Highlights:

- A Nash equilibrium weighting scheme and HDI score is estimated
- longevity and knowledge are the highest contributors to human.
- Decent living standards is considered the least contributing dimension

REBUILDING THE HUMAN DEVELOPMENT INDEX: A NASH EQUILIBRIUM WEIGHTING SCHEME

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Abstract:

A number of studies have used data envelopment analysis (DEA) or other techniques to rebuild the Human Development Index (HDI). While competition exists among the countries in human development/rankings, all these studies do not model competition among the countries. These studies find a weighting scheme that is convenient and keeps the final score below unity. Although cross efficiency goes a further step by providing a rating score based on a democratic vote by all DMUs, it is not always unique. This paper presents a new application of DEA game cross-efficiency model where each country is viewed as a player competing to set his own evaluation standards through boosting his ranking compared to his opponents. Game cross efficiency allows us to model the human development rating problem as a game, through considering rating scores as payoffs and weight selection as strategies. The current paper solves for a Nash equilibrium weighting scheme. The approach is applied to 180 countries based on the data provided by the UNDP for the year 2016. The algorithm developed by (Liang, Wu, Cook, & Zhu, 2008) has been employed to solve for the Nash equilibrium rating scores. We demonstrate that our approach can be a promising tool for aggregating composite indicators by rebuilding the HDI. The resulting Nash equilibrium weighting scheme indicates that longevity and knowledge are the highest contributors to human. Shockingly decent living standards is considered was found to be the least contributing dimension.

Key Words:

OR in societal problem analysis, Game theory, Data envelopment analysis

1. Introduction

Composite indicators have been the key evaluation criteria used by multiple world organizations such as the United Nations, the European Commission, and others to develop world policies. These organizations have been working on aggregating multiple socioeconomic indicators into a key indicator to provide comparisons of countries in a complex policy environment. Indeed, these new set of measurements especially the Human Development Index (HDI) are recognized as integral part for policy making. Even though various tools and sophisticated techniques were developed by different academics to better aggregate the HDI, disagreement on the best way to estimate the weights of the composing dimensions is widespread from a technical and economic perspectives. As a possible solution to that problem, this paper proposes a Nash equilibrium weighting scheme based on (Liang, Wu, Cook, & Zhu, 2008) to rebuild the HDI.

The Human Development Index (HDI) was developed by the United Nations Development Program (UNDP) in 1990 to capture most of the human development aspects instead of the naïve use of Gross National Income (GNI) per capita. Since its inception, the HDI has become an integral part of the annual development report of the UNDP. The HDI came after various attempts to develop a satisfactory measurement of human development. In 1979, the economic historian Morris David proposed the Physical Quality of Life Index (PQLI) which combines infant mortality, life expectancy and literacy (Desai, 1991). The PQLI is quite similar to the HDI but it lacks an income measurement (i.e. GNI).

The HDI is formed through the aggregation of three socioeconomic indicators believed to reflect most of the human development dimensions; The first dimension is healthcare, measured through life expectancy at birth. The second is educational attainment through the weighted average of the Mean Years of Schooling Index (MYSI)[1] and the Expected Years of Schooling Index (EYSI)[2]. The final dimension is income per capita measured through the gross national income (GNI).

Since the first release of the HDI by the UNDP in 1991, the index has faced various critics on two main fronts, conceptual (the dimensions included in the index) and technical (the way the index is aggregated). For the conceptual front, the main critics were presented by (Desai, 1991), in which he tackled the need to include more capabilities ((Sen, 1987)) rather than just longevity and literacy. On the technical front, (Noorbakhsh, 1998) tackled the way the HDI composite factors are derived from the raw data. (Desai, 1991) pointed out that "additivity over the three variables implies perfect substitution which can hardly be

[1] MYSI: Mean Years of Schooling Index (i.e. years that a person aged 25 or older has spent in formal education)
[2] EYSI: Expected Years of Schooling Index (i.e. total expected years of schooling for children under 18 years of age)

appropriate”. The same issue of additivity was approached by (Sagar & Najam, 1998). (Mahlberg & Obersteiner, 2001) used DEA without taking into account the equal weights. They solved for the benchmarking HDI and used arbitrary bounds on weights to tackle the flexibility of the methodology. Other development on this issue is (Despotis, 2005) approach, where he developed a DEA-like index maximizing model proceeded by goal programing as an attempt to respect the equal weighting scheme. The most recent development is from the authors of the Human Development Report. He undertook a major revision of these critiques and introduced several changes in the 2010 edition. In the new edition, the author changed the method of aggregation from an arithmetic average to a geometric average, and the upper and lower bounds for the normalization of the subdimensions.

Even though these studies yielded a better result than the equal weighting scheme of the 1991 edition, they failed to account for the competitive behavior of the countries to set their own evaluation standards. Albeit, the new edition undergone major changer through the addition of some degree of both substitutability and complementarity, the focus of the new edition is on the HDI scores rather than on the estimation of weighting scheme. In fact, the functional form of the new HDI does not provide the weights or rather the contribution per indicator. Indeed, an ideal approach would be to assess the impact of each dimension on the ultimate objective which is Human Development, and then assign the weighting scheme based on those results. Of course, this is an ideal case and quite infeasible to achieve given the current knowledge level (Chowdhury 2006).

Even though DEA provide a remedy to the issue of aggregating multi performance measure into a key indicator, it still suffers from high flexibility, thus allowing for weak discrimination among DMUs. The cross-efficiency method, developed as an extension to DEA, provides a unique ordering of DMUs and eliminates unrealistic weights schemes through peer evaluation. Even though cross efficiency has been applied successfully to multi real world problems, it still suffers from non-uniqueness (Despotis & Smirlis, 2002). Several models have been suggested as a remedy this issue. A recent development on this issue is the (Liang, Wu, Cook, & Zhu, 2008) proposal in which they adopted a non-cooperative game approach to the peer evaluation method in DEA. They viewed cross efficiency scores as payoffs, and that each DMU may choose to take a non-cooperative game stance to boost their efficiency scores given a weight selection strategy. (Liang, Wu, Cook, & Zhu, 2008) developed an algorithm converging to Nash equilibrium.

This paper attempts to address the issue of finding a satisfactory scheme that is between the simple equal weighting one and the ideal weighting system. This paper applies the game cross efficiency approach developed by (Liang, Wu, Cook, & Zhu, 2008) and solves for the Nash equilibrium weights.

As an illustration of our approach, we report a case study involving 180 countries. We have used actual data for the year 2016 provided by the UNDP. Our approach is applied as a mean for weight estimation. We demonstrate that our approach can be a promising tool for aggregating composite indicators. The paper is organized as follows. Section 2 reviews the current weighting scheme. Game cross efficiency model and algorithm is described in Section 3, followed by the development of the approach and results in Section 4. Section 5 concludes

2. The Human Development Index (HDI)

According to the UNDP, the Human Development Index (HDI) is a summary measure of human development composed of longevity, knowledge and decent living standards. The selected measurement of the three dimensions are mainly based on data availability. The first dimension is then life expectancy, the second is literacy levels through weighting the average of the Mean Years of Schooling Index and Expected Years of Schooling Index and the final dimension is the gross national income (GNI) per capita. In order to derive the final HDI, the UNDP follows two main steps (2016 Report technical note, UNDP)[3]:

Step 1: Creating the dimension indices

1) $\textbf{Life Expectancy Index (LEI)} = \frac{\mathbf{LE-20}}{\mathbf{85-20}}$, where LE is the raw life expectancy. We can note that LEI is 1 when Life expectancy at birth is 85 and 0 when Life expectancy at birth is 20.
2) $\textbf{Education Index (EI)} = \frac{\mathbf{MYSI+EYSI}}{\mathbf{2}}$
   a) $\text{Mean Years of Schooling Index (MYSI)} = \frac{\text{MYS}}{15}$, where MYS is the mean years of schooling
   b) $\text{Expected Years of Schooling Index (EYSI)} = \frac{\text{EYS}}{18}$, where MYS is the mean years of schooling
3) $\textbf{Income Index (II)} = \frac{\mathbf{ln(GNIpc)-ln(100)}}{\mathbf{ln(75000)-ln(100)}}$, we can note that II is 1 when GNI per capita is \$75,000 and 0 when GNI per capita is \$100.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{dimension}_\text{i} = \frac{\text{Actual value}_\text{i} - \text{Min}_\text{i}}{\text{Max}_\text{i} - \text{Min}_\text{i}}$$

Where, i= **LEI, EI, II.**

[3] http://dev-hdr.pantheonsite.io/sites/default/files/hdr2016_technical_notes_0.pdf

Step 2: Aggregate the HDI

The HDI is then calculated as the geometric mean of the three dimensions as follows:

$$HDI_i = \sqrt[3]{LEI.EI.II}$$

Where, HDI_i is the human development of the ith country, LEI_i is Life Expectancy Index of the ith country, EI_i is the Education Index of the ith country and II_i is Income Index of the ith country. The new HDI "attains a compromise by adopting a functional form that is between the extremes of perfect substitutability and perfect complementarity" (Klugman, Rodríguez, & Choi, 2011).The major shortcoming of this aggregation method is that it does not provide the contribution of each dimension to the final score (Ravallion, 2012). Furthermore, this functional form does not take into account the competitive behavior of the countries seeking to set their own weighting scheme.

3. DEA Game Cross-Efficiency

Given a set of **n** DMUs where a DMU_j $(j = 1,2,3 \ldots n)$ utilizes a set of **t** inputs X_{ij} $(i = 1\ 2 \ldots t, j = 1\ 2 \ldots n)$ to produce **m** outputs $Y_{rj}(r = 1,2 \ldots m, j = 1\ 2 \ldots n)$ where $X_{ij}, Y_{rj} > 0$, the standard input-oriented constant returns to scale (CRS) DEA model (Charnes, Cooper, & Rhodes, 1978) can be represented as follow:

$$\text{Max } Z = \sum_{r}^{m} \mu_r Y_{rd}$$

$$\text{Subject to} \qquad\qquad (1)$$

$$\sum_{r}^{m} \mu_r Y_{rd} - \sum_{i}^{t} \omega_i X_{id} \leq 0$$

$$\sum_{i}^{t} \omega_i X_{id} = 1$$

$$\omega_i, \mu_r \geq 0$$

Where μ_r and ω_i are the set output and input weights respectively to be determined through solving the above model. Upon solving this model, an efficiency score of DMU_d is obtained, and from which, a cross efficiency score E_{dj} for each of the other (n-1) DMUs will be determined based on DMU_d 's optimal weights.

$$E_{dj} = \frac{\sum_r^m \mu_r^d Y_{rj}}{\sum_i^t \omega_i^d X_{ij}} \quad j = 1.2 \ldots (n-1) \quad (2)$$

Where d denote the optimal weights of DMU_d . Finally, each DMU's cross efficiency score $\overleftarrow{E_j}$ will be determined through averaging its peer ratings as follows:

$$\overleftarrow{E_j} = \frac{1}{n} \sum_d^n E_{dj} \quad (3)$$

In fact, DEA gives much flexibility to each DMU. Indeed, letting each DMU choose its own set of weights will actually lead to unrealistic weighting scheme, leading to low discrimination power. This consideration justifies the use of cross efficiency evaluation. However, the latter method suffers a major shortcoming which is the non-uniqueness of weights depending on the used software (Despotis & Smirlis, 2002). For this reason, the use of game cross efficiency is justified as it provides more stable weights and a Nash equilibrium evaluation score. The Game cross efficiency model was developed by (Liang, Wu, Cook, & Zhu, 2008) through the addition of a second goal to the basic DEA model. Furthermore, the authors developed an algorithm for finding the Nash equilibrium.

Given a DMU_d with an efficiency score α_d, the other DMU_j tries to select a set of strategies (Weights selection) to maximize its own efficiency while ensuring that α_d will not decrease.

$$\text{Max } Z = \sum_r^m \mu_{rj}^d Y_{rj}$$

$$\text{Subject to} \qquad (4)$$

$$\sum_r^m \mu_{rj}^d Y_{rl} - \sum_i^t \omega_{ij}^d X_{il} \leq 0 \quad l=1, 2 \ldots n$$

$$\sum_i^t \omega_{ij}^d X_{ij} = 1$$

$$\alpha_d \sum_i^t \omega_{ij}^d X_{id} - \sum_r^m \mu_{rj}^d Y_{rd} \leq 0$$

$$\mu_{rj}^d \geq 0 \quad \forall\, r = 1,2, \ldots m,$$

$$\omega_{ij}^d \geq 0 \quad \forall\, i = 1,2, \ldots t,$$

Note $\alpha_d \leq 1$ which takes initially the value $\overleftarrow{E_d}$ from (2), is the average cross efficiency of DMU_d. When the algorithm converges, this α_d becomes the game cross efficiency. In addition, the constraint $\alpha_d \sum_i^t \omega_{ij}^d X_{id} - \sum_r^m \mu_{rj}^d Y_{rd} \ \leq 0$ in model (4) is equivalent to $\frac{\sum_r^m \mu_{rj}^d Y_{rd}}{\sum_i^t \omega_{ij}^d X_{id}} \geq \alpha_d$ which implies the restriction of DMU_d initial score to ensure that it will not deteriorate. The above model is solved once for each DMU_d thus n times. In addition, the optimal value to model (4) will represent a game cross efficiency with respect to DMU_d. Indeed, the average game cross efficiency score for DMU_j would be $\alpha_j = \frac{1}{n}\sum_{d=1}^n \sum_{r=1}^m \mu_{rj}^{d*}(\alpha_d)\, Y_{rj}$ where $\mu_{rj}^{d*}(\alpha_d)$an optimal solution to model (4).

Iterative algorithm below describes the steps to find the Nash equilibrium efficiency score. In this algorithm, α_j^t represents the efficiency of DMU_j at iteration t.

Algorithm: DEA Game Cross Efficiency

Require: ϵ

Step 1: Set t=1. For each DMU_d calculate the average cross efficiency $\overleftarrow{E_d}$ and set $\alpha_d^t = \overleftarrow{E_d}$,$\forall\ d \in \{1, \ldots, n\}$.

Step 2: For each pair of DMUs d and j, solve model (4) and obtain E_{dj}.

Step 3: Set $\alpha_d^{t+1} = \frac{1}{n}\sum_d^n E_{dj}$

Step 4: If for some d, $|\alpha_d^{t+1} - \alpha_d^t| > \epsilon$, then return to step 2. Otherwise α_d^{t+1} is the optimum game cross efficiency of DMU_d and the algorithm stops

Under game cross-efficiency evaluation, all players (DMUs) are assigned their Nash equilibrium scores. Indeed, under a cross evaluation approach, we are in what might be a democratic vote, where a set of factors is voted to be of a high importance by the majority of DMUs while the rest are of a low importance.

It is obvious that the new approach tackles the major critics and provides a highly reliable score. However, this edition lacks the weighting scheme or the contribution per indicator that can be used by countries to tackle the issue of development. In addition, the competitive behavior is neglected in the aggregation functional form. In fact, the ideal approach would be to set the weights based on the overall contribution of each dimension to human development. Even though this is theoretically correct, it still infeasible given the current level of knowledge (Chowdhury, Bertling, & Custer, 2006). In order to solve for "a solution that lies somewhere between equal and the ideal weights; and somewhere between the convenient and infeasible" (Chowdhury, Bertling, & Custer, 2006), game cross methodology seems the best fit that solves for a satisfactory economic measure, which is a Nash equilibrium weighting scheme.

The weights will be set endogenously representing the numerical characteristics of each country. The idea of peer evaluation would result in setting a democratic weighting scheme standard through the aggregation of self and peer ratings. The game cross efficiency goes a step further to solve for the Nash equilibrium weighting scheme. Now, given that the resulting weights are Nash equilibrium and exogenously determined, they are more reliable as a tool for decision making.

4. Game cross efficiency approach to the HDI

As demonstrated in the previous section, competition and absence of weighting scheme are a critical issue in estimating the HDI. On the one hand, this affects the economic value of the HDI given that the contribution per indicator to the final human development scores are unavailable and the competitive behavior is not presented. On the other hand, it affects to some extent the relative position of the countries in the HDI, especially within structurally superior groups (i.e. Norway, Australia, Switzerland, Germany, Denmark and Singapore) (Despotis, 2005).

A simple application of output oriented DEA model was presented by (Mahlberg & Obersteiner, 2001). Given that the HDI components contribute positively to the final score, (Mahlberg & Obersteiner, 2001) modeled all dimensions as outputs and created a dummy input (equal to one) for all countries. They introduced arbitrary bounds on the weights to resolve the issue of high flexibility.

In the rest of this section, we present the game cross efficiency model used to estimate the optimal weights and final HDI scores. We will adopt an approach similar to that of Mahlberg & Obersteiner in which we set the three dimensions as outputs (to be maximised indicators) and we assume a dummy input equal to one.

4.1. Game Human Development Index (GHDI)

In order to solve for the GHDI, we will use the actual minima and maxima of the data to normalize each dimension. This is done as follows

1) $\boldsymbol{Life\ Expectancy\ Index}\ (\boldsymbol{LEI}) = \frac{LE_i - Min_i^{LE}}{Max_i^{LE} - Min_i^{LE}}$, where Max_i^{LE} and Min_i^{LE} are respectively the max and min of the Life Expectancy variable.

2) $\boldsymbol{Education\ Index\ (EI)} = \frac{\boldsymbol{MYSI+EYSI}}{\boldsymbol{2}}$

a) $Expected\ Years\ of\ Schooling\ Index\ (MYSI) = \frac{EYS_i - {Min_i}^{EYS}}{{Max_i}^{EYS} - {Min_i}^{EYS}}$, where ${Max_i}^{EYS}$ and ${Min_i}^{EYS}$ are respectively the max and min of the expected year of schooling variable.

b) $Mean\ Years\ of\ Schooling\ Index\ (EYSI) = \frac{MYS_i - {Min_i}^{MYS}}{{Max_i}^{MYS} - {Min_i}^{MYS}}$, where ${Max_i}^{MYS}$ and ${Min_i}^{MYS}$ are respectively the max and min of the mean year of schooling variable.

3) $\boldsymbol{Income\ Index\ (II)} = \frac{ln(GNI_i) - ln({Min_i}^{GNI})}{ln({Max_i}^{GNI}) - ln({Min_i}^{GNI})}$, where ${Max_i}^{GNI}$ and ${Min_i}^{GNI}$ are respectively the max and min of the GNI per capita variable.

This is ideal given that we are using a benchmarking approach. This would also increase the discrimination among high performing countries.

Now, assuming that each country is a player competing to impose his evaluation standards, then according to (Liang, Wu, Cook, & Zhu, 2008) we can solve model (4) and formulate the game HDI score as follow:

$$GHDI_i = \frac{1}{N}\sum_{i=1}^{N}\frac{w_i^{LEI}Y_i^{LEI} + w_i^{EI}Y_i^{EI} + w_i^{II}Y_i^{II}}{w_i^{Dummy}X_i^{Dummy}}$$

Where,

N set of the evaluated countries, i ∈ N would stand for any country in N.

$GHDI_i$ is the game HDI of the ith country.

$w_i^{LEI},\ w_i^{EI}\ and\ w_i^{II}$ be the unknown weights of the three indicators: Life Expectancy Index (Y_i^{LEI}), Education Index (Y_i^{EI}) and Income Index (Y_i^{II}) per ith country respectively.

w_i^{Dummy} be the unknown weight of the dummy input X_i^{Dummy}, where $X_i^{Dummy} = 1\ \forall\ i = 1 \ldots N$

In this context, each country will attempt to impose its weighting scheme without causing the deterioration of the HDI score of the other peer countries. (Liang, Wu, Cook, & Zhu, 2008) showed that by implementing the preceding algorithm of section 2, the weighting scheme will converge to a Nash Equilibrium state where all countries have no incentive to deviate from the proposed weighting schemes. In other words, upon solving the game cross efficiency model, we will obtain a unique matrix N by (s+m) (in our case N by 4, a dummy input and three outputs).

$$\begin{bmatrix} w_1^{Dummy} & w_1^{LEI} & w_1^{EI} & w_1^{II} \\ w_2^{Dummy} & w_2^{LEI} & w_2^{EI} & w_2^{II} \\ w_3^{Dummy} & w_3^{LEI} & w_3^{EI} & w_3^{II} \\ & . & & \\ & . & & \\ & . & & \\ w_{N-1}^{Dummy} & w_{N-1}^{LEI} & w_{N-1}^{EI} & w_{N-1}^{II} \\ w_N^{Dummy} & w_N^{LEI} & w_N^{EI} & w_N^{II} \end{bmatrix}$$

In order to determine the final aggregate Nash weighting scheme, it suffice to determine the average weight per indicators as follows:

$$w_{LEI}^{*} = \frac{\sum_i^N w_i^{LEI}}{N}, i = 1 \dots N$$

$$w_{EI}^{*} = \frac{\sum_i^N w_i^{EI}}{N}, i = 1 \dots N$$

$$w_{II}^{*} = \frac{\sum_i^N w_i^{II}}{N}, i = 1 \dots N$$

Where, $w_{LEI}^{*}, w_{LEI}^{*}, and\ w_{LEI}^{*}$ are the Nash equilibrium weights for longevity, education and income respectively.

4.2. Results

Table 1 shows the 2010 edition of HDI, the Nash equilibrium weighting scheme and Game HDI (GHDI) the for each country. In addition, we have the raw data used to build the composite indicators; Life Expectancy at birth, the Mean Years of Schooling (MYS), the Expected Years of Schooling (EYS), and Gross National Income (GNI) per capita. Based on the weights per indicator we can conclude that longevity and knowledge are the leading contributors. In more details, the average equilibrium weight attributed to longevity is equal to 0.5968 succeeded by knowledge (0.4260) and finally the lowest weight (0.0242) was attributed to income. This is very important compared to the current HDI. Indeed, having the ability to determine the optimal weights can be very beneficial for countries to act on the right dimension to improve human development. With such weights, we can solve for the optimal GHDI per country and evaluate the impact of adding a unit while holding the rest equal on a selected dimension as follows:

$$GHDI^*{}_i = 0.5968 * \frac{LE_i - Min_i{}^{LE}}{Max_i{}^{LE} - Min_i{}^{LE}} + 0.4260 * \frac{1}{2}\left(\frac{EYS_i - Min_i{}^{EYS}}{Max_i{}^{EYS} - Min_i{}^{EYS}} + \frac{MYS_i - Min_i{}^{MYS}}{Max_i{}^{MYS} - Min_i{}^{MYS}}\right) + 0.0242 * \frac{ln(GNI_i) - ln(Min_i{}^{GNI})}{ln(Max_i{}^{GNI}) - ln(Min_i{}^{GNI})}$$

Now to evaluate the impact of a policy to increase a dimension by Δ ,while citrus paribus, we can solve the following:

- **Longevity:** $GHDI^*{}_i = 0.5968 * \frac{\Delta + LE_i - Min_i{}^{LE}}{Max_i{}^{LE} - Min_i{}^{LE}} + 0.4260 * EI_i + 0.0242 * II_i$, where EI_i and II_i are the current education index and income index of country i.
- **Knowledge:** for knowledge policy can impact tow sub dimension either the expected year of schooling or the mean year of schooling:
    - **Expected year of schooling:** $GHDI^*{}_i = 0.5968 * LEI_i + 0.4260 * \frac{1}{2}\left(\frac{\Delta + EYS_i - Min_i{}^{EYS}}{Max_i{}^{EYS} - Min_i{}^{EYS}} + MYSI_i\right) + 0.0242 * II_i$, where LEI_i, $MYSI_i$ and II_i are the current Life Expectancy Index, Mean Year of Schooling Index and Income Index of country i.
    - **Mean year of schooling:** $GHDI^*{}_i = 0.5968 * LEI_i + 0.4260 * \frac{1}{2}\left(EYSI_i + \frac{\Delta + MYS_i - Min_i{}^{MYS}}{Max_i{}^{MYS} - Min_i{}^{MYS}}\right) + 0.0242 * II_i$, where LEI_i, $EYSI_i$ and II_i are the current Life Expectancy Index, Expected Year of Schooling Index and Income Index of country i.
- **Decent Living Standards:** $GHDI^*{}_i = 0.5968 * LEI_i + 0.4260 * \frac{1}{2}(EYSI_i + MYSI_i) + 0.0242 * \frac{ln(\Delta + GNI_i) - ln(Min_i{}^{GNI})}{\ln(Max_i{}^{GNI}) - \ln(Min_i{}^{GNI})}$, where LEI_i, $EYSI_i$ and $MYSI_i$ are the current Life Expectancy Index, Expected Year of Schooling Index and Mean Year of Schooling Index of country i.

Let’s take Afghanistan as an example. This country has a Life Expectancy of 60.7. Now Citrus Paribus, a 1-year increase in life expectancy would result in a 0.233 increase in GHDI score, 5 years increase results in 0.283, 10 years in 0.367 and finally 23 years increase (making its longevity equal to the current best which is quite impossible without an increase in resources and knowledge) would result in 0.5877 increase of GHDI from 0.329 to 0.9167.

[Insert Table 1: Data and optimization results, $\epsilon = 0.001$]

Looking at the game HDI scores, we can observe a similarity with the HDI. However, we can note that high number of countries holds a score above 0.900. Indeed, these inflated scores are mainly due to the flexibility of the methodology which obviously promotes the strength of countries rather than their weaknesses.

Furthermore, Given the equilibrium weighting scheme which outweighed longevity and knowledge while degrading the value of income in human development, multiple changes have emerged compared to the classic HDI. This change affected various fronts; group sizes, composite countries per group, and ranking within each group. These major changes are mainly due to the flexibility provided by DEA.

In order to compare our methodology to that of the current HDI we will keep the same **HDR Cutoff points**. Table 2 provides the HDR cutoff points and group sizes per methodology.

[Insert Table 2: HDR cutoff points and group sizes per methodology]

We can note that given the GHDI scores, the group size of the low developed countries increased significantly from 41 to 59 countries based on the game HDI (GHDI). In addition, the group with Very high human development has decreased slightly by 2 countries. Finally, we can conclude that both High human development and Medium human development group have shrunk in favor of the low human development group. This shifts in classes can be further examined in the below world maps.

[Insert Figure 1: World Map of the Human Development Index(HDI)]

[Insert Figure 2: World Map of the Game Human Development Index (GHDI)]

We shift our attention to the composition per group. The proposed index downgraded several countries. Compared to the original HDI, we have country that shifted from second class (high Human Development to the first) while other were downgraded from the first to the second or third class. In fact, Cuba, Costa Rica, Lebanon, Albania, Panama and Barbados joined the first class of high Human development while Hungary, Bahrain, Lithuania, Latvia, Saudi Arabia, Romania and Kuwait left to the second group and Russia to the third one (Medium Human Development). This huge shift is mainly due to the overweighting of longevity and knowledge and the very low importance of income. Most of the downgraded country perform less compared to the upgraded ones on health and educational attainment while maintaining a high-income index like Saudi Arabia and Kuwait. For example, the Russian federation has been degraded to the third class mainly due to its low life expectancy of 70.3, which is the lowest among the downgraded and upgraded countries, and its moderate education and GNI per capita compared to its peer. This is also the main reason for the changes in the other classes.

Comparison of the HDI and the GHDI rankings with the rankings induced on the countries by the component indices LEI, EI and II shows that ranking is highly affected by the weighting scheme. In addition, unlike the HDI, we can determine the impact of each subdimension to develop the relevant policies.

5. Conclusion

The human development index is revisited in the light of game cross efficiency. First, an ideal Nash equilibrium value of the composite human development index is estimated for each country by a game cross efficiency model. Then, the Nash equilibrium weighting scheme have been extracted in order to evaluate the impact of each sub indictor. The new measure of human development is comparable with the HDI. The superiority of the new measure however is based on the fact that the assumed for the component indicators weights are Nash equilibrium. Furthermore, this approach takes into account the competitive behavior of the countries. In this paper, the Game HDI outweighed both longevity and Education.

References

Charnes, A., Cooper, W., & Rhodes, E. (1978). Measuring the efficiency of decision-making units. . *European Journal of Operational Research*, 429-444.

Chowdhury, A., Bertling, L., & Custer, D. (2006). Determining distribution substation transformer optimal loadings using a reliability cost-benefit approach. *Probabilistic Methods Applied to Power Systems*, 1-9.

Desai, M. (1991). Human development Concepts and measurement. *European Economic Review*, 350-357.

Despotis, D. (2005). Measuring human development via data envelopment analysis:. *Omega* , 385-390.

Despotis, D., & Smirlis, Y. (2002). Data envelopment analysis with imprecise data. *European Journal of Operational Research*, 24-36.

Doyle, J., & Green, R. (1993). Data envelopment analysis and multiple criteria decision making. *Omega*, 713-715.

Klugman, J., Rodríguez, F., & Choi, H. (2011). The HDI 2010: new controversies, old critiques. *Journal of Economic Inequality*, 249-288.

Liang, L., Wu, J., Cook, W., & Zhu, J. (2008). The DEA game cross-efficiency model and its Nash equilibrium. *Operations Research*, 1278-1288.

Mahlberg , B., & Obersteiner, M. (2001). Remeasuring the HDI by data envelopement analysis . *IIASA interim report IR-01-069*.

Noorbakhsh, F. (1998). A modified human development index. *World Development* , 517-528.

Ravallion, M. (2012). Troubling tradeoffs in the human development index. *Journal of Development Economics*, 201-209.

Sagar, A., & Najam, A. (1998). The human development index: a critical review. *Ecological Economics*, 249-264.

Sen, A. (1987). The Standard of Living. *Cambridge University Press*.

Stewart, T. (1996). Relationships between data envelopment analysis and multicriteria decision analysis. *Journal of the operational research society*, 654-665.

Table 1: Data and optimization results, ϵ=0.001

	Input	Life expectancy at birth	EYS	MYS	GNI/capita	Input weight	LEI weight	EI weight	II weight	GHDI	HDI	HDI rank	GHDI Ranking
Switzerland	1,000	83,1	16,0	13,4	56 364	1	0,553026555	0,383901704	0,280984575	1	0,939	2	1
Hong Kong, China (SAR)	1,000	84,2	15,7	11,6	54 265	1	0,706557777	0,270433964	0,177185508	0,989	0,917	12	2
Australia	1,000	82,5	20,4	13,2	42 822	1	0	1,008267796	0	0,988	0,939	2	3
Iceland	1,000	82,7	19,0	12,2	37 065	1	0,807179887	0,232014341	0	0,984	0,921	9	4
Denmark	1,000	80,4	19,2	12,7	44 519	1	0	0,986231393	0,066918876	0,982	0,925	5	5
Japan	1,000	83,7	15,3	12,5	37 268	1	0,810645321	0,228681683	0	0,982	0,903	17	5
New Zealand	1,000	82,0	19,2	12,5	32 870	1	0,801202204	0,237762995	0	0,973	0,915	13	6
Norway	1,000	81,7	17,7	12,7	67 614	1	0	0,89373844	0,347796128	0,972	0,949	1	7
Italy	1,000	83,3	16,3	10,9	33 573	1	0,810224573	0,22908631	0	0,966	0,887	26	8
Israel	1,000	82,6	16,0	12,8	31 215	1	0,808545798	0,230700764	0	0,964	0,899	19	9
Canada	1,000	82,2	16,3	13,1	42 582	1	0,811174176	0,22817309	0	0,962	0,920	10	10
Sweden	1,000	82,3	16,1	12,3	46 251	1	0,814703323	0,22477916	0	0,956	0,913	14	11
Netherlands	1,000	81,7	18,1	11,9	46 326	1	0,686554729	0,285149917	0,190913692	0,953	0,924	7	12
Korea (Republic of)	1,000	82,1	16,6	12,2	34 541	1	0,809319159	0,229957034	0	0,953	0,901	18	12
Spain	1,000	82,8	17,7	9,8	32 779	1	0,810007929	0,229294653	0	0,953	0,884	27	12
France	1,000	82,4	16,3	11,6	38 085	1	0,810726666	0,228603454	0	0,951	0,897	21	13
Ireland	1,000	81,1	18,6	12,3	43 798	1	0,657636031	0,306424985	0,210760728	0,945	0,923	8	14
Germany	1,000	81,1	17,1	13,2	45 000	1	0,639068482	0,320084862	0,223503722	0,944	0,926	4	15
Singapore	1,000	83,2	15,4	11,6	78 162	1	0,376184723	0,523519065	0,359981975	0,942	0,925	5	16
Luxembourg	1,000	81,9	13,9	12,0	62 471	1	0,576765999	0,365919897	0,266262203	0,934	0,898	20	17

United Kingdom	1,000	80,8	16,3	13,3	37 931	1	0,807384536	0,231817533	0	0,932	0,909	16	18
Austria	1,000	81,6	15,9	11,3	43 609	1	0,813468415	0,225966753	0	0,928	0,893	24	19
Chile	1,000	82,0	16,3	9,9	21 665	1	0,809782459	0,229511485	0	0,925	0,847	38	20
Slovenia	1,000	80,6	17,3	12,1	28 664	1	0,804550724	0,23454277	0	0,922	0,890	25	21
Finland	1,000	81,0	17,0	11,2	38 868	1	0,810951264	0,228387462	0	0,921	0,895	23	22
Belgium	1,000	81,0	16,6	11,4	41 243	1	0,812331998	0,227059629	0	0,92	0,896	22	23
Greece	1,000	81,1	17,2	10,5	24 808	1	0,807688089	0,23152561	0	0,917	0,866	29	24
Liechtenstein	1,000	80,2	14,6	12,4	75 065	1	0	0,867730752	0,42677477	0,915	0,912	15	25
United States	1,000	79,2	16,5	13,2	53 245	1	0	0,925620125	0,25097967	0,906	0,920	10	26
Andorra	1,000	81,5	13,5	10,3	47 979	1	0,837470321	0,20288446	0	0,905	0,858	32	27
Portugal	1,000	81,2	16,6	8,9	26 104	1	0,817703297	0,221894127	0	0,901	0,843	41	28
Cyprus	1,000	80,3	14,3	11,7	29 459	1	0,809676689	0,229613203	0	0,897	0,856	33	29
Qatar	1,000	78,3	13,4	9,8	129 916	1	0	0,898330898	0,333850018	0,894	0,856	33	30
Malta	1,000	80,7	14,6	11,3	29 500	1	0,809754219	0,229538643	0	0,89	0,856	33	31
Czech Republic	1,000	78,8	16,8	12,3	28 144	1	0,796413101	0,242368607	0	0,878	0,878	28	32
Cuba	1,000	79,6	13,9	11,8	7 455	1	0,80463115	0,234465426	0	0,87	0,775	68	33
Estonia	1,000	77,0	16,5	12,5	26 362	1	0	1,008267796	0	0,86	0,865	30	34
Costa Rica	1,000	79,6	14,2	8,7	14 006	1	0,832829762	0,20734722	0	0,851	0,776	66	35
Lebanon	1,000	79,5	13,3	8,6	13 312	1	0,840607169	0,199867799	0	0,845	0,763	76	36
Poland	1,000	77,6	16,4	11,9	24 117	1	0,793779167	0,244901624	0	0,844	0,855	36	37
Brunei Darussalam	1,000	79,0	14,9	9,0	72 843	1	0,388836071	0,512312602	0,356410967	0,839	0,865	30	38
Argentina	1,000	76,5	17,3	9,9	20 945	1	0,800524562	0,238414673	0	0,827	0,827	45	39
Albania	1,000	78,0	14,2	9,6	10 252	1	0,807266569	0,23193098	0	0,815	0,764	75	40

Slovakia	1,000	76,4	15,0	12,2	26 764	1	0,795872405	0,242888586	0	0,809	0,845	40	41
United Arab Emirates	1,000	77,1	13,3	9,5	66 203	1	0,397247444	0,504861875	0,354036748	0,806	0,840	42	42
Panama	1,000	77,8	13,0	9,9	19 470	1	0,816599231	0,222955891	0	0,806	0,788	60	42
Croatia	1,000	77,5	15,3	11,2	20 291	1	0,804527826	0,234564791	0	0,804	0,827	45	43
Montenegro	1,000	76,4	15,1	11,3	15 410	1	0,799183979	0,239703893	0	0,801	0,807	48	44
Barbados	1,000	75,8	15,3	10,5	14 952	1	0,801077541	0,237882881	0	0,8	0,795	54	45
Dominica	1,000	77,9	12,8	7,9	10 096	1	0,843746719	0,196848538	0	0,796	0,726	96	46
Hungary	1,000	75,3	15,6	12,0	23 394	1	0	1,008267796	0	0,788	0,836	43	47
Bahrain	1,000	76,7	14,5	9,4	37 236	1	0,813484706	0,225951087	0	0,787	0,824	47	48
Lithuania	1,000	73,5	16,5	12,7	26 006	1	0	1,008267796	0	0,781	0,848	37	49
Oman	1,000	77,0	13,7	8,1	34 402	1	0,832314623	0,207842622	0	0,78	0,796	52	50
Uruguay	1,000	77,4	15,5	8,6	19 148	1	0,808675752	0,230575789	0	0,78	0,795	54	50
Mexico	1,000	77,0	13,3	8,6	16 383	1	0,827815727	0,212169146	0	0,78	0,762	77	50
Bosnia and Herzegovina	1,000	76,6	14,2	9,0	10 091	1	0,807204352	0,231990814	0	0,778	0,750	81	51
Latvia	1,000	74,3	16,0	11,7	22 589	1	0	1,008267796	0	0,771	0,830	44	52
Antigua and Barbuda	1,000	76,2	13,9	9,2	20 907	1	0,80892821	0,230333004	0	0,769	0,786	62	53
Georgia	1,000	75,0	13,9	12,2	8 856	1	0,773332035	0,264565341	0	0,769	0,769	70	53
Maldives	1,000	77,0	12,7	6,2	10 383	1	0,854744131	0,186272482	0	0,765	0,701	105	54
Saudi Arabia	1,000	74,4	16,1	9,6	51 320	1	0	0,906469911	0,309133933	0,76	0,847	38	55
Bahamas	1,000	75,6	12,7	10,9	21 565	1	0,807562919	0,231645985	0	0,76	0,792	58	55
Serbia	1,000	75,0	14,4	10,8	12 202	1	0,799634596	0,239270541	0	0,76	0,776	66	55
Ecuador	1,000	76,1	14,0	8,3	10 536	1	0,815424648	0,224085472	0	0,76	0,739	89	55
Romania	1,000	74,8	14,7	10,8	19 428	1	0,799972694	0,238945396	0	0,758	0,802	50	56

Sri Lanka	1,000	75,0	14,0	10,9	10 789	1	0,800372658	0,238560757	0	0,758	0,766	73	56
Iran (Islamic Republic of)	1,000	75,6	14,8	8,8	16 395	1	0,807550734	0,231657703	0	0,756	0,774	69	57
Jamaica	1,000	75,8	12,8	9,6	8 350	1	0,806903478	0,23228016	0	0,755	0,730	94	58
China	1,000	76,0	13,5	7,6	13 345	1	0,830978232	0,20912781	0	0,75	0,738	90	59
Kazakhstan	1,000	69,6	15,0	11,7	22 093	1	0	1,008267796	0	0,748	0,794	56	60
Ukraine	1,000	71,1	15,3	11,3	7 361	1	0	1,008267796	0	0,748	0,743	84	60
Viet Nam	1,000	75,9	12,6	8,0	5 335	1	0,834232945	0,205997799	0	0,748	0,683	115	60
The former Yugoslav Republic of Macedonia	1,000	75,5	12,9	9,4	12 405	1	0,807879325	0,231341701	0	0,747	0,748	82	61
Turkey	1,000	75,5	14,6	7,9	18 705	1	0,809473126	0,229808966	0	0,745	0,767	71	62
Malaysia	1,000	74,9	13,1	10,1	24 620	1	0,808764557	0,230490387	0	0,741	0,789	59	63
Mongolia	1,000	69,8	14,8	9,8	10 449	1	0	1,008267796	0	0,74	0,735	92	64
Kuwait	1,000	74,5	13,3	7,3	76 075	1	0,243033534	0,641463324	0,397565628	0,739	0,800	51	65
Mauritius	1,000	74,6	15,2	9,1	17 948	1	0,805478016	0,233651007	0	0,739	0,781	64	65
Palau	1,000	72,9	14,3	12,3	13 771	1	0	1,008267796	0	0,733	0,788	60	66
Venezuela (Bolivarian Republic of)	1,000	74,4	14,3	9,4	15 129	1	0,80481832	0,234285427	0	0,731	0,767	71	67
Algeria	1,000	75,0	14,4	7,8	13 533	1	0,809223999	0,230048547	0	0,731	0,745	83	67
Brazil	1,000	74,7	15,2	7,8	14 145	1	0,807554455	0,231654124	0	0,73	0,754	79	68
Thailand	1,000	74,6	13,6	7,9	14 519	1	0,814863582	0,22462504	0	0,73	0,740	87	68
Tunisia	1,000	75,0	14,6	7,1	10 249	1	0,818060268	0,221550833	0	0,727	0,725	97	69
Jordan	1,000	74,2	13,1	10,1	10 111	1	0,803954565	0,235116087	0	0,724	0,741	86	70
Belarus	1,000	71,5	15,7	12,0	15 629	1	0	1,008267796	0	0,721	0,796	52	71
Peru	1,000	74,8	13,4	9,0	11 295	1	0,80672982	0,232447164	0	0,718	0,740	87	72
Nicaragua	1,000	75,2	11,7	6,5	4 747	1	0,852686841	0,188250948	0	0,718	0,645	124	72

Grenada	1,000	73,6	15,8	8,6	11 502	1	0,799763713	0,239146371	0	0,715	0,754	79	73
Tonga	1,000	73,0	14,3	11,1	5 284	1	0,522559569	0,505729667	0	0,714	0,721	101	74
Samoa	1,000	73,7	12,9	10,3	5 372	1	0,801398829	0,237573903	0	0,712	0,704	104	75
Saint Kitts and Nevis	1,000	74,0	13,7	8,4	22 436	1	0,809574866	0,229711124	0	0,707	0,765	74	76
Colombia	1,000	74,2	13,6	7,6	12 762	1	0,817715258	0,221882624	0	0,706	0,727	95	77
Seychelles	1,000	73,3	14,1	9,4	23 886	1	0,806941011	0,232244065	0	0,705	0,782	63	78
Armenia	1,000	74,9	12,7	11,3	8 189	1	0,801917912	0,237074708	0	0,698	0,743	84	79
Russian Federation	1,000	70,3	15,0	12,0	23 286	1	0	1,008267796	0	0,693	0,804	49	80
Dominican Republic	1,000	73,7	13,2	7,7	12 756	1	0,815631595	0,223886454	0	0,691	0,722	99	81
Morocco	1,000	74,3	12,1	5,0	7 195	1	0,859363616	0,181829989	0	0,689	0,647	123	82
Saint Vincent and the Grenadines	1,000	73,0	13,3	8,6	10 372	1	0,806070781	0,233080953	0	0,685	0,722	99	83
Palestine, State of	1,000	73,1	12,8	8,9	5 256	1	0,804375477	0,234711302	0	0,685	0,684	114	83
Bulgaria	1,000	74,3	15,0	10,8	16 261	1	0,787551092	0,250891076	0	0,675	0,794	56	84
Moldova (Republic of)	1,000	71,7	11,8	11,9	5 026	1	0,72322023	0,312757153	0	0,675	0,699	107	84
El Salvador	1,000	73,3	13,2	6,5	7 732	1	0,831685315	0,208447819	0	0,675	0,680	117	84
Cabo Verde	1,000	73,5	13,5	4,8	6 049	1	0,850481094	0,190372184	0	0,674	0,648	122	85
Paraguay	1,000	73,0	12,3	8,1	8 182	1	0,812272994	0,227116372	0	0,673	0,693	110	86
Fiji	1,000	70,2	15,3	10,5	8 245	1	0	1,008267796	0	0,667	0,736	91	87
Honduras	1,000	73,3	11,2	6,2	4 466	1	0,852377911	0,188548043	0	0,667	0,625	130	87
Azerbaijan	1,000	70,9	12,7	11,2	16 413	1	0	1,008267796	0	0,657	0,759	78	88
Kyrgyzstan	1,000	70,8	13,0	10,8	3 097	1	0	1,008267796	0	0,653	0,664	120	89
Trinidad and Tobago	1,000	70,5	12,7	10,9	28 049	1	0	1,008267796	0	0,647	0,780	65	90
Libya	1,000	71,8	13,4	7,3	14 303	1	0,808245989	0,230989086	0	0,644	0,716	102	91

Saint Lucia	1,000	75,2	13,1	9,3	9 791	1	0,806799621	0,232380038	0	0,642	0,735	92	92
Uzbekistan	1,000	69,4	12,2	12,0	5 748	1	0	1,008267796	0	0,638	0,701	105	93
Suriname	1,000	71,3	12,7	8,3	16 018	1	0,807140818	0,232051914	0	0,637	0,725	97	94
Vanuatu	1,000	72,1	10,8	6,8	2 805	1	0,844728935	0,195903954	0	0,637	0,597	134	94
Namibia	1,000	65,1	11,7	6,7	9 770	1	0,797642948	0,24118588	0	0,633	0,640	125	95
Belize	1,000	70,1	12,8	10,5	7 375	1	0	1,008267796	0	0,632	0,706	103	96
Turkmenistan	1,000	65,7	10,8	9,9	14 026	1	0	1,008267796	0	0,63	0,691	111	97
Zambia	1,000	60,8	12,5	6,9	3 464	1	0	1,008267796	0	0,623	0,579	139	98
Tajikistan	1,000	69,6	11,3	10,4	2 601	1	0,781696068	0,256521769	0	0,607	0,627	129	99
Micronesia (Federated States of)	1,000	69,3	11,7	9,7	3 291	1	0,7860378	0,252346387	0	0,597	0,638	127	100
Bolivia (Plurinational State of)	1,000	68,7	13,8	8,2	6 155	1	0,758112099	0,279202138	0	0,585	0,674	118	101
Indonesia	1,000	69,1	12,9	7,9	10 053	1	0,801413572	0,237559725	0	0,583	0,689	113	102
Nepal	1,000	70,0	12,2	4,1	2 337	1	0,852613909	0,188321087	0	0,575	0,558	144	103
Philippines	1,000	68,3	11,7	9,3	8 395	1	0,780565937	0,2576086	0	0,572	0,682	116	104
Iraq	1,000	69,6	10,1	6,6	11 608	1	0,842169939	0,198364905	0	0,571	0,649	121	105
Bhutan	1,000	69,9	12,5	3,1	7 081	1	0,859179152	0,182007385	0	0,568	0,607	132	106
Syrian Arab Republic	1,000	69,7	9,0	5,1	2 441	1	0,865636491	0,173933345	0	0,559	0,536	149	107
India	1,000	68,3	11,7	6,3	5 663	1	0,809066684	0,230199835	0	0,543	0,624	131	108
Cambodia	1,000	68,8	10,9	4,7	3 095	1	0,851928632	0,188980107	0	0,543	0,563	143	108
Timor-Leste	1,000	68,5	12,5	4,4	5 371	1	0,838627042	0,201772058	0	0,541	0,605	133	109
Solomon Islands	1,000	68,1	9,6	5,3	1 561	1	0,852261591	0,188659905	0	0,524	0,515	156	110
Egypt	1,000	71,3	13,1	7,1	10 064	1	0,807611897	0,231598884	0	0,519	0,691	111	111
Botswana	1,000	64,5	12,6	9,2	14 663	1	0	1,008267796	0	0,512	0,698	108	112

Guyana	1,000	66,5	10,3	8,4	6 884	1	0,798217131	0,240633697	0	0,511	0,638	127	113
Kiribati	1,000	66,2	11,9	7,8	2 475	1	0,768201295	0,269499501	0	0,51	0,588	137	114
Gabon	1,000	64,9	12,6	8,1	19 044	1	0	1,008267796	0	0,498	0,697	109	115
Sao Tome and Principe	1,000	66,6	12,0	5,3	3 070	1	0,806640793	0,232532781	0	0,495	0,574	142	116
Lao People's Democratic Republic	1,000	66,6	10,8	5,2	5 049	1	0,831627453	0,208503464	0	0,49	0,586	138	117
Senegal	1,000	66,9	9,5	2,8	2 250	1	0,87381387	0,163347742	0	0,48	0,494	162	118
Pakistan	1,000	66,4	8,1	5,1	5 031	1	0,860617177	0,180430839	0	0,472	0,550	147	119
Guatemala	1,000	72,1	10,7	6,3	7 063	1	0,850750751	0,190112859	0	0,471	0,640	125	120
Myanmar	1,000	66,1	9,1	4,7	4 943	1	0,855551143	0,18549639	0	0,468	0,556	145	121
Madagascar	1,000	65,5	10,3	6,1	1 320	1	0,805007117	0,234103864	0	0,466	0,512	158	122
Tanzania (United Republic of)	1,000	65,5	8,9	5,8	2 467	1	0,838255723	0,20212915	0	0,458	0,531	151	123
Rwanda	1,000	64,7	10,8	3,8	1 617	1	0,842550766	0,197998669	0	0,435	0,498	159	124
South Africa	1,000	57,7	13,0	10,3	12 087	1	0	1,008267796	0	0,42	0,666	119	125
Malawi	1,000	63,9	10,8	4,4	1 073	1	0,818816995	0,2208231	0	0,417	0,476	170	126
Ethiopia	1,000	64,6	8,4	2,6	1 523	1	0,87646958	0,159909928	0	0,416	0,448	174	127
Equatorial Guinea	1,000	57,9	9,2	5,5	21 517	1	0	1,008267796	0	0,414	0,592	135	128
Comoros	1,000	63,6	11,1	4,8	1 335	1	0,803546838	0,235508193	0	0,414	0,497	160	128
Yemen	1,000	64,1	9,0	3,0	2 300	1	0,868365688	0,170400404	0	0,406	0,482	168	129
Bangladesh	1,000	72,0	10,2	5,2	3 341	1	0,862721031	0,177707404	0	0,404	0,579	139	130
Ghana	1,000	61,5	11,5	6,9	3 839	1	0	1,008267796	0	0,403	0,579	139	131
Kenya	1,000	62,2	11,1	6,3	2 881	1	0	1,008267796	0	0,398	0,555	146	132
Sierra Leone	1,000	51,3	9,5	3,3	1 529	1	0	1,008267796	0	0,398	0,420	179	132
Uganda	1,000	59,2	10,0	5,7	1 670	1	0	1,008267796	0	0,395	0,493	163	133

Sudan	1,000	63,7	7,2	3,5	3 846	1	0,875197899	0,161556118	0	0,394	0,490	165	134
Mauritania	1,000	63,2	8,5	4,3	3 527	1	0,853136086	0,187818916	0	0,39	0,513	157	135
Papua New Guinea	1,000	62,8	9,9	4,3	2 712	1	0,824840783	0,215030108	0	0,385	0,516	154	136
Zimbabwe	1,000	59,2	10,3	7,7	1 588	1	0	1,008267796	0	0,361	0,516	154	137
Djibouti	1,000	62,3	6,3	4,1	3 216	1	0,873251685	0,164075489	0	0,357	0,473	172	138
Togo	1,000	60,2	12,0	4,7	1 262	1	0	1,008267796	0	0,347	0,487	166	139
Liberia	1,000	61,2	9,9	4,4	683	1	0,803520986	0,235533055	0	0,347	0,427	177	139
Niger	1,000	61,9	5,4	1,7	889	1	0,88719581	0,146024842	0	0,332	0,353	187	140
Afghanistan	1,000	60,7	10,1	3,6	1 871	1	0,809908176	0,229390584	0	0,329	0,479	169	141
Congo (Democratic Republic of the)	1,000	59,1	9,8	6,1	680	1	0	1,008267796	0	0,323	0,435	176	142
Haiti	1,000	63,1	9,1	5,2	1 657	1	0,822409224	0,217368504	0	0,321	0,493	163	143
Gambia	1,000	60,5	8,9	3,3	1 541	1	0,843012364	0,197554756	0	0,317	0,452	173	144
Benin	1,000	59,8	10,7	3,5	1 979	1	0,801715469	0,237269394	0	0,311	0,485	167	145
Congo	1,000	62,9	11,1	6,3	5 503	1	0,720497286	0,31537577	0	0,294	0,592	135	146
Guinea	1,000	59,2	8,8	2,6	1 058	1	0,849595847	0,191223514	0	0,281	0,414	183	147
Cameroon	1,000	56,0	10,4	6,1	2 894	1	0	1,008267796	0	0,279	0,518	153	148
Burkina Faso	1,000	59,0	7,7	1,4	1 537	1	0,878873126	0,156798543	0	0,266	0,402	185	149
Mali	1,000	58,5	8,4	2,3	2 218	1	0,855283597	0,185753686	0	0,26	0,442	175	150
Burundi	1,000	57,1	10,6	3,0	691	1	0	1,008267796	0	0,246	0,404	184	151
Nigeria	1,000	53,1	10,0	6,0	5 443	1	0	1,008267796	0	0,222	0,527	152	152
Angola	1,000	52,7	11,4	5,0	6 291	1	0	1,008267796	0	0,217	0,533	150	153
South Sudan	1,000	56,1	4,9	4,8	1 882	1	0,830242396	0,209835454	0	0,206	0,418	181	154
Mozambique	1,000	55,5	9,1	3,5	1 098	1	0	1,008267796	0	0,2	0,418	181	155

Guinea-Bissau	1,000	55,5	9,2	2,9	1 369	1	0	1,008267796	0	0,198	0,424	178	156
Swaziland	1,000	48,9	11,4	6,8	7 522	1	0	1,008267796	0	0,188	0,541	148	157
Lesotho	1,000	50,1	10,7	6,1	3 319	1	0	1,008267796	0	0,184	0,497	160	158
Côte d'Ivoire	1,000	51,9	8,9	5,0	3 163	1	0	1,008267796	0	0,17	0,474	171	159
Eritrea	1,000	64,2	5,0	3,9	1 490	1	0,882289183	0,152376462	0	0,137	0,420	179	160
Central African Republic	1,000	51,5	7,1	4,2	587	1	0	1,008267796	0	0,123	0,352	188	161
Chad	1,000	51,9	7,3	2,3	1 991	1	0	1,008267796	0	0,099	0,396	186	162

	Input	LEI	EI	II							
Australia	1	0	0.89374	0.3478							
Hong Kong, China (SAR)	1	0	1.00827	0							
Switzerland	1	0.55303	0.3839	0.28098							
Iceland	1	0.63907	0.32008	0.2235							
Japan	1	0	0.98623	0.06692							
Singapore	1	0.37618	0.52352	0.35998							
New Zealand	1	0.68655	0.28515	0.19091							
Norway	1	0.65764	0.30642	0.21076							

Italy	1	0.80718	0.23201	0							
Israel	1	0.81117	0.22817	0							
Canada	1	0	0.92562	0.25098							
Sweden	1	0.70656	0.27043	0.17719							
Spain	1	0.8012	0.23776	0							
Netherlands	1	0.8147	0.22478	0							
Korea (Republic of)	1	0	0.86773	0.42677							
France	1	0.80738	0.23182	0							
Ireland	1	0.81065	0.22868	0							
Germany	1	0.80932	0.22996	0							
Denmark	1	0.80855	0.2307	0							
Luxembourg	1	0.57677	0.36592	0.26626							
United Kingdom	1	0.81073	0.2286	0							
Austria	1	0.81233	0.22706	0							
Chile	1	0.81095	0.22839	0							

Slovenia	1	0.81347	0.22597	0								
Finland	1	0.80455	0.23454	0								
Belgium	1	0.81022	0.22909	0								
Greece	1	0.81001	0.22929	0								
Liechtenstein	1	0.79641	0.24237	0								
United States	1	0.80769	0.23153	0								
Andorra	1	0.38884	0.51231	0.35641								
Portugal	1	0	1.00827	0								
Malta	1	0.83747	0.20288	0								
Qatar	1	0.80968	0.22961	0								
Cyprus	1	0.80975	0.22954	0								
Czech Republic	1	0	0.89833	0.33385								
Cuba	1	0.79378	0.2449	0								
Brunei Darussalam	1	0	1.00827	0								
Costa Rica	1	0.80978	0.22951	0								

Lebanon	1	0	0.90647	0.30913							
Poland	1	0.79587	0.24289	0							
Estonia	1	0.8177	0.22189	0							
Croatia	1	0.39725	0.50486	0.35404							
Albania	1	0	1.00827	0							
Slovakia	1	0	1.00827	0							
United Arab Emirates	1	0.80052	0.23841	0							
Panama	1	0.80453	0.23456	0							
Argentina	1	0.81348	0.22595	0							
Montenegro	1	0.79918	0.2397	0							
Uruguay	1	0	1.00827	0							
Dominica	1	0.79997	0.23895	0							
Hungary	1	0.24303	0.64146	0.39757							
Bahrain	1	0	1.00827	0							
Lithuania	1	0.83231	0.20784	0							

Barbados	1	0.80108	0.23788	0								
Oman	1	0.80868	0.23058	0								
Mexico	1	0.78755	0.25089	0								
Bosnia and Herzegovina	1	0	1.00827	0								
Latvia	1	0.80756	0.23165	0								
Georgia	1	0.80876	0.23049	0								
Antigua and Barbuda	1	0	1.00827	0								
Maldives	1	0.8166	0.22296	0								
Bahamas	1	0.80893	0.23033	0								
Saudi Arabia	1	0.80694	0.23224	0								
Serbia	1	0.80548	0.23365	0								
Ecuador	1	0	1.00827	0								
Sri Lanka	1	0.83283	0.20735	0								
Romania	1	0.79963	0.23927	0								
Iran (Islamic Republic of)	1	0.80463	0.23447	0								

Jamaica	1	0.80755	0.23166	0								
China	1	0.77333	0.26457	0								
Armenia	1	0.80947	0.22981	0								
Bulgaria	1	0.80482	0.23429	0								
Viet Nam	1	0.80037	0.23856	0								
The former Yugoslav Republic of Macedonia	1	0.80957	0.22971	0								
Turkey	1	0.80727	0.23193	0								
Malaysia	1	0.84061	0.19987	0								
Saint Lucia	1	0.82782	0.21217	0								
Mauritius	1	0	1.00827	0								
Kuwait	1	0.80755	0.23165	0								
Palau	1	0.79976	0.23915	0								
Algeria	1	0.8072	0.23199	0								
Venezuela (Bolivarian Republic of)	1	0.80788	0.23134	0								

Peru	1	0.80922	0.23005	0							
Brazil	1	0.80192	0.23707	0							
Tunisia	1	0	1.00827	0							
Jordan	1	0.80395	0.23512	0							
Belarus	1	0.80673	0.23245	0							
Thailand	1	0.81486	0.22463	0							
Nicaragua	1	0.81542	0.22409	0							
Grenada	1	0.83098	0.20913	0							
Tonga	1	0	1.00827	0							
Samoa	1	0	1.00827	0							
Saint Kitts and Nevis	1	0.8068	0.23238	0							
Colombia	1	0.8069	0.23228	0							
Seychelles	1	0.81772	0.22188	0							
Ukraine	1	0.84375	0.19685	0							
Russian Federation	1	0.80714	0.23205	0							

Dominican Republic	1	0.81806	0.22155	0							
Morocco	1	0.81563	0.22389	0							
Palestine, State of	1	0.80607	0.23308	0							
Saint Vincent and the Grenadines	1	0.52256	0.50573	0							
Moldova (Republic of)	1	0.80825	0.23099	0							
Kazakhstan	1	0	1.00827	0							
El Salvador	1	0.8014	0.23757	0							
Cabo Verde	1	0.85474	0.18627	0							
Paraguay	1	0	1.00827	0							
Fiji	1	0.72322	0.31276	0							
Honduras	1	0	1.00827	0							
Azerbaijan	1	0	1.00827	0							
Kyrgyzstan	1	0.81227	0.22712	0							
Trinidad and Tobago	1	0.80761	0.2316	0							
Libya	1	0	1.00827	0							

Mongolia	1	0.80141	0.23756	0							
Uzbekistan	1	0.80438	0.23471	0							
Suriname	1	0.83423	0.206	0							
Vanuatu	1	0.78057	0.25761	0							
Guatemala	1	0.83169	0.20845	0							
Belize	1	0.75811	0.2792	0							
Egypt	1	0	1.00827	0							
Bangladesh	1	0	1.00827	0							
Tajikistan	1	0.84217	0.19836	0							
Micronesia (Federated States of)	1	0.85048	0.19037	0							
Bolivia (Plurinational State of)	1	0.85936	0.18183	0							
Indonesia	1	0.85269	0.18825	0							
Nepal	1	0.85075	0.19011	0							
Philippines	1	0.79764	0.24119	0							
Iraq	1	0.79822	0.24063	0							

Bhutan	1	0.78604	0.25235	0								
Syrian Arab Republic	1	0.7817	0.25652	0								
Cambodia	1	0.85238	0.18855	0								
India	1	0.80907	0.2302	0								
Timor-Leste	1	0.85918	0.18201	0								
Solomon Islands	1	0.83863	0.20177	0								
Turkmenistan	1	0.84473	0.1959	0								
Botswana	1	0.7205	0.31538	0								
Guyana	1	0	1.00827	0								
Kiribati	1	0.7682	0.2695	0								
Gabon	1	0.83163	0.2085	0								
Sao Tome and Principe	1	0.86272	0.17771	0								
Lao People's Democratic Republic	1	0	1.00827	0								
Senegal	1	0	1.00827	0								
Pakistan	1	0.80664	0.23253	0								

Namibia	1	0.85193	0.18898	0							
Myanmar	1	0.85261	0.18832	0							
Madagascar	1	0.85555	0.1855	0							
Tanzania (United Republic of)	1	0	1.00827	0							
Rwanda	1	0.86062	0.18043	0							
South Africa	1	0	1.00827	0							
Malawi	1	0.86564	0.17393	0							
Ethiopia	1	0	1.00827	0							
Comoros	1	0.83826	0.20213	0							
Congo	1	0	1.00827	0							
Yemen	1	0	1.00827	0							
Zambia	1	0.82484	0.21503	0							
Ghana	1	0	1.00827	0							
Kenya	1	0.85226	0.18866	0							
Eritrea	1	0.85314	0.18782	0							

Haiti	1	0.80501	0.2341	0								
Sudan	1	0.84255	0.198	0								
Mauritania	1	0.80355	0.23551	0								
Papua New Guinea	1	0	1.00827	0								
Zimbabwe	1	0.87381	0.16335	0								
Djibouti	1	0.82241	0.21737	0								
Liberia	1	0	1.00827	0								
Togo	1	0.8752	0.16156	0								
Niger	1	0	1.00827	0								
Afghanistan	1	0.80172	0.23727	0								
Congo (Democratic Republic of the)	1	0.86837	0.1704	0								
Uganda	1	0.80991	0.22939	0								
Gambia	1	0.81882	0.22082	0								
Benin	1	0	1.00827	0								
Equatorial Guinea	1	0.87325	0.16408	0								

Guinea	1	0.84301	0.19755	0								
Cameroon	1	0.87647	0.15991	0								
Burkina Faso	1	0.85528	0.18575	0								
Mali	1	0	1.00827	0								
Burundi	1	0.80352	0.23553	0								
Nigeria	1	0	1.00827	0								
Angola	1	0.88229	0.15238	0								
Mozambique	1	0	1.00827	0								
South Sudan	1	0	1.00827	0								
Guinea-Bissau	1	0.83024	0.20984	0								
Swaziland	1	0.8496	0.19122	0								
Lesotho	1	0	1.00827	0								
Côte d'Ivoire	1	0.87887	0.1568	0								
Sierra Leone	1	0	1.00827	0								
Central African Republic	1	0.8872	0.14602	0								

Chad	1	0	1.00827	0							

Table 2: HDR cutoff points and group sizes per methodology

	2014 HDR Cutoff points	**Group Size**	
		HDI	**GHDI**
Very high human development (class 1)	0.800 and above	51	49
High human development (class 2)	0.700-0.799	55	46
Medium human development (class 3)	0.550-0.699	41	34
Low human development (class 4)	Below 0.550	41	59

Figure 1: Game Human Development World Map

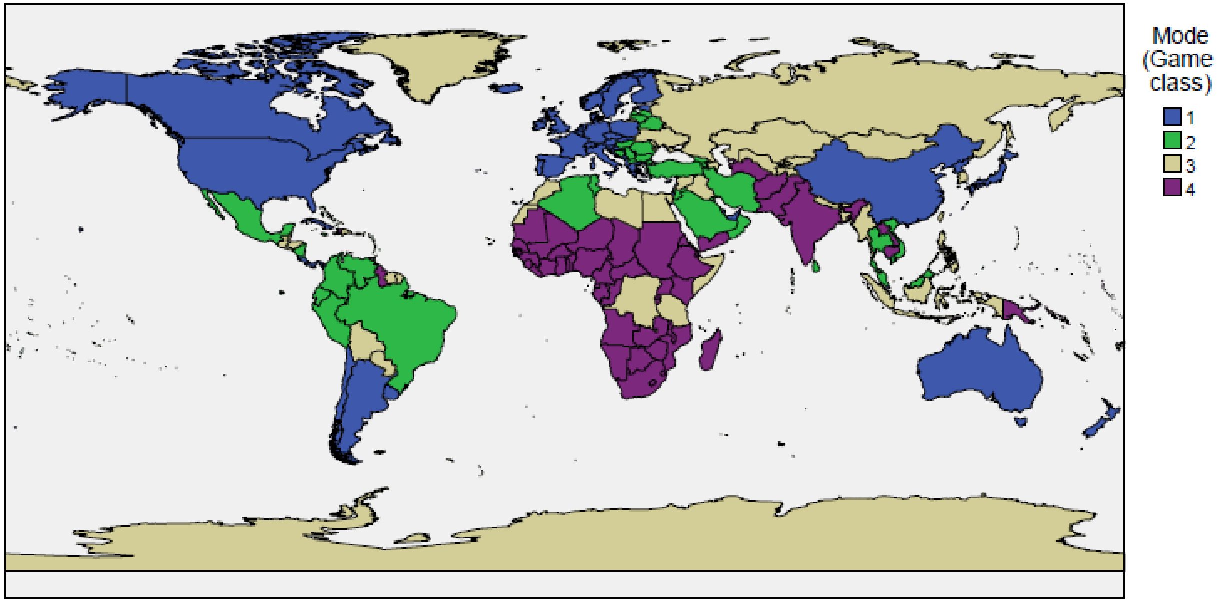

Figure 2: Human Development World Map