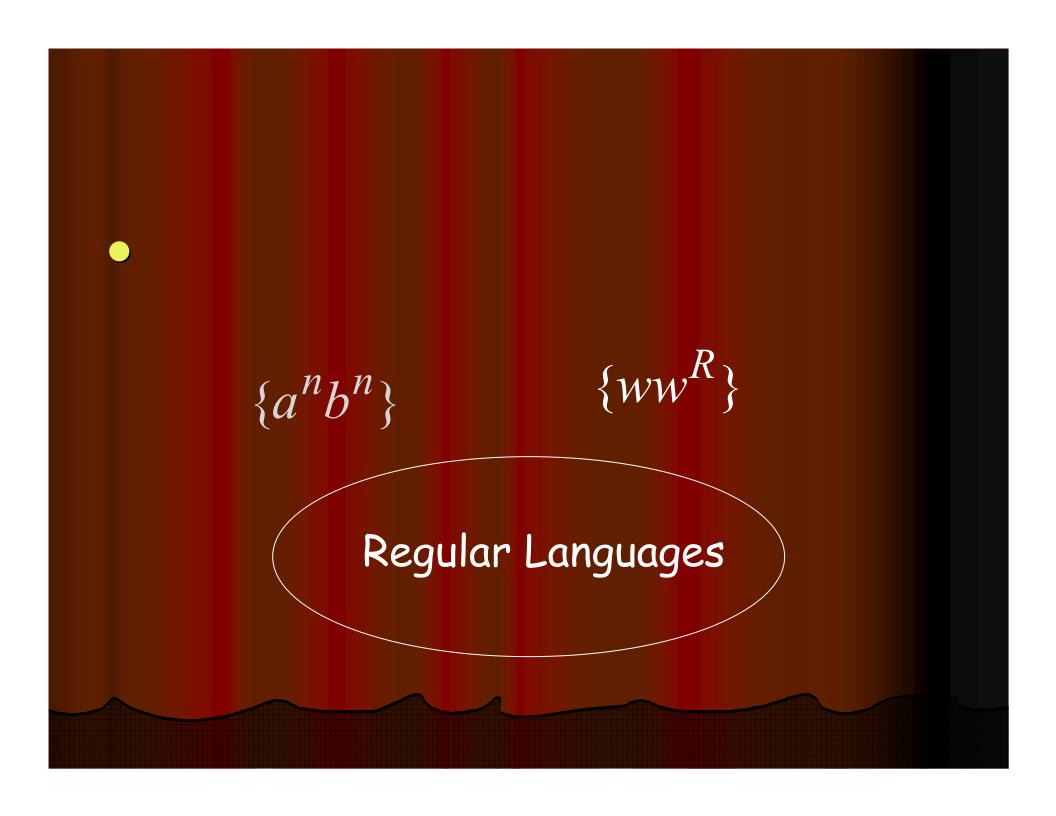
# Context-Free Languages





$$\{a^nb^n\}$$

 $\{ww^R\}$ 

Regular Languages



Context-Free Grammars

Pushdown Automata

stack

automaton

## Context-Free Grammars

#### Example

A context-free grammar  $G: S \rightarrow aSb$ 

$$S \to \lambda$$

A derivation:

 $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$ 

A context-free grammar  $G: S \rightarrow aSb$   $S \rightarrow \lambda$ 

#### Another derivation:

 $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$ 

$$S \to aSb$$
$$S \to \lambda$$

$$L(G) = \{a^n b^n : n \ge 0\}$$

#### Example

A context-free grammar G:

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \to \lambda$$

A derivation:

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$$

A context-free grammar G:

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \to \lambda$$

Another derivation:

 $S \Rightarrow aSa \Rightarrow abSba \Rightarrow abaSaba \Rightarrow abaaba$ 

$$S \to aSa$$

$$S \to bSb$$

$$S \to \lambda$$

$$L(G) = \{ww^R : w \in \{a,b\}^*\}$$

#### Example

A context-free grammar G:

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \to \lambda$$

A derivation:

$$S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow ab$$

A context-free grammar G:

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \to \lambda$$

A derivation:

 $S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow abaSb \Rightarrow abab$ 

$$S \to aSb$$

$$S \to SS$$

$$S \to \lambda$$

$$L(G) = \{w : n_a(w) = n_b(w),$$
and  $n_a(v) \ge n_b(v)$ 
in any prefix  $v\}$ 

()((()))(())

#### Definition: Context-Free

Grammars

Grammar G = (V, T, S, P)

Variables

Terminal Start symbols variables

Productions of the form:

 $A \rightarrow x$ 

x is string of variables and terminals

### Definition: Context-Free Languages

ullet A language L is context-free

if and only if

• there is a grammar G with L = L(G)