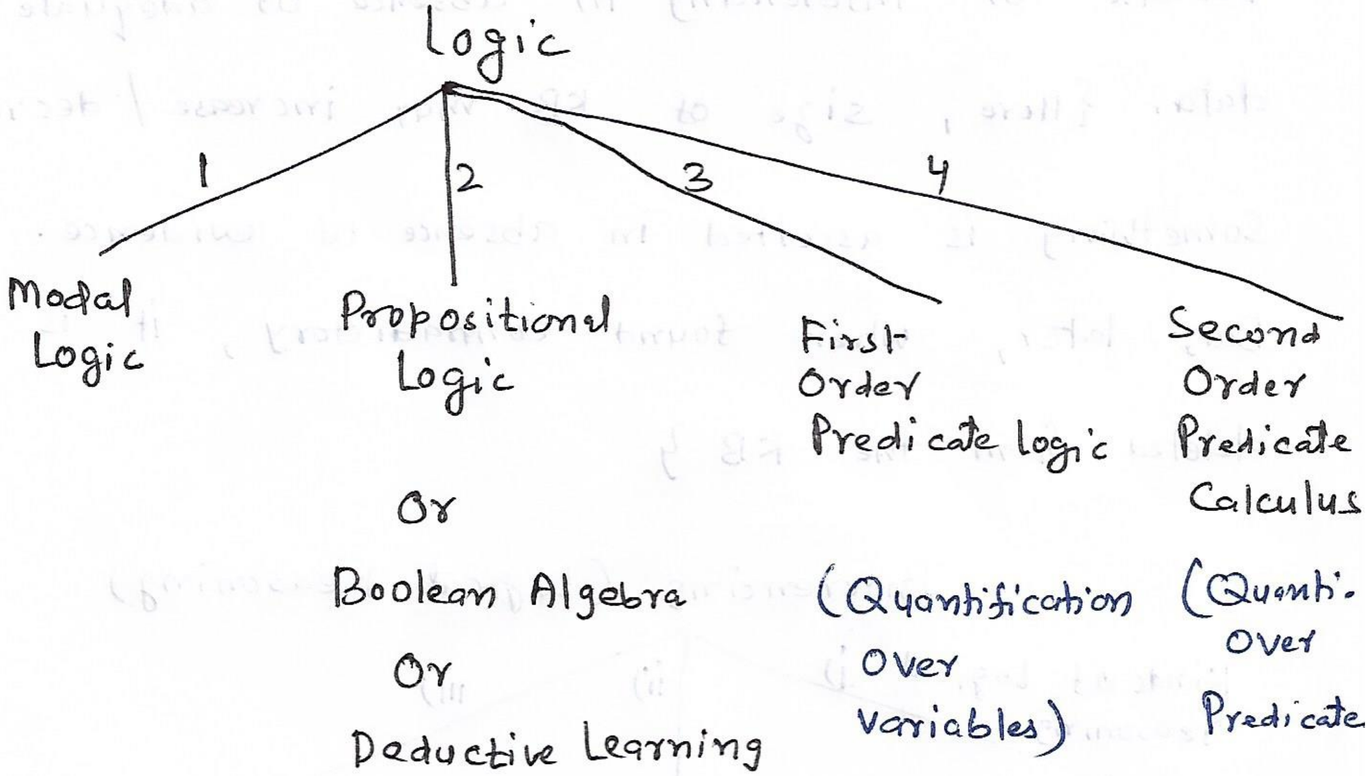
Part II Knowledge Representation



Modal Logic includes :

i) Assertive sentences (facts)

ii) Modal sentences

Possibility - e.g. If I were president ...

Belief Sentences - I suppose ...

I believe ...

I expect ...

2, 3, 4 are known as "Monotonic Logic systems". This is because the size of KB goes on increasing through inference. It never decreases.

1 is a Non-Monotonic Logic system. An asserted fact may be deleted later. This is because of inferencing in absence of adequate data. {Here, size of KB may increase / decrease

Something is asserted in absence of evidence. But, later, when found contradictory, it is deleted from the KB }

Inferencing (Logical Reasoning)

Kinds of Logical reasoning

i) Deduction

ii) Induction

iii) Abduction

Deduction

eg. All men are mortal

John is a man

⇒ John is mortal

Applications

Mathematicians use this reasoning style

Induction

(Generalizing from finite knowledge) i.e. learning a rule from several examples.

e.g. The grass has been wet every time it has rained".

Rule: When it rains, the grass gets wet

Scientists use this style of reasoning

Abduction

$A \to B$,

Suppose, we don know whether $B \to A$.

This is assumed to be true in absence of any knowledge.

This is Abductio

Diagnosticians and Detectives use this style of reasoning

Propositional Logic

Sentences	Proposition
It is raining	RAINING
It is sunny	SUNNY
It is windy	WINDY
It is raining, then it is not sunny	RAINING $\rightarrow$ $\neg$SUNNY

Predicate Calculus (First Order Logic)

Motivation

i) To represent relationship between Objects.

eg. Sky is blue : P

Screen is blue : Q

blue (sky).

blue (screen).

ii) Handling "For All" and "There exists" kind of sentences

Operators in FO PL

$\forall$ – For All [Universal Quantifier]

$\exists$ – There exists [Existential Quantifier]

$\rightarrow$ Implication

$\neg \quad \wedge \quad \vee$: NOT AND OR

Facts	Representation in Logic
1. Marcus was a man	man (Marcus)
2. Marcus was a Pompeian	pompeian (Marcus)
3. All Pompeians were Romans	$\forall x$: pompeian$(x) \rightarrow$ roman(x)
4. Caesar was a ruler	ruler (Caesar)
5. All Romans were either loyal to Caesar or hated him	$\forall x$: Roman$(x) \rightarrow$ loyalto$(x$, Caesar$) \vee$ hate$(x$, Caesar$)$
6. Everyone is loyal to someone	i) $\forall x : \exists y$: loyalto(x, y) ii) $\exists y : \forall x$: loyalto(x, y) iii) $\forall x : \exists y$: loyalto(y, x)

Person who is loyal to someone

Person to whom x is loyal

Everyone has a loya perso

There is someone to whom

possibilities

i) $\forall x : \exists y : \text{loyalto}(x, y)$

ii) $\exists y : \forall x : \text{loyalto}(x, y)$

iii) $\forall x : \exists y : \text{loyalto}(y, x)$

i) matches our interpretation

We should be careful about scope of the quantifiers and ambiguity

7. People only try to assassinate rulers they are not loyal to

Ambiguity

i) Only rulers people try to assassinate are those to whom they are not loyal

ii) Only thing people try to do is to assassinate rulers to whom they are not loyal

With interpretation i), the representation is:

$$\forall x : \forall y : \text{person}(x) \wedge \text{ruler}(y) \wedge \text{tryassassinate}(x, y) \rightarrow \neg\text{loyalto}(x, y)$$

8. Marcus tried to assassinate Caesar

tryassassinate (Marcus, Caesar)

Question : Was Marcus loyal to Caesar ?

$\neg$loyalto (Marcus, Caesar)

↑ 7, substitution

person (Marcus)
ruler (Caesar)
tryassassinate (Marcus, Caesar)

↑ 8

person (Marcus)
ruler (Caesar)

↑ 4

person (Marcus)

How to know that a ~~person~~ man is a person. Needs to be ~~b~~ explicitly specified

↑ 9, substitution

man (Marcus)

9. $\forall x$: man $(x) \rightarrow$ person (x)

↑ 1

$\square$

How should a program decide whether it should try to prove

$$loyalto(Marcus, Caesar) \text{ or}$$

$$\neg loyalto(Marcus, Caesar)$$

Possibilities

1. Use forward chaining, i.e. using available knowledge, see what are the things that can be infered.

   Problem : branching factor increases with the amount of knowledge

2. Use Heuristic knowledge to decide which answer is more likely and then try to prove that.

   If it can not be proved in some reasonable amount of time, then prove other thing.

3. Prove both things simultaneously and stop when one of the things is proved

Consider the following facts

1. Marcus was a man

man (Marcus)

2. Marcus was a Pompeian

pompeian (Marcus)

3. Marcus was born in 40 A.D.

born (marcus, 40)

4. All men are mortal

$\forall x: man(x) \rightarrow mortal(x)$

5. All Pompeians died when the volcano errupted in 79 A.D.

$errupted(volcano, 79) \wedge \forall x: pompeian(x) \rightarrow died(x, 79)$

6. No mortal lives longer than 150 years

$\forall x: \forall t_1: \forall t_2: mortal(x) \wedge born(x, t_1) \wedge gt(t_2 - t_1, 150) \rightarrow dead(x, t_2)$

7 It is now 2020

now = 2020

[Equal Quantities can be substituted for each other]

Is Marcus alive now ?

We can prove that Marcus is dead by :

i) Killed by volcano

ii) Age > 150 years

Additional knowledge (Relationship betn/ alive and dead is to be established)

8. $\forall x : \forall t :$ dead $(x, t) \rightarrow \neg$ alive (x, t)

9. If someone dies, then he is dead at all later times.

$\forall x : \forall t_1 : \forall t_2 :$ died $(x, t_1) \wedge$ gt (t_2, t_1) $\rightarrow$ dead (x, t_2)

Sol^n I

$$\neg alive(Marcus, now)$$

$\uparrow$ 8, substitution

$$dead(Marcus, now)$$

$\uparrow$ 9, substitution

$$died(Marcus, t1)$$
$$gt(now, t1)$$

$\uparrow$ 5, substitution

$$Pompeian(Marcus)$$
$$gt(now, 79)$$

$\uparrow$ 2

$$gt(now, 79)$$

$\uparrow$ 7

$$gt(2020, 79)$$

$\uparrow$ Compute gt

$$\square$$

Solution II

$\neg$alive (Marcus, now)

↑ 8, substitution

dead (Marcus, now)

↑ 6, substitution

mortal (Marcus)

born (Marcus, t_1)

gt (now $- t_1$, 150)

↑ 4, substitution

man (Marcus)

born (Marcus, t_1)

gt (now $- t_1$, 150)

↑ 1

born (Marcus, t_1)

gt (now $- t_1$, 150)

↑ 3, substitution

gt (now $-$ 40, 150)

↑ 7

gt (2020 $-$ 40, 150)

↑ Compute Minus

gt (1980, 150)

↑ Compute gt

$\boxed{}$

Resolution

- Proof by refutation (contradiction).
- It is used to infere new clause from old ones.

It works as :

"A clause may be proven to be true, in the context of a set of clauses known to be true, by adding the logical NOT of this clause to the set and seeking a contradiction".

→ Resolution provides a way of finding contradictions by trying a minimum no. of substitutions.

→ If contradiction exists, then eventually it will be found.

If resolution process fails to find a contradiction, the negative of what we seek to prove is logically consistent with the database, thus the clause can not be true.

Algo. Propositional Resolution

1. Convert all the Propositions of F, a set of axioms, to clause form.

2. Negate P, Proposition to be proved by Resolution, and convert the result to clause form, Add it to the set of clauses obtained in step 1.

3. Repeat until either a contradiction is found or no progress can be made:

   (a) Select two clauses. Call these parent clauses.

   (b) Resolve them together. The resulting clause, called the resolvent, will be the disjunction of all of the literals of all of the parent clauses with the following exception:
   If there are any pairs of literals L and ¬L such that one of the parent clauses contains L and other contains ¬L,

Then select one such pair and eliminate both L and $\neg L$ from the resolvent.

(c) If the resolvent is the empty clause, then a contradiction has been found. If it is not, then add it to the set of clauses available to the procedure.

Propositional Resolution Example

To Prove R

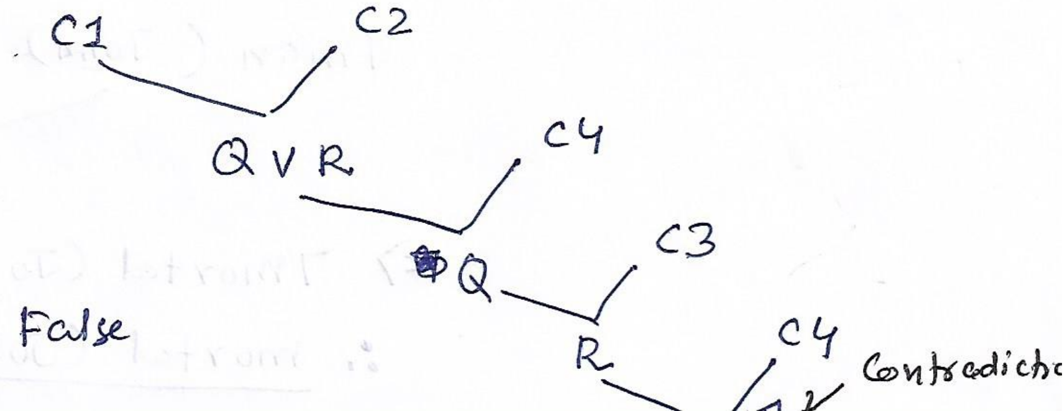
1	$P \vee Q$
2	$P \to R$
3	$Q \to R$

Clauses

C1	$P \vee Q$
C2	$\neg P \vee R$
C3	$\neg Q \vee R$

To prove R.

Negated Clause is $\neg R$. ← C4



∴ $\neg R$ is False

$\Longrightarrow$ R

Use of Resolution for inferencing in (Predicate Logic)

Example

Consider the following sentences

(a) All men are mortal

(b) John is a man

Convert the sentences into wff	Clauses for wff
1. $\forall x : man(x) \rightarrow mortal(x)$	C1 : $\neg man(x) \vee mortal(x)$
2. $man(John)$.	C2 : $man(John)$

Question : Is John mortal ?

goal : $mortal(John)$

Negated goal : $\neg mortal(John)$ C3

C3 C1

John/x

$\neg man(John)$ C2

$\square$ Contradiction

$\Rightarrow \neg mortal(John)$ is False

$\therefore$ $\underline{mortal(John)}$