x_1	0	0	0	0	1	1	1	1
x_2	0	0	1	1	0	0	1	1
y	1	0	1	0	1	0	1	0

x_1 & x_2 are features, Y is class label

x_1, x_2 & Y are Random Variables

$$\boxed{P(A \cap B) = P(A, B)}$$

$$P(Y / x_1, x_2)$$

$$P(Y=1 / x_1=0, x_2=0) \left\{ \begin{array}{l} P(y / \sim x_1, \sim x_2) \\ P(y / \bar{x}_1, \bar{x}_2) \end{array} \right.$$

$$P(Y=1 / x_1=0, x_2=1) \; \{ \; P(y / \sim x_1, x_2)$$

$$P(Y=1 / x_1=1, x_2=0)$$

$$P(Y=1 / x_1=1, x_2=1)$$

$P(Y=0 / x_1=0, x_2=0) = 1 - P(Y=1 / x_1=0, x_2=0)$

$P(Y=0 / x_1=0, x_2=1)$ ⋮

$P(Y=0 / x_1=1, x_2=0)$

$P(Y=0 / x_1=1, x_2=1)$

$$\boxed{P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}}$$

$P(y / \bar{x}_1, \bar{x}_2)$

Apply Baye's Rule

$$= \frac{\overbrace{P(\bar{x}_1, \bar{x}_2 / y)}^{A \text{ Likelihood}} \cdot \overbrace{P(y)}^{B \text{ class prior}}}{\boxed{P(\bar{x}_1, \bar{x}_2)}\ C \text{ predictor prior}}$$

x_1	0	0	0	0	1	1	1	1
x_2	0	0	1	1	0	0	1	1
Y	1	0	1	0	1	0	1	0

$P(\bar{x}_1 / y) = P(x_1 = 0 / Y = 1) = 2/4$

(B) $P(y) = 4/8 = 1/2$

(A) $P(\bar{x}_1, \bar{x}_2 / y)$

Joint Probability

$$P(X, Y) = P(X \cap Y)$$

Naive Assumption

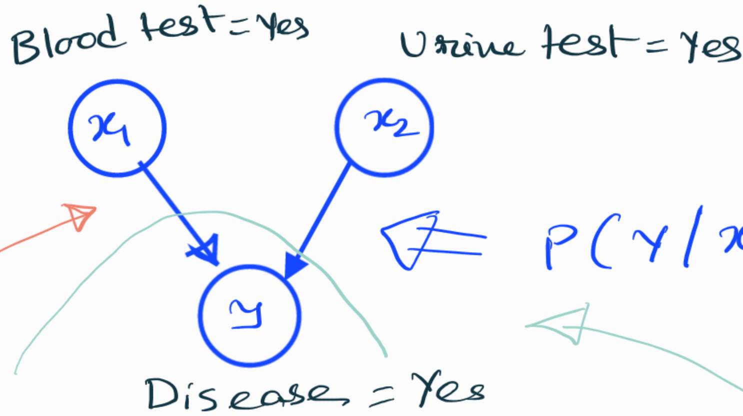
↳ $P(X) \cdot P(Y)$ — if X & Y are independent

$P(X/Y) \cdot P(Y)$

↳ if X & Y are dependent

Naive Assumption: All attributes or features are independent & identically distributed (iid assumption)

$$P(\bar{x}_1, \bar{x}_2 / y) = P(\bar{x}_1 / y) \cdot P(\bar{x}_2 / y)$$



x_1, x_2 are conditionally Independent

dependency arrow

$\Leftarrow P(Y / x_1, x_2)$

$P(\text{Disease} / \text{Blood Test, Urine Test})$

$= \boxed{P(\text{Blood Test, Urine Test} / \text{Disease}) \cdot P(\text{Disease})}$

Ⓐ Likelihood

$$P(\bar{x}_1, \bar{x}_2 / y) = P(\bar{x}_1 / y) \cdot P(\bar{x}_2 / y)$$

$$= (2/4) \cdot (2/4)$$

$$= 1/4 \text{ — } (A)$$

Ⓒ Predictor Prior (We can not calculate $P(x_1, x_2)$ without Y.)

$$\cancel{\boxed{P(\bar{x}_1, \bar{x}_2) = P(\bar{x}_1) \cdot P(\bar{x}_2) \,?}}$$

Marginalization [Y is marginalized)

$$= P(\bar{x}_1, \bar{x}_2, Y) + P(\bar{x}_1, \bar{x}_2, \bar{Y})$$

$$= P(\bar{x}_1, \bar{x}_2 / Y) \cdot P(Y) + P(\bar{x}_1, \bar{x}_2 / \bar{Y}) \cdot P(\bar{Y})$$

$$(2/4) \cdot (2/4) \cdot (1/2)$$

$$= P(\bar{x}_1 / Y) \cdot P(\bar{x}_2 / Y) \cdot P(Y) + P(\bar{x}_1 / \bar{Y}) \cdot P(\bar{x}_2 / \bar{Y}) \cdot P(\bar{Y})$$

$$(2/4) \quad (2/4) \quad (1/2)$$

$$= (1/2)(1/2)(1/2) + (1/2)(1/2)(1/2)$$

$$= 2/8 = \boxed{1/4} \text{ — } (C)$$

$$P(Y / \bar{x}_1, \bar{x}_2) = \frac{(A) \times (B)}{(C)}$$

$$= \frac{(1/2) \times (1/4)}{(1/4)}$$

$$P(Y/\bar{X}_1, \bar{X}_2) = \underline{\underline{1/2}}$$

$$P(\underline{\bar{y}/\bar{x}_1, \bar{x}_2}) = \frac{\overset{D}{P(\bar{x}_1, \bar{x}_2/\bar{y})} \cdot \overset{E}{P(\bar{y})}}{\underline{P(\bar{x}_1, \bar{x}_2)} \; C}$$

$$= 1/2$$

<u>Inference Query</u>

Q: what is Y if $x_1 = 0$ & $x_2 = 0$

if $P(Y=1/x_1=0, x_2=0)$

$>$

$P(Y=0/x_1=0, x_2=0)$

$Y = 1$

else

$Y = 0$