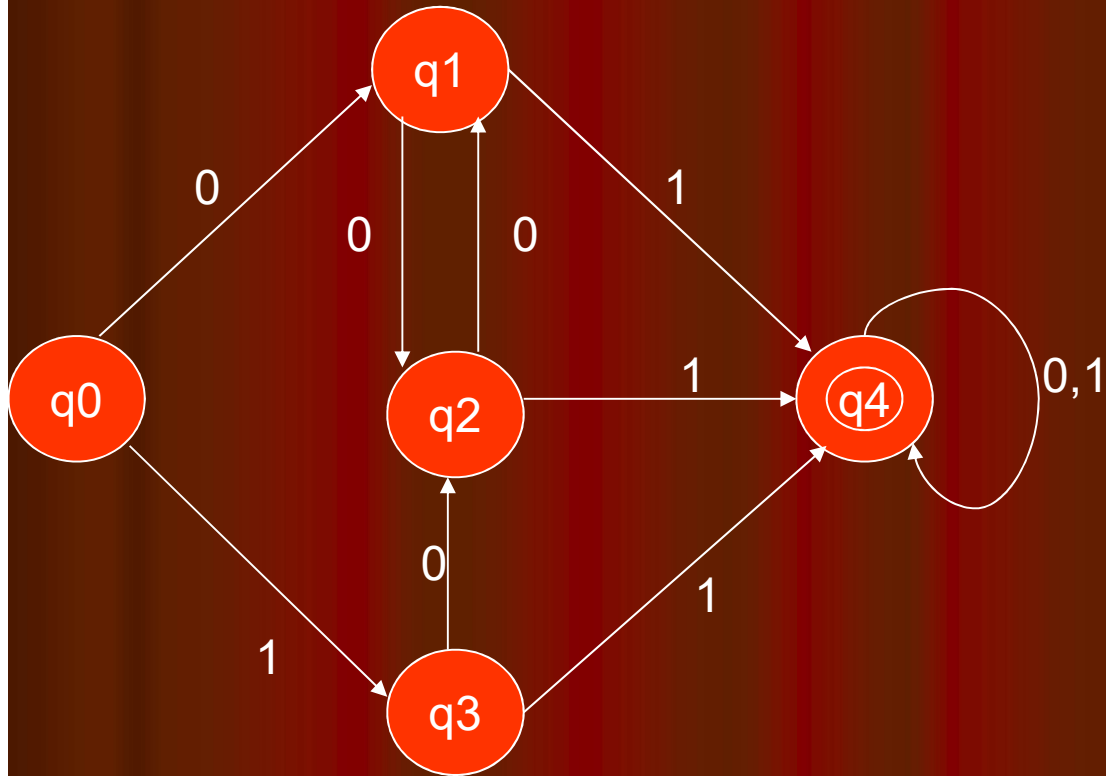


Theory of Automata & Formal Languages (Theory of Computation)

Compiled By
Prof. M. S. Bhatt

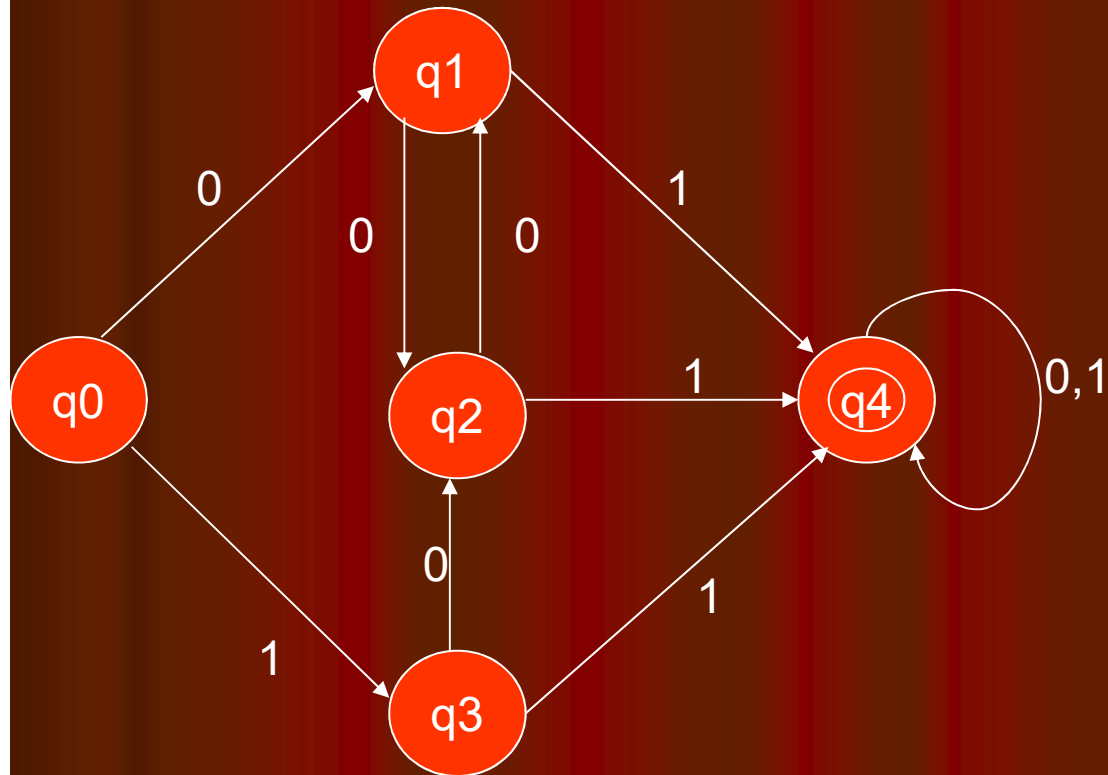
Minimization of Finite Automata



Rule :1 Remove all inaccessible States.

Can We reach to all states from the initial State ?

Minimization of Finite Automata



| | | | | |
|----|----|----|----|----|
| q1 | | | | |
| q2 | | | | |
| q3 | | | | |
| q4 | x | x | x | x |
| | q0 | q1 | q2 | q3 |

Rule :2 Consider all pairs of states (p, q).

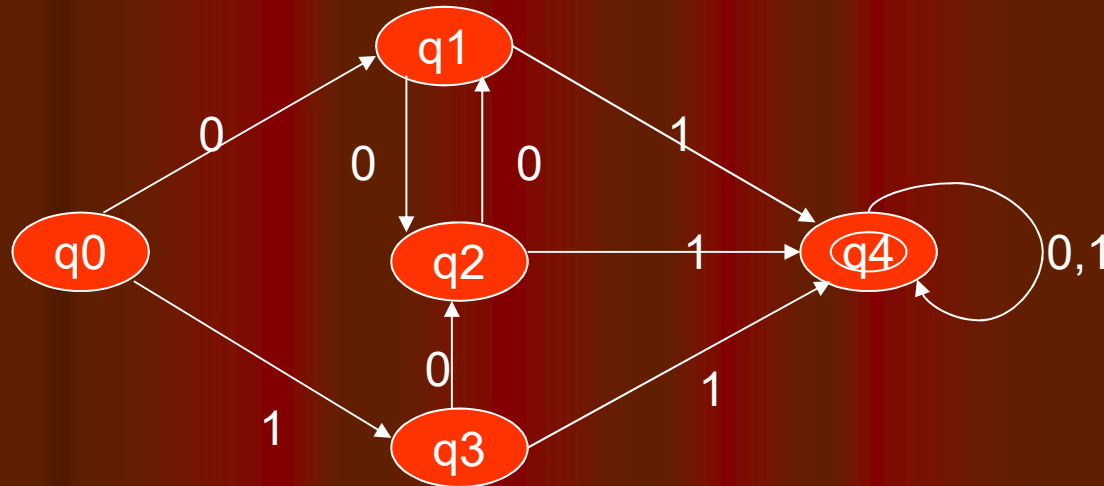
If p belongs to A and q does not belong to A

OR

If p does not belong to A and q belongs to A

MARK THE PAIR (p, q).

Minimization of Finite Automata



| | 0 | 1 |
|---------|---------|---------|
| (q0,q1) | (q1,q2) | (q3,q4) |
| (q0,q2) | (q1,q1) | (q3,q4) |
| (q0,q3) | (q1,q2) | (q3,q4) |
| (q0,q4) | (q1,q4) | (q3,q4) |
| (q1,q2) | (q2,q1) | (q4,q4) |
| (q1,q3) | (q2,q2) | (q4,q4) |
| (q2,q3) | (q1,q2) | (q4,q4) |

Rule :3

Repeat step until no previously unmarked pairs are marked

For all pairs (p, q) and all a belongs to Σ .

Compute $\delta(p, a) = m$ and $\delta(q, a) = n$

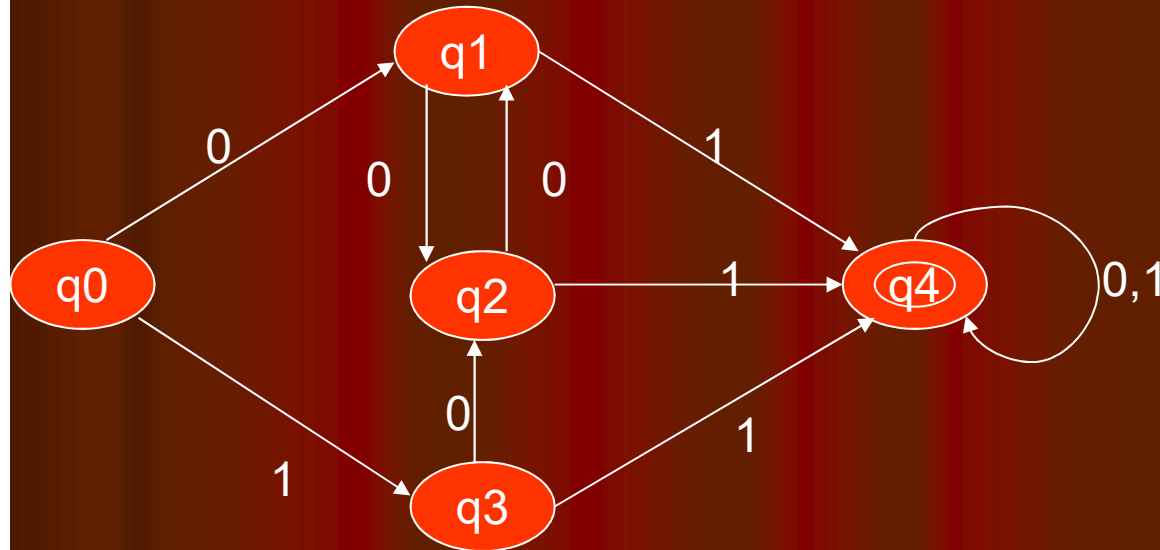
If pair (m, n) is already marked then **MARK THE PAIR (p, q).**

Here, (q0,q1) is one pair. Input alphabet is '0' and '1'

Calculating $\delta(q0, 0) = q1$ and $\delta(q1, 0) = q2$. New pair is (q1,q2) which is not marked so we can't mark (q0,q1)

Calculating $\delta(q0, 1) = q3$ and $\delta(q1, 1) = q4$. New pair is (q3,q4) which is marked so we can mark (q0,q1)

Minimization of Finite Automata



| | 0 | 1 |
|---------|---------|---------|
| (q0,q1) | (q1,q2) | (q3,q4) |
| (q0,q2) | (q1,q1) | (q3,q4) |
| (q0,q3) | (q1,q2) | (q3,q4) |
| (q0,q4) | (q1,q4) | (q3,q4) |
| (q1,q2) | (q2,q1) | (q4,q4) |
| (q1,q3) | (q2,q2) | (q4,q4) |
| (q2,q3) | (q1,q2) | (q4,q4) |

| q1 | x | | | |
|----|----|----|----|----|
| q2 | x | | | |
| q3 | x | | | |
| q4 | x | x | x | x |
| | q0 | q1 | q2 | q3 |

Rule :3

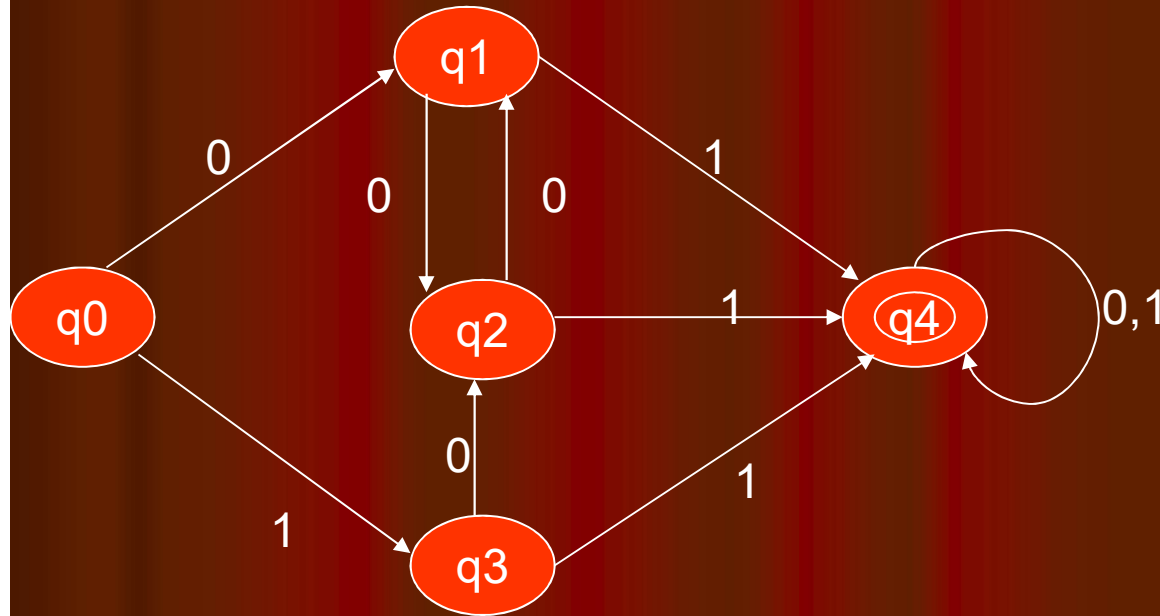
Repeat step until no previously unmarked pairs are marked

For all pairs (p, q) and all a belongs to Σ .

Compute $\delta(p, a) = m$ and $\delta(q, a) = n$

If pair (m, n) is already marked then **MARK THE PAIR** (p, q).

Minimization of Finite Automata



| | 0 | 1 |
|---------|---------|---------|
| (q1,q2) | (q2,q1) | (q4,q4) |
| (q1,q3) | (q2,q2) | (q4,q4) |
| (q2,q3) | (q1,q2) | (q4,q4) |

| q1 | x | | | |
|----|----|----|----|----|
| q2 | x | | | |
| q3 | x | | | |
| q4 | x | x | x | x |
| | q0 | q1 | q2 | q3 |

Rule :3

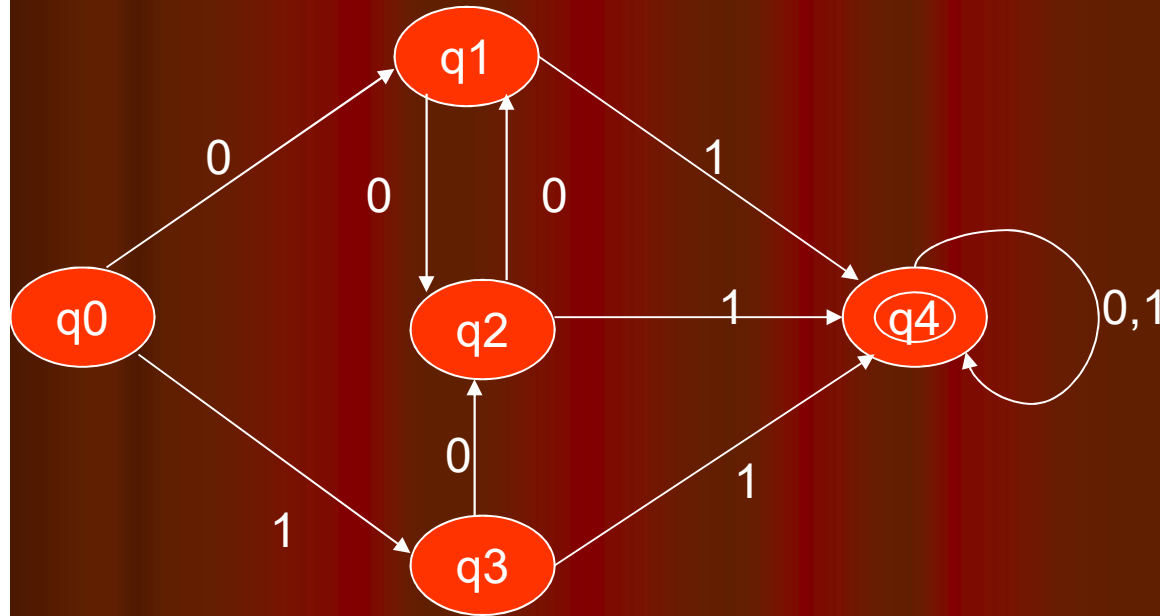
Repeat step until no previously unmarked pairs are marked

For all pairs (p, q) and all a belongs to Σ .

Compute $\delta(p, a) = m$ and $\delta(q, a) = n$

If pair (m, n) is already marked then **MARK THE PAIR (p, q).**

Minimization of Finite Automata



| | 0 | 1 |
|---------|---------|---------|
| (q1,q2) | (q2,q1) | (q4,q4) |
| (q1,q3) | (q2,q2) | (q4,q4) |
| (q2,q3) | (q1,q2) | (q4,q4) |

| q1 | x | | | |
|----|----|----|----|----|
| q2 | x | | | |
| q3 | x | | | |
| q4 | x | x | x | x |
| | q0 | q1 | q2 | q3 |

New pairs can not be marked after 2nd iteration.

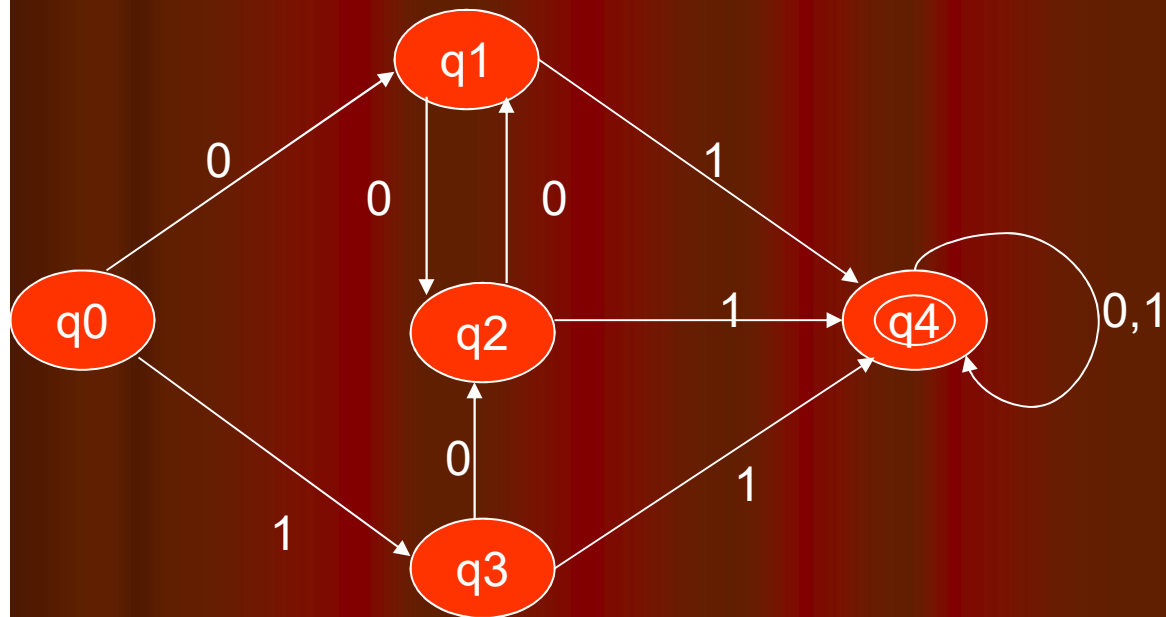
Unmarked Pairs are:

(q1,q2), (q2,q3) and (q1,q3)

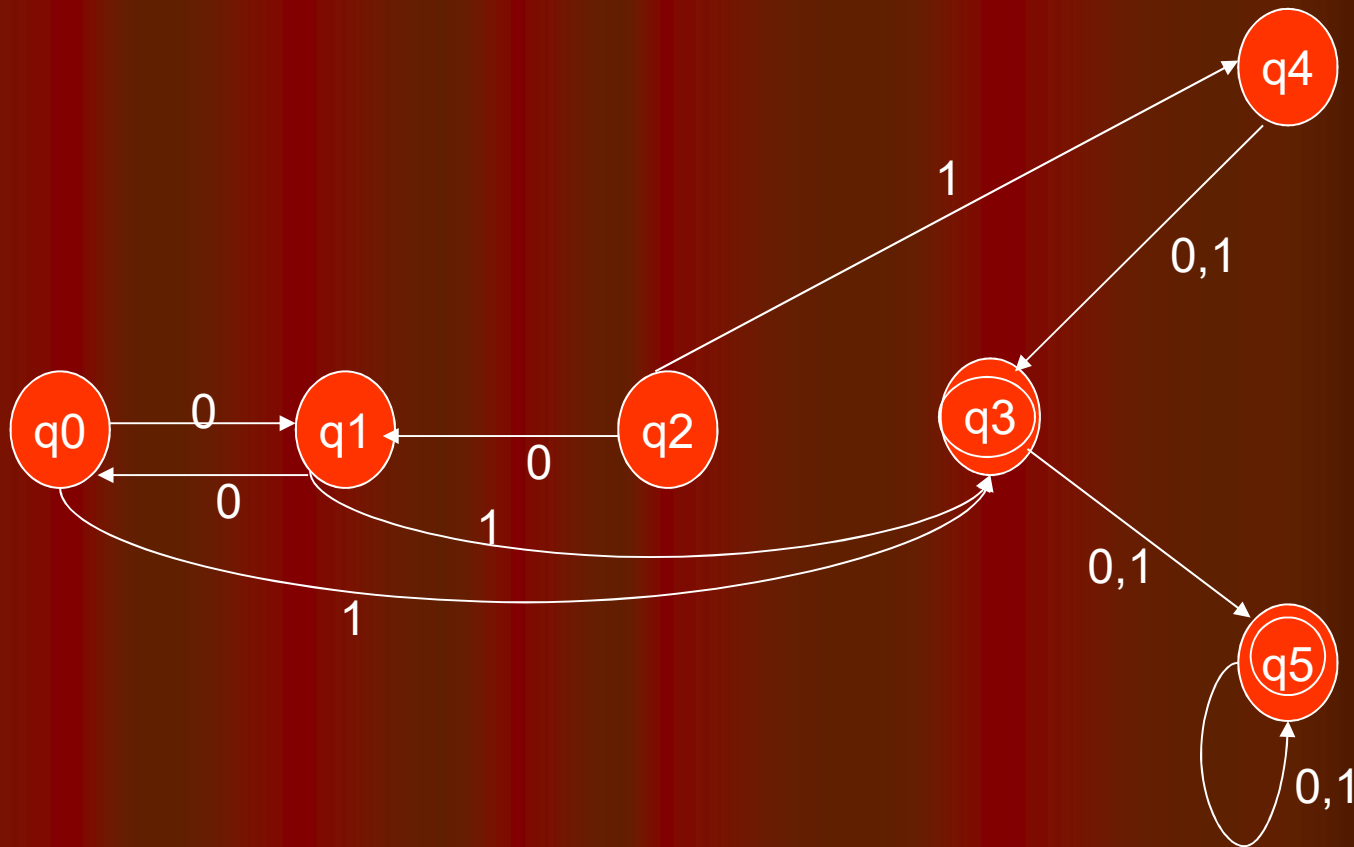
Here,

Transitivity property holds. I can combine 3 states together

Minimization of Finite Automata



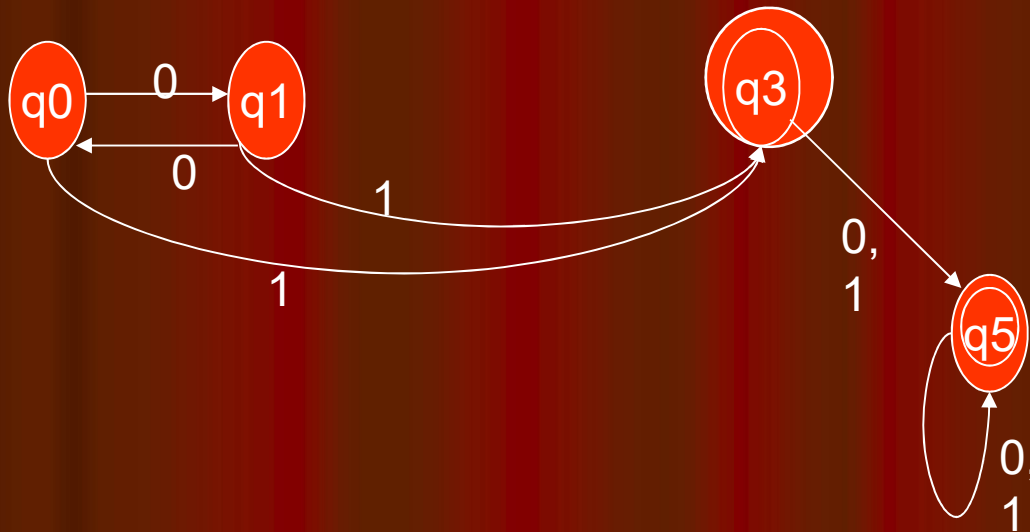
Minimization of Finite Automata



Rule :1 Remove all inaccessible States.

Can We reach to all states from the initial State ?

Minimization of Finite Automata



| | | | |
|----|----|----|----|
| q1 | | | |
| q3 | x | x | |
| q5 | x | x | |
| | q0 | q1 | q3 |

Rule :2 Consider all pairs of states (p, q).

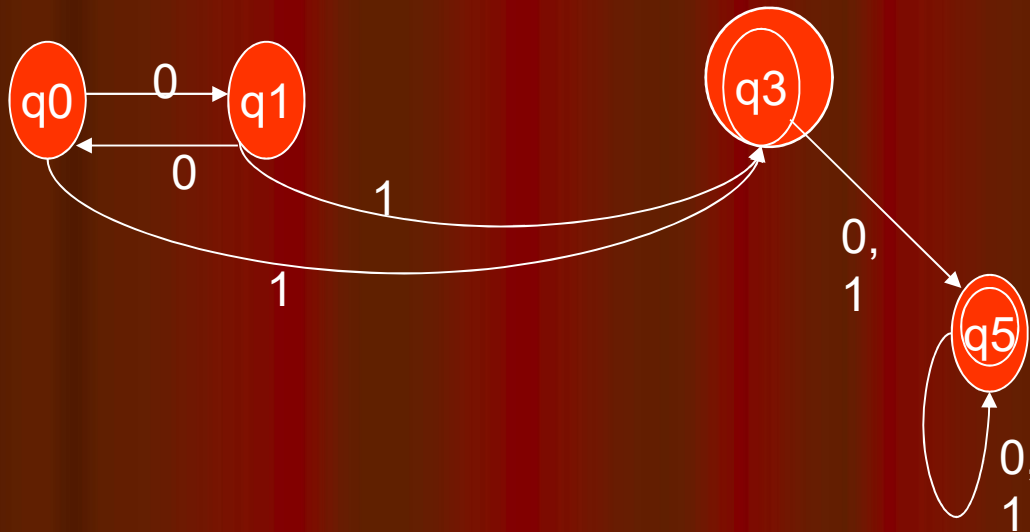
If p belongs to A and q does not belong to A

OR

If p does not belong to A and q belongs to A

MARK THE PAIR (p, q).

Minimization of Finite Automata



| | 0 | 1 |
|---------|---------|---------|
| (q0,q1) | (q1,q0) | (q3,q3) |
| (q3,q5) | (q5,q5) | (q5,q5) |

New pairs can not be marked after applying rule -3.

Unmarked Pairs are:

(q0,q1) and (q3,q5)

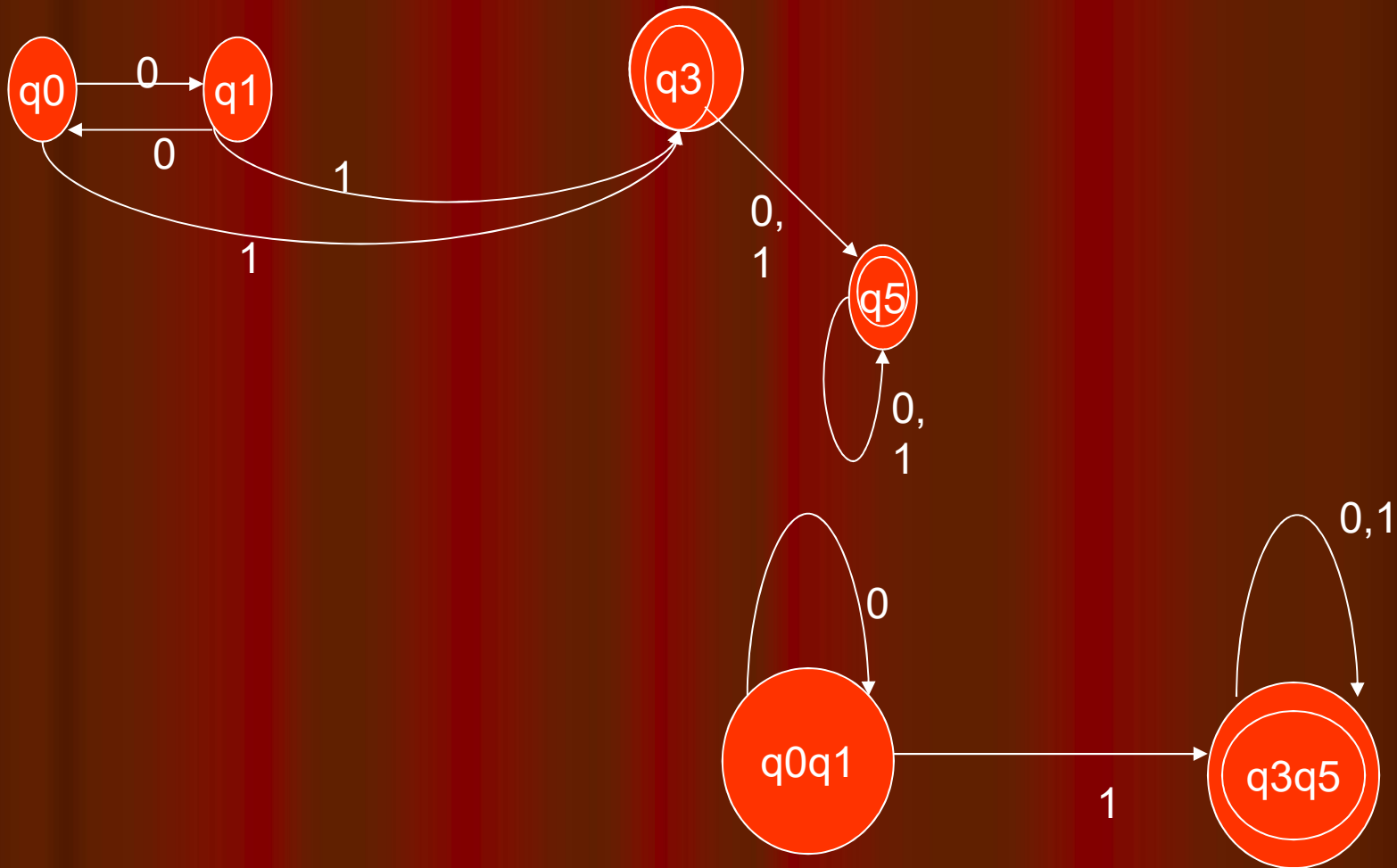
Here,

Transitivity property does not hold.

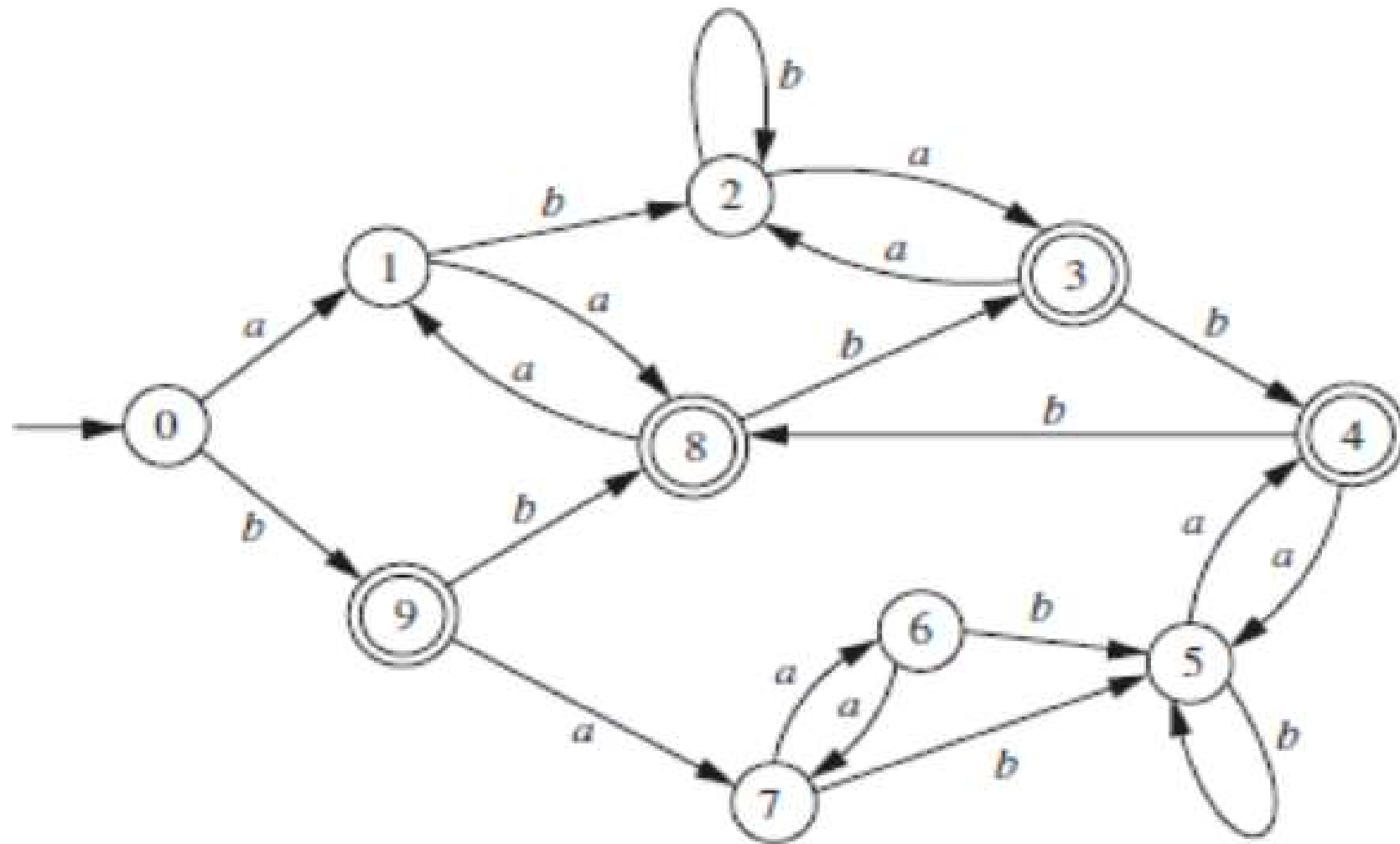
I can combine only 2 states together

| q1 | | | |
|----|----|----|----|
| q3 | x | x | |
| q5 | x | x | |
| | q0 | q1 | q3 |

Minimization of Finite Automata



Example -3



Example -3

| | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 2 | | | | | | | | |
| 2 | 2 | | | | | | | | |
| 3 | 1 | 1 | 1 | | | | | | |
| 4 | 1 | 1 | 1 | | | | | | |
| 5 | 2 | | | 1 | 1 | | | | |
| 6 | 2 | 2 | 2 | 1 | 1 | 2 | | | |
| 7 | 2 | 2 | 2 | 1 | 1 | 2 | | | |
| 8 | 1 | 1 | 1 | | | 1 | 1 | 1 | |
| 9 | 1 | 1 | 1 | 2 | 3 | 1 | 1 | 1 | 2 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

Example -3

