SET THEORY

$A = \{0, 1, 2, 3, \ldots\ldots\}$

$D = \{x \in Z / -10 \leq x \leq 10\}$

$D = \{-10, -9, -8, \ldots\ldots\ldots, 8, 9, 10\}$

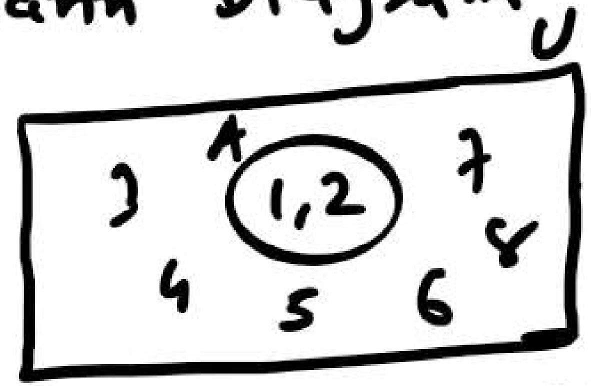
$E = \{x / i, j \geq 0, \ x = 3i + 7j\}$

U = Universal Set $\qquad \phi$ = empty set

Complement of a set is denoted by $\underline{\underline{A'}}$

$$A' = \{x \in U / x \notin A\}$$

Venn Diagram



$A' = \{3, 4, 5, 6, 7, 8\}$

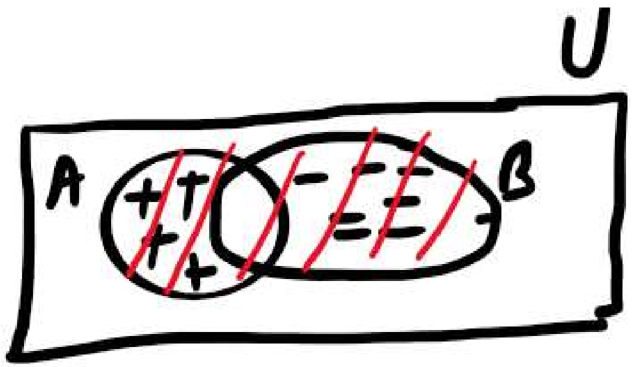
$A \cup B = \{x \,/\, x \in A \text{ or } x \in B\}$

$A \cap B = \{x \,/\, x \in A \text{ and } x \in B\}$

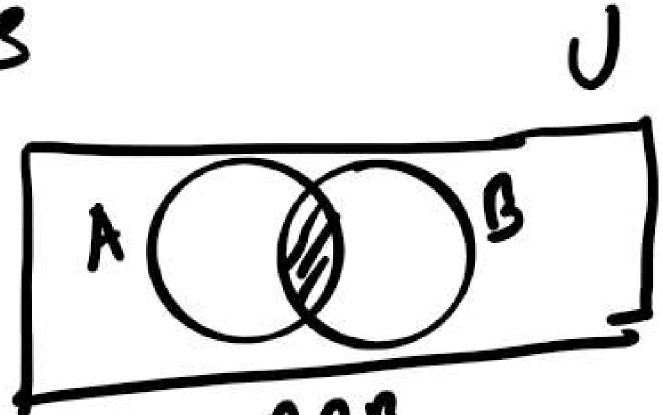
$A - B = \{x \,/\, x \in A \text{ and } x \notin B\}$

$\uparrow$

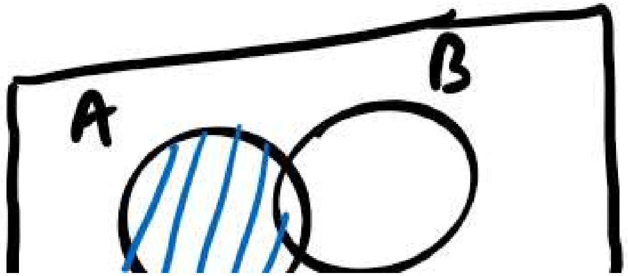
Set Difference



$A \cup B$



$A \cap B$



Laws of Set Theory

$A \cup B = B \cup A$ commutative
$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$ Associative
$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Distributive Law
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cup A = A$ Idempotent
$A \cap A = A$

$A \cup (A \cap B) = A$ Absorptive
More
$A \cap (A \cup B) = A$

Interesting Laws/Properties

$(A')' = A$

$A \cup \phi = A$

$A \cap U = A$

$A \cap A' = \phi$

$A \cap \phi = \phi$

$A \cup A' = U$

Symmetric Difference

$A \oplus B = (A - B) \cup (B - A)$

If $A_1, A_2, A_3, \ldots, A_n$ are sets then

$\bigcup\limits^{n} A_i = \{x / x \in A_i$ for atleast one i with $1 \leq i \leq n\}$

$$\bigcap_{i=1}^{n} A_i = \{x \mid x \in A_i \text{ for every } i \text{ with } 1 \le i \le n\}$$

Powerset of a set:

$A = \{1, 2\} \Rightarrow 2^A = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

$|A|$ = Number of elements present in the set

$A = \{1, 2, 3\}$ $\quad |A| = 3$

Cartesian Product of 2 sets.

$A = \{a, b\}$ $\quad B = \{c, d\}$

$A \times B = \{(a,c), (a,d), (b,c), (b,d)\}$

Ex 1.1

(a) $\{0, -1, 2, -3, 4, -5, 6, \ldots\}$

even +ive

odd -ive

$$\boxed{\{(-1)^n \cdot n \mid n \in N\}}$$

Set of positive integers including 0}

(b) $\{1/2,\ 1/4,\ 3/4,\ 1/8,\ 3/8,\ 5/8,\ 7/8, \ldots\ldots\}$

$$\boxed{\left\{ \frac{m}{2^n} \;\middle|\; n>0,\ 0<m<2^n \right\}}$$

Denominator

2	2^1
4	2^2
8	2^3

$\frac{1}{4},\ \frac{2}{4} = 1/2,\ \frac{3}{4}$

$\frac{1}{8},\ \frac{2}{8} = 1/4,\ \frac{3}{8},\ \frac{4}{8} = 1/2,\ 5/8,\ 6/8 = \frac{3}{4},\ 7/8$

$n=1 \Rightarrow 0<m<2 \Rightarrow m=1 \qquad 1/2$

$n=2 \Rightarrow 0<m<4 \Rightarrow m=1,2,3 \qquad 1/4,\ \frac{2}{4} = 1/2,\ \frac{3}{4}$

(c) $\{10, 1100, 111000, \text{------}\}$

$$\{1^n 0^n \mid n > 0\}$$

$n = 1$	10
$n = 2$	1100
$n = 3$	111000

(e) $\{\{0\}, \{0,1\}, \{0,1,2\}, \{0,1,2,3\} \text{------}\}$

$$\{\{j \mid 0 \le j \le n, \ j \in N\} \ \underline{\underline{n \in N}}\}$$

$n = 0$	$0 \le j \le 0$	$\{0\}$
$n = 1$	$0 \le j \le 1$	$\{0,1\}$
$n = 2$	$0 \le j \le 2$	$\{0,1,2\}$

1.3 Simplify the given Expressions

(4) $A - (A - B)$

$= A - (A \cap B')$

$= A \cap (A \cap B')'$

$= A \cap (A' \cup B)$

$= (A \cap A') \cup (A \cap B)$

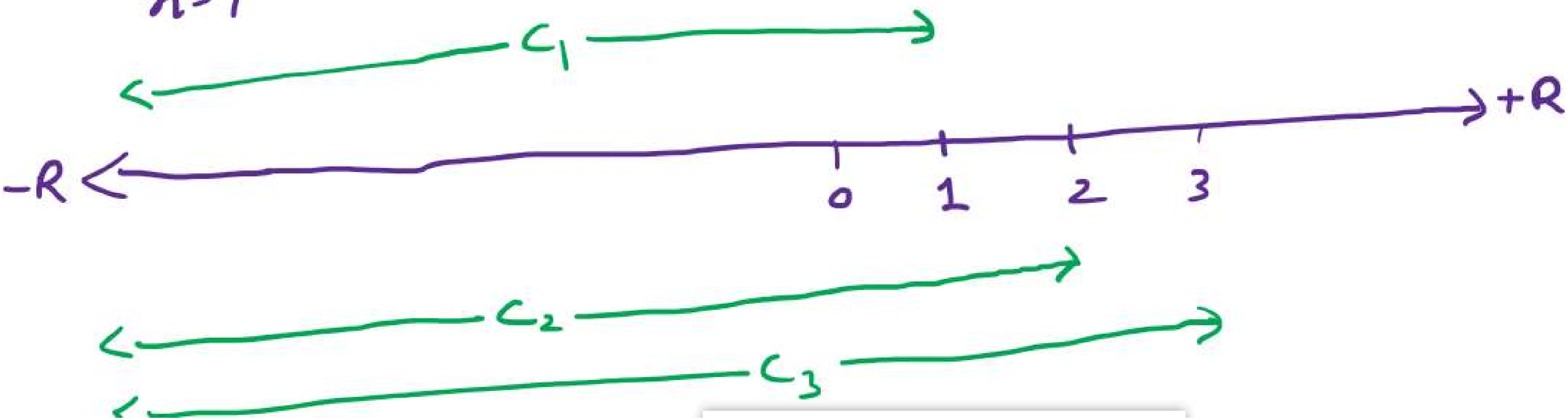
$= \phi \cup (A \cap B)$

$= A \cap B$

C_n = set of all real numbers less than n
↑
integer

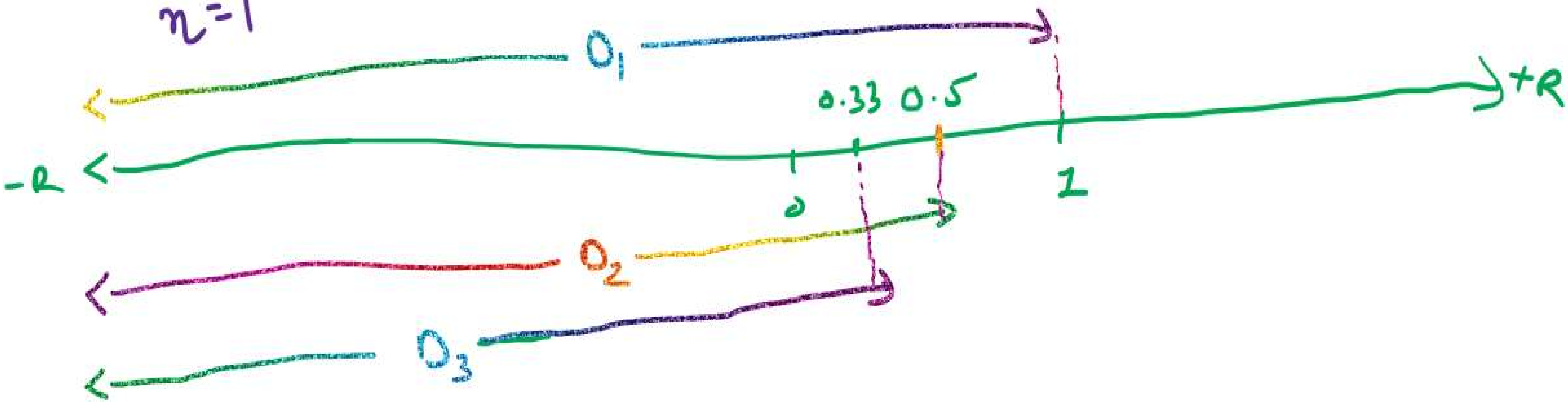
D_n = set of all real numbers less than $1/n$.

(a) $\bigcup_{n=1}^{10} C_n = C_{10}$



(c) $\bigcap_{n=1}^{10} C_n = C_1$

(D) $\bigcup_{n=1}^{10} D_n = D_1$



(d) $\bigcap_{n=1}^{10} D_n = D_{10}$

(e) $\bigcup_{n=1}^{\infty} C_n = R$

(f) $\bigcup_{n=1}^{\infty} D_n = D_1$

What is the relationship between $2^{A \cup B}$ and $2^A \cup 2^B$?

$A \cup B = \{1, 2\}$

$A = \{1\}$

$B = \{2\}$

$2^A = \{\phi, \{1\}\}$

$2^B = \{\{2\}, \phi\}$

$2^A \cup 2^B = \{\phi, \{1\}, \{2\}\}$

$2^{A \cup B} = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$2^A \cup 2^B$ is a proper subset of $2^{A \cup B}$.