

In conventional ML models involving linear regression, the "hypothesis" is a linear/affine function, i.e. of the form.

$$\cancel{z} = W^T X + b.$$

An activation layer introduces non-linearity in the model, e.g.

$$a = \sigma(z) = \sigma(W^T X + b)$$

$$\text{where } \sigma(x) = \frac{1}{1+e^{-x}}.$$

Mathematical Requirements

(1) Non-linearity $a(\lambda z_1 + \mu z_2) \neq \lambda a(z_1) + \mu a(z_2)$

(2) Differentiability

Although ReLU is not diff'ble, it is extensively used. At 0, the issue is handled by employing subgradient.

(3) Boundedness $\exists M > 0 \text{ s.t. } |a(x)| \leq M \quad \forall x \in \mathbb{R}$.
e.g. σ , \tanh etc.

However, there are activation funcs that are unbd. e.g. ReLU.

Some of the Types

(1) Sigmoid. $\sigma(x) = \frac{1}{1+e^{-x}}$

→ Used for ^{binary} classification problems.

→ $\sigma \in (0, 1)$ ~~can be +~~ ~~or~~ ~~functions~~

→ $\sigma(x)$ can be interpreted as probability.

(2) ReLU

$$\text{ReLU}(x) = \max(0, x) = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

→ Used to avoid vanishing gradients

→ Used in CNNs almost always [other alternatives → GELU
Leaky ReLU]

(3) Softmax

$$[\text{softmax}(z)]_i = \frac{e^{z_i}}{\sum_{j=1}^C e^{z_j}} \quad \rightarrow \text{no. of classes}$$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_C \end{bmatrix}$$

→ Used for multi-class classification problems

(4) Leaky ReLU

~~Leaky~~ $f(x) = \max\{\alpha x, x\}$, where $\alpha > 0$ is fixed. ~~fixed~~

→ Used in CNNs

→ Avoids dying ReLU issue, i.e. $f'(x) = \alpha \neq 0$

[as opposed to $\frac{d \text{ReLU}(x)}{dx} = 0 \forall x < 0$]