

So split the in we get

$$\sum_{i=0}^d \sum_{j=0}^i \frac{1}{2^{i+j+1}} = \sum_{i=0}^d \frac{1}{2^{i+1}} \sum_{j=0}^i \frac{1}{2^{j+1}}$$

is known with what seems to be known

- Part of the sum is a sum of sums
  - inner loop runs for a number of sums
  - middle loop runs for a number of sums
  - outer loop runs for a number of sums
- the outer loop runs for a number of sums

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is known with what seems to be known

is the same as the left series

$$\sum_{i=0}^d \frac{1}{2^{i+1}} \sum_{j=0}^i \frac{1}{2^{j+1}} = \sum_{i=0}^d \frac{1}{2^{i+1}} \left( \sum_{j=0}^i \frac{1}{2^{j+1}} \right)$$

we can separate into 2 series

from the series

we can work on each

$$\sum_{i=0}^d \frac{1}{2^{i+1}} \left( \sum_{j=0}^i \frac{1}{2^{j+1}} \right)$$

$$\sum_{i=0}^d \frac{1}{2^{i+1}} \left( \sum_{j=0}^i \frac{1}{2^{j+1}} \right)$$