1). a)

The basis functions can be defined by.

$h_1(x) = 1$

$h_2(x) = x$

$h_3(x) = x^2$

$h_4(x) = x^3$

$h_5(x) = (x-\xi_1)^3$

$h_6(x) = (x-\xi_2)^3$

$h_7(x) = (x-\xi_3)^3$

The model is specified as.

$$Y = \sum_{m=1}^{m} \theta_m h_m(x)$$

as in any regression model.

where $m = [1 \dots 7]$

in vector form,

$$Y = \theta^T X.$$

where, $X = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ (x-\xi_1)^3 \\ (x-\xi_2)^3 \\ (x-\xi_3)^3 \end{bmatrix}$ $\theta = \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_6 \end{bmatrix}$ $Y = \begin{bmatrix} y_0 \\ \vdots \\ y_{12} \end{bmatrix}$

b) To estimate the parameters of the model, we could use ~~gradient descent or anal~~ gradient descent or analytical solution. We shall describe the analytical solution below.

$$Y = X\theta + e.$$

$$J(\theta) = \| Y - X\theta \|_2^2.$$

$$J(\theta) = (Y - \theta X)^T (Y - X\theta)$$

$$-X^T Y - X^T Y + 2 X^T X \theta = 0$$

finally,

$$X^T X \theta = X^T Y$$

$$\hat{\theta} = (X^T X)^{-1} X^T Y$$

In [158]:
```
import pandas as pd
import numpy as np
import scipy.stats as stats
from tqdm import tqdm_notebook as tqdm
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import MinMaxScaler
from sklearn.metrics import confusion_matrix

%matplotlib inline
pd.set_option('display.float_format', lambda x: '%.3f' % x)
```

```
References:
https://math.dartmouth.edu/~m50f15/Lowess.html
https://gist.github.com/agramfort/850437
https://www.olamilekanwahab.com/locally-weight-regression/
https://www.antoniomallia.it/lets-implement-a-gaussian-naive-bayes-
classifier-in-python.html
```

Question 3

In [ ]:
```
ipo_df = pd.read_csv('data/hw3_dataset.txt', header = None,delim_whitespa
             names = ['Identification number', 'Venture capital funding',
                      'Face value of company', 'Number of shares offered',
                                              'Leveraged buyout'])
```

In [3]:
```
ipo_df = ipo_df.set_index('Identification number')
```

Checking the balance of the predictors

In [4]:
```
len(ipo_df.loc[ipo_df['Venture capital funding'] == 1])/ len(ipo_df)
```

Out[4]:
```
0.43983402489626555
```

Split to train and test

In [5]:
```
X_train, X_test, y_train, y_test = train_test_split(ipo_df[ipo_df.columns
```

In [6]:
```
X_train
```

Out[6]:

Identification number	Face value of company	Number of shares offered	Leveraged buyout
220	17500000	1400000	0
95	7500000	3000000	0
18	3575000	1100000	0
473	117000000	6000000	0
441	55400000	5540000	0
114	9375000	1500000	1
388	37500000	2500000	0
63	5750000	1150000	0
447	59400000	3300000	0
154	12800000	1600000	0

Scaling so a constant lambda can be set across all features

In [7]:
```
scaler = StandardScaler()
scaler.fit(X_train)
X_train.loc[:,:] = scaler.transform(X_train)
scaler.fit(X_test)
X_test.loc[:,:] = scaler.transform(X_test)


#y_train.loc = scaler.transform(y_train)
```

In [9]:
```
X_train
```

Out[9]:

	Face value of company	Number of shares offered	Leveraged buyout
Identification number			
220	-0.342	-0.570	-0.309
95	-0.697	0.492	-0.309
18	-0.836	-0.769	-0.309
473	3.186	2.485	-0.309
441	1.002	2.179	-0.309
114	-0.630	-0.504	3.240
388	0.367	0.160	-0.309
63	-0.759	-0.736	-0.309
447	1.144	0.692	-0.309
154	-0.509	-0.437	-0.309

Naive Bayes Implementation

In [47]:
```
def naive_bayes(X_train, Y_train, X_test, Y_test, lamb):

    prior_1 = np.sum(Y_train, axis = 0)/len(X_train)
    prior_0 = 1 - prior_1

    post_0 = np.zeros(len(X_test))
    post_1 = np.zeros(len(X_test))

    pred = np.zeros(len(X_test))


    for i in Y_test.index:

        for j in Y_train.index:

            if Y_train.loc[j]['Venture capital funding'] == 1:

                delta                                = (X_test.loc[i] - X
                post_1[list(X_test.index).index(i)] = post_1[list(X_test.
                                                       + 1/((lamb * len(
                                                       * stats.norm.cdf(

            if Y_train.loc[j]['Venture capital funding'] == 0:

                delta                               = (X_test.loc[i] - X_
                post_0[list(X_test.index).index(i)] = post_0[list(X_test.
                                                       + 1/((lamb * len(
                                                       * stats.norm.cdf(

        post_1[list(X_test.index).index(i)] = post_1[list(X_test.index).i
        post_0[list(X_test.index).index(i)] = post_0[list(X_test.index).i


    for i in range(len(post_1)):

        if post_1[i] > post_0[i]:

            pred[i] = 1



    accuracy = 1 - sum(abs(list(Y_test['Venture capital funding']) - pred


    return prior_1, prior_0, post_1, post_0, pred
```

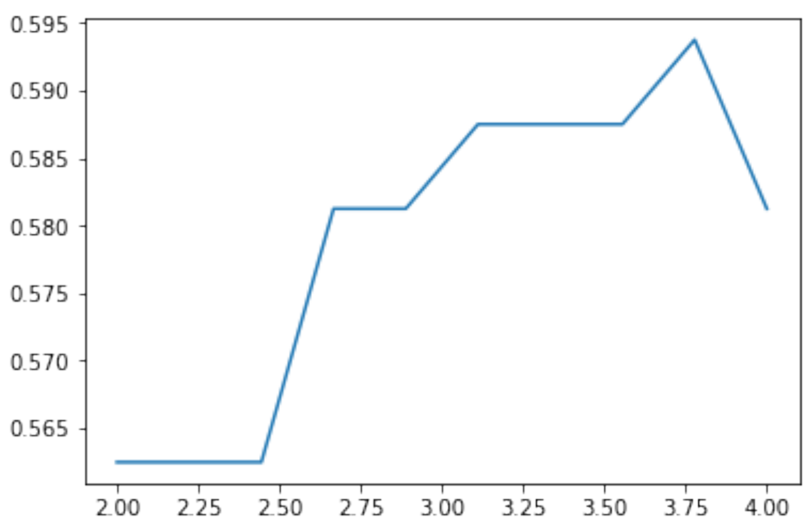
In [430]:
```
accuracies = []

for lamb in np.linspace(2.8,5, 10):

    accuracies.append(naive_bayes(X_train, y_train, X_test, y_test, lamb)
```

In [41]:
```
accuracies
```

Out[41]:
```
[0.5625,
 0.5625,
 0.5625,
 0.58125,
 0.58125,
 0.5875,
 0.5875,
 0.5875,
 0.59375,
 0.58125]
```

In [42]:
```
plt.plot(np.linspace(2,4, 10),accuracies)
```

Out[42]:
```
[<matplotlib.lines.Line2D at 0x7f7f9c8432e8>]
```



Lambda of 3.75 appears to work the best

In [48]:
```
prior_1, prior_0, post_1, post_0, pre = naive_bayes(X_train, y_train, X_t
```

In [50]:
```
1 - sum(abs(list(y_test['Venture capital funding']) - pre))/len(y_test)
```

Out[50]:
```
0.58125
```

In [51]:
```
tn, fp, fn, tp = confusion_matrix(pre, list(y_test['Venture capital fundi
```

In [52]:
```
(tn, fp, fn, tp)
```

Out[52]: (83, 60, 7, 10)

Question 4

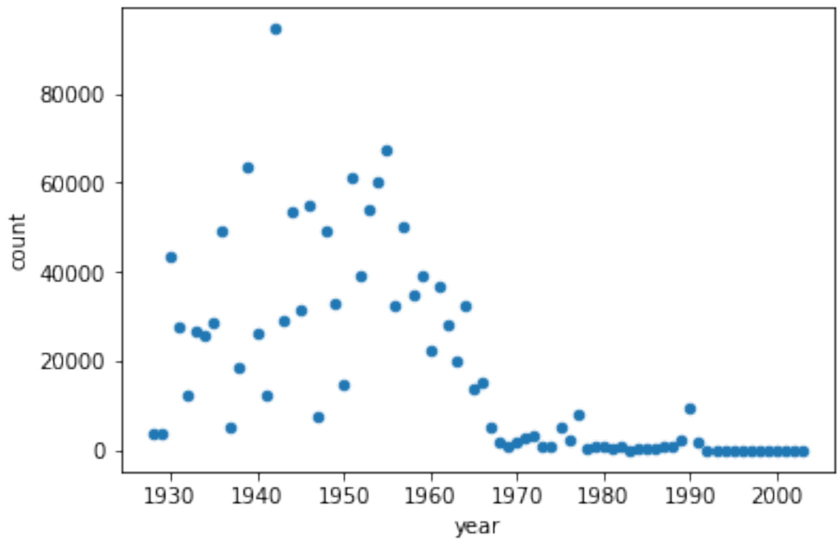
In [65]:
```
diseases = pd.read_csv("data/us_diseases.csv")
del diseases['Unnamed: 0']
diseases = diseases[(diseases.disease == 'Measles') & (diseases.state ==
diseases
```

2715	Measles	California	1993	47	63	31292538.000
2716	Measles	California	1994	45	64	31728420.000
2717	Measles	California	1995	37	38	32133526.000
2718	Measles	California	1996	42	45	32512506.000
2719	Measles	California	1997	48	35	32870358.000
2720	Measles	California	1998	31	4	33212395.000
2721	Measles	California	1999	42	18	33544208.000
2722	Measles	California	2000	38	15	33871648.000
2723	Measles	California	2001	40	34	34199784.000
2724	Measles	California	2002	33	0	34529758.000
2725	Measles	California	2003	0	0	34861711.000

76 rows × 6 columns

In [73]:
```
diseases[['year','count']].plot.scatter('year','count')
```

Out[73]: <matplotlib.axes._subplots.AxesSubplot at 0x7f7f9c420828>

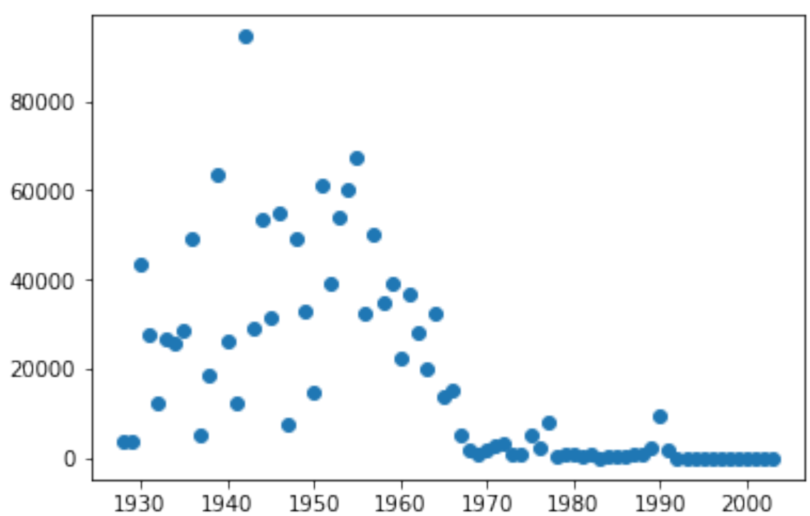


In [78]:
```
def gradientDescent(x, y, alpha, m, numIterations):

    theta = np.ones(2)

    for i in range(0, numIterations):

        loss     = (1 / (1 + np.exp(-(1 + (np.dot(x, theta)))))) - y
        gradient = np.dot(x.transpose(), loss) / m
        theta    = theta - alpha * gradient

    return theta
```

In [413]:
```
def get_local_pred(x, y, tau):

    m             = len(x)
    y_hat         = np.zeros(m)
    numIterations = 5000
    alpha         = .01

    x             = x.astype('float')
    w             = np.array([np.exp(-(x-x[i])**2/(2*tau)) for i in range

    for i in range(m):

        weights  = w[:, i]
        b        = np.array([np.sum(weights * y), np.sum(weights * y * x)
        A        = np.array([[np.sum(weights), np.sum(weights * x)],
                    [np.sum(weights * x), np.sum(weights * x * x)]])


        theta    = gradientDescent(A, b, alpha, m , numIterations)
        y_hat[i] = theta[0] + theta[1] * x[i]

    return y_hat
```

In [375]:
```
plt.scatter(diseases['year'], diseases['count'])
```

Out[375]: <matplotlib.collections.PathCollection at 0x7f7f92a0b748>



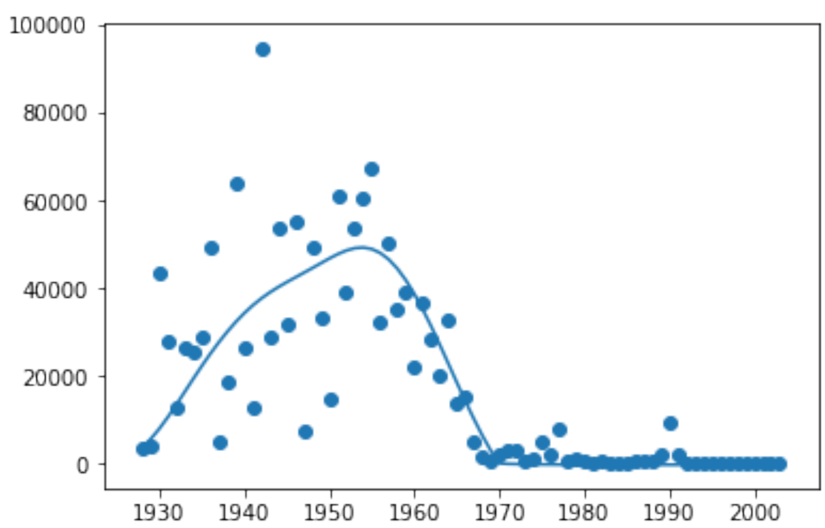
In [431]:
```
scaler = MinMaxScaler()
x      = scaler.fit_transform(diseases['year'].reshape(-1, 1))
y      = scaler.fit_transform(diseases['count'].reshape(-1, 1))
```

```
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:2: Future
Warning: reshape is deprecated and will raise in a subsequent release.
Please use .values.reshape(...) instead

/opt/conda/lib/python3.6/site-packages/sklearn/utils/validation.py:475
: DataConversionWarning: Data with input dtype int64 was converted to
float64 by MinMaxScaler.
  warnings.warn(msg, DataConversionWarning)
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:3: Future
Warning: reshape is deprecated and will raise in a subsequent release.
Please use .values.reshape(...) instead
  This is separate from the ipykernel package so we can avoid doing im
ports until
```

With Inverse Scaling and lambda as 0.005 and 0.01

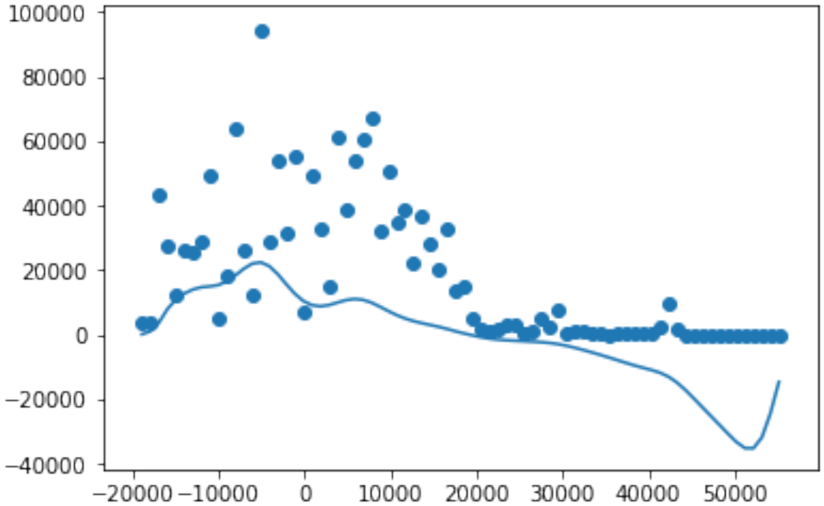
In [429]:
```
plt.plot(diseases['year'], scaler.inverse_transform(get_local_pred(x, y,
plt.scatter(diseases['year'], scaler.inverse_transform(y), label='y')
```

Out[429]: <matplotlib.collections.PathCollection at 0x7f7f91abb0f0>



In [301]:
```
plt.plot(scaler.inverse_transform(x), scaler.inverse_transform(get_local_
plt.scatter(scaler.inverse_transform(x), scaler.inverse_transform(y), lab
```

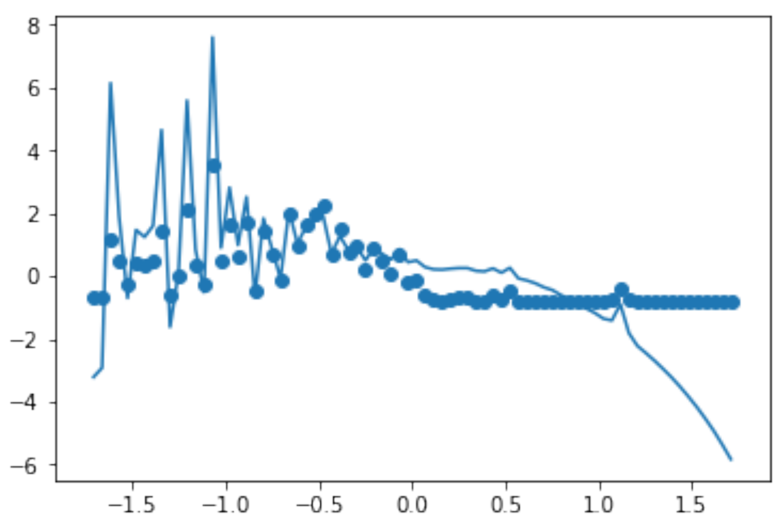
Out[301]: <matplotlib.collections.PathCollection at 0x7f7f94164b70>



Without Inverse Scaling and lambda as 0.00000005 and 0.009

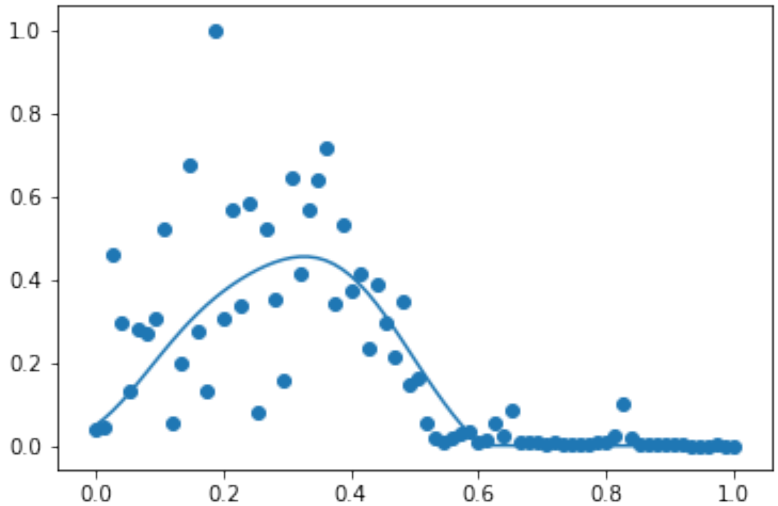
In [304]:
```
plt.plot(x, get_local_pred(x, y, 0.00000005))
plt.scatter(x, y, label='y')
```

Out[304]: <matplotlib.collections.PathCollection at 0x7f7f940e8160>



In [427]:
```
plt.plot(x, get_local_pred(x, y, 0.009))
plt.scatter(x, y, label='y')
```

Out[427]: <matplotlib.collections.PathCollection at 0x7f7f91b73e10>



Describe the role of the tuning parameter λ.

The higher the lambda, the more number of points the regression smoothes over. The lower the lamda, the more 'local' the regression is going to be.

What conclusion regarding the occurrence of the disease can you make from the local regression fit?

The number of people with measles initially increased from 1930's to 1960's and then decreased over the years from a peak of around 50k to close to 0.

Can you suggest an explanation for this pattern?

This makes obvious sense as technology in healthcare and vaccines have advanced and measles has largly been eradicated.

In [ ]: