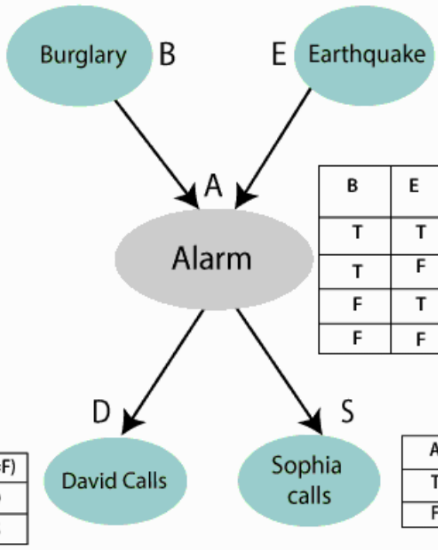
- Exact Inference

$$P(b,e,a,d,s) = P(d/a)\cdot P(s/a)\cdot P(a/b,e)\cdot P(b)\cdot P(e)$$



T	0.002
F	0.998

T	0.001
F	0.999

B	E	P(A=T)	P(A=F)
T	T	~~0.94~~	~~0.06~~
T	F	~~0.95~~	~~0.04~~
F	T	~~0.69~~	~~0.69~~
F	F	~~0.999~~	~~0.999~~

P(A=T)	P(A=F)
0.94	0.06
0.95	0.05
0.96	0.04
0.001	0.999

A	P (D=T)	P (D=F)
T	0.91	0.09
F	0.05	0.95

A	P (S=T)	P (S=F)
T	0.75	0.25
F	0.02	0.98

What is the probability of Alarm, if there was an earth quake and David called but sophia did not call ?

$$P(A=T/E=T,\ D=T,\ S=F)$$

$$\approx P(a/e,d,\bar{s}) = \frac{P(a,e,d,\bar{s})}{P(e,d,\bar{s})}$$

$$= \frac{\sum_{b\in\{0,1\}} P(a,e,d,\bar{s},b) \quad (A)}{\sum_{b\in\{0,1\}}\sum_{a\in\{0,1\}} P(a,e,d,\bar{s},b) \quad (B)}$$

(B) $\sum_{b\in\{0,1\}}\sum_{a\in\{0,1\}} P(a,e,d,\bar{s},b)$

$$= P(\bar{a},e,d,\bar{s},\bar{b}) + P(\bar{a},e,d,\bar{s},b) + P(a,e,d,\bar{s},b) + P(a,e,d,\bar{s},\bar{b})$$

$$= P(d/\bar{a})\cdot P(\bar{s}/\bar{a})\cdot P(\bar{a}/\bar{b},e)\cdot P(\bar{b})\cdot P(e)$$
$$+ P(d/\bar{a})\cdot P(\bar{s}/\bar{a})\cdot P(\bar{a}/b,e)\cdot P(b)\cdot P(e)$$
$$+ P(d/a)\cdot P(\bar{s}/a)\cdot P(a/b,e)\cdot P(b)\cdot P(e)$$
$$+ P(d/a)\cdot P(\bar{s}/a)\cdot P(a/\bar{b},e)\cdot P(\bar{b})\cdot P(e)$$

$$= (0.05)(0.98)(0.04)(0.998)(0.001)$$
$$+ (0.05)(0.98)(0.06)(0.002)(0.001)$$
$$+ (0.91)(0.25)(0.94)(0.002)(0.001)$$
$$+ (0.91)(0.25)(0.96)(0.998)(0.001)$$
$$= 220.2 \times 10^{-6}$$

$$A \quad \sum_{b \in \{0,1\}} P(a, e, d, \bar{s}, b)$$

$$= P(a, e, d, \bar{s}, b) + P(a, e, d, \bar{s}, \bar{b})$$
$$= (0.91)(0.25)(0.94)(0.002)(0.001)$$
$$+ (0.91)(0.25)(0.96)(0.998)(0.001)$$
$$= 218.3 \times 10^{-6}$$

$$P(a \mid e, d, \bar{s}) = \frac{\sum_{b \in \{0,1\}} P(a, e, d, \bar{s}, b) \quad A}{\sum_{b \in \{0,1\}} \sum_{a \in \{0,1\}} P(a, e, d, \bar{s}, b) \quad B}$$
$$= \frac{218.3 \times 10^{-6}}{220.2 \times 10^{-6}}$$
$$= 0.99$$

Inference Query 2

$$P(e \mid a, \bar{b}, \bar{d}, s) = \frac{P(a, \bar{b}, \bar{d}, s, e)}{P(a, \bar{b}, \bar{d}, s, e) + P(a, \bar{b}, \bar{d}, s, \bar{e})}$$

$$P(a, \bar{b}, \bar{d}, s, e) = P(\bar{d} \mid a) \cdot P(s \mid a) \cdot P(a \mid \bar{b}, e) \cdot P(\bar{b}) \cdot P(e)$$
$$= (0.09)(0.75)(0.96)(0.998)(0.001)$$
$$= 6.47 \times 10^{-5}$$

$$P(a, \bar{b}, \bar{d}, s, \bar{e}) = P(\bar{d} \mid a) \cdot P(s \mid a) \cdot P(a \mid \bar{b}, \bar{e}) \cdot P(\bar{b}) \cdot P(\bar{e})$$
$$= (0.09)(0.75)(0.001)(0.998)(0.999)$$
$$= 6.729 \times 10^{-5}$$

$$P(e|a,\bar{b},\bar{d},s) = \frac{P(a,\bar{b},\bar{d},s,e)}{P(a,\bar{b},\bar{d},s,e) + P(a,\bar{b},\bar{d},s,\bar{e})}$$

$$= \frac{6.47 \times 10^{-5}}{6.47 \times 10^{-5} + 6.729 \times 10^{-5}}$$

$$= 0.49$$

Inference Query 3

$$P(s|d,a,\bar{e},\bar{b}) = \frac{P(s,d,a,\bar{e},\bar{b})}{P(s,d,a,\bar{e},\bar{b}) + P(\bar{s},d,a,\bar{e},\bar{b})}$$

$$P(s,d,a,\bar{e},\bar{b}) = P(s|a) \cdot P(d|a) \cdot P(a|\bar{e},\bar{b}) \cdot P(\bar{b}) \cdot P(\bar{e})$$

$$= (0.75)(0.91)(0.001) \cdot (0.998)(0.999)$$

$$= 6.8045 \times 10^{-4}$$

$$P(\bar{s},d,a,\bar{e},\bar{b}) = P(\bar{s}|a) \cdot P(d|a) \cdot P(a|\bar{e},\bar{b}) \cdot P(\bar{b}) \cdot P(\bar{e})$$

$$= (0.25)(0.91)(0.001) \cdot (0.998)(0.999)$$

$$= 2.26 \times 10^{-4}$$

$$P(s|d,a,\bar{e},\bar{b}) = \frac{P(s,d,a,\bar{e},\bar{b})}{P(s,d,a,\bar{e},\bar{b}) + P(\bar{s},d,a,\bar{e},\bar{b})}$$

$$= \frac{6.8045 \times 10^{-4}}{6.8045 \times 10^{-4} + 2.26 \times 10^{-4}}$$

$$= 0.75$$

$P(s/d, a, \bar{e}, \bar{b}) = P(s/a) = 0.75$

$P(\bar{d}/\bar{a}, e)$? 0.95

$P(\bar{d}/\bar{e}, \bar{b})$?

is d conditionally independent of e given b?