CFG TO NPDA CONSTRUCTION

Definition 5.17 The Nondeterministic Top-Down PDA *NT*(*G*)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar. The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \{q_0, q_1, q_2\} \quad A = \{q_2\} \quad \Gamma = V \cup \Sigma \cup \{Z_0\}$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$$ — Initialization

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$$ — Acceptance

The moves from q_1 are the following:

For every $A \in V$, $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha \text{ is a production in } G\}$ Push

For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$ Pop

CFG TO NPDA CONSTRUCTION

Move Number	State	Input	Stack Symbol	Move
1	q_0	Λ	Z_0	(q_1, SZ_0)
2	q_1	Λ	S	$(q_1, [S]), (q_1, SS), (q_1, \Lambda)$
3	q_1	[	[	(q_1, Λ)
4	q_1	]	]	(q_1, Λ)
5	q_1	Λ	Z_0	(q_2, Z_0)
	(all other combinations)			none

CFG TO NPDA CONSTRUCTION

$(q_0, [\,[\,]\,[\,]\,], Z_0)$

$\vdash (q_1, [\,[\,]\,[\,]\,], SZ_0)$

$\vdash (q_1, [\,[\,]\,[\,]\,], [S]Z_0)$

$\vdash (q_1, [\,[\,]\,[\,]\,], S]Z_0)$

$\vdash (q_1, [\,[\,]\,[\,]\,], SS]Z_0)$

$\vdash (q_1, [\,[\,]\,[\,]\,], [S]S]Z_0)$

$\vdash (q_1,]\,[\,]\,[\,], S]S]Z_0)$

$\vdash (q_1,]\,[\,]\,[\,],]S]Z_0)$

$\vdash (q_1, [\,]\,[\,], S]Z_0)$

$\vdash (q_1, [\,]\,[\,], [S]]Z_0)$

$\vdash (q_1,]\,[\,], S]]Z_0)$

$\vdash (q_1,]\,[\,],]]Z_0)$

$\vdash (q_1,]\,[\,],]Z_0)$

$\vdash (q_1, \Lambda, Z_0)$

$\vdash (q_2, \Lambda, Z_0)$

S

$\Rightarrow [S]$

$\Rightarrow [SS]$

$\Rightarrow [[S]S]$

$\Rightarrow [[\,]S]$

$\Rightarrow [[\,][S]]$

$\Rightarrow [[\,][\,]]$

$\Rightarrow [[\,]\,[\,]]$

CFG TO NPDA CONSTRUCTION (EXAMPLE 2)

$S \rightarrow S + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (S) \mid a$

q_1	*	*	(q_1, Λ)
q_0	Λ	Z_0	(q_1, SZ_0)
q_1	Λ	S	$(q_1, S+T)$ (q_1, T)
q_1	Λ	T	$(q_1, T*F)$ (q_1, F)
q_1	Λ	F	$(q_1, (S))$ (q_1, a)
q_1	a	a	(q_1, Λ)
q_1	(	(	(q_1, Λ)
q_1	)	)	(q_1, Λ)
q_1	+	+	(q_1, Λ)
q_1	Λ	Z_0	(q_2, Z_0)

CFG TO NPDA CONSTRUCTION (EXAMPLE 2)

$S \rightarrow S+T \mid T$
$T \rightarrow T*F \mid F$
$F \rightarrow (S) \mid a$

$(q_0, a+a, Z_0)$
$(q_1, a+a, SZ_0)$
$(q_1, a+a, S+TZ_0)$
$(q_1, a+a, T+TZ_0)$
$(q_1, a+a, F+TZ_0)$
$(q_1, a+a, a+TZ_0)$
$(q_1, +a, +TZ_0)$
(q_1, a, TZ_0)

(q_1, a, FZ_0)
(q_1, a, aZ_0)
$(q_1, \wedge, Z_0)$
(q_2, Z_0).

CFG TO NPDA CONSTRUCTION (EXAMPLE 3)

$L = \{ a^i b^j c^k \mid j = i + k, i, k > 0 \}$

$j = i + k$

$a^i b^{i+k} c^k \Rightarrow \underbrace{a^i b^i}_{L_{11}} \underbrace{b^k c^k}_{L_{12}}$

$S \rightarrow S_1 S_2$

$S_1 \rightarrow aS_1b \mid ab$

$S_2 \rightarrow bS_2c \mid bc$

CFG TO NPDA CONSTRUCTION (EXAMPLE 3)

$L = \{ a^i b^j c^k \mid j = i + k, i, k > 0 \}$

q_0	Λ	Z_0	(q_1, SZ_0)
q_1	Λ	S	$(q_1, S_1 S_2)$
q_1	Λ	S_1	(q_1, aS_1b) (q_1, ab)
q_1	Λ	S_2	(q_1, bS_2c) (q_1, bc)
q_1	a	a	(q_1, Λ)
q_1	b	b	(q_1, Λ)
q_1	c	c	(q_1, Λ)
q_1	Λ	Z_0	(q_2, Z_0)

CFG TO NPDA CONSTRUCTION (EXAMPLE 3)

$L = \{ a^i b^j c^k \mid j = i + k, i, k > 0 \}$

$(q_0, abbc, Z_0)$

$(q_1, abbc, SZ_0)$

$(q_1, abbc, S_1 S_2 Z_0)$

$(q_1, abbc, ab S_2 Z_0)$

$(q_1, bbc, b S_2 Z_0)$

$(q_1, bc, S_2 Z_0)$

$(q_1, bc, bc Z_0)$

$(q_1, c, c Z_0)$

(q_1, Λ, Z_0)

(q_2, Z_0).

CFG TO NPDA CONSTRUCTION (EXAMPLE 4)

$S \rightarrow [S]\,S \mid \wedge$

1.	q_0	$\wedge$	Z_0	(q_1, SZ_0)
2.	q_1	$\wedge$	S	$(q_1, [S]S)$ $(q_1, \wedge)$
3.	q_1	$[$	$[$	$(q_1, \wedge)$
4.	q_1	$]$	$]$	$(q_1, \wedge)$
5.	q_1	$\wedge$	Z_0	(q_2, Z_0)