DEA Game Cross Efficiency Approach to Portfolio Selection

SOFIENE OMRI

UNDER THE SUPERVISION OF
DR. HEDI ESSID

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF RESEARCH MASTER IN MODELING.

Acknowledgement

First, I would like to express my gratitude particularly to my advisor Professor Hedi Essid for his wide knowledge and his helpful guidance throughout the development of this master thesis. I am deeply delighted and impressed by his generosity, his kindness and understanding. In addition, I am more than grateful for the help and support of my family and friends especially Meriem Matallaoui.

Finally, I wish to extend our appreciation to all of my colleagues, all the faculty and staff of the Higher Institute of Management of Tunis.

Omri Sofiene

Abstract:

In this work, we provide a new approach to portfolio selection based on game cross efficiency model. While game cross efficiency is developed as a remedy for weight multiplicity in the original cross efficiency and peer evaluation, we improve its use as a tool for portfolio selection. In fact, in our analysis we view every financial asset as a player competing for investment funds through boosting his ranking compared to his opponents. Game cross efficiency allows us to model the portfolio selection as game through considering efficiency scores as payoffs and weight selection as strategy. In this analysis, we use Data Envelopment Analysis (DEA) in a multicriteria context and employ the algorithm developed by Wu et al. (2008) to solve the Nash equilibrium rating score, which is a more reliable result for the decision maker. Furthermore, we improve on the use of cross efficiency evaluation through the development of a mean cross linear model, in which we seek to maximize the overall efficiency score of a portfolio subject to a tradeoff level with portfolio return. Finally, in addition to (average) game cross-efficiency scores, we will examine the relevant DEA literature. The main advantage accomplished by our approach is a better risk adjusted portfolio compared to benchmark index in the Euronext including; CAC40, AEX, BEL20 and PSI20 over the test period starting 2010 to 2015. We apply the proposed approach to stock portfolio selection in the Paris stock Exchange, and demonstrate that our approach can be a promising tool for stock portfolio selection by showing that the resulting portfolio yields higher risk-adjusted returns than other benchmark portfolios for a 6-year sample period from 2010 to 2015.

Key Words:

DEA, Efficiency, Cross efficiency, Game theory, Game cross efficiency, Portfolio management

Résumé :

Dans ce travail, nous proposons une nouvelle approche servant à la sélection de portefeuille basée sur le modèle « d'efficience croisée-DEA-du jeu » dans lequel chaque actif financier est perçu comme un joueur qui cherche à accroître sa propre efficience. En effet, la méthode d'enveloppement de données (DEA) repose sur l'idée que chaque « Decision Making Units » (DMU) a la possibilité de choisir subjectivement son système d'évaluation en retenant l'ensemble de pondérations, qui sont, de son point de vue, les plus favorables relativement au reste des DMUs. Bien que, l'efficience croisée se repose sur l'idée qu'une DMU doit être « pair-évaluée » au lieu d'être « auto-évaluée », cette extension ne donne pas une solution unique. En outre, en examinant la situation sous l'angle de la théorie des jeux non coopérative et en raison du fait que seuls quelques actifs financiers seront sélectionnés, nous pouvons considérer ces actifs financiers comme des acteurs en concurrence. Nous obtenons ainsi des scores d'efficience croisée de jeu qui constituent un point d'équilibre de Nash et par ailleurs des résultats plus fiables. Dans cette analyse, nous utilisons DEA dans un contexte multicritère et nous utilisons l'algorithme développé par Wu et al. (2008) pour déterminer la sélection optimale du portefeuille. En addition, nous améliorons l'utilisation de l'évaluation de l'efficacité croisée grâce au développement d'un modèle d'efficience-croisée-moyen dans lequel nous cherchons à maximiser le score d'efficience global d'un portefeuille soumis à un niveau de compromis avec le rendement du portefeuille. Le principal avantage atteint par notre approche est un meilleur portefeuille ajusté au risque par rapport à des indices de référence de l'Euronext ; CAC40, AEX, BEL20 et PSI20 au cours de la période d'essai commençant de 2010 à 2015. Nous appliquons l'approche proposée pour la sélection de portefeuille d'actions à la bourse de Paris et démontrons que notre approche peut être un outil robuste pour la résolution de construction de portefeuille

Mots-clés :

DEA, Sélection de portefeuille, théorie de jeu, efficacité croisée

Table of Contents

List of Tables:

List of Figures:

Chapter 1 Introduction and Structure

1. Introduction

16th September 2008, the world witnessed an earthquake shaking the American financial system as the journal of wall street wrote:

"Lehman Files for Bankruptcy, Merrill Sold, AIG Seeks Cash"[1]

In spite of the reasons behind the fall of this firms, portfolio selection present one of the most complex and dynamic problems. The 2008 crisis exemplify that even top highly qualified money manager can run into difficulties and make poor decisions. Portfolio selection is the decision whereby the best set of stocks, or financial assets, is selected from many different alternatives. Even though various tools, and sophisticated techniques were developed by different companies to handle these investments problems, uncertainties still govern the process and these can be undetected by even the very qualified experts. Indeed, in many cases the stakes are high because selecting the right assets is a significant resource allocation decision that can lead to high profits and in the worst case to a huge loss.

Since the original work of Markowitz, (1952), several models of portfolio selection have been suggested. According to different academia, the larger part of these models tried to improve the estimation procedure of the variance-covariance matrix thus easing the computational complexity. Markowitz modern portfolio theory, which is based on balancing the expected return and the return's variance of the portfolio, have and still the main building block for stock portfolio investment strategies. In fact, the main problem is finding the stock portfolio which will achieve the highest possible return for a given level of risk. As a possible solution of that problem, in this thesis we examine the investment strategy based on game cross efficiency which is an extension of DEA. In this context, DEA will be used as a Multi Criteria Decision Making (MCDM) tool.

[1] The Wall Street Journal, Sep 16, 2008

DEA is a linear programming technique developed by Charnes, Cooper, and Rhodes (1978) to measure the relative efficiencies of a set of decisions making units (DMUs) with multiple inputs and outputs. As an efficiency analysis tool, DEA became quickly a popular area in operations research and economics. Many studies have been published dealing with applying DEA in real world including; multiple criteria decision analysis, portfolio selection, politics, banks and organization. DEA is highly useful in the service industry as it doesn't only recognize inefficient units but also identifies a set of efficient units that can be used as benchmarks for efficiency improvement. However, a common problem in DEA is the fact of having more than 100% efficient DMU which impacts highly the discrimination power of this method. This issue is mainly due to the flexibility offered by this method, which allows the DMUs to choose optimal weights on their input and output factors. Indeed, each DMU can chose very elevated weights on some factors and very low on others to achieve optimality. As a remedy to these issues, Doyle and Green (1994) developed an extension entitled Cross-efficiency. This method provides a unique ordering of DMUs and eliminates impractical weighting. The main idea of this technique is the use of peer evaluation in addition to the self-evaluation. The weights determined by the simple DEA of each DMU is used to determine that of its peer. Cross efficiency was successfully applied to multiple real-world problems but it still suffers from a major shortcoming; the non-uniqueness of the DEA optimal weights which might reduce its usefulness. This problem was further detailed in the paper by Despotisa et al. (2002). Several models have been suggested as a remedy to the cross-efficiency issue. Javier Alcaraz et al. (2013) proposed taking all possible ranking scenarios of the DMUs based on all the possible choices of optimal weights of the DMU, thus generating a ranking range for each DMU based on the best and worst ranking. One of the most known remedies that have been suggested is to introduce a secondary goal to resolve the ambiguity of which alternative solution to the LP program to use. The well-known models using multiple objective LP are the aggressive and benevolent formulations (Sexton et al. (1986); Doyle and Green (1994)). The aggressive model maximizes the efficiency score of an arbitrary DMU as a primary goal then attempts to minimize the efficiency of the (n-1) other DMUs while the benevolent maximizes the latter. Other models were developed, some dealing with imprecise data (Despotisa et al. (2002)) and some as extensions to the Sexton et al. (1986) and Doyle (1994) formulations, one of which involved the use of game theory by (Wu et al. (2008) and Wu et al. (2009)).

Wu et al. (2008) adopted a non-cooperative game approach in which DMUs are competing among each other suggesting that the different agents (DMUs) are only interested in conflict. However, this is not the case. In game cross efficiency, an agent or a player is assigned a cross efficiency score that he will attempt to maximize but cannot let the score assigned to his peer deteriorate. Back to Green et al. (1994), the statement “It is not enough to talk yourself up; you must talk the others down too, but without being inconsistent.” holds clearly in such a situation. Adopting a non-cooperative approach, cross efficiency scores as payoffs, and each DMU may choose to take a non-cooperative game stance to the extent that it will attempt to maximize its (worst possible) payoff. The main strategies played by each DMU would be weights selection. Given that a player is assigned the usual score from the conventional cross efficiency outcome at the start, the averaged column value in the cross-efficiency matrix. Receiving a score lower than his initial score would cause him dissatisfaction thus creating an incentive to alternate his weight selection strategies. In the algorithm developed by Wu et al. (2008), they constrained the assigned algorithm score to be equal or above the initial score to result in the algorithm converging to a Nash equilibrium.

2. Scope of the Dissertation

The goal of this research is to illustrate the impact of the current advancements in efficiency modeling, more specifically DEA, towards developments in finance and investments science. In order to achieve our goals, this dissertation will analyze three DEA models (CCR, Arbitrary, Game Cross Efficiency) applied to portfolio analysis with the following primary objectives:

- Provide a detailed analysis of the data envelopment analysis, the Cross-efficiency concept and the relative mathematical models.
- Point out the drawbacks and the limits of the different DEA models relevant to portfolio management context.
- A critical comparison of the performance of game cross versus the arbitrary or simple cross.
- Provide a mathematical model to remedy some of the discussed issues

3. Thesis structure

The remaining part of this master dissertation contains the following chapters:

Chapter 2 Data Envelopment Analysis and Cross Efficiency: This chapter aims at introducing and defining some key concepts related to data envelopment analysis. We begin by giving some insights to the history and advancement in DEA and its mathematical models. The second part of the chapter is devoted to DEA extensions; Cross efficiency models. We focus on the most popular models developed through literature.

Chapter 3 Game Cross Efficiency: This chapter aims at introducing key concept of game theory and its application in data envelopment analysis. We begin by giving some insights of game theory and its theoretical approach to societal problems. Second, we review the application of DEA in game theory and review the relevant mathematical model.

Chapter 4 Empirical Analysis: DEA Cross Efficiency applied to Portfolio Management: This chapter endeavors to provide the research methodology. Furthermore, in this chapter we carry a performance analysis of the applied models. Finally, we develop a model to remedy some issues in the context of portfolio selection.

Chapter 5 Conclusion: This chapter includes some concluding remarks, some late applications of game cross efficiency and future research subjects.

Chapter 2 Data Envelopment Analysis and Cross Efficiency

1. Introduction

DEA is a non-parametric linear programming method for measuring DMUs performance through identifying the production frontier (envelopment surface). This method was first developed by Charnes et al. (1978) as a response to a study of performance of education program in the United States. DEA measures and compares each DMU efficiency score to the best practice frontier, which is the convex combination of a set of highly efficient benchmark DMUs. Indeed, any deviation from the constructed frontier would indicate technical inefficiency. DEA became quickly a popular area in operations research and economics.

One evolving research area would be multi criteria decision making, where DEA has become a key component that provides key insights to various solution approach (Stewart, 1996). In fact, a multi criteria problem can be casted in a "Multicriteria-DEA" framework through taking the to be minimized attributes as inputs and the to be maximized ones as outputs/benefits (Doyle, 1993). In more details, the DEA framework can be expressed as a multi attribute method given that the set of inputs/outputs represents the set of undesirable/desirable attributes, where an attribute is a feature of each alternative to which a numerical value can be assigned (Stewart, 1996). Project selection, supplier selection, and Olympic games are some of the applications in which DEA was used as MCDM tool.

Even though DEA can provide a remedy to the issue of aggregating multi performance measure into a key indicator, DEA suffers from high flexibility, thus allowing for weak discrimination among DMUs. The cross-efficiency method, developed as an extension to DEA, provides a unique ordering of DMUs and eliminates impractical weight structure through peer evaluation. The aggressive and benevolent formulations (Sexton et al. (1986); Doyle and Green (1994)) are one of the most popular models using second goal programming.

This chapter aims at introducing and defining some key concepts related to data envelopment analysis. We begin by giving some insights into the history and advancement of data envelopment Analysis and its mathematical models. The second part of the chapter is devoted to cross efficiency models. We focus on the most popular models developed through literature. Finally, we conclude by critical analysis of the models.

2. The basic formulations of the Data Envelopment Analysis (DEA)

The original motivation for DEA was evaluating educational programs for disadvantaged students in USA. Afterwards it has become an effective tool to compare the efficiency of similar organizations, referred to as $\mathrm{DMU_s}$. The problem is formulated as a linear programming (LP) which is solved for each DMU under evaluation. The formulation of the basic model following that of Farrell is divided into two main models; the first is input oriented and the second output oriented. In fact, under the assumption of constant return to scale (CRS) the above models are equivalent. In our analysis, we will use the input oriented one:

To proceed with the developed method (DEA), suppose we have a set of n DMUs where a $\mathrm{DMU_j}$ $(j = 1,2,3 \ldots n)$ utilizes a set of m inputs x_{ij} $(i = 1,2,3 \ldots m)$ to produce **s** y_{rj} $(r = 1,2,3 \ldots s)$ output where $x_{ij}, y_{rj} \geq 0$.

Let's denote μ_{rj} and ω_{ij} the weights assigned respectively to the output Y_{rj} and the input X_{ij}. Then, the Charner, Cooper and Rhodes (CCR) model is formulated as follow for $\mathrm{DMU_d}$, the below model is also known as the fractional model.

Input Oriented Model	Output Oriented Model
$\text{Max } Z = \frac{\sum_r^s \tilde{\mu}_r y_{rd}}{\sum_i^m \tilde{\omega}_i x_{id}}$ Subject to (2.1.a) $\frac{\sum_r^s \tilde{\mu}_r y_{rd}}{\sum_i^m \tilde{\omega}_i x_{id}} \leq 1$ $\omega_i, \mu_r \geq 0$	$\text{Min } Z = \frac{\sum_i^m \tilde{\omega}_i x_{id}}{\sum_r^s \tilde{\mu}_r y_{rd}}$ Subject to (2.1.b) $\frac{\sum_i^m \tilde{\omega}_i x_{id}}{\sum_r^s \tilde{\mu}_r y_{rd}} \geq 1$ $\omega_i, \mu_r \geq 0$

The ratio $\frac{\sum_r^s \tilde{\mu}_r y_{rd}}{\sum_i^m \tilde{\omega}_i x_{id}}$ actually represents a weighted sum of the DMU outputs to its weighted sum of inputs. This is actually very familiar to the technical efficiency also known as the "engineering ratio" which is "the ratio of the useful work performed by a machine or in a process to the total energy expended or heat taken in.” Furthermore, the optimization program (2.1.a) is a nonlinear program (NLP). In fact, it is a fractional non-convex program, thus it may generate an infinity of solutions.

As a remedy to the above issue, a rigorous mathematical development by Cooper et al. (1962) resulted in a simpler LP through the following transformation:

- $\mu_r = s\tilde{\mu}_r$
- $\omega_i = s\tilde{\omega}_i$
- where $s^{-1} = \sum_i^m \omega_i x_{id}$

Then, the CCR model is formulated as follow for DMU_d

Input Oriented	Output Oriented
$\text{Max } Z = \sum_r^s \mu_r y_{rd}$ Subject to (2.2.a) $\sum_r^s \mu_r y_{rd} - \sum_i^m \omega_i x_{id} \leq 0$ $\sum_i^m \omega_i x_{id} = 1$ $\omega_i, \mu_r \geq 0$	$\text{Min } Z' = \sum_r^m \omega_i x_{id}$ Subject to (2.2.b) $\sum_i^m \omega_i x_{id} - \sum_r^s \mu_r y_{rd} \leq 0$ $\sum_r^s \mu_r y_{rd} = 1$ $\omega_i, \mu_r \geq 0$

Illustration: A numerical example

Let’s consider 4 DMUs operating within the same sector using 2 inputs to produce 2 outputs as shown below:

	X1	X2	Y1	Y2
DMU1	7	7	4	4
DMU2	5	9	7	7
DMU3	4	6	5	7
DMU4	5	9	6	2

Table 2-1: Numerical Example

Using LPS (An LP software) we will determine the efficiency of each of them using the input oriented model.

	Efficiency	Ranking
DMU1	0.68571	3
DMU2	1	1
DMU3	1	1
DMU4	0.85714	2

Table 2-2: CCR-Efficiency and ranking

We can note from the efficiency computation that both DMU_2 and DMU_3 are 100% efficient so **which one is actually DEA efficient (100%-efficient)?**

In fact, this is a common problem in DEA. We may have more than 100% efficient DMU thus ranking DMUs can be quite hard. Furthermore, letting each DMU choose his own set of weights will actually lead to impractical weight structure, as the DMUs under evaluation heavily weights few favorable inputs/outputs and completely ignoring the others to maximize its own performance score. In order to solve such issues, some extensions of DEA were developed, mainly cross efficiency and its derivations.

3. DEA Extensions

a) Cross Efficiency

The cross-efficiency, which was developed as an extension to DEA, provides a unique ordering of DMUs and eliminates impractical weight structure. Cross efficiency was first developed by Doyle and green (1994) through which they incorporated the idea of Peer-evaluation. In other words, DMUs are treated as agents interacting and evaluating each other. Once a DMU self-evaluation is determined and the weights have been chosen. That set of weights is used to weight the input and output for the other (n-1) DMUs. Finally, to derive cross efficiency, we begin by solving the model (2.2.a or 2.2.b) then we use the set of weights obtained from self-appraisal to determine the efficiency of (n-1) left DMUs using the below formula.

Assuming a DMU_d is evaluating a peer DMU_j ,then the cross efficiency for DMU_j would be as follow:

$$E_{dj} = \frac{\sum_r^s \mu_r^d Y_{rj}}{\sum_i^m \omega_i^d X_{ij}} \qquad (2.3.a)$$

And the average cross efficiency for DMU_j would be the average of all E_{dj} $(d = 1 \ldots\ldots n)$:

$$\overleftarrow{E_j} = \frac{1}{n}\sum_d^n E_{dj} \qquad (2.3.b)$$

Solving for cross efficiency, we obtain the below matrix where cross efficiency is obtained by averaging the column cross efficiency values or using the median. Here we proceed using the mean.

	DMU_1	DMU_2	DMU_3	AVG (appraisal of peers)
DMU_1	E_{11}	E_{12}	E_{13}	A_1
DMU_2	E_{21}	E_{22}	E_{23}	A_2
DMU_3	E_{31}	E_{32}	E_{33}	A_3
AVG (Peer-appraisal)	$\overleftarrow{E_1}$	$\overleftarrow{E_2}$	$\overleftarrow{E_3}$	

Table 2-3: Cross Efficiency Matrix (CEM)

As we move along the row A of the above matrix (CEM), each entry E_{dj} is the efficiency that DMU_d accords to DMU_j. In addition, we can note that the leading diagonal is actually a special case of cross efficiency where DMU_d rates itself (self-appraisal). Averaging down column E, we get peer-appraisal. In fact, while averaging, we may neglect the diagonal values, thus not allowing DMU_d to appraise itself as in formula (2.3. c).

$$\overleftarrow{E_j} = \frac{1}{n-1} \sum_{j \neq d}^{n} E_{dj} \qquad (2.3.c)$$

Even though cross efficiency increases the DEA discrimination power, it still has a major shortcoming. Potential issues with cross efficiency may result from the non-uniqueness of the weights obtained from solving the CCR model (2.2.a). As a result, the cross-efficiency score E_{dj} can be arbitrarily generated, depending on the optimal solution generated by the software used (Despotisa 2002).

Illustration: a numerical example

In order to illustrate this issue, we will be using the numerical example by (Wu et al. (2008)), and solve for cross efficiency using LINGO and LPS and compare the results.

Let's consider 5 DMUs operating within the same sector using 3 inputs to produce 2 outputs as shown in the next table:

	X1	X2	X3	Y1	Y2
DMU1	7	7	7	4	4
DMU2	5	9	7	7	7
DMU3	4	6	5	5	7
DMU4	5	9	8	6	2
DMU5	6	8	5	3	6

Table 2-4: Numerical example by Wu et al. (2008)

Using LPS, we have generated the below cross efficiency matrix and relevant weight set for each DMU:

	ω_1	ω_2	ω_3	μ_1	μ_2	CCR-EFF
DMU1	0	0.1429	0	0.1714	0	0.6857
DMU2	0.0714	0.0714	0	0.1429	0	1
DMU3	0.25	0	0	0	0.1429	1
DMU4	0.0714	0.0714	0	0.1429	0	0.8571
DMU5	0	0	0.2	0	0.1429	0.8571

Table 2-5: LPS output for optimal weight set

	DMU1	DMU2	DMU3	DMU4	DMU5
DMU1	0.68571	0.93333	1	0.8	0.45
DMU2	0.57143	1	1	0.85714	0.42857
DMU3	0.32653	0.8	1	0.22857	0.57143
DMU4	0.57143	1	1	0.85714	0.42857
DMU5	0.40816	0.71429	1	0.17857	0.85714
$\overleftarrow{E_J}$	0.512652	0.889524	1	0.584284	0.547142

Table 2-6: LPS output for cross efficiency computation (CEM)

Using excel solver;

	ω_1	ω_2	ω_3	μ_1	μ_2	CCR-EFF
DMU1	0	0.1429	0	0.1714	0	0.6857
DMU2	0.0714	0.0714	0	0.1429	0	1
DMU3	0	0.1666	0	0	0.1428	1
DMU4	0.0714	0.0714	0	0.1429	0	0.8571
DMU5	0	0	0.2	0	0.1429	0.8571

Table 2-7: Excel solver output for optimal weight set

	DMU1	DMU2	DMU3	DMU4	DMU5
DMU1	0.68571	0.93333	1	0.8	0.45
DMU2	0.57143	1	1	0.85714	0.42857
DMU3	0.4898	0.66667	1	0.19048	0.64286
DMU4	0.57143	1	1	0.85714	0.42857
DMU5	0.40816	0.71429	1	0.17857	0.85714
$\overleftarrow{E_j}$	0.54531	0.86286	1	0.57667	0.56143

Table 2-8: Excel solver output for cross efficiency computation

Here we can note that the cross-efficiency scores are different for both softwares, precisely for DMU3. Table (2-5) and Table (2-7) indicate multiple weights generated by the CCR model. This issue is further cleared when using SAS as shown in table (2-9). Such issues do not affect the objective function values. As you may see, both diagonals are equal and so is the cross-efficiency score for each software output. This confirms the issue of selection, so **which set of weights is best suited for cross efficiency computation?**

Using SAS

	ω_1	ω_2	ω_3	μ_1	μ_2	CCR-EFF
DMU1	0	0.1429	0	0.1714	0	0.6857
DMU2	0.2	0	0	0.1429	0	1
DMU3	0	0	0.2	0	0.1428	1
DMU4	0.2	0	0	0.1429	0	0.8571
DMU5	0	0	0.2	0	0.1429	0.8571

Table 2-9: SAS output for optimal weight set

In Order to solve this issue, second goal programing was introduced. Well-known models developed by (Sexton et al. (1986)) and modified by (Doyle and Green (1994)) are *the benevolent* and *the aggressive* formulations. Depending on the choice of the weight set from the CCR, we may either maximize the efficiency of the left n-1 DMUs, known as the *benevolent model* (2.4), or minimize the latter, known as the *aggressive model* (2.5). In order to implement the above model, we start by solving a first objective of maximizing the efficiency of a $\mathrm{DMU_d}$ followed by a second objective depending on $\mathrm{DMU_d}$ behavior either peaceful or aggressive. The second objective can be either to maximize (benevolent) or minimize (aggressive) the efficiency of the left DMUs.

b) The benevolent Model "talk yourself up and the others too, be gentle"

The first goal is to solve for self-appraisal score through solving the CCR model (2.2.a). For the second goal, the efficiency of the left (n-1) DMUs is maximized while restricting the efficiency of DMU_d to the initial self-efficiency E_{dd}

$$\text{Max } Z_2 = \sum_{j \neq d} \sum_{r}^{s} \mu_r y_{rj}$$

$$\text{Subject to} \qquad (2.4)$$

$$\sum_{r}^{s} \mu_r y_{rj} - E_{dd} \sum_{i}^{m} \omega_i x_{ij} \leq 0$$

$$\sum_{i}^{m} \omega_i x_{ij} = 1$$

$$\omega_i, \mu_r \geq 0$$

Where, $E_{dd} = \frac{\sum_r^s \mu_r y_{rd}}{\sum_i^m \omega_i x_{id}}$ is a self-appraisal score of the DMU_d.

c) The Aggressive Model "Not enough to talk yourself up talk the other down"

Again, the first goal is the self-appraisal score of the CCR model (2.2.a). For the second goal, the efficiency of the left (n-1) DMUs is minimized while restricting the efficiency of DMU_d to the initial self-efficiency, E_{dd}

$$\text{Min } Z_2 = \sum_{j \neq d} \sum_{r}^{s} \mu_r y_{rj}$$

Subject to (2.5)

$$\sum_{r}^{s} \mu_r y_{rj} - E_{dd} \sum_{i}^{m} \omega_i x_{ij} \leq 0$$

$$\sum_{i}^{m} \omega_i x_{ij} = 1$$

$$\omega_i, \mu_r \geq 0$$

Where, $E_{dd} = \frac{\sum_r^s \mu_r y_{rd}}{\sum_i^m \omega_i x_{id}}$ is a self-appraisal score of the DMU_d.

d) Maverick Index

At the beginning, we pointed out the issue of impractical weight structure which was addressed by cross efficiency computation. In this section, we can go a step further identifying those DMUs using the maverick index by (Doyle and Green (1994)). This index is highly useful for the appraising party. The relevant decision maker, in appraising the different institutions under his direction, will be paying a very close attention to those who appear to be performing well but are not (maverick DMUs). More precisely, they are the units that enjoy greater relative increment when shifting from self-appraisal to peer appraisal. The index by Doyle is the following:

$$M_j = \frac{(E_{jj} - \overleftarrow{E_j})}{\overleftarrow{E_j}} \tag{2.6}$$

Where E_{jj} the self-appraisal score and $\overleftarrow{E_j}$ the relevant average cross efficiency

Numerical example

In order to illustrate this issue, we will be using the numerical example by (Wu et al 2008) same as in Table (2-4) and its cross-efficiency matrix below, then we will determine the maverick index for each DMU.

	DMU1	DMU2	DMU3	DMU4	DMU5	Arbitrary
DMU1	0.68571	0.95686	1	0.82017	0.49637	0.54531
DMU2	0.60493	1	1	0.85714	0.62087	0.86286
DMU3	0.68571	1	1	0.85714	0.85714	1
DMU4	0.68571	1	1	0.85714	0.50145	0.57667
DMU5	0.52859	0.92504	1	0.60008	0.85714	0.56143

Table 2-10: Cross efficiency matrix "Quantitative Models for Performance Evaluation and Benchmarking. By Zhu, Joe. (2009)"

	Arbitrary	Self-Appraisal	Rank	Maverick index
DMU5	0.56143	0.85714	4	0.526708583
DMU4	0.57667	0.85714	3	0.486361351
DMU1	0.54531	0.68571	5	0.257468229
DMU2	0.86286	1	2	0.15893656
DMU3	1	1	1	0

Table 2-11: Ranking and Maverick index

Sorting by the maverick index immediately draws attention to DMU5 which self-appraised held the same position as DMU4 (second position) but now dropped to the 4th position while DMU4 to the 3rd position when evaluated by peers. Here we can interpret that DMU5 used an impractical weight structure, in other words this DMU is only efficient at very few inputs and outputs. As a result, this DMU5 should be monitored very closely and the appraising party should identify potential issues within its operations.

4. Conclusion

The mathematical programming nature of DEA led to the existing of multiple weights and multi-100% efficient DMUs, thus we ended up with a uniqueness and selection problem. Furthermore, in real life, some inefficient DMUs are actually better in regard to overall performance than the efficient ones which is mainly due to the impractical weight structure. In order to solve such issue, second goal programing was introduced. However, the proposed solution does not take into account the behavioral nature of the analyzed organizations (DMUs) which can be either competitive or cooperative.

Chapter 3 Game Cross Efficiency

1. Introduction

DEA suffers from high flexibility, thus allowing for weak discrimination among DMUs. In fact, we may have more than 100% efficient DMU, thus ranking DMUs can be quite hard. In addition, DMUs can be performant only on some factors which make them mavericks (Doyle, 1994). The cross-efficiency method, developed as an extension to DEA, provides a unique ordering of DMUs and eliminates impractical weight structure through peer evaluation. Even though cross efficiency was applied successfully to multiple real-world problems, it still has some issues as the non-uniqueness of the DEA optimal weights which may reduce its usefulness, specifically the possibility of multiple optimal weights in the DEA model depending on the used optimization software (Despotisa, 2002).

Several models have been suggested as a remedy to the cross-efficiency issue. The most known remedy that has been suggested is to introduce a secondary goal to resolve the ambiguity of which alternative solution to the linear program (LP) to use. The well-known models using secondary goal are the aggressive and benevolent formulations (Sexton et al (1986); Doyle and Green (1994)).

Recently, a very interesting remedy was developed by Wu et al. (2008) in which they adopt a non-cooperative game approach to the peer evaluation method in DEA. In fact, DMUs are viewed as agents competing among each other. They viewed cross efficiency scores as payoffs, and that each DMU may choose to take a non-cooperative game stance to boost its efficiency scores given a weight selection strategy. Wu et al. (2008) developed an algorithm converging to Nash equilibrium. In this study, we seek to apply the latter to the portfolio selection problem.

This chapter aims at introducing key concept of game cross efficiency model and its algorithm through giving some insights of game theory and its theoretical approach to societal problems.

2. Game Theory: A theoretical approach to Data Envelopment Analysis

Game theory is the science concerned with decision making in strategic settings, where an organization (DMU) must encompass the rational of other players into its Decision-making process in order to make the best choices. The basic components of a game are the following:

- ✓ There is a set of participants/agents, whom we call the players.
- ✓ Each player has a set of possible strategies
- ✓ For each choice of strategies, each player receives a payoff that can depend on the strategies selected by everyone.

Our interest is reasoning about how players will behave in a given game. In many societal problems, we have a set of n organizations, each have k criteria for evaluating their capabilities where each criterion is expressed by a positive score. DEA is an evaluation technique in which each player has the right to choose a set of non-negative weights of the evaluation criteria(input/output) that are most preferable to the player as to amplify his own performance. In addition, organization can judge the competencies or abilities of their peer in a cross-efficiency sense, thus selecting weights to evaluate their peers with a certain objective depending on the situation. Here we may recognize two main situations, either competitive (degrade that of each competitors) or cooperative (collaborate with its peer for coalition formation). In this part, we will focus on the competitive situation. Further information regarding cooperation in DEA can be found in Ken Nakabayashi, Kaoru Tone (2006).

3. Potential competition among different DMUs

Competition is an essential part even within the same organization, this is the case also in many societal problems. Indeed, in a setting where resources are scarce then competition is clear among the different DMUs. For example, scholarship application submitted by different students in a university can be viewed as DMUs, and subject to a DEA analysis. These applications are clearly competing for available funds. R&D project submitted by departments within the same organization, research proposal by academia are all forms of direct competition that are subject to a DEA analysis. On the other hand, DEA analyses of public healthcare institution, universities and local municipalities are all examples of indirect competition. Specifically, the relevant ministry or government, in appraising the different institutions under its direction, will be paying a very close attention to the least performers with the possibility of shutting them down.

In this situation, we can view DMUs as competing agents and the scores for evaluating their performance as payoffs. Hence, all players are supposed to be selfish insisting on their own advantage on the scores, thus each of the DMUs will attempt to maximize its worst payoff without taking a full conflict stance by not letting its peer score deteriorate. The game cross efficiency can be defined as follows:

<table>
<tr><th>Players</th><th>Strategy set</th><th>Payoffs</th></tr>
<tr><td>A set of n DMUs (s=1…. n)</td><td>Weight selection {different virtual weight to be assigned to inputs/outputs}</td><td>Cross efficiency score</td></tr>
<tr><td colspan="3">Weight selection will be restrained by the fact that a selected set shouldn’t cause the deterioration of a peer score.</td></tr>
</table>

To make the above assumption concrete, Wu et al. (2008) developed the so-called game cross efficiency. The model came in response to the non-unicity of DEA efficient DMUs. Assuming an agent/player DMU_d is given an efficiency score α_d, and another player DMU_j then tries to select a set of strategies (Weights selection) to maximize its own efficiency while ensuring that α_d wont decrease, then the game cross efficiency model is defined as follows:

$$\alpha_{dj} = \frac{\sum_{r=1}^{s} \mu_{rj}^{d} y_{rj}}{\sum_{i=1}^{m} \omega_{ij}^{d} x_{ij}} \quad d=1, 2 \ldots n \tag{3.1}$$

Where

$$\begin{cases} x_{ij}\text{: the observed input of type i for entity j } \left(x_{ij} \geq 0,\ \ i = 1, 2 \ldots m, j = 1, 2, \ldots, n\right) \\ y_{rj}\text{: the observed output of type r for entity j } \left(y_{rj} \geq 0,\ \ r = 1, 2 \ldots s, j = 1, 2, \ldots, n\right) \\ \mu_{rj}^{d} \text{ and } \omega_{ij}^{d}\text{: the optimal weights obtained in the game cross efficiency model} \end{cases}$$

Note that the subscript dj is intended to indicate that DMU_j is only permitted to pursue a set of strategies that will not result in the deterioration of the currently estimated efficiency of DMU_d.

Furthermore, in order to ensure availability of a set of strategies for DMU_j, the weights in (3.1) are not necessarily optimal as in (2.2) but are feasible solutions to the CCR model. Here we have ensured that each DMU is able to negotiate a set of strategies (weights) thus a form of cross efficiency.

Now in order to determine the game d-cross efficiency in (3.1) (wu et al., 2008) developed the below LP problem for each DMU_j :

$$\text{Max } Z = \sum_{r}^{s} \mu_{rj}^{d} y_{rj}$$

$$\text{Subject to} \qquad\qquad\qquad\qquad (3.2)$$

$$\sum_{r}^{s} \mu_{rj}^{d} y_{rl} - \sum_{i}^{m} \omega_{ij}^{d} x_{il} \leq 0 \quad \forall\, l = 1,2, \dots n$$

$$\sum_{i}^{m} \omega_{ij}^{d} x_{ij} = 1$$

$$\alpha_d \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \leq 0$$

$$\mu_{rj}^{d} \geq 0 \ \ \forall\, r = 1,2, \dots s,$$

$$\omega_{ij}^{d} \geq 0 \ \ \forall\, i = 1,2, \dots m,$$

$\alpha_d \leq 1$ takes initially the value $\overleftarrow{E_d}$ from (2.3. c), which is the average cross efficiency of DMU_d. When the algorithm converges, this α_d becomes the game cross efficiency.

In comparison to the benevolent model, α_d would be equal to E_{dd} optimal score of DMU_d and the objective would be replaced by the average of the efficiency ratio of the (n-1) DMU_j.

Clearly the constraint $\alpha_d \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \leq 0$ in model (3.2) is equivalent to $\frac{\sum_{r}^{s} \mu_{rj}^{d} y_{rd}}{\sum_{i}^{m} \omega_{ij}^{d} x_{id}} \geq \alpha_d$ which implies the restriction of DMU_d initial score to ensure that it won't deteriorate.

The above model is solved once for each DMU_d thus n times, in addition the optimal value to model (3.2) will represent a game cross efficiency with respect toDMU_d. Now assuming that $\mu_{rj}^{d*}$is an optimal solution to model (3.2) then the average game cross efficiency for DMU_j is $\alpha_j = \frac{1}{n}\sum_{d=1}^{n}\sum_{r=1}^{s}\mu_{rj}^{d*}\, y_{rj}$.

The game cross efficiency table would be as follow for 3 DMUs:

	DMU_1	DMU_2	DMU_3	AVG (appraisal of peers)
DMU_1	$\sum_{r=1}^{s}\mu_{r1}^{1*}\, y_{r1}$	$\sum_{r=1}^{s}\mu_{r2}^{1*}\, y_{r2}$	$\sum_{r=1}^{s}\mu_{r3}^{1*} y_{r3}$	A_1
DMU_2	$\sum_{r=1}^{s}\mu_{r1}^{2*}\, y_{r1}$	$\sum_{r=1}^{s}\mu_{r2}^{2*}\, y_{r2}$	$\sum_{r=1}^{s}\mu_{r3}^{2*}\, y_{r3}$	A_2
DMU_3	$\sum_{r=1}^{s}\mu_{r1}^{3*} y_{r1}$	$\sum_{r=1}^{s}\mu_{r2}^{3*} y_{r2}$	$\sum_{r=1}^{s}\mu_{r3}^{3*} y_{r3}$	A_3
AVG (Peer-appraisal)	α_1	α_2	α_3	

Table 3-1: Game Cross Efficiency Matrix

Iterative algorithm below describes the steps to find the Nash equilibrium efficiency score. In this algorithm, α_j^t represents the efficiency of DMU_j at iteration t.

Algorithm: DEA Game Cross Efficiency

Require: ϵ

Step 1: Set t=1. For each $\mathrm{DMU_d}$ calculate the average cross efficiency $\overleftarrow{\mathrm{E_d}}$ and set $\alpha_d^t = \overleftarrow{\mathrm{E_d}}, \forall\, d \in \{1, \dots, n\}$.

Step 2: For each pair of DMUs d and j, solve model (3.2) and obtain $\mathrm{E_{dj}}$.

Step 3: Set $\alpha_d^{t+1} = \frac{1}{n}\sum_d^n \mathrm{E_{dj}}$

Step 4: If for some d, $|\alpha_d^{t+1} - \alpha_d^t| > \epsilon$, then return to step 2. Otherwise α_d^{t+1} is the optimum game cross efficiency of $\mathrm{DMU_d}$ and the algorithm stops

Numerical example:

	X1	X2	Y1	Y2
DMU1	7	7	4	4
DMU2	4	6	5	7

Table 3-2: Numerical example for Game cross efficiency computation

Using excel solver, we set $\boldsymbol{\varepsilon = 0.001}$

Step1: Cross Efficiency Matrix (CEM) computation.

	DMU1	DMU2
DMU1	0.68571	0.68571
DMU2	0.85714	1
$\overline{E_J}$	0.77143	0.84286

Table 3-3: Cross Efficiency Matrix (CEM)

- Set α_J $(0.77143, 0.84286)$

	DMU1	DMU2
DMU1	0	0
DMU2	0.68571	1
$\boldsymbol{\alpha_J^2}$	0.34286	0.5
$\lvert \boldsymbol{\alpha_J^2} - \alpha_J \rvert$	0.42857	0.34286

Table 3-4: Game Cross Efficiency Matrix (GCEM) iteration 1 output

- We have $\lvert \boldsymbol{\alpha_J^2} - \alpha_J \rvert \geq \boldsymbol{\varepsilon}$ thus we set α_J $(0.42857, 0.34286)$ and proceed

	DMU1	DMU2
DMU1	0.68571	1
DMU2	0.68571	1
$\boldsymbol{\alpha_J^3}$	0.68571	1
$\lvert \boldsymbol{\alpha_J^3} - \boldsymbol{\alpha_J^2} \rvert$	0.34285	0.5

Table 3-5: GCEM iteration 2 output

- We have $\lvert \boldsymbol{\alpha_J^3} - \boldsymbol{\alpha_J^2} \rvert \geq \boldsymbol{\varepsilon}$ thus we set α_J $(0.68571, 1)$

	DMU1	DMU2
DMU1	0.68571	1
DMU2	0.68571	1
$\boldsymbol{\alpha_J^4}$	0.68571	1
$\lvert \boldsymbol{\alpha_J^4} - \boldsymbol{\alpha_J^3} \rvert$	0	0

Table 3-6: GCEM iteration 3 output

- We have $\lvert \boldsymbol{\alpha_J^4} - \boldsymbol{\alpha_J^3} \rvert < \varepsilon$ thus $(0.68571, 1)$ are the best average game cross efficiency given to DMU1 and DMU2 respectively.

Here the algorithm converged to an equilibrium after 3 iterations. Both DMUs have received an improved score compared to their initial cross efficiency score: DMU1 became worst while DMU2 became 100% efficient but now we have a more stable and representative score.

4. Preference incorporation based on (Allen et al.,1997)

We have discussed in the earlier chapter the issue of nonrealistic weighting scheme which is a major issue. In fact, the issue is due to the fact that each DMU, on pursuit of maximizing their efficiency, can choose a set of preferable weights to assign to a set of inputs/outputs and totally neglect the others which result in some weights equal to zero. In other words, this means that the input or output with zero weight is not accounted even though it may be an important part of evaluation. To make this more concrete, assume that a university is assessing its professors to make salary adjustments. The main input would be:

- (I1) Salary
- (I2) Housing

Outputs can be

- (O1) Average number of article published during past years
- (O2) Research innovation
- (O3) Monetary contribution to the school through research funding
- (O4) Teaching score (can be assigned by the administration based on how performing the teacher is with his student)

In this situation, the evaluating foundation may favor some outputs in regard to others. For example.

- favor (O1) to (O3)
- favor (O2) to (O1)

This constraint can be incorporated as follows assuming $\mu_i (i = 1,2,3,4)$ are the respective weights for each output then the below constraints shall be added to the initial model.

$$\mu_1 \geq \mu_3 \text{ or } \frac{\mu_1}{\mu_3} \geq 1$$

$$\mu_2 \geq \mu_1 \text{ or } \frac{\mu_2}{\mu_1} \geq 1$$

This is one kind of weights constraints that may be applied when accounting for preference. Many others are available in Allen et al. (1997). Also, adding such constraints may require further investigation on their impact on the final solution. This suggestion can be of great impact when working on portfolio selection given that we can stress investor preference on input/output set.

5. Conclusion

After the discussion of the cross-efficiency evaluation method and its weakness, we have pointed out the key concept of the game cross efficiency developed by (Wu et Al 2008) to obtain unique scores, which are constructed from the perspective of non-cooperative game. Then we suggested the incorporation of preference for potential application in portfolio selection.

Chapter 4 Empirical Analysis: DEA Cross Efficiency applied to Portfolio Selection

1. Introduction

A naïve usage of DEA cross efficiency in portfolio selection is the selection of the top M stocks with the highest efficiency score. Even though this simple use yielded a better result than simple DEA, it still has some issues as the non-uniqueness of the DEA optimal weights which may reduce its usefulness, specifically the possibility of multiple optimal weights in the DEA model depending on the used optimization software (Despotisa, 2002). For this reason, we seek to apply the game cross efficiency model developed by Wu et al. (2008). In fact, DMUs are viewed as agents competing among each other. We can model the competitive behavior of firms through taking cross-efficiency scores as payoffs and weight selection as strategies. Each firm may choose to take a non-cooperative game stance to boost its efficiency scores (a better chance for investment opportunity) given a weight selection strategy. In the current chapter, we present a new application of game cross efficiency in order to solve for the optimal portfolio. The findings will be compared to benchmark indices and that of arbitrary model (cross efficiency). Around 527 companies listed on the French stock market were used in this study over the course of 6 years starting 2010. This empirical analysis has been motivated and justified by the proven lack of previous DEA studies that incorporate the competitive behavior of these firms.

2. Portfolio selection: A DEA approach

In the past decade, DEA methodology application fields have grown tremendously reaching financial applications; hedge fund, portfolio selection, and financial performance. This new application approach, especially to portfolio selection witnessed a scientific controversy given that DEA has been designed for production theory. Actually, DEA focuses on comparing a set of decision making units (DMUs), which consume different quantities of inputs and outputs produced. Going back to financial applications, we can point out the reason behind this controversy which is the numerous choices of inputs and outputs. Indeed, we can identify two major schools of thought; the “Production-DEA” and “Multicriteria-DEA”. In the former, the set of inputs/outputs choices exhibit the characteristics of a production process thus we are talking about the so called “financial production process or the underlying financial technology”. However, the latter ignores the main assumption of DEA approach, that of a production process, thus we are in a multi criteria evaluation approach. This issue of non-clarity in regard to the choice of inputs/outputs was pointed by (Cook, Tone and Zhu (2014)) who argued that as long as the underlying technology can relate to some kind of “production process”, then inputs/outputs can be easily identified. We will go through this issue in details in the next section proceeded by the analysis of cross efficiency application to portfolio selection strategy.

3. Identification of input and output variables

As discussed earlier, the criteria proposed in the literature implicitly or explicitly do not relate to any form of production process. In fact, going through DEA applications to portfolio selection, in literature we can identify two main criteria for inputs/outputs identification. One criterion is the finding of an analogy between the interaction of inputs/outputs and that of production process. Indeed, this analogy is more of a “causal relationship of the production kind” Tarnaud and Leleu, (2017). The identification of such relationship would definitely ease the choice of the inputs/outputs variables. Another criterion is tied to the conduct of decision makers towards the different factors (attributes), which are the set of inputs/outputs. In the case of portfolio selection, this choice is inferred from the assumed investor preferences.

In spite of the implicit consensus on the choice of return/risk as attributes, it is unclear whether risk attributes should be identified as inputs or as undesirable outputs. The multiplicity of the chosen factors in literature leads to the conclusion that the theoretical framework to be used in the study is yet to be defined. For this reason, there should be a prior analysis to the choice of model, in other words the validation of the inputs/outputs choice.

A bare of the DEA-portfolio literature reveals on the one hand, that several investment expenses and undesirable outcomes have been the corner stone of input choice. Murthi, Choi and Desai (1997) adopted a philosophy that relate to “a production process”, they identified the standard deviation and the set of factors that contribute to the generation of returns (expenses ratios, management turnover, operational expenses and other related expenses) as inputs. Following this track, McMullen et al (1998) identified a similar set of inputs; standard deviation, sales expenses, minimum investment cost and expenses ratio. Basso and Funari (2007) did not only choose a similar set of inputs but also added different risk measures including standard deviation and the Beta coefficient. To elaborate on the importance of risk as building block of the theoretical framework, Eling (2006) and Branda (2015) added different measures of downside risk consisting of standard deviation of drawdown, Value-at-Risk, and conditional value at risk.

On the other hand, various desired outcomes have always been selected as the set of outputs. In fact, one major choice is a set of various measures of average return (mean) of historical return distributions. Gregoriou (2003) considered higher moments as outputs. In addition, other authors (Wilkens and Zhu (2001), Gregoriou and Zhu (2007)) took minimum returns or consecutive gains among the set of outputs. In addition to skewness of return distribution (see Wilkens and Zhu, (2001)), several traditional performance indicators including the Sharpe ratio, Treynor index, and Jensen alpha were considered among the output measures (see Basso and Funari, (2005)).

From the aforementioned contribution, we can infer that measures of risk have been identified as the main set of inputs while those of returns as outputs. This conclusion can be explained by the incorporation of investor's preferences given that it is common to seek the reduction of risk/input and the increase of returns.

4. Financial technology

As mentioned earlier, the controversy exhibited by the numerous applications of DEA to portfolio selection pauses a modeling issue of the right framework. One major approach applied is the Mean-Variance (MV) framework, which is similar to the philosophy of Markowitz (1959). In this framework, a "production" like relationship is defined between the second order risk (Standard Deviation) and the expected return (mean). This approach is facing some disagreement: some see it is fit and appropriate to apply an MV framework while others concluded that such a relationship can lead to invalid representation of the underlying technology (Tarnaud and Leleu, (2017)). Indeed, there is no mathematical form that demonstrates that "higher risk results mean higher returns". Furthermore, this assumed positive correlation between risk/return have been proven incorrect for some kind of assets (leverage effect for instance). This fact was further reminded by Devaney, Morillon and Weber (2016). As an alternative for this representation, Tarnaud and Leleu, (2017) suggest that return and risk should be both considered as outputs given that they have the same source which is the return distribution. Another similar opinion can be found in Anderson et al. (2004) who consider that gains resulting from an initial investment are to be considered outputs while risk is an input as it is related to the investment. Of course, this view can be valid if the risk can be quantified a priori, which is not the case. Another framework would be the expected utility. This framework is similar to that of the mean variance with regard to the decision maker's preference. In fact, the utility framework can exhibit a mean variance only or multi moments (skewness, kurtosis) modeling approach. Of course, similar to the prior analysis, the simple analogy with production process can be misleading or rather erroneous. This is mainly due to the same argument that moments are the characteristics of the return distribution thus all moments should be considered as outputs for an adequate representation of the production technology. However, in a simple multi criteria analysis problem, we can model the moments as simple preference criteria meaning a set of desirable (outputs) and undesirable (inputs) factors.

5. Methodology

In order to determine the optimal portfolio while considering the competitive behavior of the firms, we will use the model developed by Wu et al. (2008) in a multi criteria context. We include a set of decision maker's preference characteristics which are the four moments of the return distribution. This choice shall be motivated in later section. The problem takes a multicriteria context where we seek to find the optimal portfolio in order to capture the tradeoff between the set of preferences. Furthermore, taking the preferences of the investor as inputs/outputs in our model would bring us to the expected utility framework, thus we assume rationality. A rational investor would prefer odd moments (mean and skewness) and dislike the even ones (variance and kurtosis).

The problem will be solved using the CIPLEX platform in R software. Furthermore, we will assess the 5-year performance of the game portfolio and compare it to those of four benchmark portfolios, which are listed in the Paris stock exchange, CAC40, AEX, BEL20 and PSI20 over the period starting 2010 to 2015. We compare the game cross efficiency selected portfolio performance with that of DEA-cross efficiency model. For all approaches applied, firms are ranked in a decreasing order of their efficiency scores and the relevant portfolio size is selected. The intent of this comparison is to prove that the game cross efficiency is a more coherent tool compared to the DEA-cross efficiency which provides arbitrary efficiency scores depending on the software, thus various different rankings.

6. Data

a) Data source and choice

This study uses a dataset ranging from 493~527 composed of firms listed in the Paris exchange. We exploit original database, which is a compilation of daily prices over the course of 6 years starting from 2010 until 2015. The component of this database is published on free financial databases such as Yahoo finance and Google finance. Using this data, our approach aims to select stocks to include in a portfolio. Furthermore, even though most of the Western European stock markets endured a harsh twenties without a complete shutdown, Paris stock exchange, a very well-developed market (Market capitalization to GDP), only recovered in the past 30 years to become one of the leading stock markets worldwide. In fact, prices were set up by Walrus-style auctioneers in 1986. The auctioneers determined an equilibrium price at a fixed moment in the morning after weighting demand against supply. In addition, other prices could be cited later in the day based on the importance of transactions incurred. Finally, the Paris exchange is characterized by the heterogeneity of listed companies and their size. The last reason is the fact that we are using a non-parametric method thus we are faced with the so-called curse of dimensionality as it requires a high number of DMUs for a robust analysis.

b) Data descriptions

We have a raw financial data provided on 527 firms. The data is composed of daily prices (opening and closing price) since the start of 2010 until 2015. The selected set of firms is heterogeneous and has fluctuating prices which has led us to use the closing price of the day in our analysis. From this raw data, we have managed to generate a final database with monthly return of each firm over the course of 6 year starting 2010. The choice of monthly return was motivated by a search for simplicity and a better strategic long-term inference. Below is a sample of the 2015 data

	Jan 2015	Feb 2015	Mar 2015	Apr 2015	Mai 2015	Jun 2015	Jul 2015	Aug 2015	Sep 2015	Oct 2015	Nov 2015	Dec 2015
ACTEOS	0.05116	0.01688	-0.02703	-0.19196	-0.18803	-0.16017	-0.21368	-0.29958	-0.38397	-0.37387	-0.38496	-0.46835
ACTIA GROUP	0.03955	0.26126	-0.07500	-0.01805	0.09217	-0.11111	0.03308	-0.12030	-0.15789	0.14592	-0.03656	-0.01509
ADA	0.00241	0.01200	0.00712	0.00589	-0.02576	-0.03005	0.08118	-0.05378	0.02061	-0.01425	0.16029	-0.19796
ADL PARTNER	-0.04965	0.11039	0.03130	-0.05946	-0.03438	-0.08824	0.11391	-0.03143	0.00540	-0.15119	0.05688	0.00325
ADOMOS	0.00000	-0.25000	0.00000	1.00000	0.00000	0.25000	0.00000	0.00000	0.00000	-0.16667	0.00000	0.00000
ADTHINK MEDIA	-0.10599	0.00000	-0.02778	0.01163	0.03529	-0.14286	0.00000	-0.08108	-0.15328	0.20536	0.00826	0.07563
AFFINE R.E.	0.19987	0.01998	-0.00053	-0.00107	-0.12862	0.01329	-0.03034	-0.03639	0.00256	0.07969	-0.03197	0.00926
AFONE	-0.12270	-0.07343	-0.05505	-0.02549	-0.02132	-0.14629	0.05682	-0.00866	-0.04348	0.09091	-0.07099	0.22000
AGTA RECORD	0.03239	0.03226	0.00503	0.11750	0.02170	-0.02444	0.08656	-0.02495	0.02752	-0.00755	-0.01795	0.00000
AIR FRANCE -KLM	-0.02896	-0.06658	0.14254	-0.08072	-0.03308	-0.15631	0.03594	-0.04217	0.04285	0.11499	-0.06101	0.11980
AIRBUS GROUP	0.13666	0.18050	0.07755	0.01790	0.00649	-0.05856	0.07007	-0.11623	-0.06371	0.18319	0.07627	-0.07807
AIR LIQUIDE	0.10454	0.04421	0.01741	-0.02829	-0.01096	-0.03406	0.02419	-0.10772	0.00332	0.12823	-0.03307	-0.05558
AKKA TECHNOLOGIES	0.12042	-0.02625	-0.03194	0.00882	0.05397	0.06440	-0.08459	-0.03575	-0.18912	0.09130	0.14781	0.03048
ALBIOMA	0.00919	0.18616	-0.04866	0.04781	-0.00215	-0.26179	0.03354	-0.00523	-0.00931	-0.00604	0.01080	-0.00067
ALCATEL-LUCENT	0.04668	0.14506	0.00199	-0.09480	0.17251	-0.10269	0.03244	-0.12838	0.11869	0.14202	0.01442	-0.03439
ALES GROUPE	-0.02793	0.04300	0.03556	-0.01182	-0.02935	-0.01120	0.08215	-0.00157	-0.11458	-0.04762	0.00613	0.01250
ALSTOM	0.08353	0.01704	-0.03445	-0.03177	0.00018	-0.10245	0.04003	0.03208	0.00840	0.06614	-0.00474	-0.03146
ALTAMIR	0.00193	0.08529	0.05263	-0.01818	-0.01235	-0.06619	0.05268	-0.03176	-0.04808	0.04379	0.04510	0.04977
ALTAREA	0.14798	0.00000	0.12739	-0.02627	-0.07750	-0.03342	0.00259	0.06718	-0.00909	0.04933	-0.00619	0.04605
ALTAREIT	0.04723	0.02587	0.03767	0.03024	-0.02930	-0.00018	-0.01753	-0.00383	0.02783	-0.02322	0.04784	-0.02242
ALTEN	0.06707	0.08204	0.04336	0.01597	-0.01595	-0.00750	0.03883	-0.06708	0.08522	0.01505	0.09350	0.01443

Table 4-1: Firms monthly return in 2015

c) Univariate profiling of firms' returns

The strating point of understanding a variable is by characterising its distribution. However, in our case we have a set of 524 firms thus only a sample will be selected for the current analysis. We will select randomly a set of 12 firms to characterise their return distribution. We will start by a summary of the descriptive statistics.

Descriptive Statistics								
	N	Range	Min	Max	Mean	MSE	Std. Dev	Variance
ACTEOS	72	.9075	-.4684	.4391	-.0296	.0210	.1782	.0318
ACTIA.GROUP	72	.8379	-.1831	.6548	.0198	.0152	.1290	.0166
ADA	72	.7245	-.3035	.4211	.0051	.0114	.0967	.0094
ADL.PARTNER	72	.5747	-.3706	.2041	.0047	.0096	.0815	.0066
ADOMOS	72	1.3333	-.3333	1.0000	-.0219	.0232	.1970	.0388
ADTHINK.MEDIA	72	1.2238	-.2267	.9971	.0146	.0204	.1733	.0300
AFFINE.R.E.	72	.3903	-.1904	.1999	.0036	.0076	.0643	.0041
AFONE	72	.5530	-.1768	.3761	.0093	.0117	.0995	.0099
AGTA.RECORD	72	.4458	-.0685	.3772	.0164	.0069	.0586	.0034
AIR.FRANCE.KLM	72	.4952	-.2220	.2732	-.0012	.0141	.1194	.0143
AIRBUS.GROUP	72	.3830	-.1994	.1836	.0211	.0096	.0817	.0067
AIR.LIQUIDE	72	.2360	-.1077	.1282	.0049	.0055	.0470	.0022

Table 4-2: Descriptive Statistics

The above table contains the main descriptive feature of the first twelve companies, including the mean , variance, minimum, maximum and range. We can note that the mean returns varies between a minimum of -0.0296 and a maximum of 0.0198, thus some firms have a postive expected return over the course of 6 years while others have a loss one. Furthermore, we can note alow variance, which indicates the level of dispersion around the mean, for some companies and a high one for others. Based on the above descriptive statistics and the density plots in figure (4-1), we can conclude that some firms' return distributions are within the bellshape of the normal data while others are not. For further clearance in regard to the shape of the return distribution, we have the Kolmogorov-Smirnov and Shapiro-Wilk normality tests in Table (4-3) which proves the prioir findings.

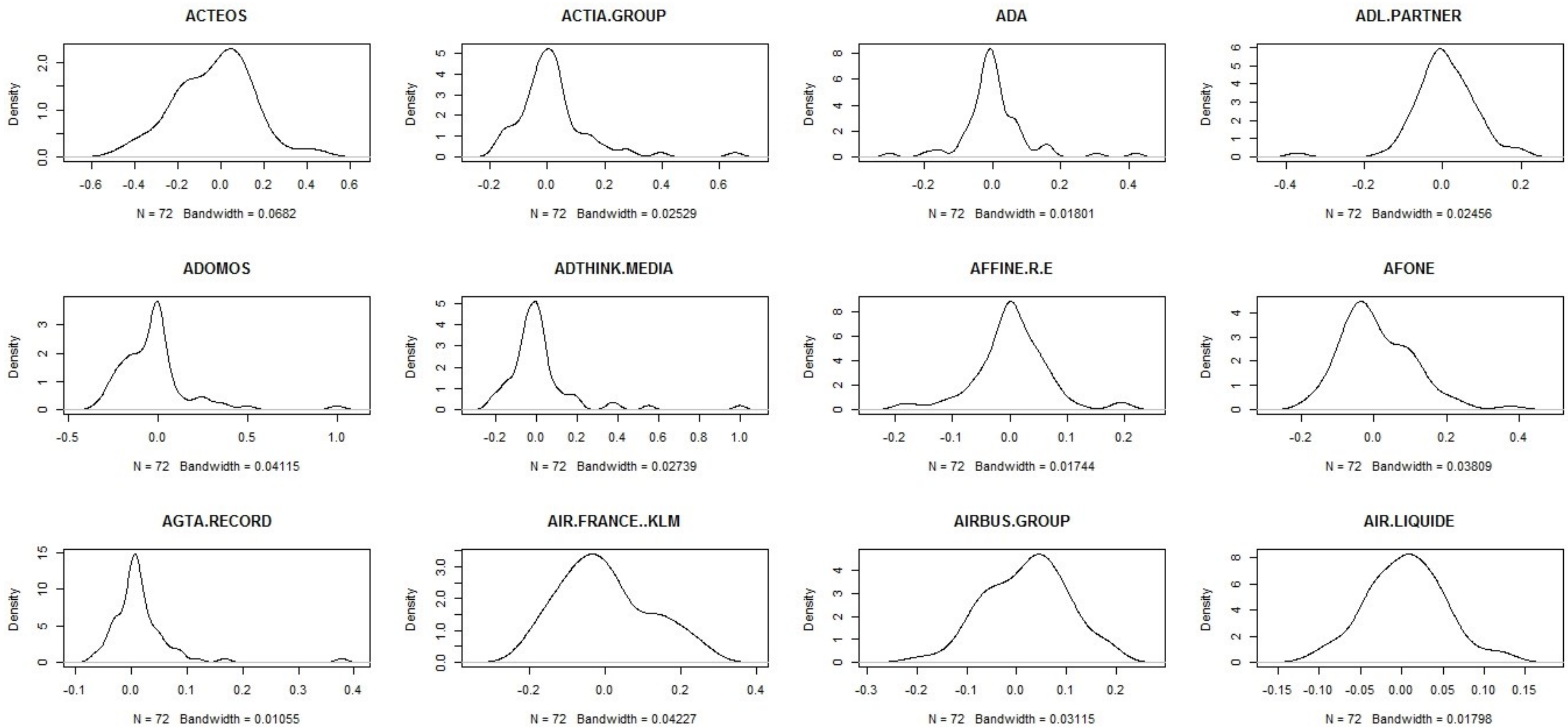

Figure 4-1: Density curve

Tests of Normality						
	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
ACTEOS	.083	72	.200*	.981	72	.362
ACTIA.GROUP	.193	72	.000	.837	72	.000
ADA	.173	72	.000	.864	72	.000
ADL.PARTNER	.081	72	.200*	.921	72	.000
ADOMOS	.233	72	.000	.810	72	.000
ADTHINK.MEDIA	.230	72	.000	.716	72	.000
AFFINE.R. E	.103	72	.057	.949	72	.006
AFONE	.135	72	.002	.950	72	.006
AGTA.RECORD	.198	72	.000	.705	72	.000
AIR.FRANCE.KLM	.082	72	.200*	.970	72	.084
AIRBUS.GROUP	.064	72	.200*	.988	72	.704
AIR.LIQUIDE	.047	72	.200*	.991	72	.908

Table 4-3: Kolmogorov-Smirnov and Shapiro-Wilk normality tests

7. Input/output selection

In the previous section, we have discussed the controversy between “Production-DEA” and “Multicriteria-DEA”. In this section, we will validate our choice of approach and input/output selection. In our analysis, we seek to apply an expected utility framework, thus exhibiting investor preference given a set of axioms (that of expected utility). To carry on with this approach, we have selected the “Multicriteria-DEA” approach as we consider investor preference a set of attributes in a decision-making problem. The problem can be casted in a “Multicriteria-DEA” framework through taking the to be minimized attributes as inputs or expected costs and that to be maximized as outputs or expected benefits. Using the efficiency ratio, each firm(stock) will seek to maximize its desirability for investment through weighing its benefits and costs. In fact, the literature on expected utility is very rich in regard to preference modeling. In 1991, 1992, and 1993, Kimball, while building on the work of Arrow and Pratt (1971), published three papers providing new concept of decision maker’s behavior. Kimball (1992) developed two main concepts; prudence and temperance, tow measure of risk intensity. Following this work, numerous papers have been published in regard to risk and its relation to higher order moments. Indeed, most of these papers suggest not only to consider the mean and the variance but also the skewness and kurtosis to model investor preference. A rational decision maker is believed to favor odd moments (mean and skewness) and dislike even ones (variance and Kurtosis). In fact, a well-known expected utility approximation is the following equation:

$$E[\mu(x+\varepsilon)] \cong \mu(x) + \frac{\sigma_\varepsilon^2}{2}\mu''(x) + \frac{Sk_\varepsilon}{3}\mu'''(x) + \frac{Ku_\varepsilon}{4}\mu''''(x) + \qquad (4.1)$$

where ε is a zero-mean risk while σ_ε^2, $Sk_\varepsilon = \frac{1}{N}\sum(\frac{x_i - \bar{x}}{\sigma_\varepsilon{}^3})^3$ and $Ku_\varepsilon = \frac{1}{N}\sum(\frac{x_i - \bar{x}}{\sigma_\varepsilon{}^4})^4$ are respectively variance, skewness and the kurtosis of this risk. From the above equation, it can be concluded that the n derivative is the nth moment which proves the link between expected utility and distribution characteristics. Furthermore, we can infer from Kimball work and the above utility approximation that the third moment (skewness) would be "prudence" while kurtosis is "temperance". Based on the prior analysis and decision theory, an investor is a prudent and temperate individual who seeks to maximize his wealth while minimizing the overall risk associated with his investments. Based on this definition, we will take the mean and skewness (prudence) as benefits (outputs) and the variance and kurtosis (temperance) as inputs.

To better characterize temperance, we need to point out the characteristics of a kurtotic distribution. In fact, looking at figure 4-1, we can note that most of the distributions have slightly different characteristics than that of a normal distribution including; greater elongation in the tails and peaks near the mean. In terms of probability mass, kurtosis represents the reallocation of dispersion from the center, thus creating more peaks and more prominent tails. Menezes and Wang (2005) showed that temperance can be characterized as outer risk and temperate individuals are outer risk averse. They dislike reallocation of probability mass to the tails.

Prudence is related to the distribution skewness, thus requiring an analysis of the distribution shape. Actually, skewness mainly characterizes the level of symmetry of a distribution around its mean. Prudence, which is characterized by preference for skewness, was associated by Menzes et al. (1980) as being analogous to an aversion to "increase in downside risk". Given an Expected Utility (EU) context, prudence measures the tradeoff between risk and skewness for decision under uncertainty, Chiu (2005). From this, it can be confirmed that a prudent investor would prefer skewness, in hope to minimize downside risk.

Table (4-4) presents the descriptive statistics over the 6-year- test period of the inputs (Variance and Kurtosis) and outputs (Mean and Skewness). We have the main descriptive feature per year, including the number of observations, Standard deviation, the minimum, and the maximum. We can note that the mean (output) varies between a minimum of -0.19 and a maximum of 0.75, thus some firms have a postive expected return over the 2010 horizon while others have a negative one.

Descriptive Statistics					
2010	N	Minimum	Maximum	Mean	Std. Deviation
Mean	505	-.1926893748152710	.7525929518264140	.009908618517329	.050372721732632
Variance		.0000655981128720	7.4163867848365100	.025691347193212	.330949746903930
Skewness		-2.8614928111597200	3.4544836287850300	.307450770903675	.876362588125746
Kurtosis		-1.8671267738319300	11.9518077026077000	.693284549314482	2.046230297767620
2011					
Mean	508	-.2066289656455110	.2853311371247790	-.009208929869709	.037534177870417
Variance		.0001865136989900	.4428111350846240	.012994054126502	.031581639936036
Skewness		-2.6664884633910900	3.3853774603737000	.171725207990373	.902964301687253
Kurtosis		-1.9485888695929700	11.5881470691676000	.826829862770566	2.031636532955700
2012					
Mean	500	-.1621995426108220	.2846467299932080	.006363950685434	.034327352634797
Variance		.0000050030680492	.1544656635802470	.010727423588765	.015084179574527
Skewness		-3.4383155711514500	3.1712128620485800	.237934950025216	.933950618173685
Kurtosis		-1.7383121483374100	11.8712933519002000	.889678656491946	2.100292846591450
2013					
Mean	500	-.2180801032193350	1.3251856856331300	.020416423165790	.072961341246391
Variance		.0000114066104826	.7186009878097490	.012567295606190	.044516467599537
Skewness		-2.7022709288108600	3.1138005979239900	.337196681709857	.909637919817350
Kurtosis		-1.9187776878860400	10.3493812737241000	.836996468225845	2.121558134081400
2014					
Mean	502	-.1853502710610920	.4030529501185170	.011079568326561	.038111236165614
Variance		.0000056765649768	.3993874599774040	.012402799070133	.030693896175640
Skewness		-2.4552426153382900	3.4537325418792200	.462710595044396	.929976671874269
Kurtosis		-1.8372935025512800	11.9465796011431000	1.072297739138150	2.334031689572900
2015	N				
Mean	501	-.2186301505882400	.2405269794857730	.014092125128014	.040008621348881
Variance		.0000474849225387	.3119681400965420	.013104169820121	.030749641664885
Skewness		-1.9887268846023500	3.1052731769750900	.483923386372495	.858720661966491
Kurtosis		-2.0695696881426600	10.2069153454613000	.835991029163110	2.167712970671260

Table 4-4: Input/output descriptive statistics

The same goes for the other years with the exception of 2013 during which the mean output picked, reaching a maximum of 1.325. For the skewness, we can note that the range over the 6 year period is quite the same ranging from 2.86 to 3.5. The same goes for the kurtosis ranging from -1.70 to 11.95. For the variance input, we can note a low range with a minimum of 0.00006 and a maximum of 0.72 except for 2010 during which the variance reached a maximum of 7.41.

8. Dealing with negative data

Since the usage of the distribution moments will lead to the rise of negative values both in the inputs and outputs, it is appropriate to use data transformation. Indeed, applying data transformation when constant return to scale (CRS) model is used would result in altering the efficiency values. However, in the context of our analysis, we seek a benchmarking ("multicriteria-DEA") among the stocks, thus, as long as efficiency classification is preserved, portfolio construction won't be affected. In order to transform negative data to positive ones, the following formula will be applied to each variable (input/output):

Given an input or output variable X^k and θ^k the transformed variable of input/output k we have

$$\theta^k = X^k + |\min\{X_i^k; i = 1 \dots n\}| \text{ with n the number of DMUs}$$

The above transformation would conserve each variable scale without impacting efficiency classification.

9. Portfolio selection strategy

In order to construct the different portfolios, we will consider a buy-and-hold strategy, where this year optimal portfolio is selected through solving our model using the prior year data. The constructed portfolio will be held for an investment horizon of one year and revised (new stock selection based on model solutions) each new investment horizon.

In this study, we seek to empirically evaluate the performance of the constructed portfolios based on each model. The resulting portfolios will be compared to the benchmark indices. For each investment horizon, the portfolio sizes will be fixed to a certain level with equally weighted stocks. More specifically, at the beginning of each investment horizon, the set of stocks shall be classified based on each model's solution in a decreasing order of efficiency. Once a portfolio is selected, we assume that the same dollar amount will be invested in each of the stocks constituting the portfolio, with no more transactions to be made until the end of the investment horizon. This strategy will imply that investment cost will only be incurred at the end of each investment horizon.

10. Portfolio performance

We examine the 6-year performance of the portfolio generated through game cross and that of arbitrary (simple cross) and compare it to those of four market indexes, CAC40, AEX, BEL20 and PSI20 over the test period starting 2010 to 2015. The CAC40 is the index of top 40 performant stocks traded in Paris Exchange. It is the representative stock market index of France, similar to the S&P 500 in the U.S. Similarly, the AEX (Amsterdam Exchange index) is a stock market composed of 25 securities, the most traded on the exchange. Also, BEL20 and PSI20 are composed of the best 20 companies traded at the Brussels Stock Exchange and that of the main stock exchange of Portugal respectively. These benchmark indices are the main national indices of the stock exchange group Euronext. The intent of this comparison is to show that the selected model outperforms the best Euronext indices and beats that of Arbitrary model (simple cross).

	CAC40_EX	AEX_EX	BEL20_EX	PSI20_EX	Game Portfolio	Arbitrary Portfolio
2010	-2.44%	-2.33%	-8.56%	0.01%	-2.78%	-15.14%
2011	-4.10%	-4.01%	-22.15%	-16.32%	-23.99%	-31.84%
2012	12.60%	13.27%	10.75%	5.93%	**14.85%**	-1.47%
2013	5.87%	2.72%	12.62%	12.30%	13.81%	10.42%
2014	10.99%	10.78%	0.24%	5.33%	13.22%	-28.36%
2015	12.32%	13.53%	8.05%	3.63%	11.76%	8.44%
Annualized Return (Geometric)	0.66%	2.56%	4.48%	-9.39%	4.70%	4.89%
Annualized Std Dev	16.61%	15.28%	13.27%	19.23%	5.19%	5.45%
Annualized Sharpe	0.0395	0.1674	0.3379	-0.4884	0.9059	0.8961

Table 4-5: Performance Summary

Table (4-5) shows the annual excess return of each portfolio and benchmark indices over the 6-year test period. As a proxy for the risk-free return, we will use INTGSTFRM193N[2] monthly rates, which range from -0.36% to 1.08% over the test period. The "Game Portfolio" attains the highest geometric mean excess return **4.89%**, which is quite higher than those of the CAC40 (**0.66%**),

[2] INTGSTFRM193N: Interest Rates of Government Securities and Treasury Bills for France (Percent Monthly)

AEX (**2.56%**), BEL20 (**4.48%**), PSI20 (**-9.39%**) and "Arbitrary Portfolio" (**4.70%**). To proceed with our performance analysis, we calculate the Sharpe ratio to examine the risk-adjusted performance of the portfolios followed by a performance chart of cumulative returns and the underwater drawdown curve of monthly return over the 6-year test period. As can be seen at the bottom of Table (4-5), the "Game Portfolio" attains the highest annualized Sharpe ratio of **0.9059** succeeded by that of "Arbitrary Portfolio" (**0.8961**), BEL20 (**0.3379**), AEX (**0.1674**), CAC40 (**0.0395**) and PSI20 (**-0.4884**).

Figure (4.2) and (4-3) present the cumulative returns and underwater drawdown curves of "Game Portfolio" vs CAC40, AEX, BEL20, PSI20 and "Arbitrary Portfolio". We can note that the "Game Portfolio" beats the cumulative returns of the four market indices while having a maximum drawdown[3] of **-0.12** compared to CAC40 (**-0.28**), AEX (**-0.22**), BEL20 (**-0.25**) and PSI20 (**-0.48**). Furthermore figure (4.3), demonstrates that "Game Portfolio" has lower and more stable drawdown compared to "Arbitrary Portfolio". In addition, our portfolio still performs quite good relatively to the cumulative return which highly exceeds that of "Arbitrary Portfolio".

[3] The maximum amount of loss from a portfolio through the drawdown and back to the initial return point of the portfolio.

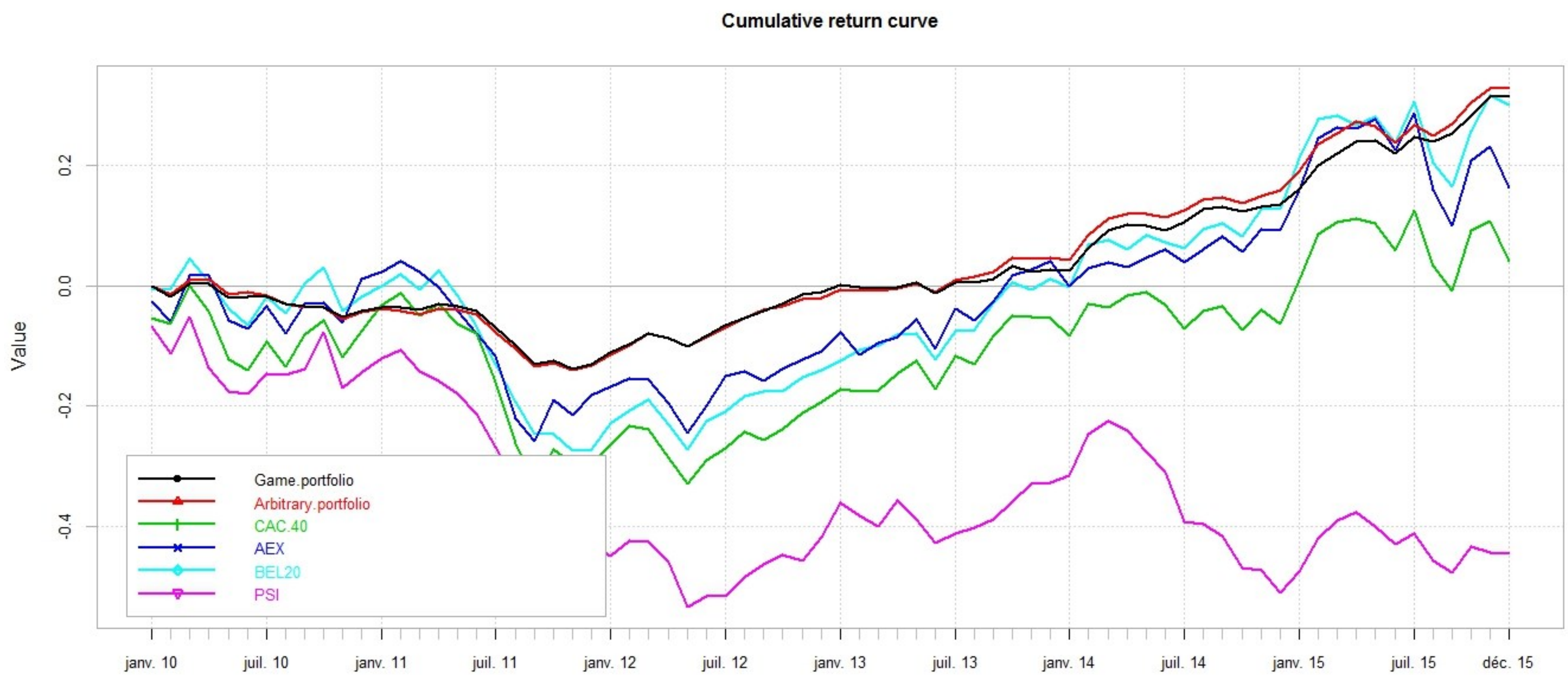

Figure 4-2: Cumulative returns

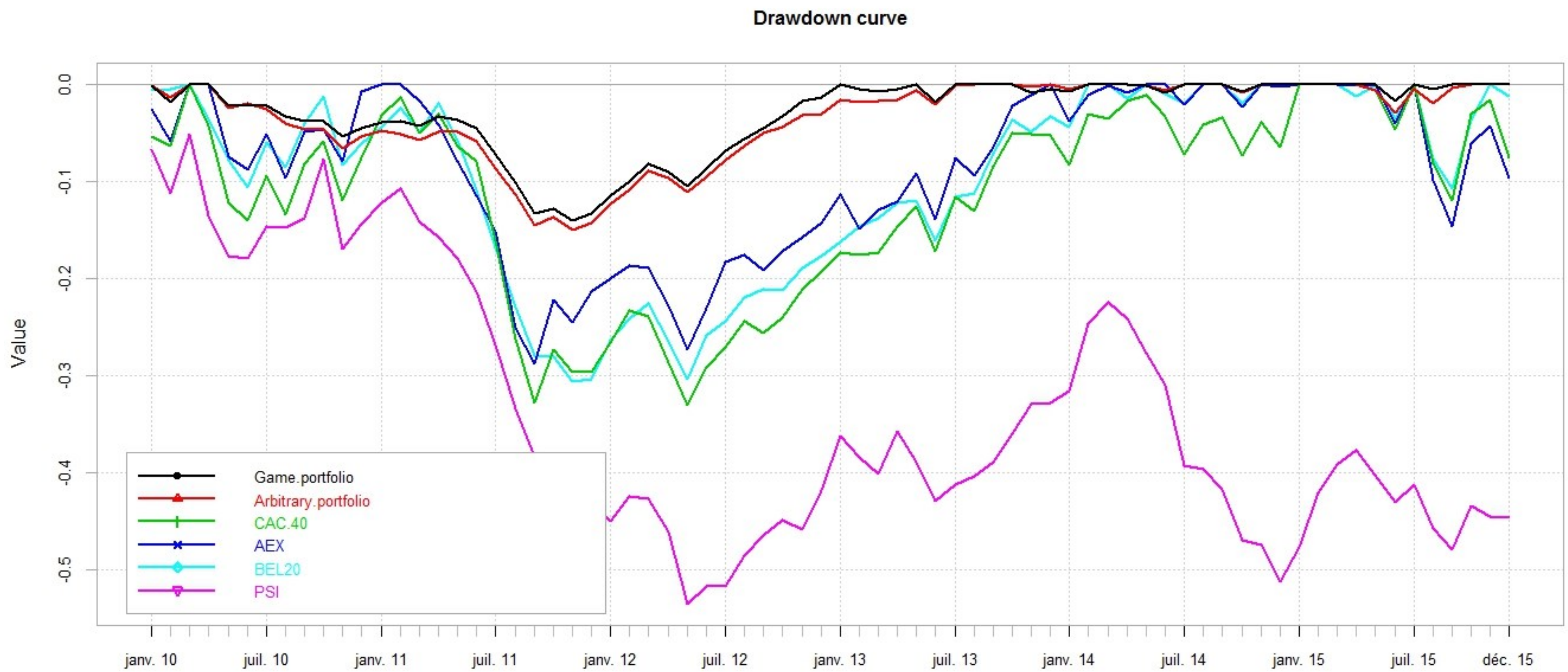

Figure 4-3: Drawdown curve

Finally, we examine whether the “Game Cross Efficiency ranking” and “Arbitrary Cross Efficiency ranking” are identical based on Wilcoxon Signed-Rank Test. Furthermore, we verify the statistical significance of the difference of two Sharpe (“Game Portfolio” compared to each of the other indices and arbitrary portfolio). We use the studentized circular block bootstrap (SCBB) developed by Ledoit and Wolf (2008), which takes into consideration the skewness, kurtosis and autocorrelation effects when comparing tow Sharpe ratios. We formulate the following five two-sided hypotheses as follows:

$$\begin{aligned}\mathrm{H}_0(\text{“Game Portfolio” Vs “Arbitrary Portfolio”})\text{: Sharpe ratio(“Game Portfolio”)}\\ -\text{Sharpe ratio(“Arbitrary Portfolio”)} = 0\end{aligned}$$

$$\mathrm{H}_0(\text{“Game Portfolio” Vs CAC40}): \text{Sharpe ratio“Game Portfolio”} - \text{Sharpe ratio(CAC40)} = 0$$

$$\mathrm{H}_0(\text{“Game Portfolio” Vs AEX}): \text{Sharpe ratio(“Game Portfolio”)} - \text{Sharpe ratio(AEX)} = 0$$

$$\mathrm{H}_0(\text{“Game Portfolio” Vs BEL20}): \text{Sharpe ratio(“Game Portfolio”)} - \text{Sharpe ratio(BEL20)} = 0$$

$$\mathrm{H}_0(\text{“Game Portfolio” Vs PSI20}): \text{Sharpe ratio(“Game Portfolio”)} - \text{Sharpe ratio(PSI20)} = 0$$

We use the R implementation of Michael Wolf to test the above hypothesis. We apply the test on pairs of monthly excess returns. Table (4-6) summarizes the test statistics resulting from the R code using the default parameter setting. As shown in the table, based on the studentized circular block bootstrap (SCBB) Pvalues, we reject the null hypothesis implying that the Sharpe ratio of the “Game Portfolio” is significantly greater than that of PSI20 at 5% significance level and greater than that of CAC40 at 10% significance level. However, we fail to reject that versus the “Arbitrary Portfolio” with a Pvalue=0.9398 even though the Wilcoxon rank test proves that the “Game Cross Efficiency ranking” and “Arbitrary Cross Efficiency ranking” are not identical for the test period as shown in Table (4-7).

VS		CAC 40	AEX	BEL20	PSI20	Game Cross	Arbitrary Portfolio
Game	Difference	0.2282	0.1938	0.1489	0.3832	---------	0.0026
	P value	**(0.087)** *	(0.2244)	(0.1722)	**(0.0788)** **	---------	(0.9398)
Arbitrary	Difference	0.2256	0.1912	0.1463	0.3806	-0.0026	---------
	P value	**(0.0768)** *	(0.175)	(0.167)	**(0.0482)** **	(0.9392)	---------

Table 4-6: Tow sided Sharpe difference test [Significance level 1%***, 5%**, 10%*]: the Studentized Circular Block Bootstrap (B=10. M=4999)

Wilcoxon	2010	2011	2012	2013	2014	2015
Statistic	127760	129290	125250	125250	126250	125750
Pvalue	2.20E-16	2.20E-16	2.20E-16	2.20E-16	2.20E-16	2.20E-16

Table 4-7: Wilcoxon Signed-Rank Test “Game Cross Efficiency ranking” vs “Annual return based ranking”

This further supports our claim that the “Game Portfolio” outperforms some of Euronext benchmark market indices and provides a more stable and meaningful ranking that incorporates firms’ behavior.

The results above demonstrate the effectiveness of our approach as a tool for portfolio selection. Our results also show that the game cross efficiency approach is more effective and stable than the one based on the simple use of cross efficiency at least for this particular study. In fact, the simple cross efficiency approach is unstable and unpredictable depending on the software and problem context compared to the Nash equilibrium score provided by the game cross efficiency. However, a major shortcoming of this approach is the lack of investor control over the expected return. Indeed, a rational investor would have a reserve utility, in other words he would want to set a minimum expected return level. This motivates our development of a Mean Cross (MC) framework of portfolio selection based on DEA game cross efficiency evaluation, in which we seek to maximize the overall portfolio efficiency score subject to a tradeoff level of expected return, which will be detailed in the subsequent section

11. A Mean-Cross framework for portfolio selection based on game cross-efficiency evaluation

While the simple use of game cross-efficiency can result in moderately consistent portfolio compared to the market indices, it still lacks the control over the minimum achievable return. For this reason, we seek to develop a Mean Cross model in which we seek to maximize the overall efficiency of the portfolio subject to a tradeoff level of expected return, to be set by the investor. In fact, under a cross evaluation approach, we are in what might be a democratic vote, where a set of factors are voted to be of high importance by the majority of DMUs while the rest are of low importance. Moreover, as we are in a multicriteria context, we are in a tradeoff between the different attributes (Criteria), in our case the four moments; Mean, Variance, Skewness and Kurtosis. Indeed, the round players in game cross efficiency are not necessarily the ones holding the highest return in that time horizon, given that the fact of having high return does not imply good performance on the other attributes. This enables us to draw the conclusion that ranking based on returns is different than that based on game cross efficiency. This is further confirmed by the Wilcoxon Rank test provided in table (4-8).

Wilcoxon	2010	2011	2012	2013	2014	2015
Statistic	92509	118340	82225	74464	97264	77298
Pvalue	2.20E-16	2.20E-16	1.331E-09	0.0002497	2.20E-16	8.649E-06

Table 4-8: Wilcoxon Signed-Rank Test “Game Cross Efficiency ranking” vs “Annual return based ranking”

This motivates our development of a Mean Cross framework of portfolio selection based on DEA game cross-efficiency evaluation as below.

For a $\mathrm{DMU_j}$, the return and efficiency characteristics are defined as its annual return r_j and its efficiency score e_j respectively. Similarly, for a portfolio δ with individual DMUs being weighted using a weight vector $W \in R^{+}$, where $\sum_i^n w_i = 1$, the return and efficiency characteristics are defined as $R^{\delta} = \sum_i^n w_i r_i$ and $E_{\delta} = \sum_i^n w_i e_i$. An optimal portfolio δ * with an optimal weight vector w* is determined by solving the following linear optimization model

$$\max \; E_{\delta}$$

$$\text{Subject to} \quad R^{\delta} \geq (1-\gamma) R_m^{\delta} \qquad (4.2)$$

$$\sum_i^n w_i = 1$$

$$w_i \geq 0, i = 1 \dots n$$

where γ is the return-efficiency trade-off parameter and R_m^δ represents the maximum achievable return, can be determined by maximizing R^δ under the constraints of $\sum_i^n w_i = 1$ and $w_i \geq 0, i = 1 \ldots n$. Model (4.3) maximizes the portfolio overall efficiency subject to a minimum achievable return set by the investor.

As an illustration of our approach, we will use the same prior sample. In addition, we will change the normalization constraint $\sum_i^n w_i = 1$ and $w_i \geq 0, i = 1 \ldots n$, by a cardinality constraint $\sum_i^n w_i = K$ and $w_i \in \{0, 1\}$ where K represents the size of the portfolio. We will set K=30 for the Mean Cross portfolio, same as the Game and Arbitrary in order to compare the different portfolios. Furthermore, we will explore the results at three different tradeoff levels, 10%, 20%, and 30%.

Table (4-9) shows the geometric mean excess return of each portfolio and benchmark indices over the 6-year test period. The "MC Portfolio" with $\gamma = 10\%, 20\%$ and 30% portfolio attains the highest geometric mean excess return respectively, **122.94%, 91.94% and 68.24%**, which are quite higher than those of the CAC40 (**0.66%**), AEX (**2.56%**), BEL20 (**4.48%**), PSI20 (**-9.39%**), "Arbitrary Portfolio" (**4.89%**) and "Game Portfolio" (**4.7%**). Furthermore, the respective Sharpe ratios of the "MC Portfolio", (**5.8942, 5.1616, and 4.0720**) are quite higher than those of the market indices(CAC40(**0.0395**), AEX (**0.1674**), BEL20(**0.3379**) and PSI20(**-0.4884)**), "Arbitrary Portfolio" (**0.8961**) and "Game Portfolio" (**0.9059**). Table (4-10) further confirms the significance of the attained results. Based on the Pvalues, we reject the null hypothesis implying that the Sharpe ratio of the "MC Portfolio" for $\gamma = 10\%$ and 20% is significantly greater than all the market indices, "Arbitrary Portfolio" and "Game Portfolio" for 1% and 5% significance level. However, we fail to prove the significance of the Sharpe ratio difference for $\gamma = 30\%$ compared to the other indices. We can infer from the failure to reject the null hypothesis for $\gamma = 30\%$ that as we increase the parameter, γ, we approach similar portfolio's characteristics to that of the game and arbitrary portfolios.

	10%	20%	30%	CAC.40	AEX	BEL20	PSI20	Game Portfolio	Arbitrary Portfolio
Annualized Return	1.2294	0.9194	0.6824	0.0066	0.0256	0.0448	-0.0939	0.0470	0.0489
Annualized Std Dev	0.2086	0.1781	0.1676	0.1661	0.1528	0.1327	0.1923	0.0519	0.0545
Annualized Sharpe	5.8942	5.1616	4.0720	0.0395	0.1674	0.3379	-0.4884	0.9059	0.8961

Table 4-9: Performance Summary

VS		**CAC 40**	**AEX**	**BEL20**	**PSI20**	**Game Portfolio**	**Arbitrary Portfolio**
	Difference	1.1383	1.1039	1.059	1.2933	0.9101	0.9127
γ=10%	P value	**(0.0016)** ***	**(0.004)** ***	**(0.0078)** ***	**(0.0018)** ***	**(0.008)** ***	**(0.0098)** ***
	Difference	1,0804	1,046	1,0011	1,2354	0,8522	0,8548
γ=20%	P value	**(0.0104)** **	**(0.0204)** **	**(0.0258)** **	**(0.0084)** ***	**(0.0346)** *	**(0.0598)** *
	Difference	0.9011	0.8667	0.8218	1.0561	0.6729	0.6755
γ=30%	P value	(0.1258)	(0.1826)	(0.332)	**(0.0692)** **	(0.378)	(0.3938)
	Difference	0.2282	0.1938	0.1489	0.3832	---------	0.0026
Game	P value	**(0.087)** *	(0.2244)	(0.1722)	**(0.0788)** **	---------	(0.9398)
	Difference	0.2256	0.1912	0.1463	0.3806	-0.0026	---------
Arbitrary	P value	**(0.0768)** *	(0.175)	(0.167)	**(0.0482)** **	(0.9392)	---------

Table 4-10: Tow sided Sharpe difference test [Significance level 1%***, 5%**, 10%*]: the Studentized Circular Block Bootstrap (B=10. M=4999)

In this section, we have managed to improve on the naïve cross based approach to portfolio selection and proved that the above described methodology can be a promising tool for portfolio selection at least for this study. However, we can point out that this methodology needs further examination, more specifically finding the optimal number of stocks for a given tradeoff factor.

12. Conclusion

The current dissertation has extended the DEA game cross-efficiency evaluation to portfolio selection. This development is motivated by the observation that the traditional simple use of cross-efficiency scores is unstable and unpredictable depending on the software. This problem is mainly due to DEA flexibility and the nature of the model which is fractional. We have addressed this issue by extending the DEA Game Cross Efficiency to portfolio selection. Finally, we have addressed a shortcoming of the naïve cross approach which is the lack of control over the expected mean return over the investment horizon through developing a Mean Cross framework of portfolio selection based on DEA game cross-efficiency evaluation, which has been proven to be effective at least for this study.

Chapter 5 Conclusion

Through this thesis, we have represented a significant version of the cross-efficiency method. We have examined in details the idea of simple efficiency, cross efficiency and game cross efficiency both intuitively and mathematically. We have pointed out the main issues and drawbacks of using some cross-efficiency models. Furthermore, we have concluded that the solution to the non-cooperative model by (Wu et al (2008)) is a stable solution due to its uniqueness and being a Nash equilibrium. We have managed to prove that game cross can be a promising tool in portfolio selection since the Nash equilibrium solution itself is a meaningful solution to the investor as it incorporates the behavioral nature of firms. Finally, we have developed a Mean Cross framework of portfolio selection based on DEA game cross-efficiency evaluation, which has been proven to be effective at least for this study.

Finally, we would like to mention some potential application of the game cross efficiency approach. This method can be widely used in the field of portfolio selection and performance assessment in banking, government etc. The late application of game cross efficiency was mainly to the Olympic games in which competition is clear and direct (Wu et al. (2009)), during which each player seeks to maximize his own desirability.

If we address further applications, we can point out the below set of possible ones.

- Advance the game cross efficiency to deal with the concept of economies of scale and imprecise data.
- Study the potential cooperation within the same portfolio
- Design of a decision support system for portfolio selection based on game cross efficiency and modern portfolio theory
- Solve for the optimal tradeoff and number of stocks
- Study the cross-efficiency matrix to determine efficient ways of incorporating the maximum information in it.

- **References**

Charnes, A., Cooper, W.W and Rhodes, E. (1978). Measuring the efficiency of DMUs. *European Journal of Operational Research*, 2, p. 429-444.

Charnes, A., Clark, C.T., Cooper, W.W., Golany, B., (1985a). A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in the US air forces. *Annals of Operations Research* 2, 95–112.

Charnes, A., Cooper, W.W., Golany, B., Seiford, L., Stutz, J., (1985b). Foundations of data envelopment analysis for Pareto–Koopmans efficient empirical production functions. *Journal of Econometrics* 30, 91–107.

Charnes, A., Cooper, W.W., Huang, Z.M., Sun, D.B., (1990). Polyhedral cone-ratio data envelopment analysis models with an illustrative application to large commercial banks. *Journal of Econometrics* 46, 73–91.

Charnes, A., Cooper, W.W., Li, S., (1989). Using data envelopment analysis to evaluate the efficiency of economic performance by Chinese cities. *Socio-Economic Planning Science* 23, 325–344.

Charnes, A., Cooper, W.W., Rhodes, E., (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research* 2, 429–444.

Charnes, A., Cooper, W.W., Seiford, L., Stutz, J., (1982). A multiplicative model for efficiency analysis. *Socio-Economic Planning Sciences* 6, 223–224.

Charnes, A., W.W. Cooper and E. Rhodes, (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2: 429-444.

Charnes, A., W.W. Cooper. (1962). Programming with linear fractional functionals. *Naval Research Logistics Quarterly*, 9, pp. 67–88

Dimitris K. Despotisa, Yiannis G. Smirlisb, (2002). Data envelopment analysis with imprecise data. *European Journal of Operational Research*, 1, 24–36

Doyle, J.R. and R. Green, (1994). Efficiency and cross-efficiency in data envelopment analysis: Derivatives meaning and uses. *Journal of the Operational Research Society*, 45 (5): 567-578.

Eling (2009). Performance measurement of hedge funds using data envelopment analysis. *Financial Markets and Portfolio Management*, 20:442

Green, R.H., Doyle, J.R., (1995). On maximizing discrimination in multiple criteria decision making. *Journal of the Operational Research Society* 46, 192–204.

Greg N. Gregoriou and Joe Zhu, (2007). Data Envelopment Analysis. The Journal of Portfolio Management Winter, 2, 120-132.

Javier Alcaraz, Nuria Ramón, José L. Ruiz, Inmaculada Sirvent, (2013). Ranking ranges in cross-efficiency evaluations. *European Journal of Operational Research*, 3, 516–521

Jie Wu · Liang, (2012). A multiple criterion ranking method based on game cross-evaluation approach. *Ann. Oper. Res*. 197:191–200.

Ken Nakabayashi, Kaoru Tone (2006), Egoist's dilemma: a DEA game. *Omega* 34, 135 – 148.

Kimball Miles S, (1992). Background Risk, Prudence, and the Demand for Insurance Contributions to Insurance Economics, Huebner International Series on Risk. *Insurance and Economic Security*, 13, 239-254

Kimball Miles S, (1993). Standard risk aversion, *Econometrica*, 3, 589-611

Liang, L., J. Wu, W.D. Cook and J. Zhu, (2008b). The DEA game cross-efficiency model and its Nash equilibrium. *Operations Research*, 56: 1278-1288.

Branda Martin, (2015). Diversification-consistent data envelopment analysis based on directional-distance measures. *Omega* 52, Pages 65–76.

Markowitz H. (1952). Portfolio Selection. *The Journal of Finance*, 1, 77-91.

Menezes, C., C. Geiss and J. Tressler (1980). Increasing Downside Risk. *American Economic Review* 5, 921-932.

Menezes C. F., Wang X. Henry, (2005). Increasing outer risk, *Journal of Mathematical Economic*, 7, 875–886

McMullen, Patrick R; Strong, Robert A, (1998). Selection of mutual funds using data envelopment analysis. *The Journal of Business and Economic Studies*, 4, 1-12.

Michael Devaney, Thibaut Morillon, William Weber, (2016). Mutual fund efficiency and tradeoffs in the production of risk and return. *Managerial Finance*, 3, 225-243.

Murthi, B.P.S., Yoon K. Choi, Preyas Desai, (1997). Efficiency of mutual funds and portfolio performance measurement: A non-parametric approach, *European Journal of Operational Research*, 98, 408-418.

Orala Muhittin, Amar Oukilb, Jean-Louis Malouinc, Ossama Kettanid, (2014). The appreciative democratic voice of DEA: A case of faculty academic performance evaluation, *Socio-Economic Planning Sciences*, 1, 20–28.

Qifa Qu, (2012). Determination of Weights for the Ultimate Cross Efficiency: A Use of Principal Component Analysis. *Technique journal of software*, 10.

Robert A. Becker1 and Subir K. Chakrabarti, (2005). Satisficing behavior, Brouwer's Fixed Point Theorem and Nash Equilibrium. *Economic Theory* 26, 63–83

Sexton, T.R., (1986). The methodology of data envelopment analysis. In Silk man, R.H. (Ed.) Measuring efficiency: An assessment of Data Envelopment Analysis, San Francisco: Jossey-Bass, 7-29.

Sexton, T.R., Silkman, R.H., Hogan, A.J., (1986). Data envelopment analysis: Critique and extensions. In: Silkman, R.H. (Ed.), Measuring Efficiency: An Assessment of Data Envelopment Analysis. Jossey-Bass, San Francisco. 73–105.

Tarnaud Albane Christine, Leleu H. (2017). Portfolio analysis with DEA: Prior to choosing a model, *Omega*. In Press, Corrected Proof, https://doi.org/10.1016/j.omega.2017.02.003.

Wade D. Cook, Kaoru Tone and Joe Zhu, (2014). Data envelopment analysis: Prior to choosing a model *Omega*, 44,1-4

Wenbin Liua, Zhongbao Zhoua, Debin Liua, Helu Xiao, (2015). Estimation of portfolio efficiency via DEA. *Omega*, 52, 107–118

Wu, J., L. Liang and F. Yang, (2009c). Determination of the weights for the ultimate cross efficiency using shapely value in cooperative game. *Expert Systems with Applications*, 36: 872-876.

Wu, J., L. Liang and Y. Chen, (2009a). DEA game cross-efficiency approach to Olympic rankings. *Omega*, 37: 909-918.

Zohrehbandian M. and S. Sadeghi Gavgani, (2013). Cross-efficiency Evaluation under the Principle of Rank Priority of DMUs. *World Applied Sciences Journal* 21 (Special Issue of Applied Math): 46-49.

Appendix A: Tests of normality

Tests of Normality						
	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
ACTEOS	0,083	72	,200*	0,981	72	0,362
ACTIA.GROUP	0,193	72	0	0,837	72	0
ADA	0,173	72	0	0,864	72	0
ADL.PARTNER	0,081	72	,200*	0,921	72	0
ADOMOS	0,233	72	0	0,81	72	0
ADTHINK.MEDIA	0,23	72	0	0,716	72	0
AFFINE.R. E	0,103	72	0,057	0,949	72	0,006
AFONE	0,135	72	0,002	0,95	72	0,006
AGTA.RECORD	0,198	72	0	0,705	72	0
AIR.FRANCE.KLM	0,082	72	,200*	0,97	72	0,084
AIRBUS.GROUP	0,064	72	,200*	0,988	72	0,704
AIR.LIQUIDE	0,047	72	,200*	0,991	72	0,908
AKKA.TECHNOLOGIES	0,078	72	,200*	0,985	72	0,54
ALBIOMA	0,087	72	,200*	0,979	72	0,278
ALCATEL.LUCENT	0,108	72	0,037	0,954	72	0,011
ALES.GROUPE	0,111	72	0,027	0,981	72	0,346
ALSTOM	0,084	72	,200*	0,937	72	0,001
ALTAMIR	0,103	72	0,055	0,968	72	0,066
ALTAREA	0,131	72	0,004	0,961	72	0,026
ALTAREIT	0,282	72	0	0,483	72	0
ALTEN	0,068	72	,200*	0,984	72	0,497
ALTRAN.TECHN	0,075	72	,200*	0,991	72	0,87
ANF.IMMOBILIER	0,064	72	,200*	0,986	72	0,625
ANTEVENIO	0,097	72	0,092	0,966	72	0,047
APRIL	0,069	72	,200*	0,986	72	0,632
AQUILA	0,125	72	0,007	0,943	72	0,003
ARCELORMITTAL	0,072	72	,200*	0,985	72	0,548
ARGAN	0,059	72	,200*	0,986	72	0,6
ARKEMA	0,073	72	,200*	0,983	72	0,461
ARTPRICE.COM	0,276	72	0	0,499	72	0
ASSYSTEM	0,059	72	,200*	0,993	72	0,965
ATOS	0,102	72	0,06	0,949	72	0,006
AUBAY	0,101	72	0,066	0,981	72	0,356

AUFEMININ	0,092	72	,200*	0,98	72	0,302
AUGROS.COSMETICS	0,188	72	0	0,805	72	0
AUPLATA	0,204	72	0	0,657	72	0
AUREA	0,105	72	0,047	0,969	72	0,076
AURES.TECHNOLOGIES	0,101	72	0,068	0,979	72	0,26
AUSY	0,056	72	,200*	0,985	72	0,553
AVANQUEST	0,149	72	0	0,834	72	0
AXA	0,086	72	,200*	0,979	72	0,259
BACCARAT	0,065	72	,200*	0,987	72	0,697
BAINS.MER.MONACO	0,144	72	0,001	0,896	72	0
BARBARA.BUI	0,208	72	0	0,6	72	0
BASTIDE.LE. CONFORT	0,111	72	0,028	0,968	72	0,064
BD. MULTI.MEDIA	0,048	72	,200*	0,988	72	0,759
BELIER	0,117	72	0,015	0,912	72	0
BENETEAU	0,131	72	0,004	0,924	72	0
BERNARD.LOISEAU	0,054	72	,200*	0,987	72	0,68
BIC	0,075	72	,200*	0,981	72	0,351
BIGBEN.INTERACTIVE	0,134	72	0,003	0,957	72	0,015
BILENDI	0,115	72	0,02	0,958	72	0,017
BIOMERIEUX	0,066	72	,200*	0,992	72	0,921
BNP.PARIBAS.ACT.A	0,05	72	,200*	0,991	72	0,899
BOIRON	0,076	72	,200*	0,964	72	0,037
BOLLORE	0,089	72	,200*	0,949	72	0,006
BONDUELLE	0,08	72	,200*	0,978	72	0,228
BOURBON	0,09	72	,200*	0,979	72	0,257
BOUYGUES	0,09	72	,200*	0,98	72	0,319
BRICORAMA	0,084	72	,200*	0,987	72	0,644
BUDGET.TELECOM	0,08	72	,200*	0,964	72	0,035
BUREAU.VERITAS	0,066	72	,200*	0,983	72	0,454
BURELLE	0,062	72	,200*	0,984	72	0,515
BUSINESS.ET.DECIS	0,049	72	,200*	0,979	72	0,276
CA.TOULOUSE.31.CCI	0,089	72	,200*	0,986	72	0,583
CAFOM	0,14	72	0,001	0,925	72	0
CAP.GEMINI	0,065	72	,200*	0,987	72	0,657
CAPELLI	0,047	72	,200*	0,989	72	0,767
CARREFOUR.PRO.DEV	0,18	72	0	0,807	72	0
CARREFOUR	0,056	72	,200*	0,985	72	0,567

CASINO.GUICHARD	0,069	72	,200*	0,986	72	0,592
CAST	0,106	72	0,044	0,95	72	0,007
CATANA.GROUP	0,153	72	0	0,817	72	0
CATERING.INTL. SCES	0,103	72	0,056	0,971	72	0,096
CBO.TERRITORIA	0,091	72	,200*	0,965	72	0,04
CEGEREAL	0,086	72	,200*	0,977	72	0,213
CEGID.GROUP	0,052	72	,200*	0,993	72	0,958
CELLECTIS	0,243	72	0	0,768	72	0
CFAO	0,097	72	0,092	0,972	72	0,102
CGG	0,079	72	,200*	0,931	72	0,001
CHARGEURS	0,129	72	0,005	0,952	72	0,008
CHAUF.URB	0,132	72	0,003	0,913	72	0
CHINA.SUPER. POWER	0,238	72	0	0,77	72	0
CHRISTIAN.DIOR	0,052	72	,200*	0,987	72	0,675
CIBOX.INTER.A. CTIV	0,251	72	0	0,873	72	0
CIC	0,107	72	0,041	0,963	72	0,033
CIE.DU. MONT.BLANC	0,128	72	0,005	0,973	72	0,129
CLASQUIN	0,068	72	,200*	0,989	72	0,81
CNIM.CONSTR.FRF.10	0,13	72	0,004	0,953	72	0,009
CNP.ASSURANCES	0,043	72	,200*	0,991	72	0,88
COFIDUR	0,13	72	0,004	0,92	72	0
SAFT	0,084	72	,200*	0,96	72	0,023
SAINT.GOBAIN	0,056	72	,200*	0,991	72	0,875
SALVEPAR	0,186	72	0	0,786	72	0
SAMSE	0,139	72	0,001	0,941	72	0,002
SANOFI	0,094	72	0,19	0,979	72	0,273
SARTORIUS.STED.BIO	0,076	72	,200*	0,973	72	0,124
SAVENCIA	0,093	72	0,197	0,967	72	0,054
SBT	0,118	72	0,014	0,946	72	0,004
SCBSM	0,098	72	0,084	0,96	72	0,021
SCHLUMBERGER	0,08	72	,200*	0,985	72	0,578
SCHNEIDER.ELECTRIC	0,094	72	0,187	0,974	72	0,14
SCOR.SE	0,097	72	0,089	0,975	72	0,162
SECHE.ENVIRONNEM	0,058	72	,200*	0,984	72	0,512
SELECTIRENTE	0,062	72	,200*	0,979	72	0,266
SEQUANA	0,103	72	0,056	0,956	72	0,013
SES	0,097	72	0,092	0,968	72	0,065

SI. PARTICIPATIONS	0,096	72	0,099	0,966	72	0,046
SIDETRADE	0,207	72	0	0,869	72	0
SIGNAUX.GIROD	0,059	72	,200*	0,986	72	0,626
SII	0,091	72	,200*	0,971	72	0,101
SIMO.INTERNATIONAL	0,16	72	0	0,915	72	0
SMALTO	0,27	72	0	0,826	72	0
SMTPC	0,142	72	0,001	0,923	72	0
SOCIETE.GENERALE	0,041	72	,200*	0,986	72	0,587
SODEXO	0,098	72	0,085	0,968	72	0,062
SODITECH.ING	0,245	72	0	0,533	72	0
SOFT.COMPUTING	0,113	72	0,023	0,947	72	0,004
SOGECLAIR	0,09	72	,200*	0,954	72	0,011
SOITEC	0,102	72	0,059	0,922	72	0
SOLOCAL.GROUP	0,486	72	0	0,12	72	0
SOLUCOM	0,07	72	,200*	0,986	72	0,621
SOLUTIONS.30.SE	0,124	72	0,008	0,968	72	0,06
SOLVAY	0,06	72	,200*	0,991	72	0,87
SOMFY.SA	0,084	72	,200*	0,978	72	0,241
SOPRA.STERIA. GROUP	0,045	72	,200*	0,99	72	0,862
SPIR.COMMUNICATION	0,162	72	0	0,811	72	0
SPOREVER	0,114	72	0,021	0,961	72	0,025
SQLI	0,071	72	,200*	0,987	72	0,646
ST. DUPONT	0,223	72	0	0,758	72	0
STEF	0,099	72	0,077	0,964	72	0,038
STMICROELECTRONICS	0,122	72	0,01	0,962	72	0,028
STORE.ELECTRONICS	0,094	72	0,19	0,945	72	0,004
STRADIM.ESPAC.FIN	0,079	72	,200*	0,968	72	0,061
STREAMWIDE	0,131	72	0,004	0,907	72	0
SUEZ.ENVIRONNEMENT	0,076	72	,200*	0,982	72	0,398
SWORD.GROUP	0,101	72	0,066	0,965	72	0,045
SYNERGIE	0,104	72	0,052	0,975	72	0,166
TECHNICOLOR	0,141	72	0,001	0,888	72	0
TECHNIP	0,067	72	,200*	0,984	72	0,47
TELEPERFORMANCE	0,049	72	,200*	0,992	72	0,945
TERREIS	0,1	72	0,07	0,975	72	0,158
TESSI	0,055	72	,200*	0,985	72	0,533
TF1	0,06	72	,200*	0,99	72	0,818

TFF.GROUP	0,127	72	0,006	0,913	72	0
THALES	0,076	72	,200*	0,987	72	0,678
THERMADOR.GROUPE	0,091	72	,200*	0,968	72	0,06
TIPIAK	0,082	72	,200*	0,957	72	0,016
TIVOLY	0,159	72	0	0,914	72	0
TONNA.ELECTRONIQUE	0,196	72	0	0,807	72	0
TOTAL.GABON	0,08	72	,200*	0,988	72	0,738
TOTAL	0,058	72	,200*	0,987	72	0,656
TOUAX	0,095	72	0,174	0,973	72	0,123
TOUPARGEL.GROUPE	0,129	72	0,005	0,941	72	0,002
TOUR.EIFFEL	0,091	72	,200*	0,981	72	0,339
TRANSGENE	0,137	72	0,002	0,847	72	0
TRAQUEUR	0,154	72	0	0,901	72	0
TRIGANO	0,055	72	,200*	0,988	72	0,757
TRILOGIQ	0,094	72	0,186	0,97	72	0,088
TURENNE.INV	0,138	72	0,002	0,894	72	0
UNIBAIL.RODAMCO	0,072	72	,200*	0,982	72	0,374
VALEO	0,086	72	,200*	0,974	72	0,134
VALLOUREC	0,085	72	,200*	0,972	72	0,103
VALNEVA	0,074	72	,200*	0,962	72	0,03
VALTECH	0,077	72	,200*	0,966	72	0,046
VDI.GROUP	0,184	72	0	0,894	72	0
VELCAN	0,118	72	0,014	0,96	72	0,022
VEOLIA.ENVIRON	0,06	72	,200*	0,99	72	0,829
VET.AFFAIRES	0,214	72	0	0,627	72	0
VETOQUINOL	0,072	72	,200*	0,982	72	0,383
VICAT	0,088	72	,200*	0,977	72	0,202
VIDELIO	0,108	72	0,037	0,894	72	0
VIEL.ET.COMPAGNIE	0,068	72	,200*	0,982	72	0,377
VILMORIN	0,046	72	,200*	0,991	72	0,872
VINCI	0,094	72	0,193	0,982	72	0,413
VIRBAC	0,077	72	,200*	0,99	72	0,854
VISIODENT	0,195	72	0	0,79	72	0
VIVENDI	0,07	72	,200*	0,979	72	0,262
VOLTALIA	0,178	72	0	0,784	72	0
VOYAGEURS.DU. MONDE	0,191	72	0	0,914	72	0
VRANKEN.POMMERY	0,108	72	0,037	0,979	72	0,278

WEBORAMA	0,094	72	0,185	0,971	72	0,099
WENDEL	0,085	72	,200*	0,967	72	0,053
ZCCM	0,181	72	0	0,819	72	0
ZODIAC.AEROSPACE	0,094	72	0,193	0,938	72	0,002
EASYVISTA	0,221	72	0	0,708	72	0
ECA	0,108	72	0,037	0,873	72	0
EDF	0,056	72	,200*	0,98	72	0,325
EFESO.CONSULTING	0,081	72	,200*	0,977	72	0,201
EGIDE	0,184	72	0	0,829	72	0
EIFFAGE	0,08	72	,200*	0,987	72	0,67
ELEC.MADAGASCAR	0,109	72	0,033	0,946	72	0,004
ENCRES.DUBUIT	0,124	72	0,008	0,948	72	0,005
ENGIE	0,082	72	,200*	0,963	72	0,032
ENTREPARTICULIERS	0,1	72	0,071	0,892	72	0
ENVIRONNEMENT.SA	0,092	72	,200*	0,971	72	0,096
EO2	0,059	72	,200*	0,985	72	0,56
ERAMET	0,083	72	,200*	0,977	72	0,221
ESI.GROUP	0,066	72	,200*	0,955	72	0,012
ESKER	0,134	72	0,003	0,959	72	0,019
ESPERITE	0,168	72	0	0,814	72	0
ESSILOR.INTL	0,07	72	,200*	0,989	72	0,762
ESSO	0,071	72	,200*	0,977	72	0,199
ETAM.DEVELOPPEMENT	0,1	72	0,073	0,974	72	0,134
EULER.HERMES. GROUP	0,07	72	,200*	0,988	72	0,728
EURAZEO	0,065	72	,200*	0,99	72	0,821
EURO.DISNEY	0,252	72	0	0,615	72	0
EURO.RESSOURCES	0,161	72	0	0,893	72	0
EUROFINS.CEREP	0,486	72	0	0,111	72	0
EUROFINS.SCIENT	0,081	72	,200*	0,963	72	0,032
EUROGERM	0,188	72	0	0,832	72	0
EUROMEDIS.GROUPE	0,144	72	0,001	0,918	72	0
EUROPACORP	0,149	72	0	0,937	72	0,001
EUROSIC	0,164	72	0	0,912	72	0
EUTELSAT.COMMUNIC	0,098	72	0,083	0,95	72	0,006
EVOLIS	0,09	72	,200*	0,97	72	0,084
EXACOMPTA.CLAIREF	0,147	72	0,001	0,913	72	0
EXEL.INDUSTRIES	0,1	72	0,071	0,964	72	0,04

EXPLOS.PROD.CHI.PF	0,103	72	0,056	0,97	72	0,086
EXPLOSIFS.PROD.CHI	0,136	72	0,002	0,901	72	0
FAIVELEY.TRANSPORT	0,116	72	0,018	0,915	72	0
FASHION.B.AIR	0,141	72	0,001	0,93	72	0,001
LECTRA	0,18	72	0	0,847	72	0
LEGRAND	0,071	72	,200*	0,989	72	0,766
LES.HOTELS. BAVEREZ	0,133	72	0,003	0,964	72	0,036
LESNXCONSTRUCTEURS	0,107	72	0,039	0,958	72	0,016
LEXIBOOK.LINGUIST	0,197	72	0	0,581	72	0
LINEDATA.SERVICES	0,142	72	0,001	0,956	72	0,013
LIONAX	0,214	72	0	0,902	72	0
LISI	0,08	72	,200*	0,988	72	0,707
LOCINDUS	0,101	72	0,064	0,977	72	0,204
LOGIC.INSTRUMENT	0,141	72	0,001	0,908	72	0
LOGIN.PEOPLE	0,157	72	0	0,894	72	0
L. 039.OREAL	0,079	72	,200*	0,974	72	0,145
LVMH	0,071	72	,200*	0,988	72	0,74
M.R.M	0,125	72	0,007	0,885	72	0
MAISONS.FRANCE	0,086	72	,200*	0,982	72	0,413
MAKHEIA.GROUP	0,188	72	0	0,791	72	0
MALTERIES.FCO.BEL	0,119	72	0,014	0,932	72	0,001
MANITOU.BF	0,107	72	0,041	0,961	72	0,025
MANUTAN.INTL	0,069	72	,200*	0,978	72	0,231
MAROC.TELECOM	0,087	72	,200*	0,977	72	0,209
MASTRAD	0,202	72	0	0,734	72	0
MAUREL.ET.PROM	0,062	72	,200*	0,977	72	0,215
MBWS	0,18	72	0	0,795	72	0
MEDASYS	0,146	72	0,001	0,843	72	0
MEDIA.6	0,183	72	0	0,864	72	0
MEDICREA.INTERNAT	0,107	72	0,039	0,936	72	0,001
MEMSCAP.REGPT	0,119	72	0,013	0,868	72	0
MERCIALYS	0,079	72	,200*	0,978	72	0,252
MERCK.AND.CO.INC	0,05	72	,200*	0,991	72	0,873
MERSEN	0,068	72	,200*	0,984	72	0,5
METABOLIC.EXPLORER	0,112	72	0,026	0,922	72	0
METROPOLE.TV	0,105	72	0,048	0,976	72	0,192
MG. INTERNATIONAL	0,14	72	0,001	0,878	72	0

MGI.DIGITAL. GRAPHI	0,098	72	0,082	0,964	72	0,038
MGI.COUTIER	0,081	72	,200*	0,99	72	0,818
MICHELIN	0,101	72	0,065	0,99	72	0,835
MICROPOLE	0,087	72	,200*	0,973	72	0,122
MICROWAVE.VISION	0,111	72	0,029	0,973	72	0,124
MIGUET.ET.ASSOCIES	0,193	72	0	0,8	72	0
MONTEA.C.V. A	0,081	72	,200*	0,985	72	0,541
MONTUPET	0,095	72	0,181	0,979	72	0,268
MR. BRICOLAGE	0,159	72	0	0,917	72	0
NATIXIS	0,054	72	,200*	0,992	72	0,93
NATUREX	0,159	72	0	0,899	72	0
NEOPOST	0,083	72	,200*	0,989	72	0,799
NETBOOSTER	0,145	72	0,001	0,851	72	0
NETGEM	0,084	72	,200*	0,955	72	0,011
NEURONES	0,064	72	,200*	0,995	72	0,997
NEWSINVEST	0,163	72	0	0,885	72	0
NEXANS	0,056	72	,200*	0,995	72	0,991
NEXITY	0,078	72	,200*	0,987	72	0,687
NEXTEDIA	0,104	72	0,05	0,971	72	0,093
NEXTRADIOTV	0,099	72	0,075	0,948	72	0,005
NICOX	0,166	72	0	0,865	72	0
NORBERT.DENTRESS	0,089	72	,200*	0,963	72	0,033
NRJ.GROUP	0,084	72	,200*	0,985	72	0,576
NSC.GROUPE	0,103	72	0,055	0,942	72	0,002
O2I	0,067	72	,200*	0,991	72	0,877
OCTO.TECHNOLOGY	0,142	72	0,001	0,855	72	0
OENEO	0,091	72	,200*	0,963	72	0,031
OL. GROUPE	0,146	72	0,001	0,861	72	0
ONXEO	0,129	72	0,005	0,776	72	0
ORANGE	0,098	72	0,085	0,952	72	0,008
ORAPI	0,057	72	,200*	0,992	72	0,927
ORCO.PROPERTY.GRP	0,075	72	,200*	0,977	72	0,219
OREGE	0,091	72	,200*	0,986	72	0,606
OROLIA	0,156	72	0	0,902	72	0
ORPEA	0,091	72	,200*	0,982	72	0,404
OXIS.INTL	0,378	72	0	0,712	72	0
PAREF	0,109	72	0,034	0,963	72	0,032

PARROT	0,068	72	,200*	0,986	72	0,586
PASSAT	0,075	72	,200*	0,986	72	0,619
PATRIMOINE.ET.COMM	0,156	72	0	0,877	72	0
PCAS	0,095	72	0,173	0,934	72	0,001
PERNOD.RICARD	0,063	72	,200*	0,993	72	0,966
PERRIER.GERARD	0,076	72	,200*	0,977	72	0,206
PEUGEOT	0,071	72	,200*	0,948	72	0,005
PHARMAGEST.INTER	0,102	72	0,061	0,966	72	0,049
PHILIP.MORRIS. INTL	0,081	72	,200*	0,985	72	0,564
PIERRE.VACANCES	0,091	72	,200*	0,991	72	0,874
PISCINES.DESJOYAUX	0,123	72	0,009	0,954	72	0,011
PLAST.VAL.LOIRE	0,1	72	0,074	0,955	72	0,011
PLASTIC.OMNIUM	0,064	72	,200*	0,979	72	0,278
POUJOULAT	0,096	72	0,1	0,967	72	0,053
PRECIA	0,111	72	0,028	0,963	72	0,034
PRISMAFLEX.INTL	0,159	72	0	0,771	72	0
PROCTER.GAMBLE	0,059	72	,200*	0,988	72	0,723
PRODWARE	0,093	72	0,196	0,961	72	0,025
PSB.INDUSTRIES	0,071	72	,200*	0,992	72	0,923
PUBLICIS.BSA	0,052	72	,200*	0,993	72	0,956
PUBLICIS.GROUPE.SA	0,07	72	,200*	0,989	72	0,777
QUANTEL	0,26	72	0	0,478	72	0
RADIALL	0,181	72	0	0,867	72	0
RALLYE	0,083	72	,200*	0,971	72	0,095
RAMSAY.GEN.SANTE	0,118	72	0,014	0,961	72	0,024
RECYLEX.S. A	0,254	72	0	0,536	72	0
REMY.COINTREAU	0,058	72	,200*	0,991	72	0,91
RENAULT	0,061	72	,200*	0,988	72	0,753
REXEL	0,056	72	,200*	0,99	72	0,857
RIBER	0,111	72	0,027	0,961	72	0,025
ROBERTET	0,11	72	0,03	0,937	72	0,001
ROTHSCHILD	0,104	72	0,053	0,958	72	0,016
ROUGIER.S. A	0,101	72	0,068	0,963	72	0,033
RUBIS	0,094	72	0,183	0,973	72	0,13
S.E. B	0,069	72	,200*	0,972	72	0,11
SABETON	0,06	72	,200*	0,991	72	0,891
SAFRAN	0,113	72	0,024	0,977	72	0,198

GROUPE.GORGE	0,199	72	0	0,586	72	0
GROUPE.GUILLIN	0,099	72	0,08	0,957	72	0,016
GROUPE.OPEN	0,08	72	,200*	0,968	72	0,061
GROUPE.PARTOUCHE	0,071	72	,200*	0,988	72	0,735
GROUPE.SFPI	0,192	72	0	0,741	72	0
GROUPIMO	0,148	72	0	0,824	72	0
GUERBET	0,106	72	0,045	0,938	72	0,002
GUILLEMOT	0,173	72	0	0,865	72	0
GUY.DEGRENNE	0,197	72	0	0,906	72	0
HARVEST	0,122	72	0,009	0,962	72	0,027
HAULOTTE.GROUP	0,076	72	,200*	0,98	72	0,304
HAVAS	0,065	72	,200*	0,988	72	0,75
HERMES.INTL	0,084	72	,200*	0,986	72	0,6
HEURTEY.PETROCHEM	0,09	72	,200*	0,978	72	0,241
HF	0,079	72	,200*	0,982	72	0,396
HIGH.CO	0,058	72	,200*	0,974	72	0,134
HI. MEDIA	0,089	72	,200*	0,952	72	0,008
HIOLLE.INDUSTRIES	0,07	72	,200*	0,987	72	0,683
HOLOSFIND	0,418	72	0	0,192	72	0
HOPSCOTCH.GROUPE	0,157	72	0	0,897	72	0
HOTELS.DE.PARIS	0,07	72	,200*	0,983	72	0,455
HSBC.HOLDINGS	0,066	72	,200*	0,98	72	0,316
HUBWOO	0,125	72	0,008	0,949	72	0,006
HYBRIGENICS	0,238	72	0	0,678	72	0
I2S	0,115	72	0,019	0,978	72	0,227
ICADE	0,069	72	,200*	0,99	72	0,855
IDI	0,07	72	,200*	0,985	72	0,54
IDS	0,127	72	0,005	0,885	72	0
IDSUD	0,193	72	0	0,882	72	0
IGE...XAO	0,062	72	,200*	0,988	72	0,726
ILIAD	0,096	72	0,099	0,979	72	0,26
IMERYS	0,09	72	,200*	0,966	72	0,051
IMMOB.DASSAULT	0,142	72	0,001	0,913	72	0
IMPRIMERIE.CHIRAT	0,161	72	0	0,816	72	0
INFOTEL	0,109	72	0,034	0,977	72	0,212
INGENICO.GROUP	0,106	72	0,044	0,985	72	0,544
FAURECIA	0,072	72	,200*	0,966	72	0,049

FDL	0,158	72	0	0,94	72	0,002
FFP	0,078	72	,200*	0,983	72	0,418
FIDUCIAL.OFF.SOL	0,399	72	0	0,532	72	0
FIDUCIAL.REAL.EST	0,102	72	0,061	0,978	72	0,233
FILAE	0,14	72	0,001	0,889	72	0
FIMALAC	0,084	72	,200*	0,988	72	0,732
INNATE.PHARMA	0,221	72	0	0,75	72	0
INNELEC.MULTIMEDIA	0,108	72	0,037	0,976	72	0,188
INSTALLUX	0,055	72	,200*	0,99	72	0,842
INTERPARFUMS	0,047	72	,200*	0,992	72	0,936
INTLE.PLANT. HEVEAS	0,084	72	,200*	0,982	72	0,384
IPSEN	0,071	72	,200*	0,979	72	0,258
IPSOS	0,065	72	,200*	0,975	72	0,157
IRDNORDPASDECALAIS	0,245	72	0	0,803	72	0
IT. LINK	0,17	72	0	0,85	72	0
ITESOFT	0,122	72	0,01	0,929	72	0,001
ITS.GROUP	0,137	72	0,002	0,862	72	0
IVALIS	0,3	72	0	0,587	72	0
JACQUET.METAL.SCE	0,067	72	,200*	0,986	72	0,61
JC.DECAUX.SA	0,067	72	,200*	0,981	72	0,354
KAUFMAN.ET.BROAD	0,117	72	0,017	0,969	72	0,07
KERING	0,107	72	0,039	0,969	72	0,069
KEYRUS	0,171	72	0	0,879	72	0
KEYYO	0,147	72	0,001	0,897	72	0
KINDY	0,083	72	,200*	0,962	72	0,03
KLEPIERRE	0,112	72	0,025	0,987	72	0,674
KORIAN	0,129	72	0,005	0,94	72	0,002
LAFUMA	0,152	72	0	0,833	72	0
LAGARDERE.S.C. A	0,071	72	,200*	0,988	72	0,708
LANSON.BCC	0,075	72	,200*	0,981	72	0,337
LAURENT.PERRIER	0,098	72	0,084	0,971	72	0,098
LDC	0,138	72	0,002	0,933	72	0,001
LDLC.COM	0,076	72	,200*	0,984	72	0,473
LE.NOBLE.AGE	0,123	72	0,008	0,952	72	0,008
LE. TANNEUR	0,152	72	0	0,947	72	0,004
LEBON	0,046	72	,200*	0,994	72	0,987
FIN.ETANG. BERRE	0,13	72	0,004	0,969	72	0,071

FINANCIERE.ODET	0,073	72	,200*	0,973	72	0,128
FINATIS	0,141	72	0,001	0,925	72	0
FLEURY.MICHON	0,084	72	,200*	0,961	72	0,026
FONC.DES.REGIONS	0,077	72	,200*	0,992	72	0,949
FONCIERE.ATLAND	0,205	72	0	0,918	72	0
FONCIERE.DE.PARIS	0,108	72	0,038	0,966	72	0,051
FONCIERE.DES.MURS	0,083	72	,200*	0,976	72	0,193
FONCIERE.EURIS	0,117	72	0,016	0,926	72	0
FONCIERE.INEA	0,153	72	0	0,934	72	0,001
FONCIERE.LYONNAISE	0,115	72	0,02	0,961	72	0,027
FOUNTAINE.PAJOT	0,11	72	0,032	0,919	72	0
FREY	0,144	72	0,001	0,892	72	0
FROMAGERIES.BEL	0,076	72	,200*	0,897	72	0
FUTUREN	0,17	72	0	0,83	72	0
GAMELOFT.SE	0,106	72	0,045	0,908	72	0
GAUMONT	0,067	72	,200*	0,977	72	0,212
GAUSSIN	0,1	72	0,072	0,935	72	0,001
GEA.GRENOBL. ELECT	0,09	72	,200*	0,957	72	0,016
GECINA.NOM	0,067	72	,200*	0,968	72	0,061
GEMALTO	0,057	72	,200*	0,989	72	0,784
GENERAL.ELECTRIC	0,068	72	,200*	0,979	72	0,255
GENERIX	0,117	72	0,016	0,933	72	0,001
GENFIT	0,173	72	0	0,842	72	0
GENOWAY	0,123	72	0,009	0,894	72	0
GEVELOT	0,137	72	0,002	0,815	72	0
GFI.INFORMATIQUE	0,085	72	,200*	0,973	72	0,121
GIORGIO.FEDON	0,212	72	0	0,688	72	0
GL. EVENTS	0,093	72	,200*	0,977	72	0,207
GPE.GROUP. PIZZORNO	0,101	72	0,064	0,969	72	0,073
GRAINES.VOLTZ	0,183	72	0	0,838	72	0
GRAND.MARNIER	0,175	72	0	0,816	72	0
GROUPE.CRIT	0,062	72	,200*	0,986	72	0,609
GROUPE.EUROTUNNEL	0,09	72	,200*	0,972	72	0,103
GROUPE.FLO	0,078	72	,200*	0,98	72	0,322
COHERIS	0,148	72	0	0,929	72	0,001
COIL	0,108	72	0,037	0,942	72	0,002
COLAS	0,072	72	,200*	0,98	72	0,303

COURTOIS	0,138	72	0,002	0,87	72	0
CRCAM.ALP.PROV.CCI	0,08	72	,200*	0,985	72	0,544
CRCAM.ATL.VEND.CCI	0,097	72	0,087	0,967	72	0,056
CRCAM.BRIE.PIC2CCI	0,088	72	,200*	0,985	72	0,539
CRCAM.ILLE.VIL.CCI	0,069	72	,200*	0,984	72	0,476
CRCAM.LANGUED.CCI	0,071	72	,200*	0,977	72	0,2
CRCAM.LOIRE.HTE.L	0,136	72	0,002	0,95	72	0,006
CRCAM.MORBIHAN.CCI	0,065	72	,200*	0,991	72	0,878
CRCAM.NORD.CCI	0,05	72	,200*	0,99	72	0,82
CRCAM.NORM. SEINE	0,077	72	,200*	0,952	72	0,008
CRCAM.SUD.R.A.CCI	0,079	72	,200*	0,983	72	0,418
CRCAM.TOURAINE.CCI	0,071	72	,200*	0,981	72	0,334
CREDIT.AGRICOLE	0,079	72	,200*	0,982	72	0,383
CYBERGUN	0,098	72	0,082	0,984	72	0,511
D.L.S. I	0,117	72	0,015	0,924	72	0
DALENYS	0,059	72	,200*	0,978	72	0,247
DALET	0,094	72	0,192	0,929	72	0,001
DAMARTEX	0,139	72	0,001	0,944	72	0,003
DANONE	0,054	72	,200*	0,985	72	0,537
DARTY.PLC	0,069	72	,200*	0,966	72	0,046
DASSAULT.AVIATION	0,136	72	0,002	0,9	72	0
DASSAULT.SYSTEMES	0,079	72	,200*	0,988	72	0,736
DELFINGEN	0,102	72	0,06	0,942	72	0,002
DELTA.PLUS. GROUP	0,126	72	0,007	0,934	72	0,001
DEMOS	0,123	72	0,009	0,929	72	0,001
DERICHEBOURG	0,099	72	0,076	0,891	72	0
DEVOTEAM	0,076	72	,200*	0,988	72	0,752
DEXIA	0,206	72	0	0,763	72	0
DIAGNOSTIC.MEDICAL	0,145	72	0,001	0,864	72	0
DIAXONHIT	0,101	72	0,068	0,93	72	0,001
DIRECT.ENERGIE	0,185	72	0	0,903	72	0
DL.SOFTWARE	0,164	72	0	0,944	72	0,003
DNXCORP	0,118	72	0,014	0,854	72	0
DOM.SECURITY	0,095	72	0,175	0,915	72	0

Appendix B: Stock ranking based on game cross and Arbitrary scores

2012			2013		
	Arbitrary	Game cross		Arbitrary	Game cross
L' OREAL	0,9575	0,993	EULER HERMES GROUP	0,979	1
IVALIS	0,9557	0,9918	MAROC TELECOM	0,7624	0,8447
CNIM CONSTR.FRF 10	0,7365	0,7985	TURENNE INV	0,7453	0,8299
SABETON	0,7353	0,7843	VIEL ET COMPAGNIE	0,6872	0,8093
VILMORIN	0,6828	0,7304	HI-MEDIA	0,6616	0,7232
SOFRAGI	0,6793	0,7544	RUBIS	0,6099	0,6736
ZODIAC AEROSPACE	0,4392	0,4704	VETOQUINOL	0,6008	0,6512
TFF GROUP	0,3554	0,3853	CIC	0,5784	0,6146
BERNARD LOISEAU	0,3551	0,3796	QUANTEL	0,57	0,643
LEBON	0,353	0,3693	PROCTER GAMBLE	0,563	0,6016
MERCIALYS	0,3469	0,3694	LEGRAND	0,5572	0,5884
DASSAULT SYSTEMES	0,3422	0,3699	UNIBAIL-RODAMCO	0,5434	0,6238
SES	0,3406	0,3603	EUROGERM	0,5397	0,641
GAMELOFT SE	0,3308	0,3505	SELECTIRENTE	0,5322	0,5537
BRICORAMA	0,3245	0,3496	AXA	0,5247	0,5556
INSTALLUX	0,3192	0,3392	AIR LIQUIDE	0,5178	0,5899
SARTORIUS STED BIO	0,3161	0,338	ATOS	0,5109	0,5547
PERRIER (GERARD)	0,3138	0,3335	HOPSCOTCH GROUPE	0,5091	0,5537
STEF	0,3103	0,3603	SCOR SE	0,5037	0,5694
ADL PARTNER	0,3032	0,3286	AFFINE R.E.	0,4997	0,5469
TRILOGIQ	0,2814	0,3031	GROUPE PARTOUCHE	0,4993	0,5833
HOTELS DE PARIS	0,2809	0,3008	CRCAM ATL.VEND.CCI	0,4708	0,5077
SIDETRADE	0,2798	0,3113	BOURBON	0,4683	0,5371
RAMSAY GEN SANTE	0,2701	0,2928	NATUREX	0,4616	0,5215
ENVIRONNEMENT SA	0,2696	0,2846	NETBOOSTER	0,4513	0,5736
ICADE	0,2623	0,2891	VDI GROUP	0,4451	0,5155
BIC	0,2509	0,2709	FONCIERE LYONNAISE	0,4378	0,5134
BONDUELLE	0,2492	0,268	GIORGIO FEDON	0,4342	0,4606
PRODWARE	0,2453	0,2607	ESSO	0,4259	0,4919
DANONE	0,2447	0,2672	INTERPARFUMS	0,4193	0,449
UNIBAIL-RODAMCO	0,2404	0,261	MEDIA 6	0,4181	0,4799
S.E.B.	0,2391	0,26	EURO DISNEY	0,4165	0,4397
BUREAU VERITAS	0,2367	0,254	IDI	0,4164	0,4469
AUFEMININ	0,235	0,2574	NEURONES	0,4145	0,469

PUBLICIS GROUPE SA	0,2325	0,2553	FONCIERE ATLAND	0,4131	0,4692
COFIDUR	0,2302	0,2471	NSC GROUPE	0,4035	0,464
VALTECH	0,2245	0,2458	NEOPOST	0,3981	0,4427
GEA GRENOBL.ELECT.	0,2231	0,2448	CAP GEMINI	0,396	0,4268
EXPLOSIFS PROD.CHI	0,2223	0,2412	FONCIERE EURIS	0,3901	0,432
SOLUCOM	0,2213	0,2479	LEBON	0,3858	0,4412
ESSILOR INTL.	0,2208	0,2378	COURTOIS	0,3838	0,4511
MGI DIGITAL GRAPHI	0,2142	0,2359	MAKHEIA GROUP	0,3829	0,438
EUROGERM	0,2106	0,2336	ST DUPONT	0,3728	0,4479
VALEO	0,209	0,2213	FONCIERE INEA	0,368	0,4352
ROTHSCHILD	0,2067	0,2236	SES	0,3678	0,4179
ESKER	0,2057	0,2254	FIPP	0,3654	0,3998
ILIAD	0,2049	0,2175	THALES	0,3639	0,3954
MALTERIES FCO-BEL.	0,2041	0,2207	NEXTRADIOTV	0,3637	0,4202
ANTEVENIO	0,203	0,2232	CBO TERRITORIA	0,3625	0,4209
FDL	0,2001	0,2089	LANSON-BCC	0,3615	0,426
FINANCIERE ODET	0,1997	0,2163	IRDNORDPASDECALAIS	0,3602	0,4781
NSE	0,1963	0,2114	TOTAL	0,3599	0,408
BIGBEN INTERACTIVE	0,1953	0,2102	DANONE	0,359	0,4065
GAUMONT	0,1947	0,2105	AUBAY	0,3578	0,4055
FONCIERE INEA	0,1934	0,2135	L' OREAL	0,3567	0,4232
SAFRAN	0,1904	0,2019	SCHLUMBERGER	0,3562	0,3971
TELEPERFORMANCE	0,1885	0,207	EIFFAGE	0,3548	0,3871
AUSY	0,1884	0,2045	PHARMAGEST INTER.	0,3545	0,4073
IPSOS	0,1883	0,2074	CFAO	0,3537	0,3882
BOLLORE	0,1878	0,2026	VALTECH	0,3531	0,4176
SI PARTICIPATIONS	0,1856	0,1988	SWORD GROUP	0,349	0,3691
IMMOB.DASSAULT	0,185	0,2045	VINCI	0,3481	0,3712
OCTO TECHNOLOGY	0,1842	0,1968	LVMH	0,3479	0,4011
VOYAGEURS DU MONDE	0,183	0,1975	INFOTEL	0,3475	0,3639
LES HOTELS BAVEREZ	0,1829	0,1972	KAUFMAN ET BROAD	0,3381	0,4133
MONTEA C.V.A.	0,1793	0,1954	S.E.B.	0,3348	0,3512
GUERBET	0,1782	0,1926	GPE GROUP PIZZORNO	0,3346	0,3689
PHILIP MORRIS INTL	0,1761	0,1896	VICAT	0,333	0,3645
GENERAL ELECTRIC	0,1747	0,1889	SI PARTICIPATIONS	0,3315	0,3745
CFAO	0,1738	0,1919	SABETON	0,3304	0,3812
FIMALAC	0,1731	0,1844	FDL	0,3298	0,3902

GIORGIO FEDON	0,1719	0,1884	DELTA PLUS GROUP	0,3286	0,362
FROMAGERIES BEL	0,1714	0,1861	DASSAULT SYSTEMES	0,3283	0,3559
SELECTIRENTE	0,1699	0,1857	CEGID GROUP	0,3281	0,3833
SODEXO	0,1696	0,187	OENEO	0,3272	0,3489
ATOS	0,1693	0,1843	TIVOLY	0,3267	0,3733
CASINO GUICHARD	0,1692	0,1886	EURO RESSOURCES	0,3249	0,3533
SOGECLAIR	0,1691	0,182	LISI	0,3244	0,3406
LECTRA	0,1681	0,18	GRAINES VOLTZ	0,323	0,3677
RIBER	0,1672	0,1788	SAVENCIA	0,3201	0,3658
GUILLEMOT	0,167	0,1805	AUSY	0,3159	0,3622
MERCK AND CO INC	0,1667	0,1821	ORPEA	0,3157	0,3443
PROCTER GAMBLE	0,1667	0,1794	IPSOS	0,3141	0,3327
DNXCORP	0,1661	0,1819	ZODIAC AEROSPACE	0,3122	0,3865
OROLIA	0,1655	0,1796	TESSI	0,3107	0,3322
CIE DU MONT BLANC	0,1646	0,1729	ADL PARTNER	0,3068	0,3361
TOTAL	0,1616	0,1772	LINEDATA SERVICES	0,3057	0,3612
VIEL ET COMPAGNIE	0,1608	0,1752	MICHELIN	0,3047	0,3435
REMY COINTREAU	0,1588	0,174	KEYYO	0,3036	0,356
INFOTEL	0,158	0,1713	HERMES INTL	0,301	0,3478
ALBIOMA	0,1574	0,1672	CIE DU MONT BLANC	0,3009	0,3482
HAVAS	0,1567	0,1715	NRJ GROUP	0,2984	0,333
SPOREVER	0,1564	0,1731	JC DECAUX SA.	0,2939	0,3147
CA TOULOUSE 31 CCI	0,1559	0,1694	VOYAGEURS DU MONDE	0,2917	0,3526
EUROMEDIS GROUPE	0,1556	0,1732	FFP	0,2917	0,3207
BURELLE	0,1555	0,1694	BACCARAT	0,2913	0,3126
FREY	0,1549	0,1646	PSB INDUSTRIES	0,2899	0,3416
CRCAM SUD R.A.CCI	0,1545	0,1693	INSTALLUX	0,2891	0,3192
ALTEN	0,1541	0,167	GAUSSIN	0,2891	0,3062
AGTA RECORD	0,1539	0,1673	CHAUF.URB.	0,2889	0,3219

2014			2015		
	Arbitrary	Game cross		Arbitrary	Game cross
MERCK AND CO INC	0,975	1	ALTAREIT	0,9788	1
DANONE	0,8362	0,9069	CFAO	0,7314	0,8052
BNP PARIBAS ACT.A	0,7291	0,824	CRCAM NORM.SEINE	0,5922	0,6544
LE NOBLE AGE	0,7256	0,8238	CHARGEURS	0,5718	0,6453
SIGNAUX GIROD	0,6738	0,783	SELECTIRENTE	0,5378	0,6009
SABETON	0,6561	0,7219	CHAUF.URB.	0,525	0,6528
PAREF	0,6457	0,7202	INSTALLUX	0,5138	0,6008
EXEL INDUSTRIES	0,6305	0,7515	PRECIA	0,4755	0,6102
CRCAM TOURAINE CCI	0,6292	0,7132	CRCAM LANGUED CCI	0,4686	0,5616
NETBOOSTER	0,6081	0,696	LINEDATA SERVICES	0,4587	0,4787
GROUPE EUROTUNNEL	0,6012	0,6562	FLEURY MICHON	0,4584	0,5089
SOFRAGI	0,5986	0,6766	SAINT GOBAIN	0,4451	0,4734
ALES GROUPE	0,5859	0,6387	COURTOIS	0,4326	0,5219
CFAO	0,585	0,6492	EUROMEDIS GROUPE	0,4302	0,5429
MONTEA C.V.A.	0,5771	0,6434	SMTPC	0,4279	0,4828
BIOMERIEUX	0,5771	0,6217	EFESO CONSULTING	0,4238	0,478
ADL PARTNER	0,5731	0,6615	VEOLIA ENVIRON.	0,4222	0,4959
TFF GROUP	0,5596	0,6166	TURENNE INV	0,4208	0,5008
BIC	0,5405	0,5798	CRCAM SUD R.A.CCI	0,4145	0,4719
DASSAULT SYSTEMES	0,5313	0,5826	NEURONES	0,4115	0,4618
BERNARD LOISEAU	0,5294	0,6077	LAFUMA	0,4109	0,4688
SELECTIRENTE	0,5241	0,5895	EURAZEO	0,406	0,4617
COURTOIS	0,5219	0,5886	SCBSM	0,4042	0,4872
HSBC HOLDINGS	0,5159	0,5493	SALVEPAR	0,3902	0,4615
PROCTER GAMBLE	0,5138	0,5774	ATOS	0,3839	0,4356
ESPERITE	0,5035	0,5788	PATRIMOINE ET COMM	0,3824	0,4434
SIMO INTERNATIONAL	0,4911	0,5661	GROUPE EUROTUNNEL	0,3814	0,4285
SAFT	0,4889	0,5665	TERREIS	0,3793	0,4078
CASINO GUICHARD	0,4788	0,5498	THERMADOR GROUPE	0,376	0,4136
EULER HERMES GROUP	0,4779	0,5231	FONCIERE DE PARIS	0,3755	0,4207
CA TOULOUSE 31 CCI	0,4757	0,5263	SYNERGIE	0,3724	0,4188
ARCELORMITTAL	0,4752	0,5487	ROBERTET	0,3706	0,4079
VIRBAC	0,4681	0,5336	CAPELLI	0,3696	0,4401
ORAPI	0,4643	0,5415	MAKHEIA GROUP	0,3688	0,4101
AURES TECHNOLOGIES	0,4628	0,5327	SES	0,3589	0,4027

JC DECAUX SA.	0,4624	0,5259	OBER	0,3575	0,3941
HAVAS	0,4598	0,5331	MALTERIES FCO-BEL.	0,3564	0,4897
ROTHSCHILD	0,4586	0,5177	ECA	0,3558	0,4077
AKKA TECHNOLOGIES	0,4531	0,4986	HARVEST	0,3543	0,3852
INFOTEL	0,4515	0,5346	FONCIERE LYONNAISE	0,3531	0,3855
CRCAM NORM.SEINE	0,451	0,5062	BNP PARIBAS ACT.A	0,352	0,3946
ORPEA	0,4486	0,4945	GEVELOT	0,3453	0,3766
WENDEL	0,4448	0,521	WENDEL	0,3423	0,3789
THALES	0,4441	0,4831	ALTAMIR	0,333	0,359
KORIAN	0,4374	0,4934	SQLI	0,3295	0,3535
MANITOU BF	0,4346	0,4892	TIPIAK	0,3256	0,3662
MICROWAVE VISION	0,4336	0,4982	BRICORAMA	0,3252	0,4976
AGTA RECORD	0,4285	0,4734	SOLUCOM	0,3208	0,3647
REMY COINTREAU	0,4276	0,4894	SARTORIUS STED BIO	0,3184	0,3388
ESI GROUP	0,4178	0,4609	CAFOM	0,311	0,3459
EUTELSAT COMMUNIC.	0,4136	0,463	MR BRICOLAGE	0,3099	0,3852
ZODIAC AEROSPACE	0,4119	0,4615	ORPEA	0,3084	0,3464
SARTORIUS STED BIO	0,4086	0,45	CRCAM TOURAINE CCI	0,3084	0,3296
SAVENCIA	0,3983	0,448	AGTA RECORD	0,3053	0,3574
ST DUPONT	0,3976	0,4659	CRCAM NORD CCI	0,3044	0,3282
AFONE	0,3965	0,4963	LEBON	0,3043	0,3506
OCTO TECHNOLOGY	0,3965	0,4346	RIBER	0,3006	0,3525
AXA	0,3947	0,449	KORIAN	0,3	0,3232
NEURONES	0,3945	0,4322	VINCI	0,2999	0,3377
SI PARTICIPATIONS	0,3918	0,4537	OENEO	0,2975	0,3287
NATUREX	0,3907	0,4493	VETOQUINOL	0,2949	0,3354
SES	0,3873	0,4346	EUROSIC	0,2949	0,3251
TURENNE INV	0,3861	0,4355	MAISONS FRANCE	0,2934	0,3221
KERING	0,3854	0,4438	CRCAM ATL.VEND.CCI	0,2909	0,3295
PRECIA	0,3841	0,4617	FONCIERE ATLAND	0,2872	0,3248
MERSEN	0,3825	0,436	HOPSCOTCH GROUPE	0,2859	0,3137
M.R.M	0,3816	0,4353	SWORD GROUP	0,2831	0,3104
DOM SECURITY	0,3779	0,431	GECINA NOM.	0,2825	0,3202
CHARGEURS	0,375	0,4212	IDSUD	0,2818	0,3399
FONCIERE LYONNAISE	0,3733	0,4582	ANF IMMOBILIER	0,2791	0,3101
TOUAX	0,3727	0,4146	KEYYO	0,2789	0,3276
FIDUCIAL REAL EST.	0,3723	0,4034	FOUNTAINE PAJOT	0,2786	0,3102

EUROFINS SCIENT.	0,3676	0,4084	IDI	0,278	0,3482
LEBON	0,365	0,3987	BOUYGUES	0,2776	0,3086
ALTAREA	0,3649	0,4093	ST DUPONT	0,2774	0,3043
VRANKEN-POMMERY	0,3644	0,3826	BELIER	0,2762	0,3093
SUEZ ENVIRONNEMENT	0,3636	0,3833	COLAS	0,2759	0,3072
CRCAM ILLE-VIL.CCI	0,3595	0,4055	FIMALAC	0,2756	0,3065
GECINA NOM.	0,3591	0,4063	TELEPERFORMANCE	0,273	0,3173
FONCIERE DE PARIS	0,3554	0,407	COHERIS	0,2728	0,2817
PATRIMOINE ET COMM	0,355	0,3996	FONCIERE INEA	0,271	0,319
ANTEVENIO	0,351	0,3984	INFOTEL	0,271	0,3031
FONCIERE INEA	0,3505	0,404	ILIAD	0,2701	0,3052
RAMSAY GEN SANTE	0,3501	0,385	AURES TECHNOLOGIES	0,269	0,3027
CIC	0,3491	0,3833	NEXITY	0,2669	0,2954
KAUFMAN ET BROAD	0,3487	0,3843	THALES	0,2663	0,3067
ROUGIER S.A.	0,3486	0,398	VRANKEN-POMMERY	0,265	0,2935
OENEO	0,3461	0,3569	ENVIRONNEMENT SA	0,2648	0,2945
COLAS	0,3425	0,3736	SANOFI	0,2646	0,2931
SCHNEIDER ELECTRIC	0,3409	0,37	GRAND MARNIER	0,2638	0,3028
FONCIERE DES MURS	0,3398	0,3679	ROUGIER S.A.	0,262	0,3407
IDI	0,3395	0,3773	GASCOGNE	0,2614	0,2724
IPSEN	0,3354	0,3921	CBO TERRITORIA	0,2604	0,2936
ELEC.MADAGASCAR	0,3339	0,3842	LEGRAND	0,2596	0,2856
VICAT	0,3335	0,3842	CARREFOUR PRO DEV	0,2582	0,2936
EUROGERM	0,3331	0,3796	ARGAN	0,2576	0,2927
COHERIS	0,3326	0,349	FDL	0,2569	0,3009
BONDUELLE	0,3311	0,3814	LE NOBLE AGE	0,2549	0,2656
TELEPERFORMANCE	0,327	0,3749	MONTEA C.V.A.	0,2535	0,2919
ENGIE	0,3269	0,3705	LDC	0,2532	0,3049