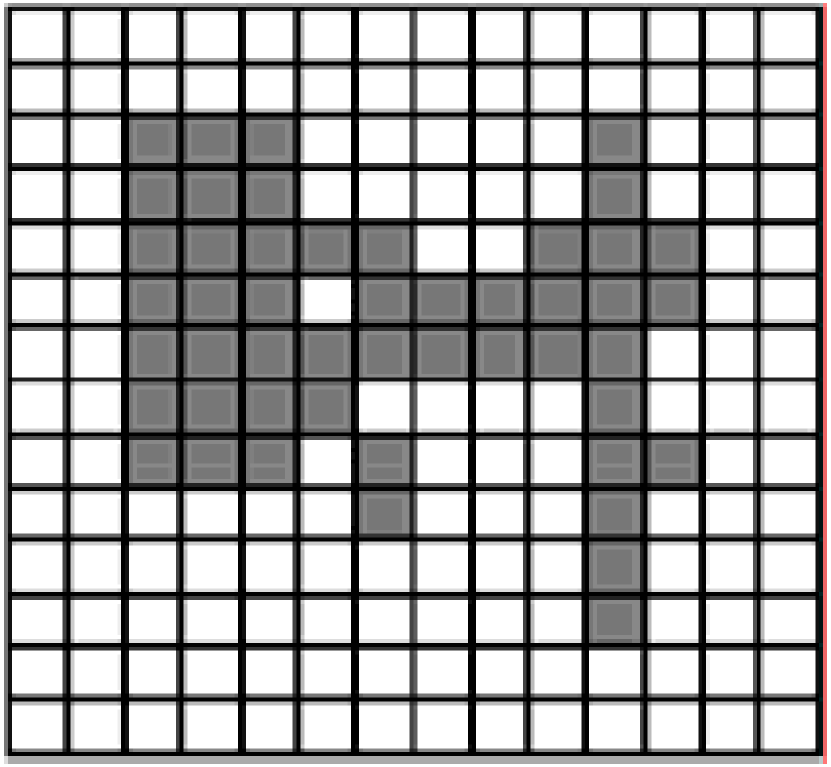
A



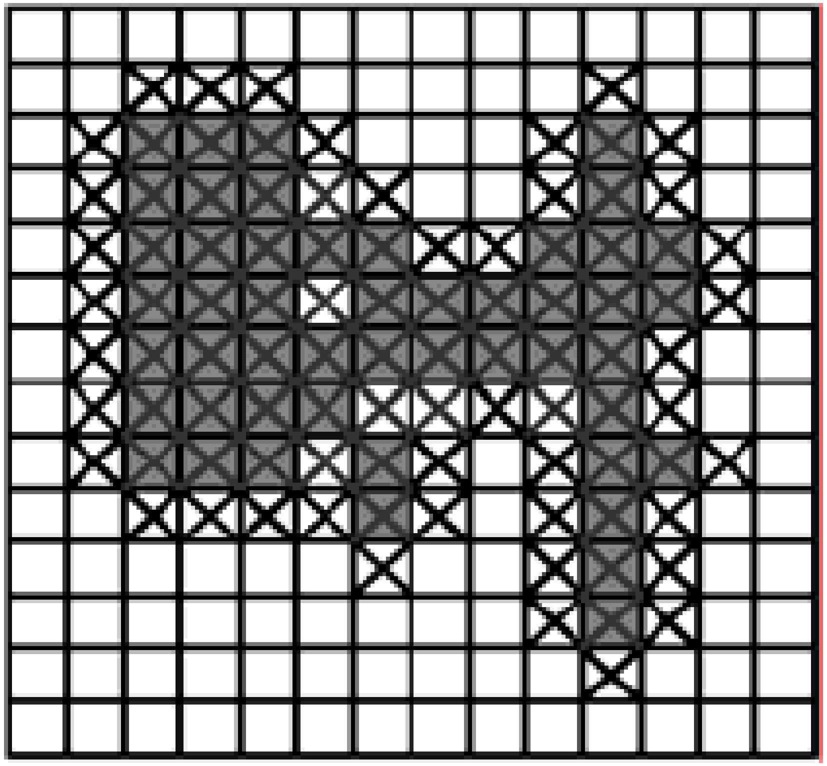
$A \oplus B$ $\quad$ $A \bullet B$

Closing

B= 

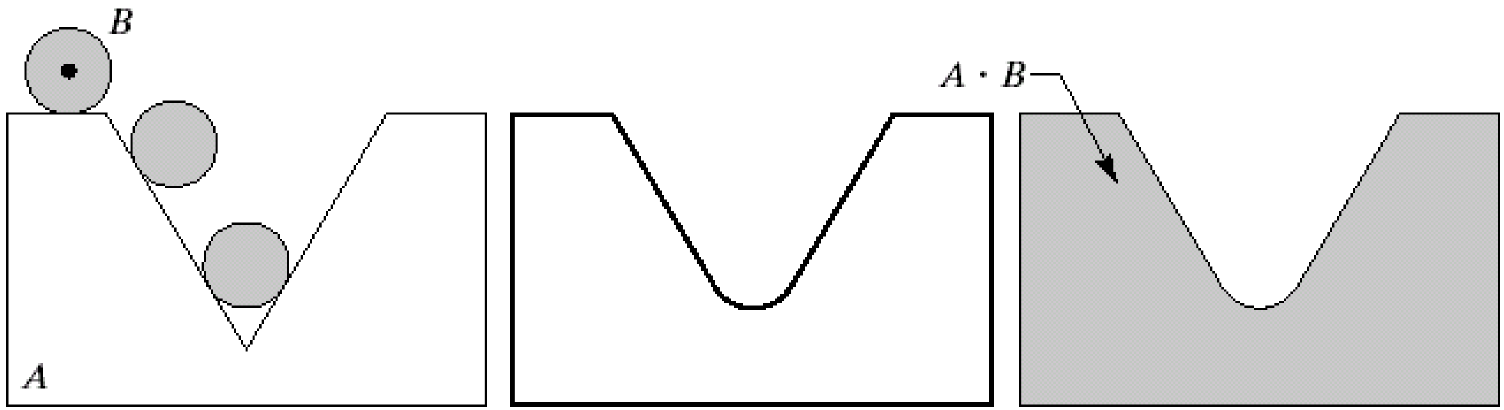


A



$A \oplus B \quad A \bullet B$

Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$A \bullet B = (A \oplus B) \ominus B$$

Properties

Opening

(i) $A \circ B$ is a subset (subimage) of A

(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

(iii) $(A \circ B) \circ B = A \circ B$

Closing

(i) A is a subset (subimage) of $A \bullet B$

(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

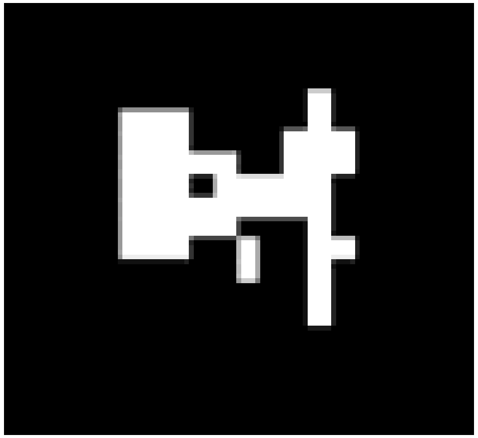
(iii) $(A \bullet B) \bullet B = A \bullet B$

Note: repeated openings/closings has no effect!

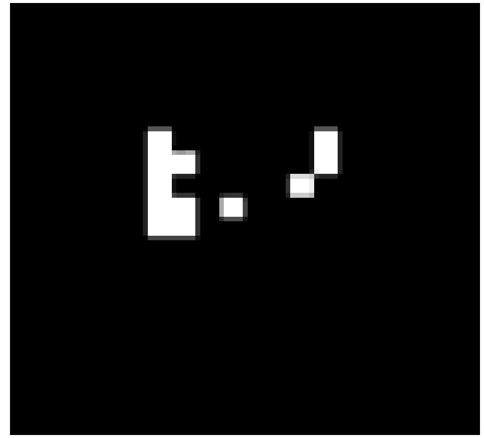
Duality

- Opening and closing are dual with respect to complementation and reflection

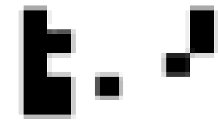
$$(A \bullet B)^c = (A^c \circ \hat{B})$$



A

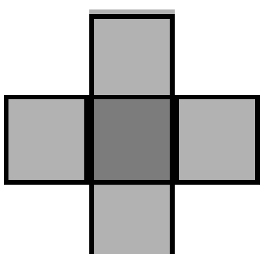


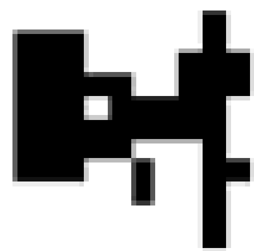
$A \ominus B$



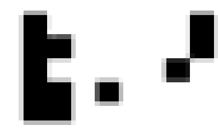
$(A \ominus B)^C$

$$B = \hat{B}$$





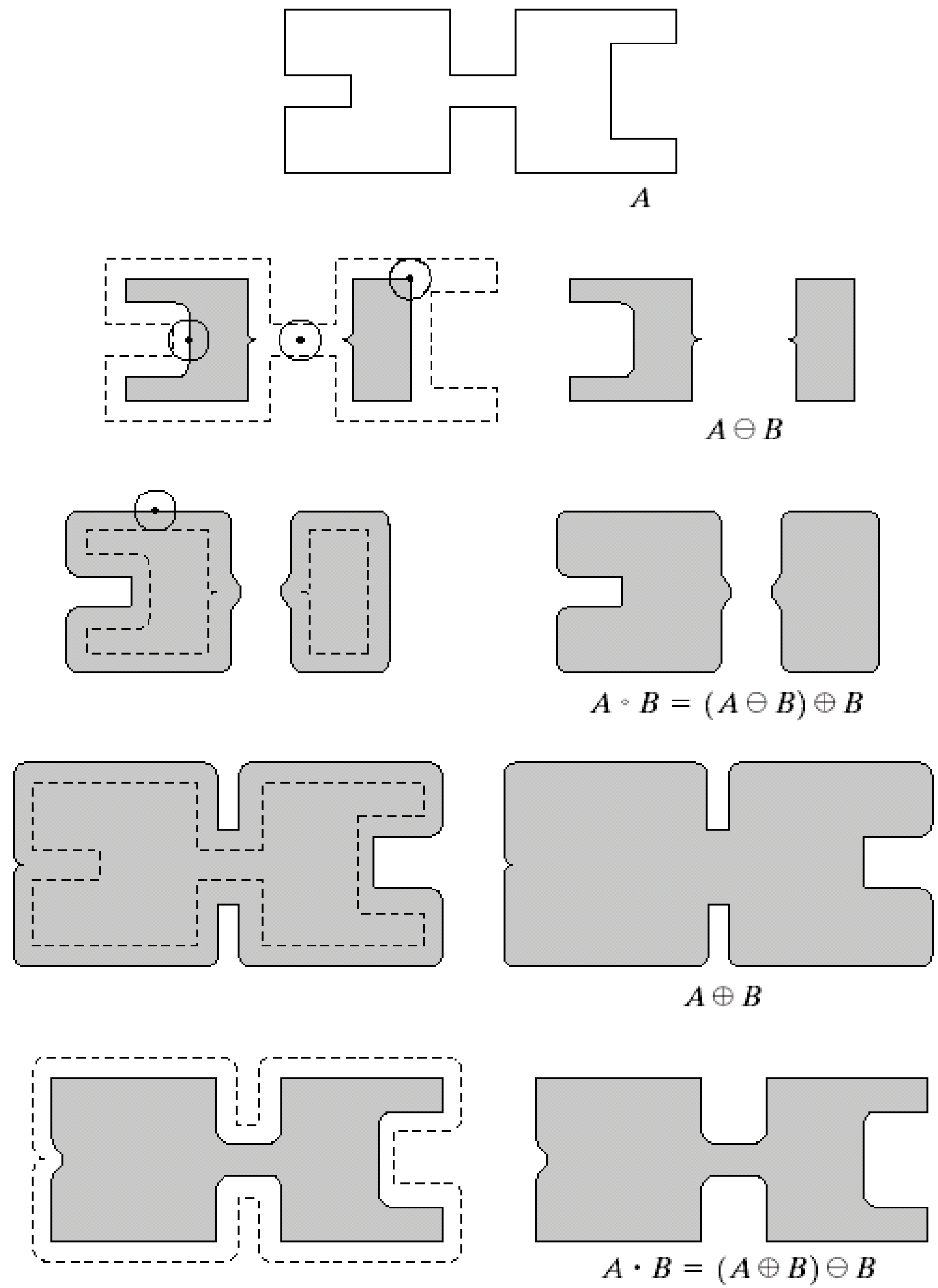
A^C



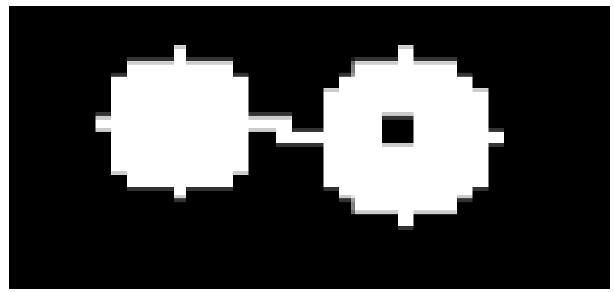
$A^C \oplus B$

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Useful: open & close

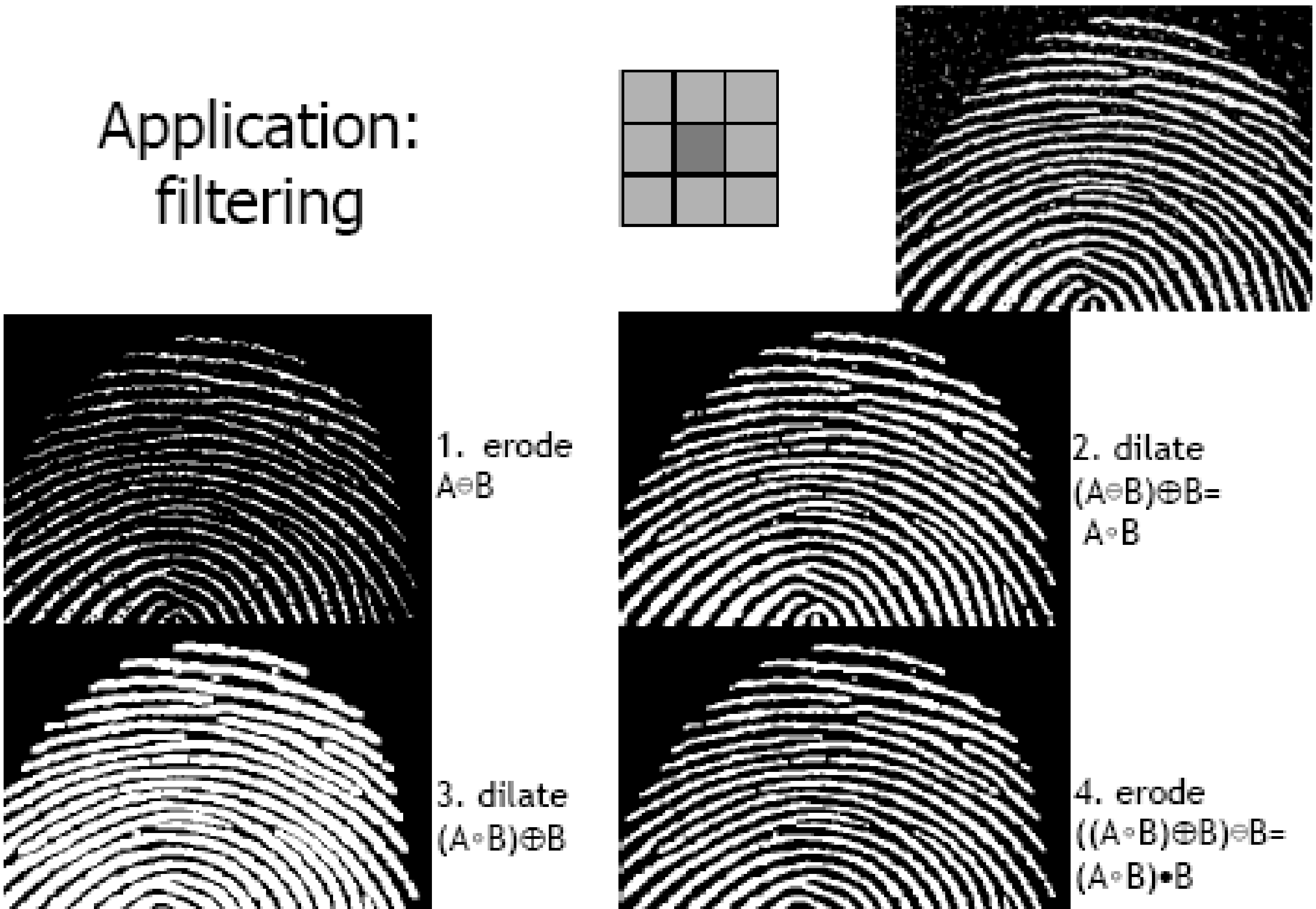


A

opening of A
→removal of small protrusions, thin connections, ...

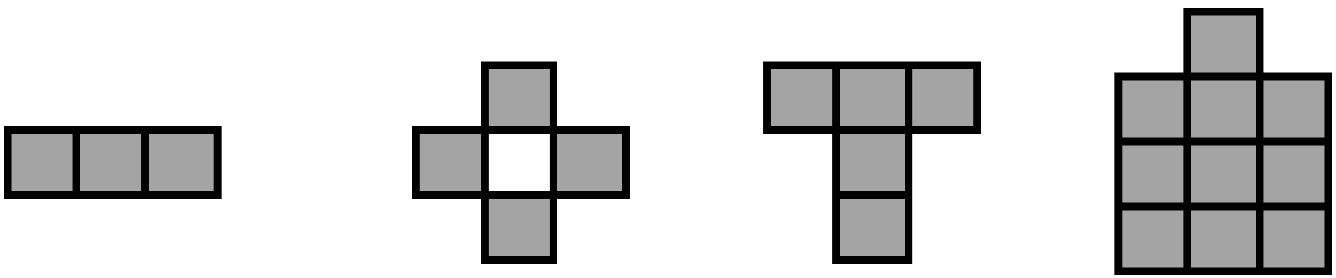
closing of A
→ removal of holes

Application: filtering



Hit-or-Miss Transformation ⊛ (HMT)

- find location of one shape among a set of shapes ”template matching



- composite SE: object part (B1) and background part (B2)
- does B1 ***fits the object while, simultaneously,*** B2 misses the object, i.e., ***fits the background?***

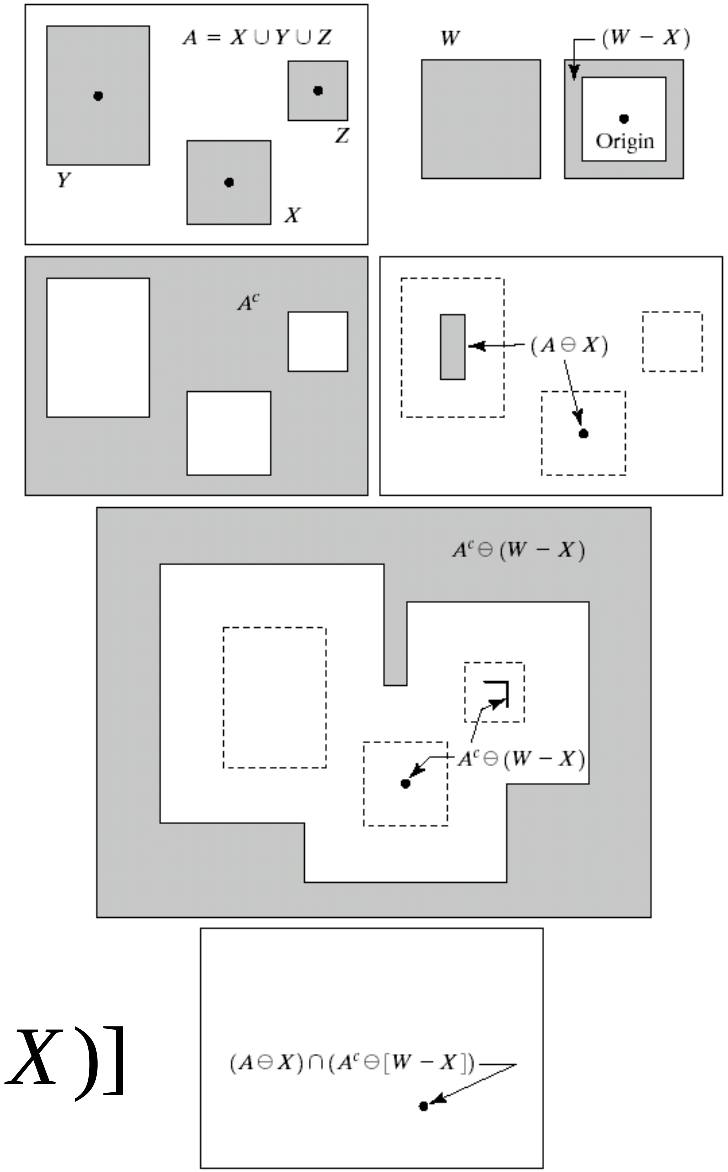
Hit-or-Miss Transformation

a	b
c	d

e

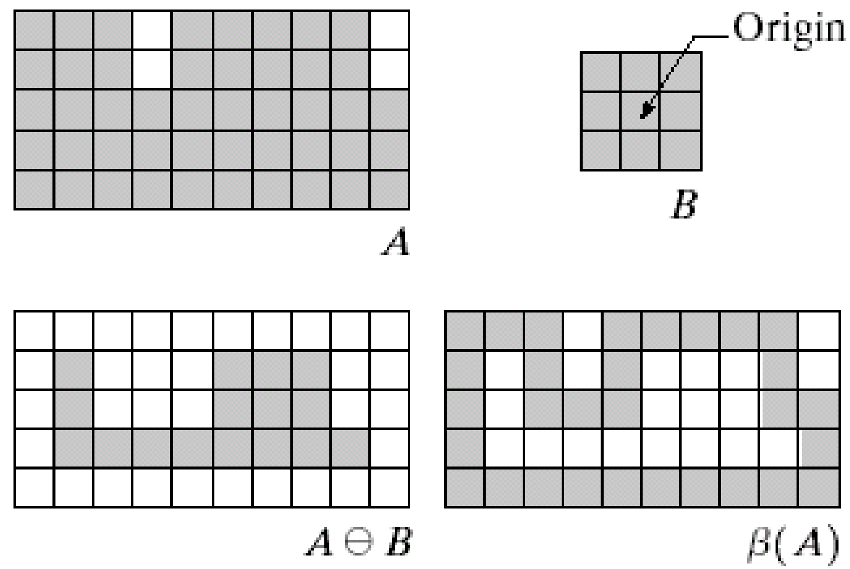
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.



$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

Boundary Extraction

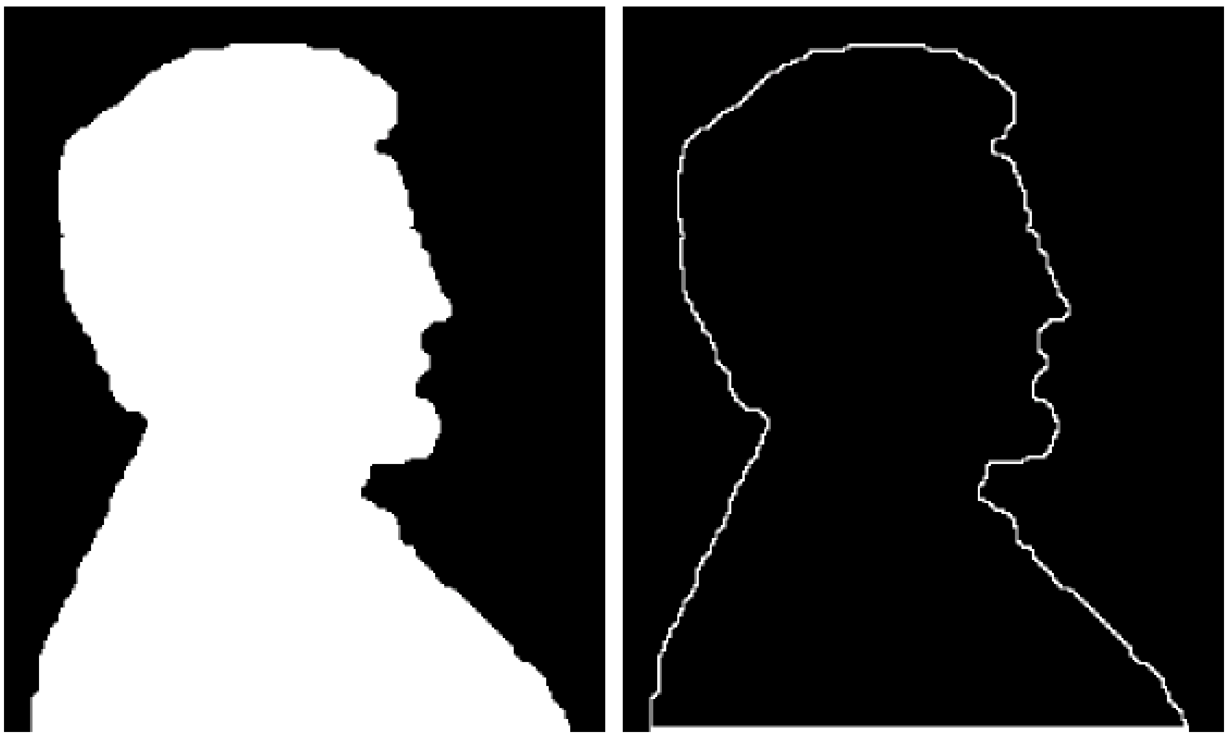


a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Example



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$

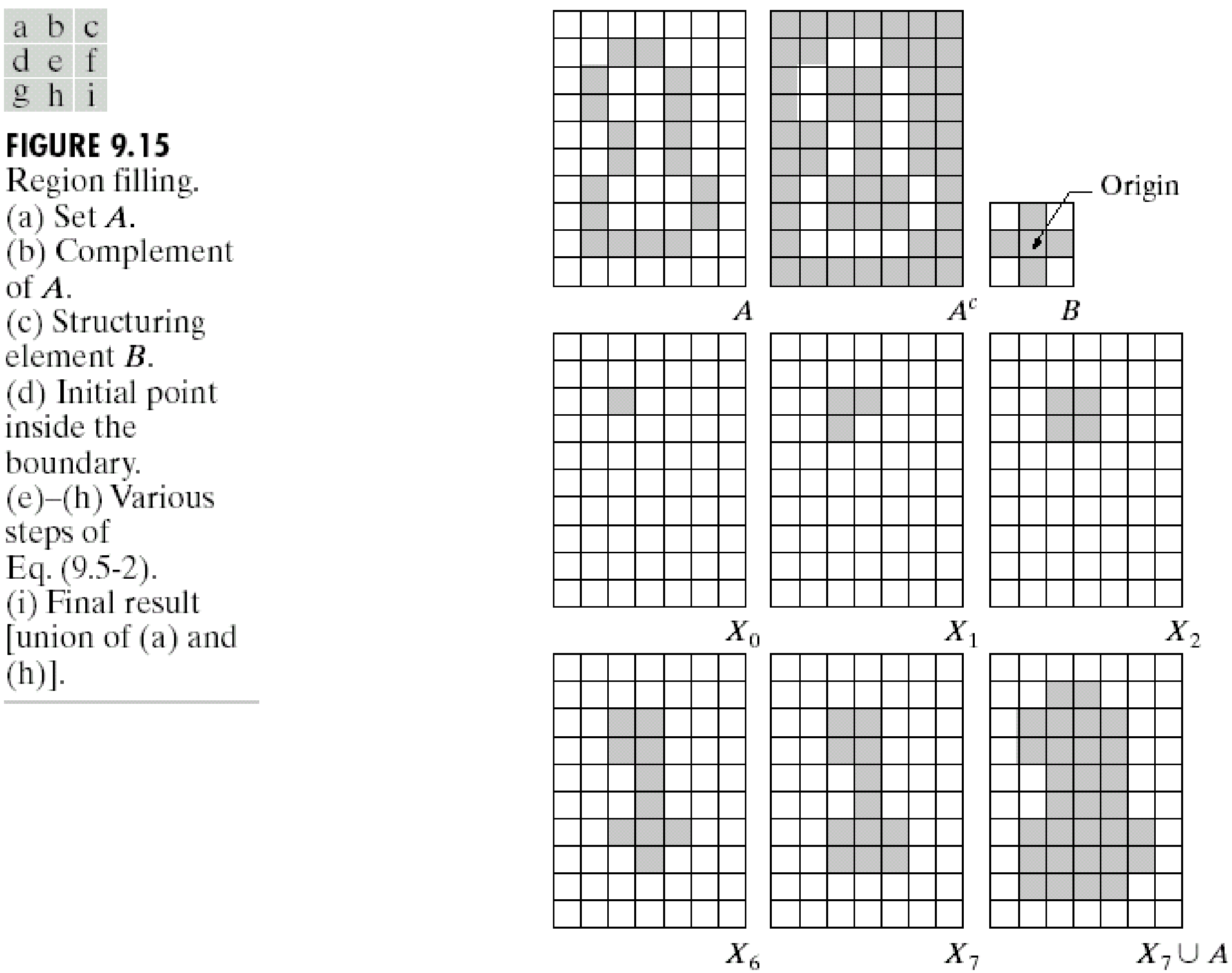
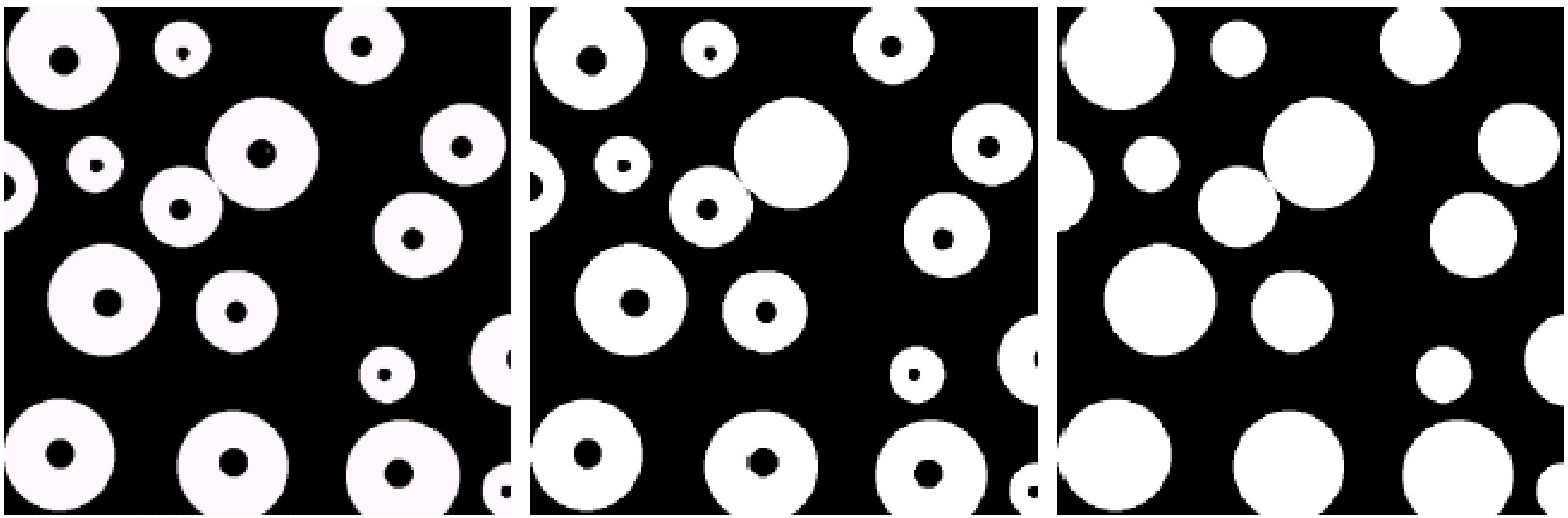


FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extraction of connected components

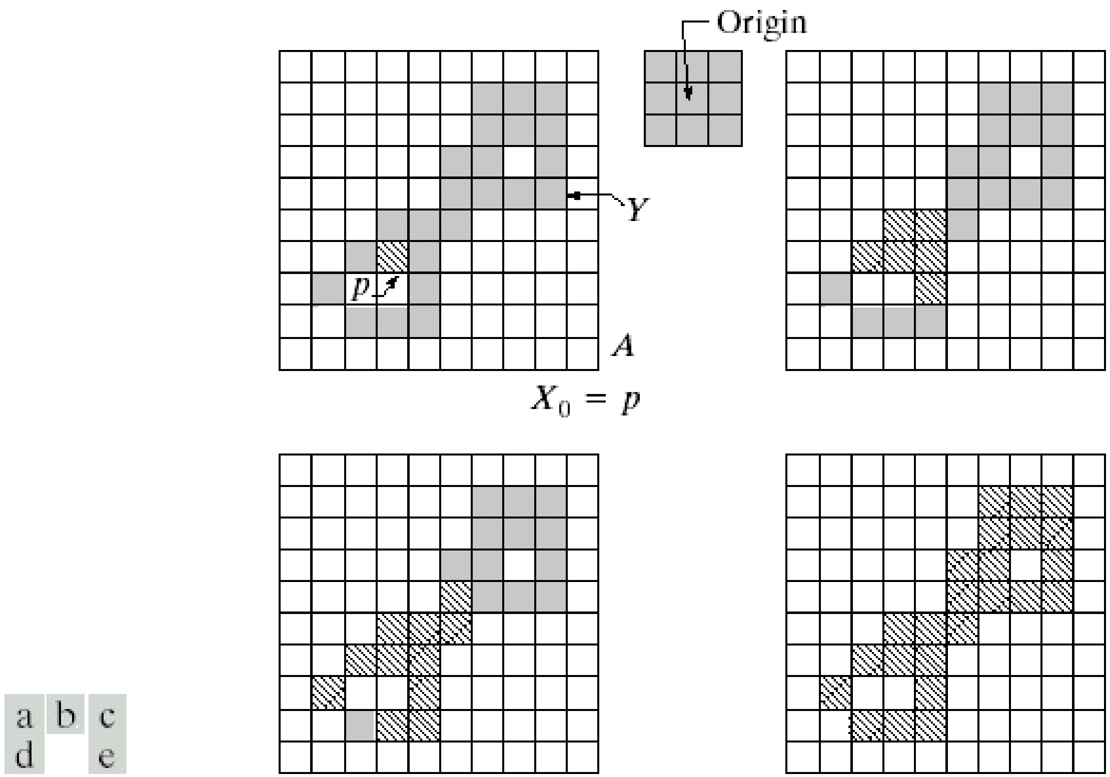
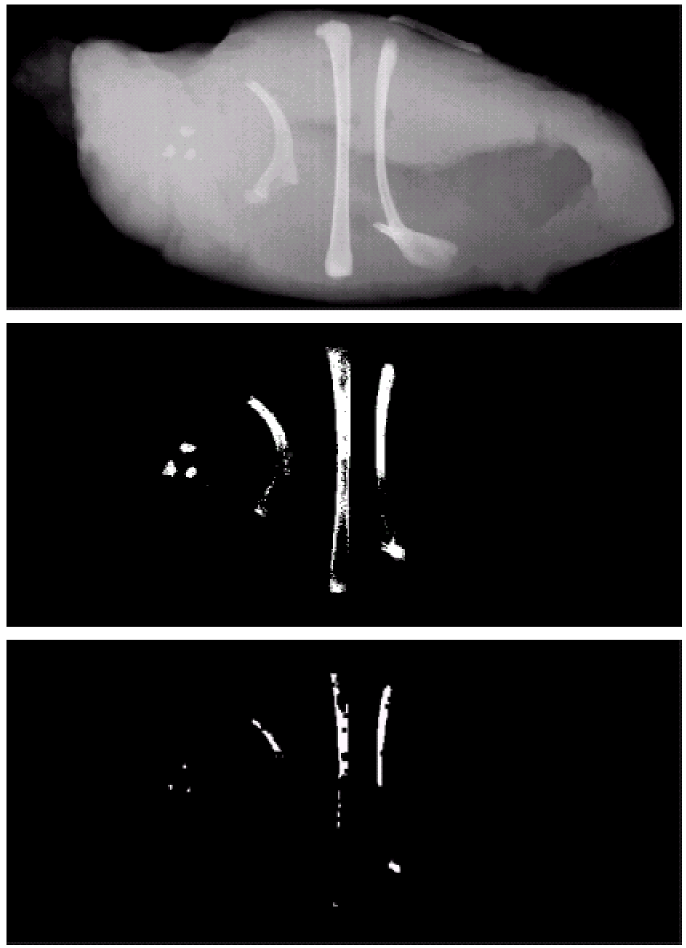


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

Example



a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex hull

$$X_k^i = (X_k^i \circledast B^i) \cup A \quad i = 1,2,3,4 \ \text{and}\ k = 1,2,3,\ldots$$

- A set *A* is is said to be convex if the straight line segment joining any two points in A lies entirely within A.

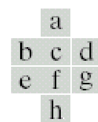
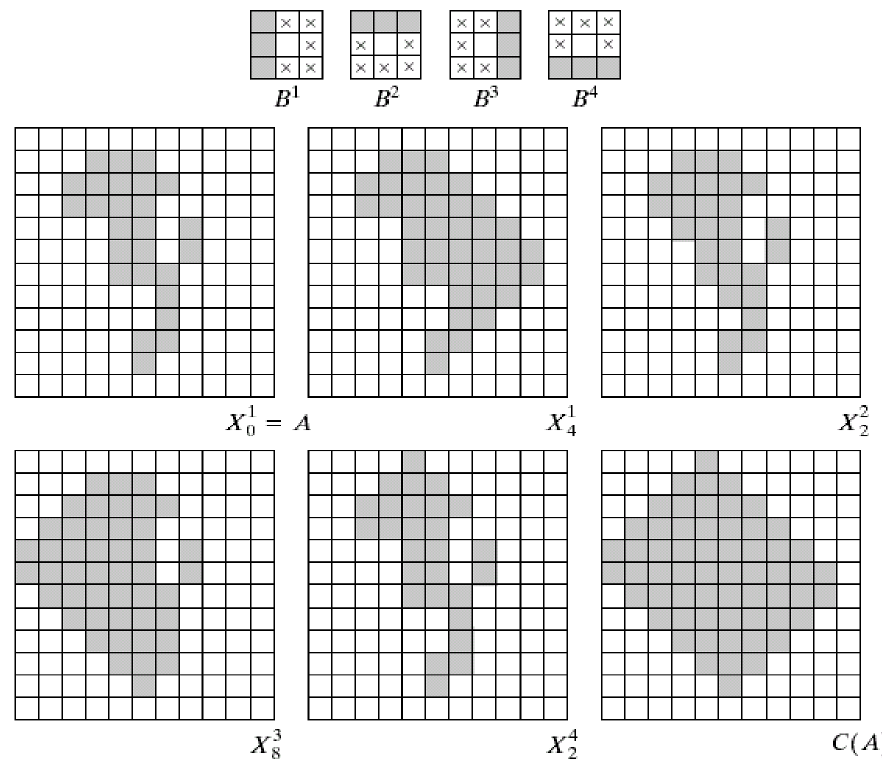
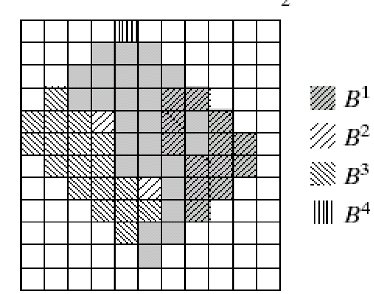


FIGURE 9.19 (a) Structuring elements. (b) Set *A*. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



$$C(A) = \bigcup_{i=1}^{4} D^i$$



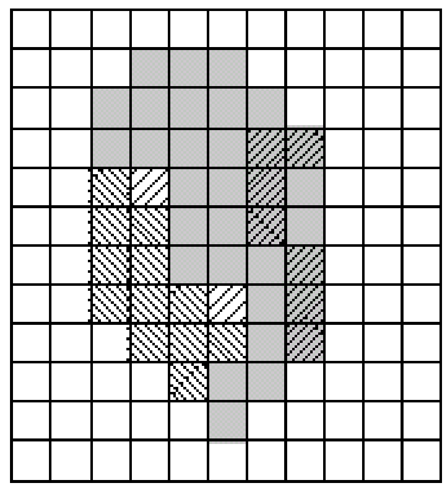
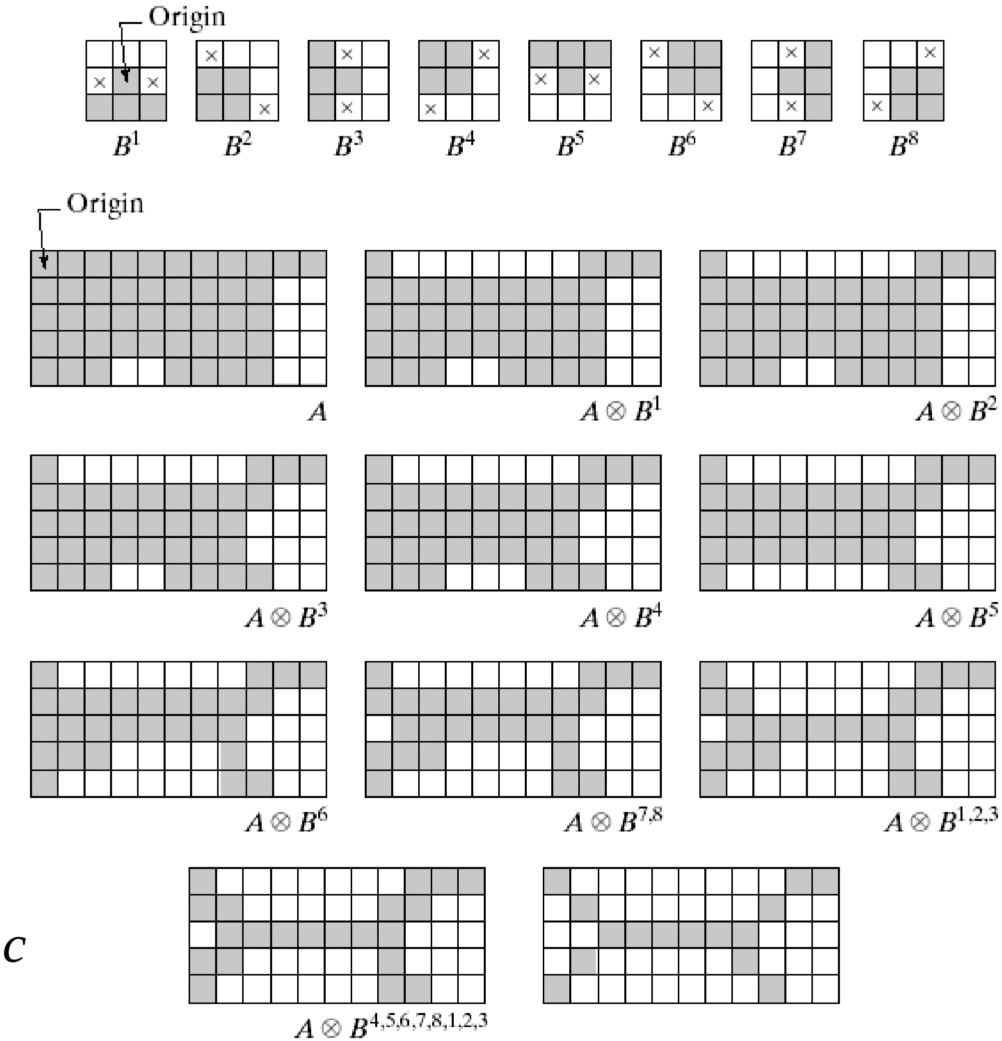


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.



$$A \otimes B = A - (A \circledast B)$$

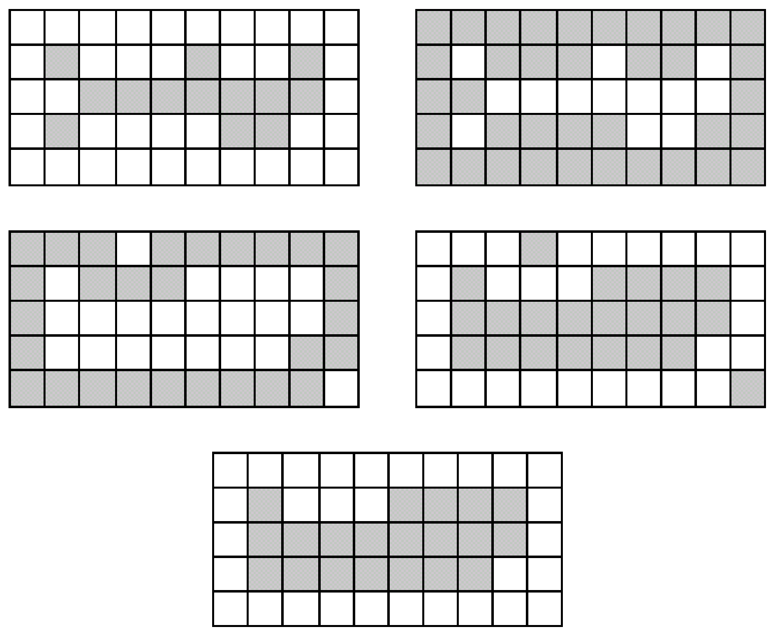
$$= A \cap (A \circledast B)^c$$

	a	
b	c	d
e	f	g
h	i	j
k	l	

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

$$A \odot B = A \cup (A \circledast B)$$



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

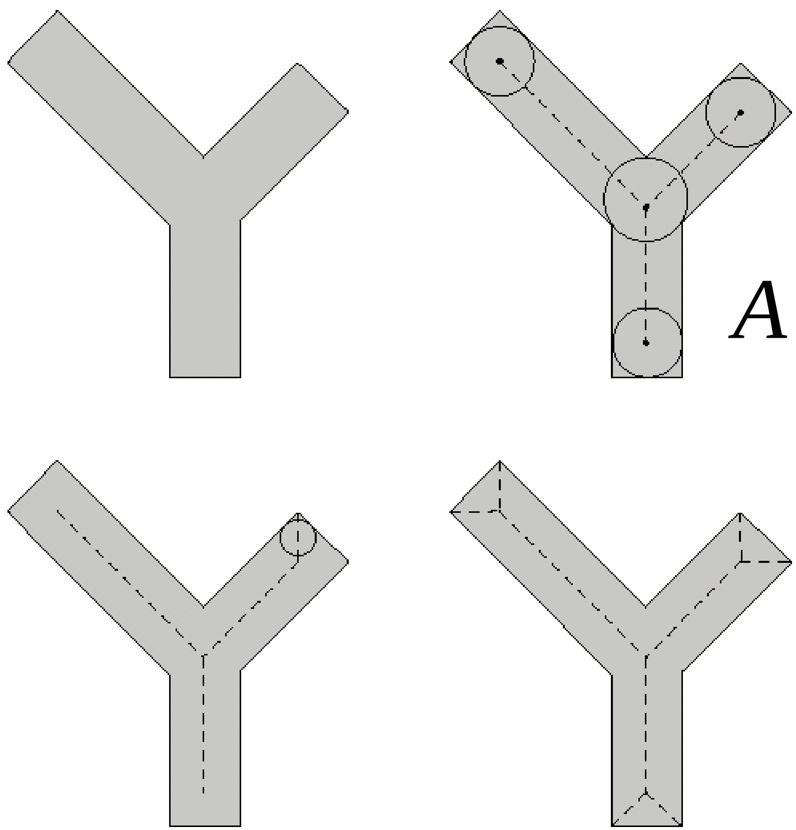
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max\{k \mid (A - kB) \neq \Phi\}$$

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.



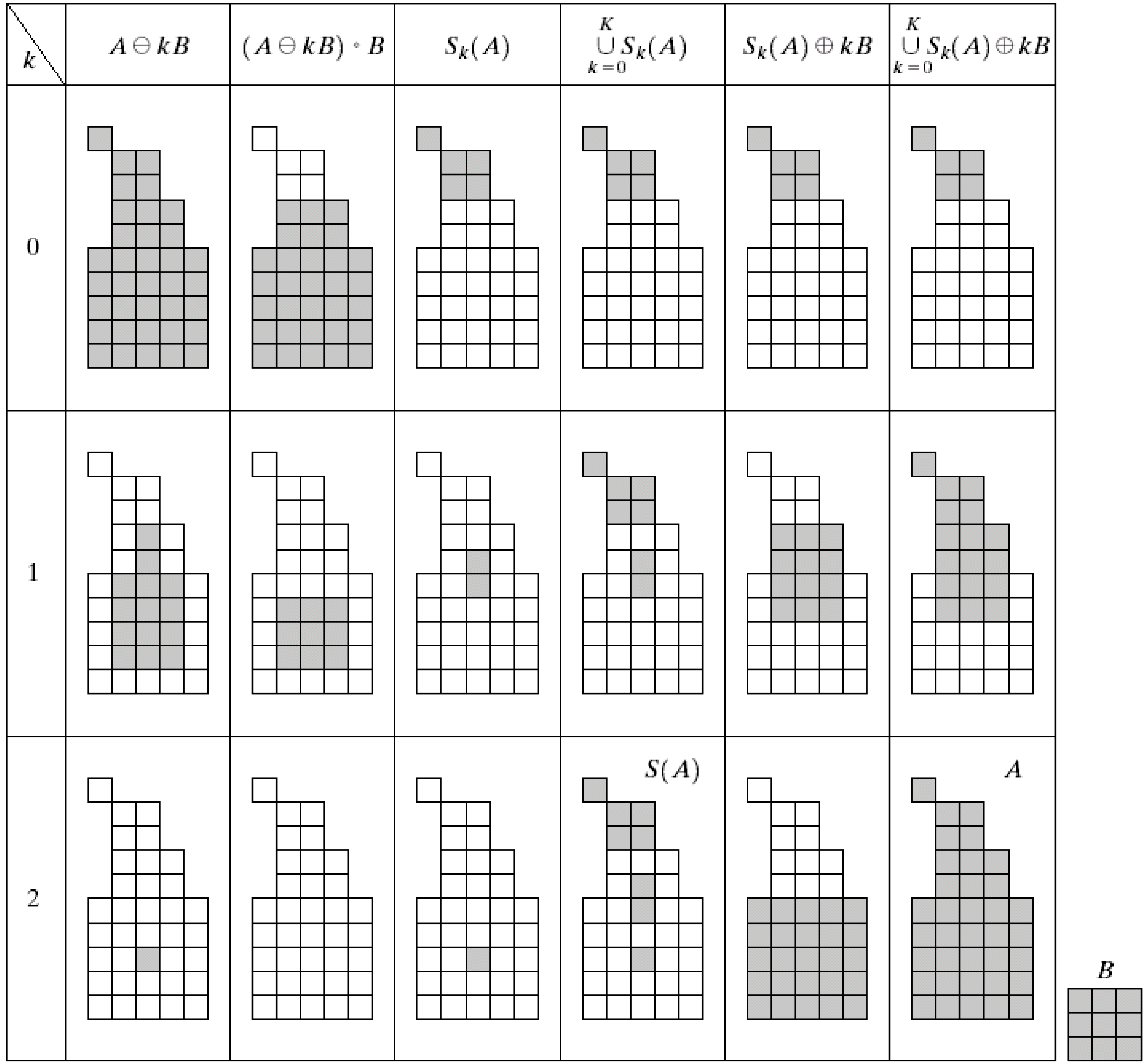


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Pruning

H = 3x3 structuring element of 1's

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

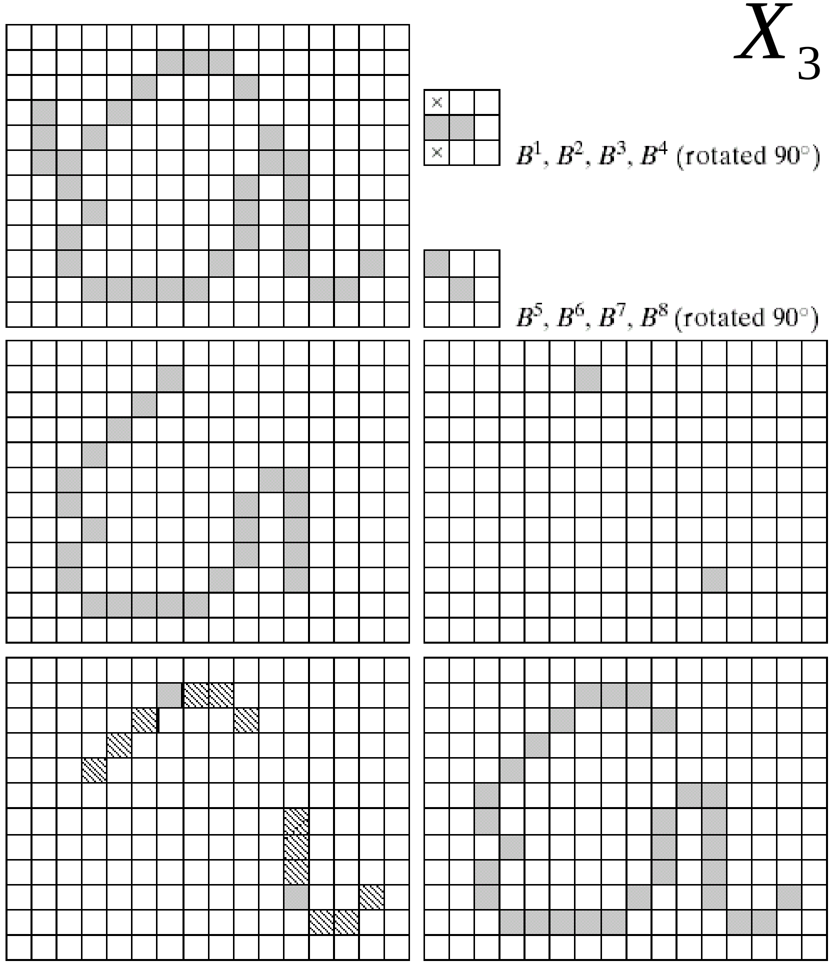


FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

5 basic structuring elements

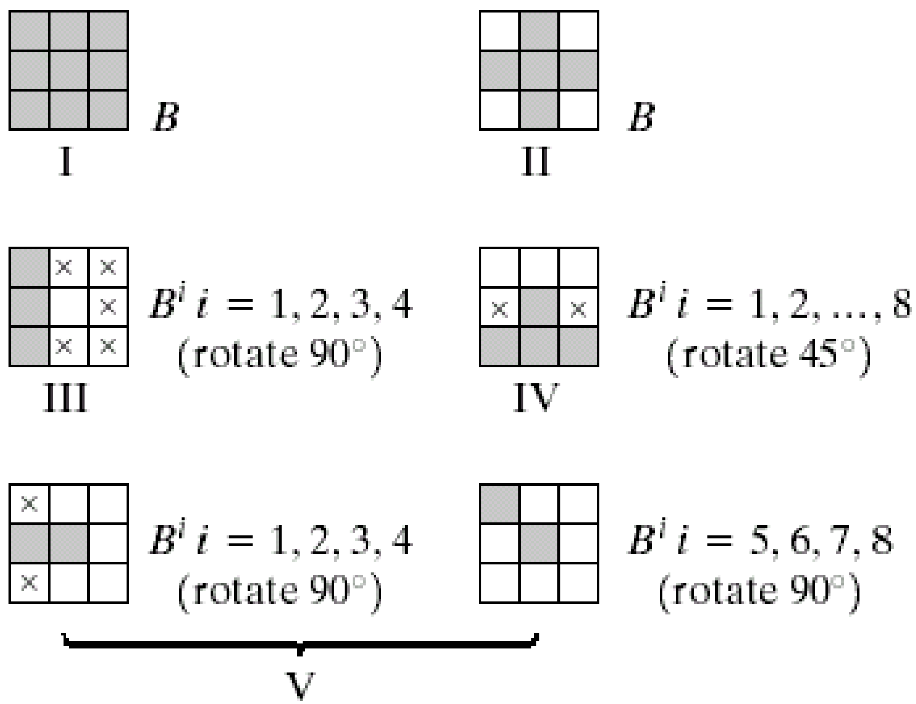


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.