

Theory of Automata & Formal Languages (Theory of Computation)

Compiled By
Prof. M. S. Bhatt

Languages

Examples

- Find a language over $\{a, b\}$ which is neither $\{\lambda\}$ nor $\{a, b\}^*$ and satisfies $L=L^*$

Examples

- Find a language over $\{a, b\}$ which is neither $\{\lambda\}$ nor $\{a, b\}^*$ and satisfies $L=L^*$
- $L = \{\lambda, a, aa, aaa, aaaa, \dots, \} = \{a^n \mid n \geq 0\}$
- $L = \{x \text{ belongs to } \{a, b\}^* \mid \text{length of } x \text{ is even}\}$

Examples

- Find an infinite language over $\{a, b\}$ which is neither $\{\lambda\}$ nor $\{a, b\}^*$ and satisfies $L \neq L^*$

Examples

- Find an infinite language over $\{a, b\}$ which is neither $\{\lambda\}$ nor $\{a, b\}^*$ and satisfies $L \neq L^*$
- $L = \{a, aaa, aaaaa, \dots, \} = \{a^n \mid n \text{ is odd} \}$
- $L = \{x \text{ belongs to } \{a, b\}^* \mid \text{length of } x \text{ is odd}\}$

Examples

- Find languages L_1 and L_2 satisfying $L_1L_2 = L_2L_1$ and neither language is subset of the other and neither language is $\{\lambda\}$

Examples

- Find languages L_1 and L_2 satisfying $L_1L_2 = L_2L_1$ and neither language is subset of the other and neither language is $\{\lambda\}$
- $L_1 = \{a, aaa, aaaaa, \dots, \} = \{a^n \mid n \text{ is odd} \}$
- $L_2 = \{\lambda, aa, aaaa, \dots, \} = \{a^n \mid n \text{ is even} \}$
- $L_1L_2 = a.aa = aaa = aa.a = L_2L_1$
- $L_1L_2 = a.\lambda = a = \lambda.a = L_2L_1$

Examples

- Find languages L_1 and L_2 satisfying $L_1L_2 = L_2L_1$ and L_1 is a proper non-empty subset of L_2 and $L_1 \neq \{\lambda\}$

Examples

- Find languages $L1$ and $L2$ satisfying $L1L2 = L2L1$ and $L1$ is a proper non-empty subset of $L2$ and $L1 \neq \{\lambda\}$
- $L1 = \{a, aaa, aaaaa, \dots, \} = \{a^n \mid n \text{ is odd} \}$
- $L2 = \{\lambda, a, aa, aaa, aaaa, \dots, \} = \{a^n \mid n \geq 0 \}$
- $L1L2 = a.aa = aaa = aa.a = L2L1$
- $L1L2 = a.\lambda = a = \lambda.a = L2L1$

Examples

- Let languages L_1 and L_2 be subset of $\{a,b\}^*$ and consider two languages $L_1^* \cup L_2^*$ and $(L_1 \cup L_2)^*$. Which of the two is always a subset of the other ?

Examples

- Let languages $L1$ and $L2$ be subset of $\{a,b\}^*$ and consider two languages $L1^* \cup L2^*$ and $(L1 \cup L2)^*$. Which of the two is always a subset of the other ?
- $L1 = \{a\}$, $L1^* = \{\lambda, a, aa, aaa, aaaa, \dots\}$
- $L2 = \{b\}$, $L2^* = \{\lambda, b, bb, bbb, bbbb, \dots\}$
- $L1^* \cup L2^* = \{\lambda, a, b, aa, bb, aaa, bbb, \dots\}$
- $(L1 \cup L2) = \{a, b\}$, $(L1 \cup L2)^* = \{\lambda, a, b, ab, ba, aba, \dots\}$

Examples

- For a finite language L , $|L|$ denotes number of elements in L , for finite languages A and B , is $|A.B| = |A| * |B|$ always true ?

Examples

- For a finite language L , $|L|$ denotes number of elements in L , for finite languages A and B , is $|A.B| = |A|*|B|$ always true ?
- $A = \{ab, a\}$, $|A| = 2$
- $B = \{ba, a\}$, $|B| = 2$
- $AB = \{abba, aba, aba, aa\} = \{abba, aba, aa\}$
- $|AB| = 3$
- $|A|*|B| = 2*2 = 4$
- Statement is not always true

Examples

- Let L_1 , L_2 and L_3 be languages over some alphabet Σ . Is $L_1(L_2 \cap L_3) = L_1.L_2 \cap L_1.L_3$ always true ?

Examples

- Let L_1 , L_2 and L_3 be languages over some alphabet Σ . Is $L_1(L_2 \cap L_3) = L_1.L_2 \cap L_1.L_3$ always true ?
- $L_1 = \{ab, a\}$
- $L_2 = \{a\}$
- $L_3 = \{ba\}$
- $L_1(L_2 \cap L_3) = \{ab, a\} . \Phi = \Phi$
- $L_1.L_2 \cap L_1.L_3 = \{aba, aa\} \cap \{abba, aba\} = \{aba\}$

Let L_1, L_2 be languages, then the concatenation $L_1 \circ L_2 = \{w \mid w = xy, x \in L_1, y \in L_2\}$. If $L_2 = \emptyset$, then there is no string $y \in L_2$ and so there is no possible w such that $w = xy$. Thus for any L_1 , we'll have $L_1 \circ \emptyset = \emptyset$.

Are $(L1 \cap L2)^*$ and $L1^* \cap L2^*$ always Equal ?

$L1 = \{a, ba\}$

$L2 = \{ab, a\}$

$L1^* = \{^, a, ba, aa, aba, baa, baba, \dots\}$

$L2^* = \{, ab, a, abab, aba, aab, aa, \dots\}$

$L1^* \cap L2^* = \{^, a, aba, aa, \dots\}$

$L1 \cap L2 = \{a\}$

$(L1 \cap L2)^* = \{^, a, aa, aaa, \dots\}$

They are not always equal