Load Balancing

Problem Input: m identical machines, n jobs with ith job having processing time ti

Goal: Schedule jobs to computers such that
- Jobs run contiguously on a machine
- A machine processes only one job a time
- Makespan or maximum load on any machine is minimized

Definition Let A(i) be the set of jobs assigned to machine i.
The load on i is Ti = Summation of time of every job executed on i.
The makespan of A is T = maxi Ti

Load Balancing is NP-Complete.

1. Greedy Algorithm

- Consider the jobs in some fixed order
- Assign job j to the machine with lowest load so far

Consider 6 jobs whose processing times is given as follows
Jobs 1 2 3 4 5 6
ti 2 3 4 6 2 2

1 job 1(2) - - job 4(6) - - - - - - : 8
2 job 2(3) - - - job 5(2) - - : 5
3 job 3(4) - - - - job 6 (2) - - : 6

The loads are: T1 = 8, T2 = 5, and T3 = 6.
So makespan of schedule is 8

Is the greedy algorithm optimal?

Modified Greedy Sort the jobs in descending order of processing time, and process jobs using greedy algorithm

Consider 6 jobs whose processing times is given as follows

Jobs	1	2	3	4	5	6
ti	2	3	4	6	2	2

Jobs	4	3	2	1	5	6
time	6	4	3	2	2	2

Machine 1 J4 - - - - - - : 6
Machine 2 J3 - - - - J2 - - : 6
Machine 3 J2 - - - J1 - - J3 - - : 7

Makespan : 7

1	2	3	4	5
5	4	3	3	3

Machine 1 J1-----J4 : 8
Machine 2 J2-----J3-------J5 : 10

1	2	3	4	5	6	7
7	7	6	6	5	5	5

Machine 1 J1----J5----J7 :17
Machine 2 J2----J6 :12
Machine 3 J3----J4 :12

Is the modified greedy algorithm always optimal?

Greedy algorithm :

Jobs	1	2	3	4	5	6	7	8	9
Time	5	8	5	7	7	1	5	8	1

Machine 1 J1 - - - - - J4 - - - - - - - J8 - - - - - - - - : 20
Machine 2 J2 - - - - - - - - J6 – J7 - - - - - : 14
Machine 3 J3 - - - - - J5 - - - - - - - J9 - : 13

Makespan : 20

Modified greedy algorithm :

Jobs	2	8	4	5	1	3	7	6	9
Time	8	8	7	7	5	5	5	1	1

Machine 1 J2 - - - - - - - - J1 - - - - - J7 - - - - - : 18
Machine 2 J8 - - - - - - - - J3 - - - - - J6 - : 14
Machine 3 J4 - - - - - - - J5 - - - - - - - J9 - : 15

Makespan : 18

Machine 1	5	5	7		: 17
Machine 2	8	7	1	1	: 17
Machine 3	8	5			: 13

Machine 1	8	7	1 :	16
Machine 2	8	7	1 :	16
Machine 3	5	5	5 :	15

Greedy algorithm :

Jobs	1	2	3	4	5	6	7	8	9
Time	6	9	6	9	9	2	6	9	2

6	9	9	24
9	2	6	17
6	9	2	17

9	9	9	9	6	6	6	2	2

9	9	2	20
9	6	6	21
9	6	2	17

9	9	9	9	6	6	6	2	2

9	9	2	20
6	6	6	18
9	9	2	20

Machine 1	J1 - - - - - J4 - - - - - - - J8 - - - - - - - - : 20	
Machine 2	J2 - - - - - - - - J6 – J7 - - - - -	: 14
Machine 3	J3 - - - - - J3 - - - - - - - J9 -	: 13

Makespan : 20