Extended Euclidean Algorithm

```
int ext_gcd (int a,int b,int &x,int &y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    int x1, y1;
    int d = ext_gcd (b%a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}
```

Ext_GCD provides a pair of {x,y} such that |x|+|y| is minimum and in case of tie, x <= y.

Diophantine Equation

http://www.alpertron.com.ar/METHODS.HTM#Linear

Linear Equation

Ax + By = C --- (i)

Using Extended Euclidean Algorithm, we can solve ax + by = 1 --- (ii), if and only if gcd (a, b) = 1. So in order to solve equation (i), we have to first find g = gcd (A,B). Next, there is no solution if (C % g != 0). If it is possible to divide C with g, then the equation becomes, (A/g)x+(B/g)y=(C/g). Or we can rewrite as A'x + B'y = C'. Now, we simply find the solution for A'x + B'y = 1, and then multiply C' with the pair of solution (x,y).

Hyperbolic Equation

Bxy + Dx + Ey + F = 0

Bxy + Dx + Ey + F = 0
Bxy + Dx + Ey = -F
B2xy + BDx + BEy = -BF
B2xy + BDx + BEy + DE = DE - BF
(Bx + E) (By + D) = DE - BF

There are two cases: DE - BF = 0 (two lines parallel to x and y axes respectively) and DE - BF != 0 (a hyperbola whose asymptotes are parallel to x and y axes).

In the first case a necessary condition to have solutions occurs when one of the parentheses equal zero, i.e., Bx + E = 0 or By + D = 0. Since B != 0, we have solutions for:

If (Bx + E) = 0 then, x = -E/B and y = any integer
or (By + D) = 0 then, x = any integer and y = -D/B

In the second case the values of x and y are found by finding all divisors of DE - BF. Let d1, d2, ..., dn be the set of divisors of DE - BF.So, (Bx+E)=d1 and (By+D) = (DE-BF)/d1 and solve accordingly

Harmonic Sequence

In mathematics, the harmonic series is the divergent infinite series:

$$\sum_{n=1}^{\infty} \frac{1}{n} \;=\; 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots.$$

There is no closed form formula for partial sum of the sequence, but an approximate formula exists: $H_n \approx \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2}$ is quite good, where γ≈0.5772156649 is the Euler-Mascheroni constant.

This is particularly useful for calculating complexities of algorithm. O(n+n/2+n/3+n/4+…+1) = O(n*(1+½+⅓+...+1/n)) = O(n*ln(n)).

Combinatorics

Binomial Coefficient

ncr (n, k) = ncr (n - 1, k - 1) + ncr (n - 1, k), n, k > 0
k = 0, ncr (n, 0) = 1
n = 0, ncr (0, k) = 0

$$\binom{n}{k} = \frac{n!}{(n-k)! \times k!}$$

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}, \quad \binom{n}{k} = \frac{n-k-1}{k}\binom{n}{k-1}$$

When we have to find nck such that the final result will fit into integer, but intermediate result will overflow, we need to factorize each factorial and simplify to calculate the result.

Horizontal Sum

Sum of all terms in a row is 2 ^ (row)
Sum of alternate terms of a row is 2^(row-1).

Vertical Sum of Pascal Triangle

$$\sum_{j=0}^{n} \binom{j}{k} = \sum_{j=k}^{n} \binom{j}{k} = \binom{n+1}{k+1}$$

Ball and Urn Theorem

$a_1 + a_2 + ... a_k = N$, where “a” is non negative integer. The solution is $\binom{n+k-1}{k-1}$.

Variation

Lower Bound / Upper Bound

If there are lower bounds on each of the partition, then simply subtracting the lower bounds from N will give the same results. If there are upper bounds, then we must perform Dynamic Programming.

Less than or Equal to N

Another variation could be the following: $a_1 + a_2 + ... + a_k \leqslant N$. Notice that this is equivalent to $\sum_{x=0}^{N} \binom{x+k-1}{k-1}$ which is simply the vertical sum in Pascal Triangle.

Reference

http://www.artofproblemsolving.com/Wiki/index.php/Ball-and-urn

Integer Partition

In number theory and combinatorics, a partition of a positive integer n, also called an integer partition, is a way of writing n as a sum of positive integers. Two sums that differ only in the order of their summands are considered the same partition. (If order matters, the sum becomes a composition.) For example, 4 can be partitioned in five distinct ways:

4
3 + 1
2 + 2
2 + 1 + 1
1 + 1 + 1 + 1

The # of ways of making \$i with coins of value \$j or less is the same as the # of ways of making \$i with at most j coins.

The number of partitions of n into no more than k parts (complexity O(n*k*last)) is the same as the number of partitions of n into parts no larger than k (complexity O(n*last)).

Proof is evident from **Ferrers Diagram**.

http://en.wikipedia.org/wiki/Partition_(number_theory)

Derangement

In combinatorial mathematics, a derangement is a permutation of the elements of a set, such that no element appears in its original position.

$D(n) = (n-1) * (D(n-1)+D(n-2))$, where $D(1) = 0$ and $D(2) = 1$.

Resource

http://www.shafaetsplanet.com/planetcoding/?p=600

Contest

NSU From Scratch 2: http://acm.hust.edu.cn/vjudge/contest/view.action?cid=69368#overview
NSU From Scratch 3: http://acm.hust.edu.cn/vjudge/contest/view.action?cid=69368#overview