Integral Image-Based Filtering for Computer Vision

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Technical Approach

Integral Image Computation

For the given input image $I(x,y) \in [0,255]$, we compute its integral, i.e. summed-area table $S(x,y)$ as per the following rule:

$$S(x,y) = I(x,y) + S(x-1,y) + S(x,y-1) - S(x-1,y-1) \tag{1}$$

where $S(0,y) = S(x,0) = 0$ are the initial conditions. This can be achieved pretty fast using vectorisation through NumPy arrays using the `cumsum` method.

The central advantage of this approach is that after this, we can calculate any rectangular sum in **O(1) time complexity**. For example, to find the sum of all the numbers in a rectangular region defined by (x_1,y_1)-(x_2,y_2):

$$\sum_{\substack{x_1 \le x \le x_2 \\ y_1 \le y \le y_2}} I(x,y) = S(x_2,y_2) - S(x_1-1,y_2) - S(x_2,y_1-1) + S(x_1-1,y_1-1) \tag{2}$$

Filtering Algorithm

1. Convert the given image, as shown in Figure 1 to grayscale (0-255) pixels. This is done to make integral computations efficient.
2. Compute integral image in $\mathbf{O(}mn\mathbf{)}$ **time** for $m \times n$ image. This pre-computation is expensive computationally, but makes subsequent queries O(1).
3. Apply predefined 4x4 filters using the obtained integral image.
4. Carry out Min-max normalization: $\hat{I}(x,y) = 255 \times \frac{I(x,y)-I_{min}}{I_{max}-I_{min}}$
5. The $\hat{I}(x,y)$ obtained at the end of the procedure is the required filtered image. Find this for all the six filters, and store them.

Figure 1: Unfiltered Image (iitk.png)

Filters used and Results

We used 6 4×4 kernels for pattern detection in the unfiltered image. Those filters are as mentioned below:

$$F_1 = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \quad F_2 = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad F_3 = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \quad F_5 = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \quad F_6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

On applying these filters, we were able to detect the occurrence of patterns contained in the filter in the original unfiltered image. The detailed filtered images can be seen in the runtime of the Python Notebook, which has been attached with this writeup.

Overall, this shows how **Haar filters** can be used in edge/pattern detection in a computationally convenient manner, using the idea of Integral Images.

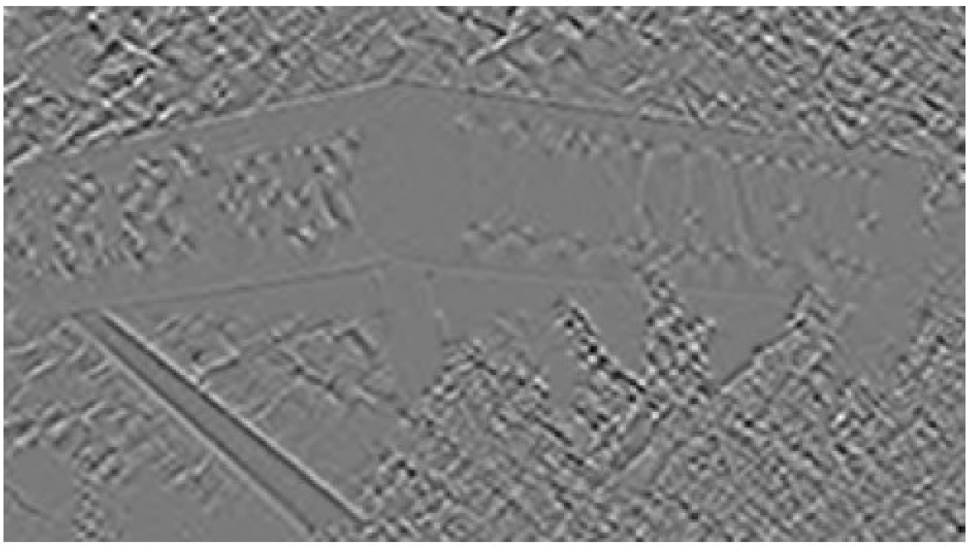
```python
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt

# Load the image and convert it to a numpy array
def load_image(image_path):
    img = Image.open(image_path).convert('L')
    return np.array(img)

# Compute integral image using NumPy cumulative sum
def compute_integral_numpy(image_array):
    return image_array.astype(np.int64).cumsum(axis=0).cumsum(axis=1)

# Compute sum over a rectangular region using the integral image
def get_sum(integral_image, x1, y1, x2, y2):
    A = integral_image[y1-1, x1-1] if x1 > 0 and y1 > 0 else 0
    B = integral_image[y1-1, x2] if y1 > 0 else 0
    C = integral_image[y2, x1-1] if x1 > 0 else 0
    D = integral_image[y2, x2]
    return D - B - C + A

# Apply a filter using the integral image
def apply_filter(integral_image, filter_def, filter_size=4):
    H, W = integral_image.shape
    output = np.zeros((H - filter_size, W - filter_size))

    for y in range(H - filter_size):
        for x in range(W - filter_size):
            total_sum = 0
            for dy in range(filter_size):
                for dx in range(filter_size):
                    weight = filter_def[dy, dx]
                    region_sum = get_sum(integral_image, x + dx, y +
dy, x + dx + 1, y + dy + 1)
                    total_sum += weight * region_sum
            output[y, x] = total_sum
    return output

# Normalize array to grayscale (0-255)
def normalize_to_grayscale(array):
    if array.size == 0:
        return array
    array_min, array_max = array.min(), array.max()
    if array_max == array_min:
        return np.full_like(array, 128, dtype=np.uint8)  # Handle
uniform images
    normalized = (array - array_min) * (255.0 / (array_max -
array_min))
    return normalized.astype(np.uint8)
```

```python
# Display grayscale image
def display_grayscale_image(image, path, title="Filtered Image"):
    plt.imshow(image, cmap='gray', vmin=0, vmax=255)
    plt.title(title)
    plt.axis('off')
    plt.savefig(path)
    plt.show()

if __name__ == "__main__":
    image_path = "iitk.png"

    # Load and process image
    pixel_array = load_image(image_path)
    integral_image = compute_integral_numpy(pixel_array)

    print("Original Image Shape:", pixel_array.shape)
    print("Top-left 5x5 pixels:\n", pixel_array[:5, :5], "\n")
    print("Top-left 5x5 pixels of Integral Image:\n",
integral_image[:5, :5], "\n")

    # Define filters
    filters = [
        np.array([[-1, -1,  1,  1], [-1, -1,  1,  1], [ 1,  1, -1, -
1], [ 1,  1, -1, -1]]),
        np.array([[-1, -1, -1, -1], [-1, -1, -1, -1], [ 1,  1,  1,
1], [ 1,  1,  1,  1]]),
        np.array([[-1, -1,  1,  1], [-1, -1,  1,  1], [-1, -1,  1,
1], [-1, -1,  1,  1]]),
        np.array([[ 1,  1, -1, -1], [ 1,  1, -1, -1], [-1, -1,  1,
1], [-1, -1,  1,  1]]),
        np.array([[-1, -1, -1, -1], [-1, -1, -1, -1], [-1, -1,  1,
1], [-1, -1,  1,  1]]),
        np.array([[ 1,  1,  1,  1], [ 1, -1, -1, -1], [ 1, -1, -1, -
1], [ 1, -1, -1, -1]])
    ]

    # Apply filters and display results
    for i, f in enumerate(filters):
        filtered_output = apply_filter(integral_image, f)
        normalized_output = normalize_to_grayscale(filtered_output)
        display_grayscale_image(normalized_output,
f"filtered_{i}.png", title=f"Filtered Image {i+1}")
```

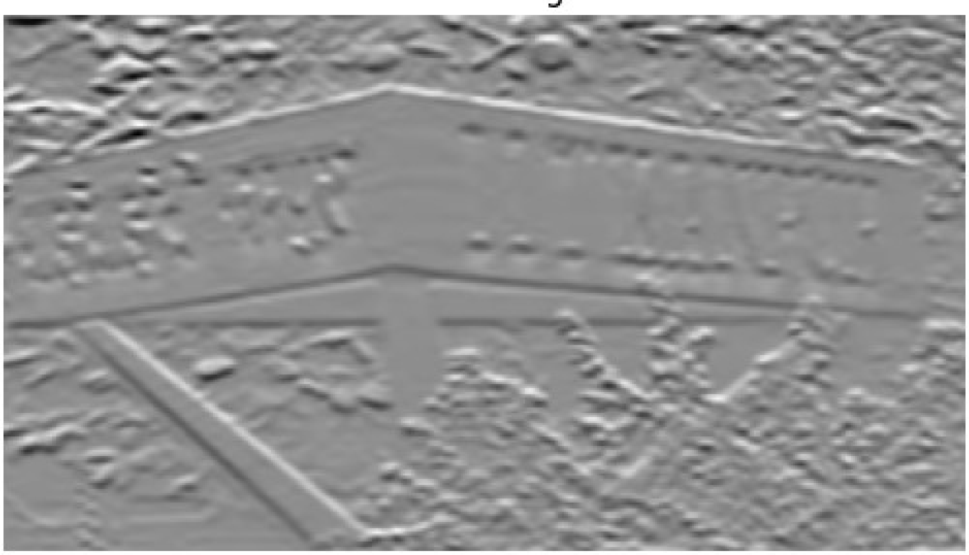
```
Original Image Shape: (168, 299)
Top-left 5x5 pixels:
 [[ 56   8  66  51  91]
 [ 36  33 167 221 224]
 [ 22  50 152 176 161]
 [ 25  14  59 143 196]
 [ 58  62  42  52  44]]
```

```
Top-left 5x5 pixels of Integral Image:
 [[  56   64  130  181  272]
 [  92  133  366  638  953]
 [ 114  205  590 1038 1514]
 [ 139  244  688 1279 1951]
 [ 197  364  850 1493 2209]]
```

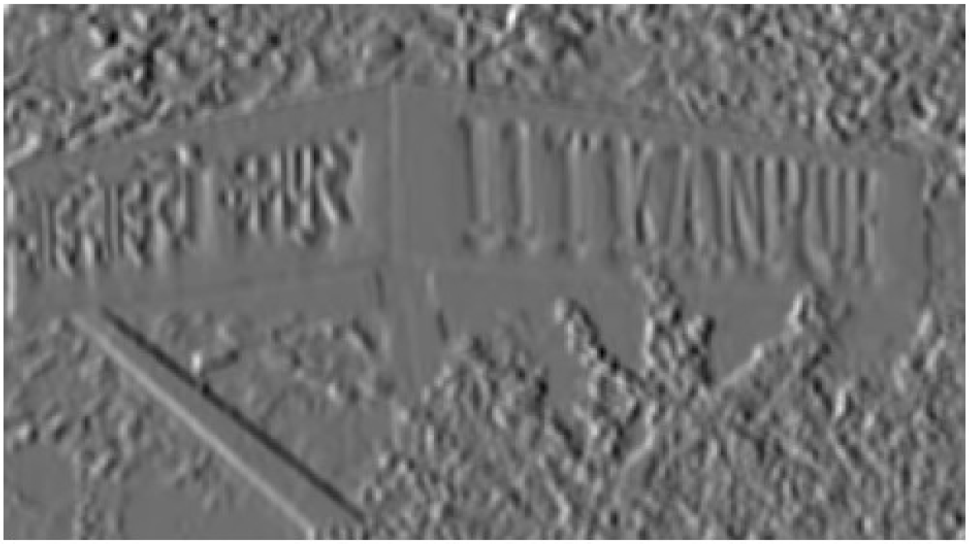
Filtered Image 1



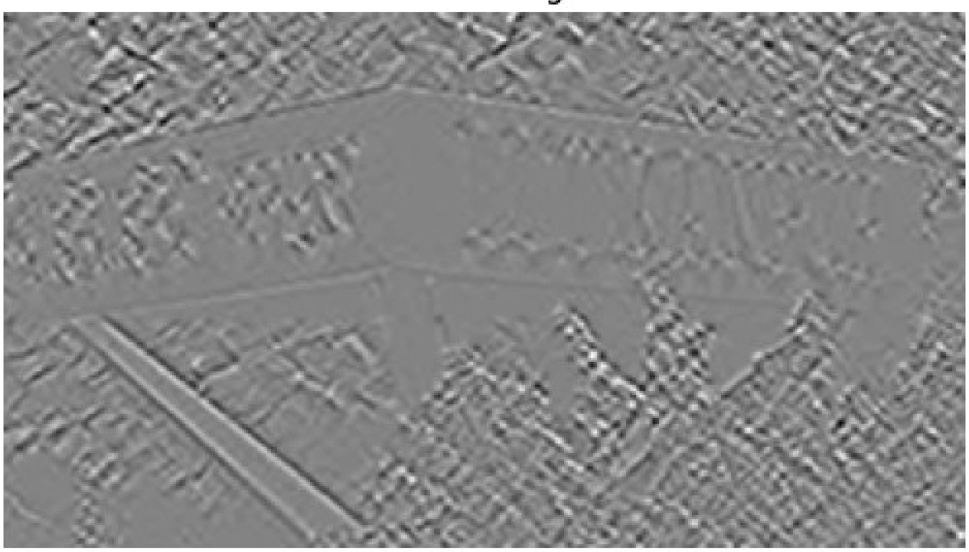
Filtered Image 2



Filtered Image 3



Filtered Image 4



Filtered Image 5

Filtered Image 6