PUSH DOWN AUTOMATA

Definition 5.1 A Pushdown Automaton

A *pushdown automaton* (PDA) is a 7-tuple $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, where

Q is a finite set of states.

Σ and Γ are finite sets, the *input* and *stack* alphabets.

q_0, the initial state, is an element of Q.

Z_0, the initial stack symbol, is an element of Γ.

A, the set of accepting states, is a subset of Q.

δ, the transition function, is a function from $Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma$ to the set of finite subsets of $Q \times \Gamma^*$.

PUSH DOWN AUTOMATA

Definition 5.2 Acceptance by a PDA

If $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ and $x \in \Sigma^*$, the string x is *accepted* by M if

$$(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \alpha)$$

for some $\alpha \in \Gamma^*$ and some $q \in A$. A language $L \subseteq \Sigma^*$ is said to be accepted by M if L is precisely the set of strings accepted by M; in this case, we write $L = L(M)$. Sometimes a string accepted by M, or a language accepted by M, is said to be accepted *by final state*.

DETERMINISTIC PUSH DOWN AUTOMATA

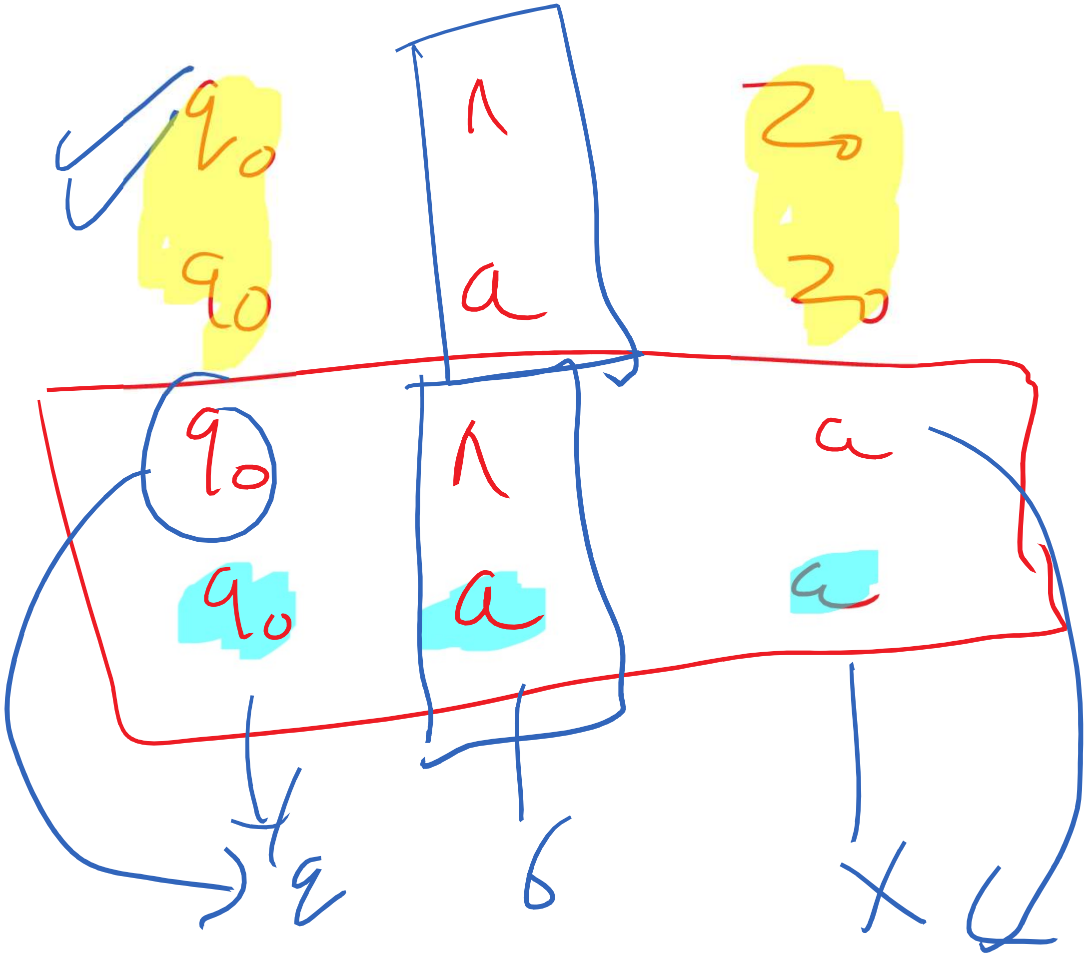
Definition 5.10 A Deterministic Pushdown Automaton

A pushdown automaton $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ is *deterministic* if it satisfies both of the following conditions.

1. For every $q \in Q$, every $\sigma \in \Sigma \cup \{\Lambda\}$, and every $X \in \Gamma$, the set $\delta(q, \sigma, X)$ has at most one element.
2. For every $q \in Q$, every $\sigma \in \Sigma$, and every $X \in \Gamma$, the two sets $\delta(q, \sigma, X)$ and $\delta(q, \Lambda, X)$ cannot both be nonempty.

A language L is a *deterministic context-free language* (DCFL) if there is a deterministic PDA (DPDA) accepting L.

DETERMINISTIC PUSH DOWN AUTOMATA



(q_1, Z_0) — 1

(q_0, aZ_0) — 2

(q_1, q) ✓

(q_0, aa)

(q_0, aa, Z_0)

(q_1, aa, Z_0) $(q_0, q, q$

DETERMINISTIC PUSH DOWN AUTOMATA

S-> [S] | SS | ^

q_0	[	Z_0	$(q_1, [Z_0)$	
q_1	[	[	$(q_1, [[)$	— Push
q_1	]	[	$(q_1, \wedge)$	— Pop
q_1	$\wedge$	Z_0	(q_0, Z_0)	

DETERMINISTIC PUSH DOWN AUTOMATA

S-> [S] | SS | ^

Move Number	State	Input	Stack Symbol	Move
1	q_0	[	Z_0	$(q_1, [Z_0)$
2	q_1	[	[	$(q_1, [[)$
3	q_1	]	[	(q_1, Λ)
4	q_1	Λ	Z_0	(q_0, Z_0)
	(all other combinations)			none

DETERMINISTIC PUSH DOWN AUTOMATA

S-> [S] | SS | ^

$(q_0, [\,][\,], z_0)$ 1

$(q_1,][\,], [z_0)$

3

$(q_1, [\,], z_0)$

4

$(q_0, [\,], z_0)$

$(q_0, [\,], z_0)$ 1

$(q_1,], [z_0)$ 3

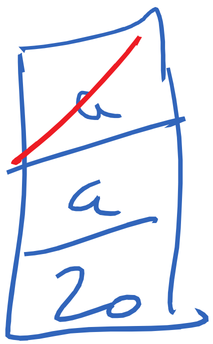
$(q_1, \wedge, z_0)$ 4

$(q_0, \underline{z_0})$

DETERMINISTIC PUSH DOWN AUTOMATA

S-> [S] | SS | ^

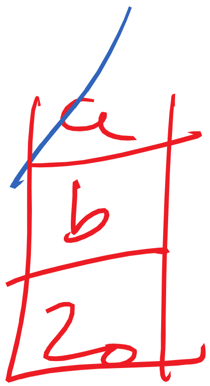
aabb



~~b~~ b

Accepted

babb



~~b~~ b

Rejected

DETERMINISTIC PUSH DOWN AUTOMATA

L= {x belongs to ∑* | $n_a(x) = n_b(x)$}

1	q_0	a	Z_0	(q_1, aZ_0)	
2	q_0	b	Z_0	(q_1, bZ_0)	– Push
3	q_1	a	a	(q_1, aa)	
4	q_1	b	b	(q_1, bb)	
5	q_1	a	b	(q_1, Λ)	Pop
6	q_1	b	a	(q_1, Λ)	
7	q_1	Λ	Z_0	(q_0, Z_0)	

DETERMINISTIC PUSH DOWN AUTOMATA

L= {x belongs to ∑* | $n_a(x) = n_b(x)$}

Move Number	State	Input	Stack Symbol	Move
1	q_0	a	Z_0	(q_1, aZ_0)
2	q_0	b	Z_0	(q_1, bZ_0)
3	q_1	a	a	(q_1, aa)
4	q_1	b	b	(q_1, bb)
5	q_1	a	b	(q_1, Λ)
6	q_1	b	a	(q_1, Λ)
7	q_1	Λ	Z_0	(q_0, Z_0)
	(all other combinations)			none

DETERMINISTIC PUSH DOWN AUTOMATA

L= {x belongs to ∑* | $n_a(x) = n_b(x)$}

$(q_0, aabbab, z_0)$

$\vdash 1$

$(q_1, abbab, az_0)$

$\vdash 3$

$(q_1, bbab, aaz_0)$

$\vdash 6$

(q_1, bab, az_0)

$\vdash 6$

(q_1, ab, z_0)

$\vdash 7$

(q_0, ab, z_0)

$\vdash 1$

(q_1, b, az_0)

$\vdash 6$

$(q_1, \wedge, z_0)$

$\vdash 7$

(q_0, z_0)

METHOD - 2

DETERMINISTIC PUSH DOWN AUTOMATA

L= {x belongs to ∑* | $n_a(x) = n_b(x)$}

Move Number	State	Input	Stack Symbol	Move
1	q_0	a	Z_0	(q_1, AZ_0)
2	q_0	b	Z_0	(q_1, BZ_0)
3	q_1	a	A	(q_1, aA)
4	q_1	b	B	(q_1, bB)
5	q_1	a	a	(q_1, aa)
6	q_1	b	b	(q_1, bb)
7	q_1	a	b	(q_1, Λ)
8	q_1	b	a	(q_1, Λ)
9	q_1	a	B	(q_0, Λ)
10	q_1	b	A	(q_0, Λ)
	(all other combinations)			none

Σ Γ

DETERMINISTIC PUSH DOWN AUTOMATA

$(q_0, abab, Z_0)$ — 1

(q_1, bab, AZ_0) — 10

(q_0, ab, Z_0) — 1

(q_1, b, AZ_0) — 10

(q_0, Z_0)

Accepted

DETERMINISTIC PUSH DOWN AUTOMATA

- Language of Palindrome cannot be accepted by Deterministic Push Down Automata

- Every Context- Free Languages are accepted by Push- Down Automata

- Every Context-Free Languages **cannot** be accepted by Deterministic Push Down Automata

- PDA is more powerful as compared to DPDA. i.e. PDA accepts more CFLs

Try This

1. $\{x \in \{a,b\}^* \mid n_a(x) > n_b(x)\}$

2. $S \to [S] \mid \{S\} \mid (S) \mid SS \mid \Lambda$

3. $\{x \in \{a,b\}^* \mid n_a(x) \neq n_b(x)\}$

4. $\{ x c y \,,\ |x| = |y| \,,\ x, y \in \{a,b\}^* \}$