1

a) Prove E step updates on membership achieves minimum objective given current centroids

Proof by Contradiction:

Assuming that the E step update does not achieve the minimum objective given the current centroids, for point x, cluster I and membership π_{Ix} such that $\pi_{Ix} = 1$, then there exists another cluster J and membership $\pi_{Jx} = 1$ such that:

$$\pi_{ix} \cdot \| x_i - \mu_x \|^2 < \pi_{Jx} \cdot \| x_j - \mu_x \|^2$$

This then contradicts our initial assumption that the E step does not achieve the minimum objective, given the centroids.

1

b) $J = \sum_i \sum_k \pi_{ik} \left(||x_i - \mu_k|| \right)^2$

Taking derivative and solving for 0, wrt to μ

$$0 = \frac{dJ}{d\mu} = \frac{d}{d\mu} \sum_i \sum_k \pi_{ik} \left(||x_i - \mu_k|| \right)^2$$

For ~~Assuming~~ $\pi_{ik} = 1$

$$\frac{dJ}{d\mu} = \frac{d\, \pi_{ik} \left(||x_i - \mu_k|| \right)^2}{d\mu} = 0$$

$$= \frac{d \sum_i \pi_{ik} \sum_i (x_i - \mu_k)^2}{d\mu} = 0$$

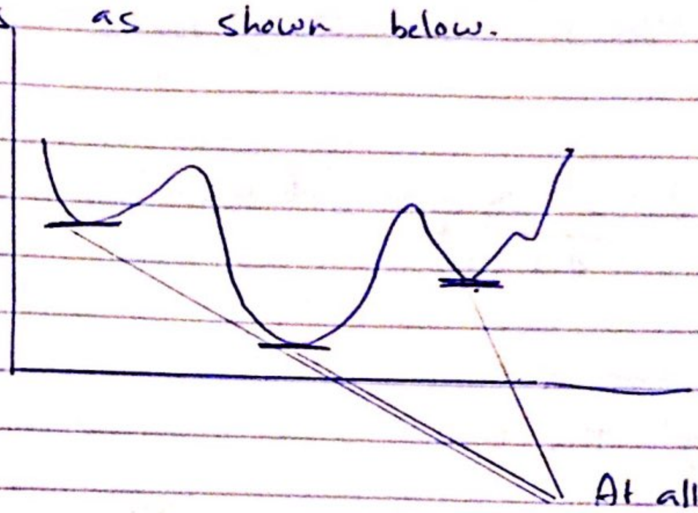
$$= \sum_i \pi_{ik} \cdot 2 (x_i - \mu_k) = 0$$

$$\sum_i x_i \pi_{ik} - \sum_i \pi_{ik} \mu_k = 0$$

$$\sum_i x_i \pi_{ik} = \sum_i \pi_{ik} \mu_k$$

$$\mu_k = \frac{\sum_i x_i \pi_{ik}}{\sum_i \pi_{ik}}$$

c) Differentiation of a concave function and equating it to 0 gives us all points on the concave function where the tangent is 0. While these points indicate minimums, they may not necessarily be global minimus as shown below.



At all these points, $f'(x) = 0$.
but only one of the points is the global minimum.