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Q.1

(e) $X_i = 1$, if i^{th} Trial is Even $\Big\}$ $i = 1$ to 100
$= 0$, otherwise

X = Number of times an even number appeared

$$= \sum_{i=1}^{100} X_i$$

$$\therefore E[X] = E\left[\sum X_i\right]$$

$$= \sum E[X_i]$$

$$= \sum_{i=1}^{100} Pr(i^{th} \text{ trial is Even})$$

$$= \sum_{i=1}^{100} \frac{1}{2}$$

$$= \frac{100}{2} = \boxed{50}$$

(f) $X_{ij} = 1$ if $i < j$ and $A[i] > A[j]$
$= 0$ Otherwise

Find Expected Number of Inversions

→ Number of Inversions $X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}$

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$$\therefore E[X] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E[X_{ij}]$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Pr(i<j \text{ and } A[i]>A[j])$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{2}$$

$$= \frac{1}{2} \sum_{i=1}^{n-1} (n-i)$$

$$= \frac{1}{2} \left[n \sum_{i=1}^{n-1} 1 - \sum_{i=1}^{n-1} i \right]$$

$$= \frac{1}{2} \left[n(n-1) - \frac{(n-1)n}{2} \right]$$

$$= \frac{n(n-1)}{4}$$

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Q.3 (a) (ii) → Min cut Size = 3

→ After (n-2) contructions, prob. that the algorithm returns the correct answer is $\geq \frac{2}{n(n-1)}$

Here $n = 12$

$\therefore$ Prob. $\geq \frac{2}{12 \times 11} = \frac{1}{66}$

→ If you repeat (n-2) contructions for 100n(n-1) times then prob. that algo. returns the correct answer is $\geq 1 - e^{-200}$
$\rightarrow 1$

$\boxed{OR}$

(c)

→ Pr. that the randomly selected pivot creates the worst partition

$\geq \frac{2}{50}$ ($\because$ Min Rank and Max Rank elements)

→ $Pr\left(\frac{2^{nd} \text{ is not the worst}}{\text{First was the worst Partition}}\right) = 1 - \frac{2}{49} = \frac{47}{49}$

($\because$ 49 elements for 2^{nd} partition)

Just calculation part

OR

Q.3 (a) $n = 97$

$a = 29$

Compute $a^{n-1} \bmod n$

$= 29^{96} \text{ MOD } 97$

$$96 : \begin{array}{ccccccc} 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ (1 & 1 & 0 & 0 & 0 & 0 & 0)_2 \end{array}$$

→ $res = 1$

MSB i.e. Bit 7 is 1

$\therefore res = (res * a) \% n$

$= (1 * 29) \% 97$

$= 29$

$res = res^2 \% n = 841 \% 97$

$= 65$

→ Bit 6 is 1

$\therefore res = (res * a) \% n$

$= (65 * 29) \% 97$

$= 42$

→ Bit 5 is 0

$\therefore res = 42^2 \% 97$

$= 18$

Bit 4 is 0

$\therefore res = 18^2 \% 97$
$= 33$

Bit 3 is 0

$\therefore res = 33^2 \% 97$
$= 22$

Bit 2 is 0

$\therefore res = 22^2 \% 97$
$= 96$

Bit 1 is 0

$\therefore res = 96^2 \% 97$
$= 1$

→ Here $a = 29$ is F-witness, as, We know that 97 is prime

→ $\therefore$ For every $1 < a < 96$, $a^{n-1} \bmod 97 = a^{96} \bmod 97 = 1$
(Fermat's Little Theorem)

→ Just to avoid possibility of incorrect answer, it will (algo) be repeated k times.