In [1]:

```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import random
from sklearn import linear_model
import scipy.stats
from sklearn.metrics import accuracy_score
np.set_printoptions(suppress=True)
```

In [2]:

```
def genData(numPoints):
    x = np.zeros(shape=(numPoints, 3))
    y = np.zeros(shape=numPoints)

    x[:,0] = 1
    #x[:,1] = [i for i in range(numPoints)]
    x[:,1] = np.random.uniform(0, 3, size = numPoints)
    x[:,2] = np.random.uniform(0, 3, size = numPoints)



    for i in range(0, numPoints): y[i] =  1/(1 + np.exp( - (-3 + (x[i][1]) + (x[
i][2]))))
    #for i in range(0, numPoints): y[i] =  1/(1 + np.exp( - (-3 + (x[i][1]))))


    y = np.array([1 if i > 0.5 else 0 for i in y])
    return x, y
```

In [3]:

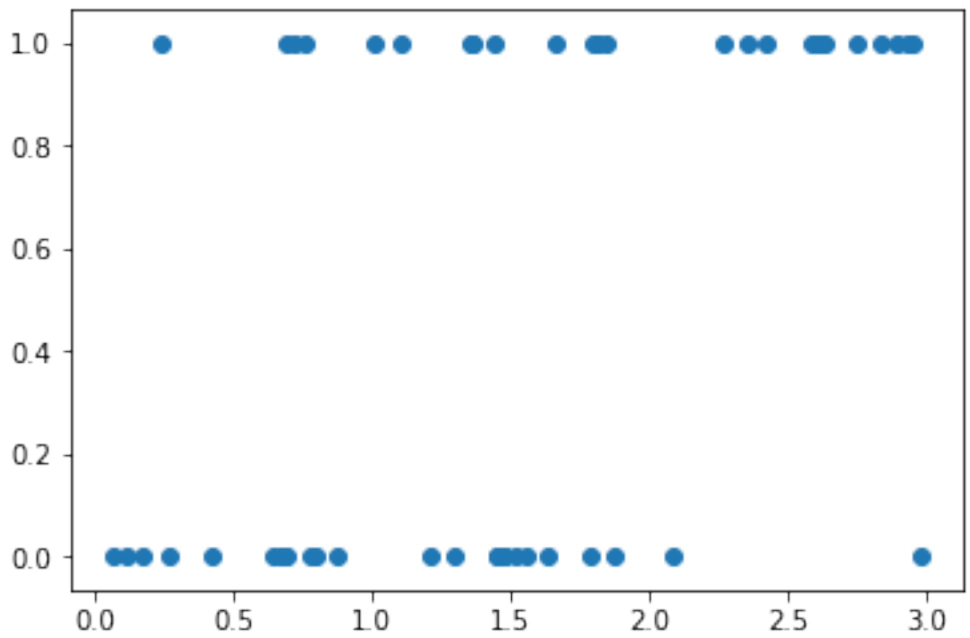
```
x,y = (genData(50))
#y_temp = np.array([1 if i > 0.5 else 0 for i in y])
```

In [4]:

```
plt.scatter(x[:,1], y)
```

Out[4]:

```
<matplotlib.collections.PathCollection at 0x7ff2ba97cd30>
```



In [5]:

```
def gradientDescent(x, y, theta, alpha, m, numIterations):

    xTrans = x.transpose()

    for i in range(0, numIterations):

        loss = (1 / (1 + np.exp(-(1 + (np.dot(x, theta)))))) - y

        #print (np.mean(loss))

        #gradient = np.dot(xTrans,  np.dot(x, theta) - y) / m #Partial derivativ
e
        gradient = np.dot(xTrans, loss) / m
        theta = theta - alpha * gradient


    return theta
```

In [6]:

```
m, n = np.shape(x)

theta = gradientDescent(x,y, np.ones(n), 1, m, 70000)
```

In [7]:

```
theta
```

Out[7]:

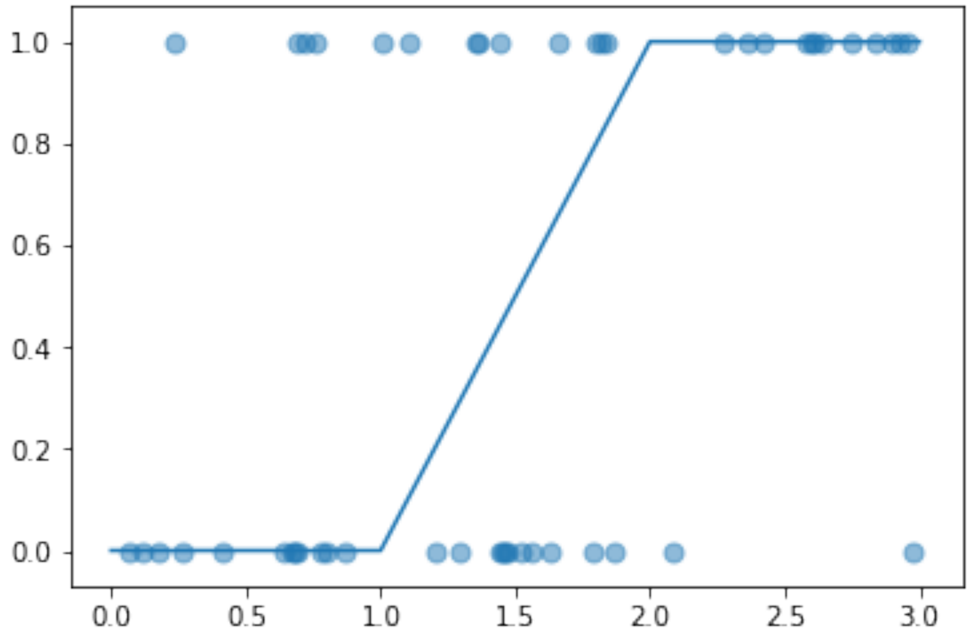
```
array([-66.12729013,  20.52518735,  21.32420216])
```

In [8]:

```
hypothesis = lambda x: 1 / (1 + (np.exp(-(x * theta[1] + x * theta[2]+ theta[0])
)))

plt.plot([i for i in range(0,4)],[hypothesis(i) for i in range(0,4)])

plt.scatter([i[1] for i in x], y, s=50, alpha = 0.5)
plt.show()
```



Decision boundaries for X[1] vs X[2] and X[1] vs X[0] respectively for Batch GD

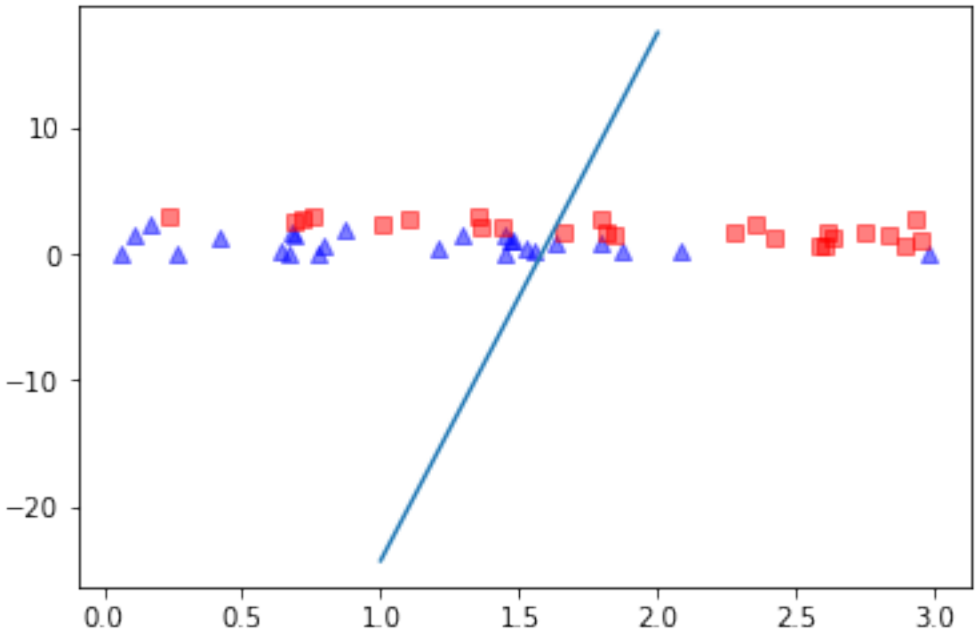
In [9]:

```
d_boundary_plot = lambda x: x*theta[1] + x*theta[2] + theta[0]
plt.plot([i for i in range(1,3)],[d_boundary_plot(i) for i in range(1,3)])

#plt.scatter([i[1] for i in x], [i[2] for i in x], s=50, alpha = 0.5)

for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')):

        plt.scatter(x=x[:,1].real[y == label],
                y=x[:,2].real[y == label],
                marker=marker,
                color=color,
                alpha=0.5
                )
```



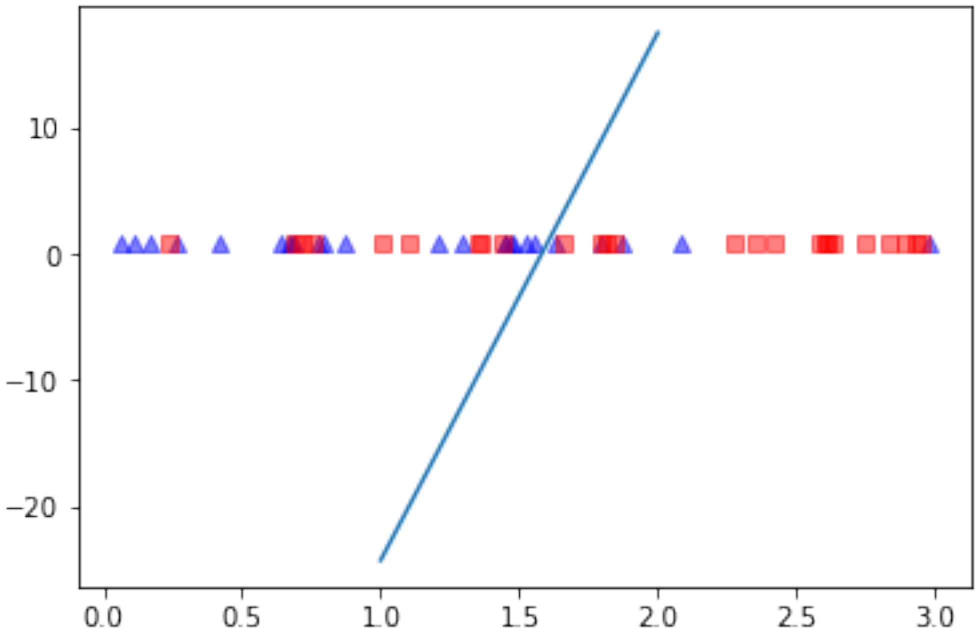
In [10]:

```
d_boundary_plot = lambda x: x*theta[1] + x*theta[2] + theta[0]
plt.plot([i for i in range(1,3)],[d_boundary_plot(i) for i in range(1,3)])

#plt.scatter([i[1] for i in x], [i[2] for i in x], s=50, alpha = 0.5)

for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')):

        plt.scatter(x=x[:,1].real[y == label],
                y=x[:,0].real[y == label],
                marker=marker,
                color=color,
                alpha=0.5
                )
```



In [11]:

```
def gradientDescentSto(x, y, theta, alpha, m, numIterations, delta=2, conv=0.000
00001):

    xTrans = x.transpose()
    count = 0
    theta_prev = theta + delta + 1

    while (count < numIterations):

            count += 1

            for i in range(m):

                hypothesis = x[i][0]*theta[0] + x[i][1]*theta[1] + x[i][2]*theta
[2]

                loss = 1/(1 + np.exp(- hypothesis)) - y[i]

                #loss = hypothesis - y[i]

                gradient1 = x[i][0] * loss
                gradient0 = x[i][1] * loss

                theta_prev = theta
                theta[1] = theta[1] - alpha * gradient0
                theta[0] = theta[0] - alpha * gradient1


    return theta
```

In [12]:

```
theta = gradientDescentSto(x, y, theta, 0.08, m, 10000)
```

In [13]:

```
theta
```

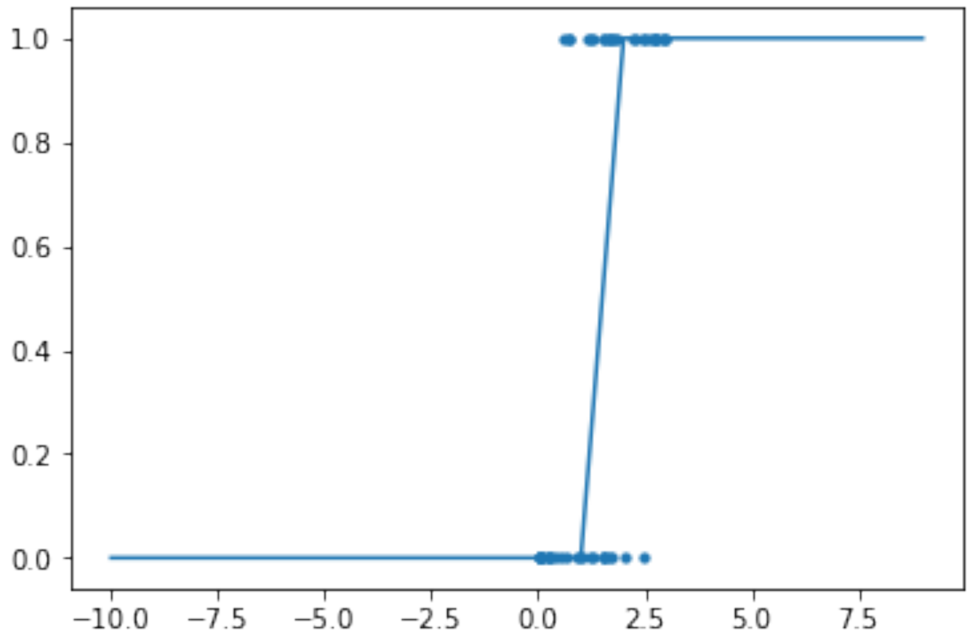
Out[13]:

```
array([-65.34984189,  20.63300455,  21.32420216])
```

In [14]:

```
hypothesis = lambda x: 1 / (1 + (np.exp(-(x * theta[1] + x * theta[2] + theta[0]
))))

plt.plot([i for i in range(-10,10)],[hypothesis(i) for i in range(-10,10)])
plt.scatter([i[2] for i in x], y, s=10)
plt.show()
```



Decision boundaries for X[1] vs X[2] and X[1] vs X[0] respectively for Stochastic GD

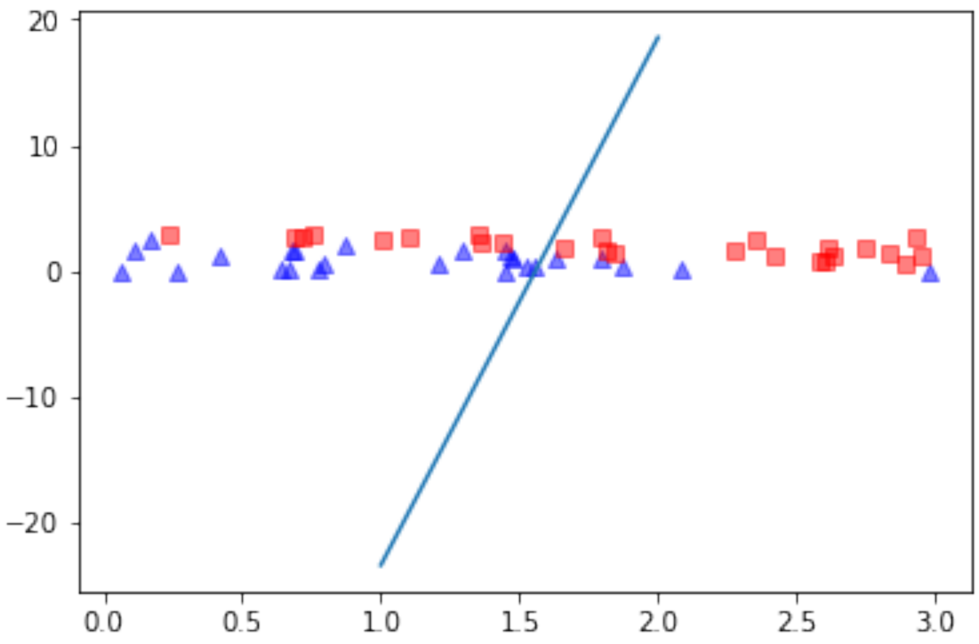
In [15]:

```
d_boundary_plot = lambda x: x*theta[1] + x*theta[2] + theta[0]
plt.plot([i for i in range(1,3)],[d_boundary_plot(i) for i in range(1,3)])

#plt.scatter([i[1] for i in x], [i[2] for i in x], s=50, alpha = 0.5)

for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')):

        plt.scatter(x=x[:,1].real[y == label],
                y=x[:,2].real[y == label],
                marker=marker,
                color=color,
                alpha=0.5
                )
```



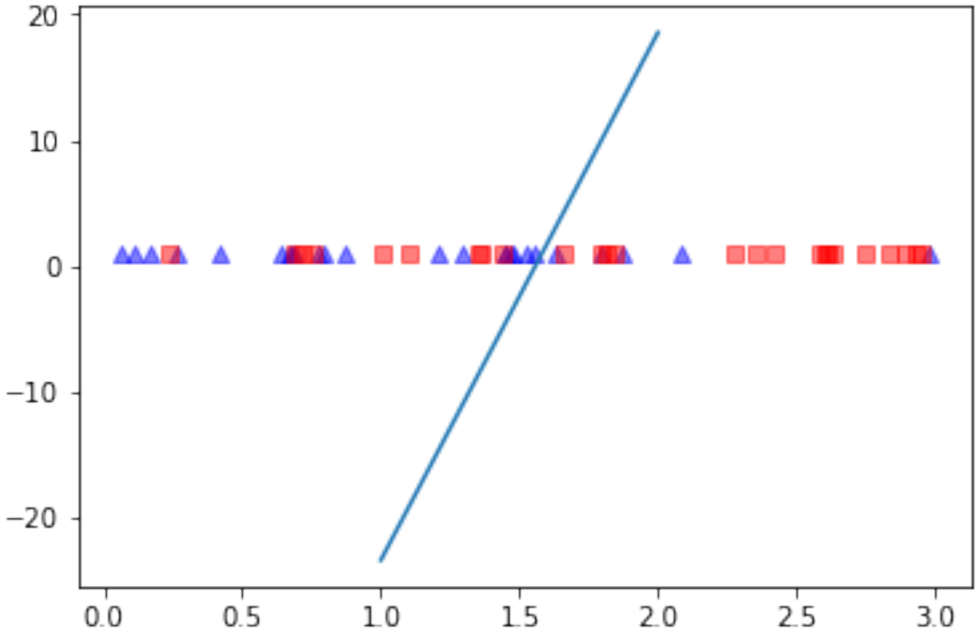
In [16]:

```
d_boundary_plot = lambda x: x*theta[1] + x*theta[2] + theta[0]
plt.plot([i for i in range(1,3)],[d_boundary_plot(i) for i in range(1,3)])

#plt.scatter([i[1] for i in x], [i[2] for i in x], s=50, alpha = 0.5)

for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')):

        plt.scatter(x=x[:,1].real[y == label],
                y=x[:,0].real[y == label],
                marker=marker,
                color=color,
                alpha=0.5
                )
```



In [17]:

```
d_boundary = lambda x: x[1]*theta[1] + x[2]*theta[2] + theta[0] * x[0]
y_hat = [1 if i > 0 else 0 for i in [d_boundary(i) for i in x]]
```

In [18]:

```
accuracy_score(y, y_hat)
```

Out[18]:

```
1.0
```

Linear discriminant analysis

In [19]:

```
x_old = x
x = x[:, 1:]
```

In [20]:

```
mean_vs = []
y_temp = np.array([1 if i > 0.5 else 0 for i in y])
for i in range(0,2):mean_vs.append(np.mean(x[y_temp==i], axis = 0))
print(mean_vs)
```

```
[array([1.11687638, 0.83202265]), array([1.90450382, 1.97216123])]
```

In [21]:

```
within_class = np.zeros((2,2))

for cl,mv in zip(range(0,2), mean_vs):

    class_scatter_matrix = np.zeros((2,2))
    for row in x[y_temp == cl]:
        row, mv = row.reshape(2,1), mv.reshape(2,1)
        class_scatter_matrix += (row-mv).dot((row-mv).T)
    within_class += class_scatter_matrix

within_class
```

Out[21]:

```
array([[ 28.46191249, -14.49320429],
       [-14.49320429,  26.81805413]])
```

In [22]:

```
mean = np.mean(x, axis=0)
between_class = np.zeros((2,2))

for i,mean_vec in enumerate(mean_vs):
    n = x[y_temp==i+1,:].shape[0]
    mean_vec = mean_vec.reshape(2,1)
    overall_mean = mean.reshape(2,1)
    between_class += n * (mean_vec - mean).dot((mean_vec - mean).T)

between_class
```

Out[22]:

```
array([[ 5.91092904, 10.74652962],
       [10.74652962, 19.63921255]])
```

In [23]:

```
eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(within_class).dot(between_class
))
for i in range(len(eig_vals)): eigvec_sc = eig_vecs[:,i].reshape(2,1)
print(eig_vals, eig_vecs)
```

```
[0.00058142 1.85935676] [[-0.87693265 -0.6255713 ]
 [ 0.48061328 -0.780167  ]]
```

In [29]:

```
eig_pairs = sorted([(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len(eig_
vals))]
                   , key=lambda k: k[0], reverse=True)
eig_pairs
```

Out[29]:

```
[(1.859356764932039, array([-0.6255713, -0.780167 ])),
 (0.0005814232087433258, array([-0.87693265,  0.48061328]))]
```

In [33]:

```
W = eig_pairs[0][1].reshape(2,1)
x_lda = x.dot(W)
```

In [41]:

```
from matplotlib import pyplot as plt

def plot_step_lda(feature):

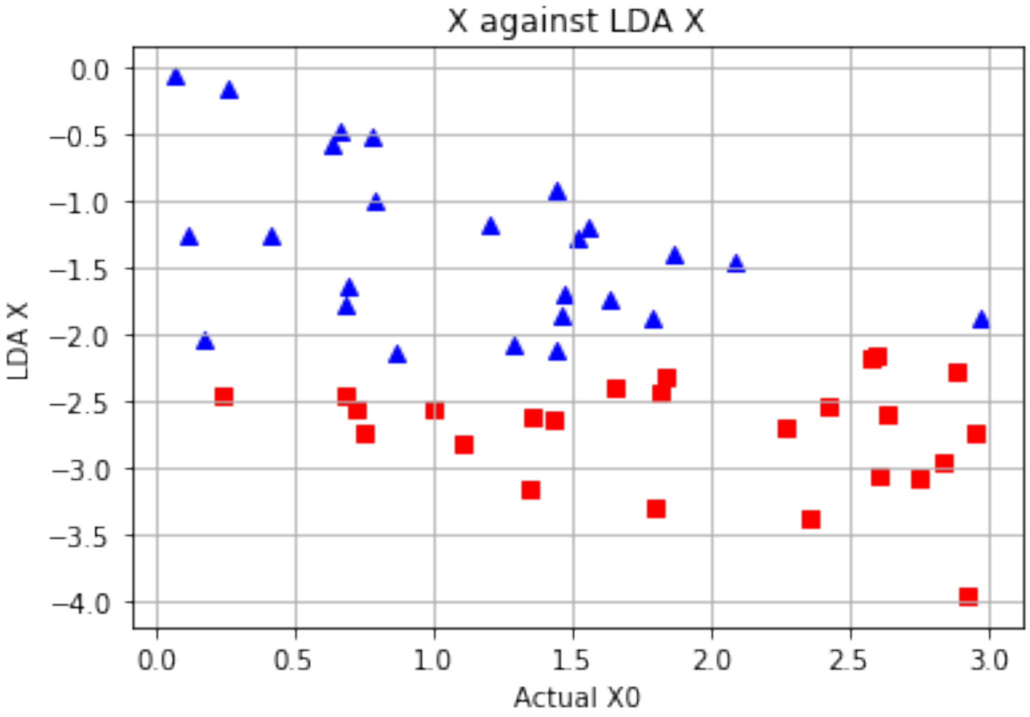
    ax = plt.subplot(111)
    for label,marker,color in zip(range(0,2),('^', 's', 'o'),('blue', 'red', 'gr
een')):

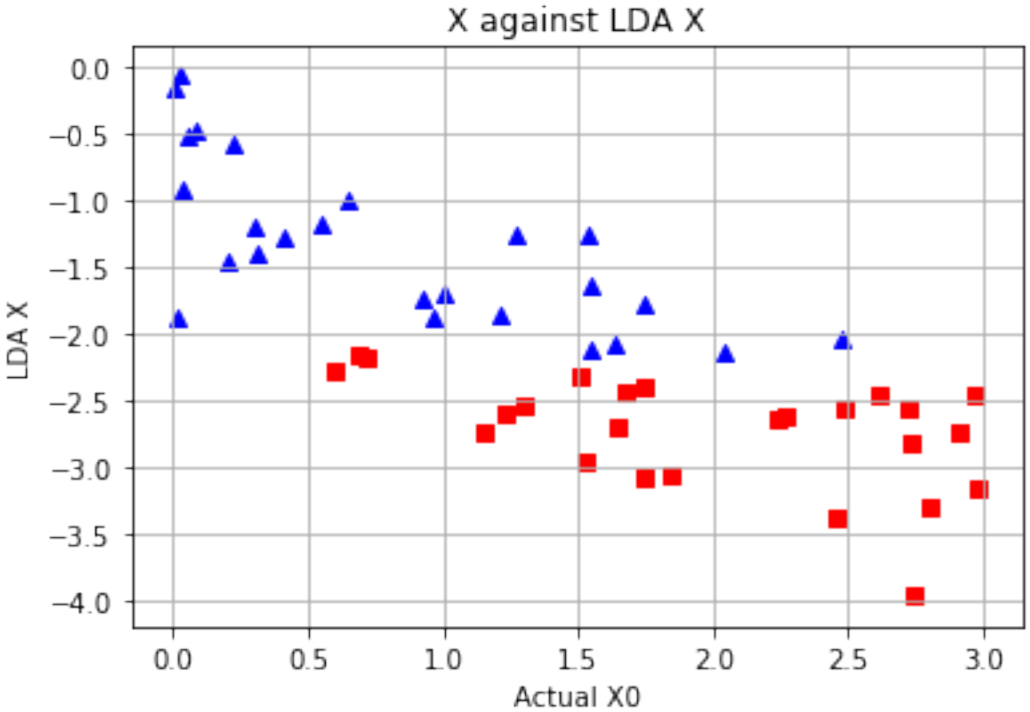
        plt.scatter(x=x[:,feature].real[y == label],
                y=x_lda[:,0].real[y == label],
                marker=marker, color=color)

    plt.xlabel('Actual X0')
    plt.ylabel('LDA X')
    plt.title('X against LDA X')


    plt.grid()
    plt.tight_layout
    plt.show()

plot_step_lda(0)
plot_step_lda(1)
```





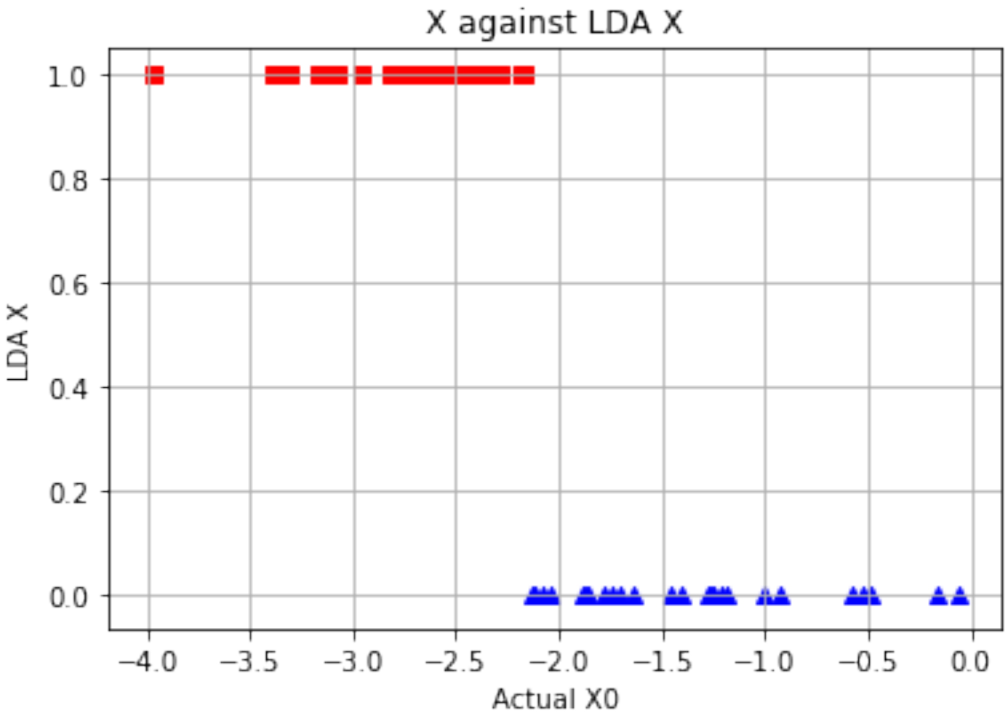
The above plot shows the relationship between actual X and LDA transformed X. Both X0 and X1 are shown to be linearly seperated at X ~ 2.0

In [42]:

```
ax = plt.subplot(111)
for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')): plt.scatte
r(x=x_lda[:,0].real[y == label],y=y.real[y == label],marker=marker,color=color)

plt.xlabel('Actual X0')
plt.ylabel('LDA X')
plt.title('X against LDA X')


plt.grid()
plt.tight_layout
plt.show()
```



There is a clean seperation once LDA is applied to X

In [584]:

```
y_hat = [1 if i>=2 else 0 for i in x_lda]
accuracy_score(y, y_hat)
```

Out[584]:

```
0.96
```

Histograms of the three methods

In [609]:

```
accuracies_gd = []

for run in range(50):

    x,y = (genData(50))
    m, n = np.shape(x)
    theta = gradientDescent(x,y, np.ones(n), 1, m, 70000)

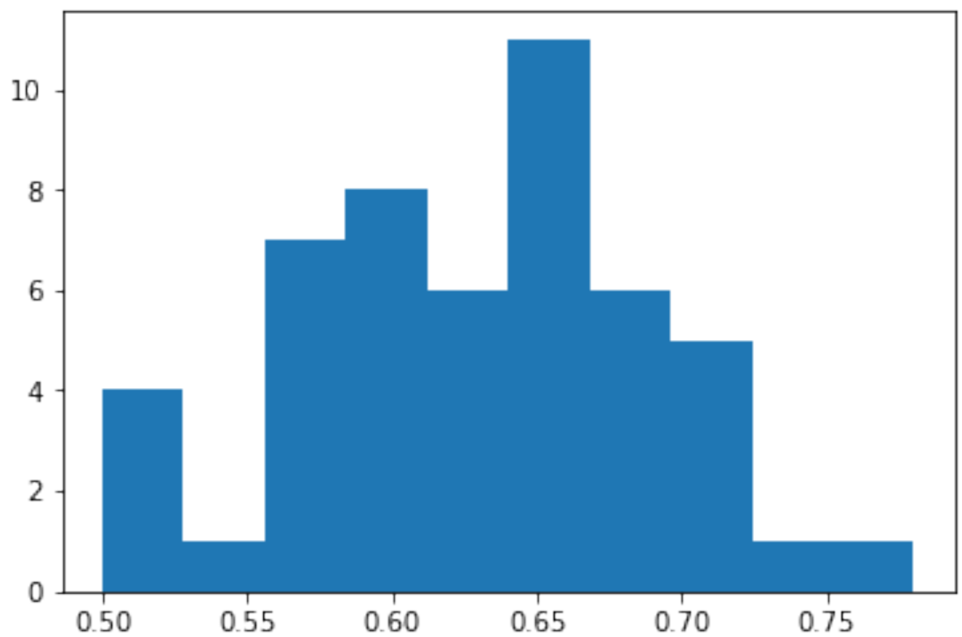
    d_boundary = lambda x: x[0]*theta[1] + x[1]*theta[2] + x[2]*theta[0]


    y_hat = [0 if i > 0 else 1 for i in [d_boundary(i) for i in x]]
    accuracies_gd.append(accuracy_score(y, y_hat))

plt.hist(np.array(accuracies_gd), label = 'GD')
```

Out[609]:

```
(array([ 4.,  1.,  7.,  8.,  6., 11.,  6.,  5.,  1.,  1.]),
 array([0.5  , 0.528, 0.556, 0.584, 0.612, 0.64 , 0.668, 0.696, 0.72
4,
        0.752, 0.78 ]),
 <a list of 10 Patch objects>)
```



In [620]:

```
accuracies_sto = []

for run in range(20):

    print (run)
    x,y = (genData(50))
    m, n = np.shape(x)
    theta = gradientDescentSto(x,y, np.ones(n), 1, m, 10000)

    d_boundary = lambda x: x[0]*theta[1] + x[1]*theta[2] + x[2]*theta[0]

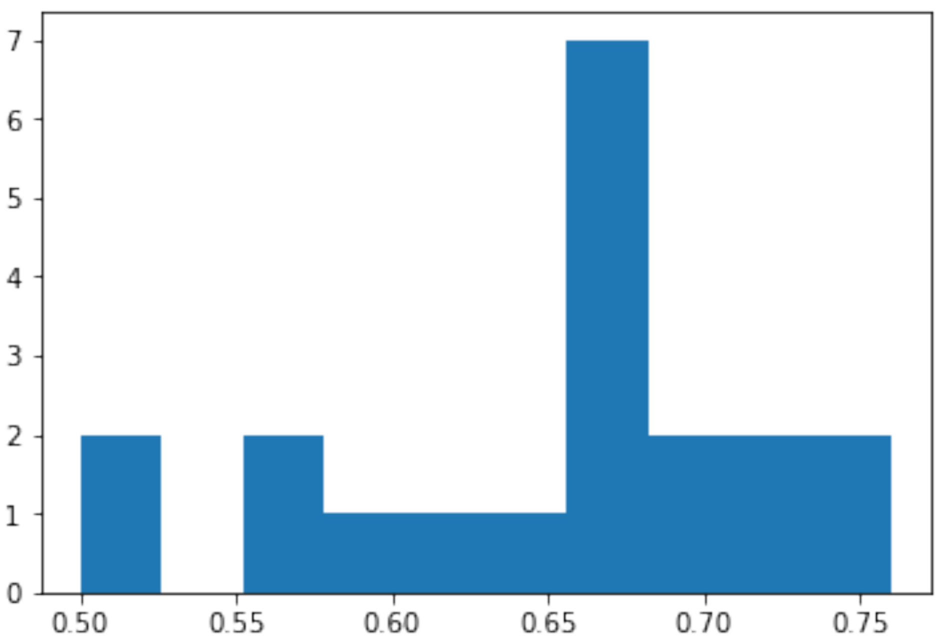

    y_hat = [0 if i > 0 else 1 for i in [d_boundary(i) for i in x]]
    accuracies_sto.append(accuracy_score(y, y_hat))

plt.hist(np.array(accuracies_sto), label = 'GD')
```

```
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
```

Out[620]:

```
(array([2., 0., 2., 1., 1., 1., 7., 2., 2., 2.]),
 array([0.5  , 0.526, 0.552, 0.578, 0.604, 0.63 , 0.656, 0.682, 0.70
8,
        0.734, 0.76 ]),
 <a list of 10 Patch objects>)
```



In [45]:

```
accuracies_lda = []

for run in range(200):

    x,y = (genData(50))
    x = x[:, 1:]

    mean_vs = []
    y_temp = np.array([1 if i > 0.5 else 0 for i in y])
    for i in range(0,2):mean_vs.append(np.mean(x[y_temp==i], axis = 0))
    print(mean_vs)

    within_class = np.zeros((2,2))

    for cl,mv in zip(range(0,2), mean_vs):

        class_scatter_matrix = np.zeros((2,2))
        for row in x[y_temp == cl]:
            row, mv = row.reshape(2,1), mv.reshape(2,1)
            class_scatter_matrix += (row-mv).dot((row-mv).T)
        within_class += class_scatter_matrix


    overall_mean = np.mean(x, axis=0)

    mean = np.mean(x, axis=0)
    between_class = np.zeros((2,2))

    for i,mean_vec in enumerate(mean_vs):
        n = x[y_temp==i+1,:].shape[0]
        mean_vec = mean_vec.reshape(2,1)
        overall_mean = mean.reshape(2,1)
        between_class += n * (mean_vec - mean).dot((mean_vec - mean).T)

    eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(within_class).dot(between_c
lass))
    for i in range(len(eig_vals)): eigvec_sc = eig_vecs[:,i].reshape(2,1)

    eig_pairs = sorted([(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len(
eig_vals))]
                   , key=lambda k: k[0], reverse=True)


    W = eig_pairs[0][1].reshape(2,1)
    x_lda = x.dot(W)

    y_hat = [1 if i>=2 else 0 for i in x_lda]
    accuracies_lda.append(accuracy_score(y, y_hat))

plt.hist(np.array(accuracies_lda), label = 'GD')
```

```
[array([0.83929704, 1.00614434]), array([2.26066898, 1.98306645])]
[array([1.06232932, 1.05398051]), array([1.69221586, 2.20116009])]
[array([0.8595049 , 1.12125005]), array([1.90860151, 2.04333948])]
[array([0.66273389, 1.12640724]), array([2.0469637 , 1.79945824])]
[array([1.39749888, 0.72031956]), array([1.97557825, 1.95054578])]
[array([1.16347381, 1.01042746]), array([2.07469372, 1.86183587])]
[array([0.98200943, 0.92036327]), array([1.80268477, 2.2628307 ])]
[array([1.13439717, 1.09243617]), array([2.17336398, 2.28375346])]
[array([1.25256252, 0.97733217]), array([1.93141848, 2.28168542])]
[array([0.84419823, 0.91675769]), array([2.26498206, 1.89756332])]
[array([0.73760576, 1.41559017]), array([2.01185166, 1.8043372 ])]
[array([1.19567031, 0.6815785 ]), array([2.25383607, 1.77951421])]
[array([1.13134132, 1.07579967]), array([1.98603291, 1.97414516])]
[array([1.08014185, 0.9773905 ]), array([2.1687166 , 2.05693904])]
[array([0.92437414, 1.27888806]), array([1.77244258, 2.20451583])]
[array([0.94577157, 0.82670936]), array([2.10106905, 1.89203975])]
[array([1.04056687, 0.86694862]), array([1.84393481, 2.1233452 ])]
[array([1.1376197 , 1.01674285]), array([1.95185591, 1.80288235])]
[array([0.79833992, 1.04388146]), array([1.99347243, 2.13186886])]
[array([1.11348563, 0.92840595]), array([2.16059205, 1.77703197])]
[array([1.17397058, 0.7901681 ]), array([2.05593733, 1.98202127])]
[array([1.02757846, 1.06142999]), array([2.05920861, 1.95480949])]
[array([1.09639654, 0.80886487]), array([1.92884076, 1.97198275])]
[array([1.03332626, 0.88547872]), array([2.0162863, 2.060518 ])]
[array([1.01642599, 0.94183089]), array([1.96869499, 2.05641464])]
[array([0.8334427 , 0.79823615]), array([2.21331414, 1.83948053])]
[array([1.0974606 , 0.92377971]), array([2.01187447, 2.04512918])]
[array([0.90334048, 1.13076672]), array([2.11443585, 1.97782092])]
[array([0.94844166, 0.98771563]), array([2.23343228, 1.69019859])]
[array([1.02636027, 1.28305872]), array([1.97338957, 2.02634463])]
[array([1.00490103, 0.99833925]), array([1.96769705, 1.75880355])]
[array([1.04004681, 0.98802631]), array([1.90451316, 2.00244706])]
[array([0.8371501 , 1.10444209]), array([1.84189095, 2.16167476])]
[array([1.14778684, 0.95436929]), array([2.13162302, 1.83861561])]
[array([0.76344194, 1.33282895]), array([1.96857004, 1.95690778])]
[array([0.93710826, 1.15980332]), array([1.88983318, 2.02440109])]
[array([1.07358488, 1.07455568]), array([1.95049053, 2.13105952])]
[array([0.99597244, 1.20381687]), array([1.91977011, 2.21975561])]
[array([0.98242316, 0.89770331]), array([2.11831111, 2.1170304 ])]
[array([0.81203613, 1.10178435]), array([1.94710593, 1.90929428])]
[array([0.89721168, 1.0803754 ]), array([1.9731049, 1.9711887])]
[array([0.95514714, 0.96270942]), array([2.05381239, 1.7455297 ])]
[array([1.01984562, 0.9033315 ]), array([1.91527404, 2.12371993])]
[array([0.94577072, 1.02794188]), array([1.72712178, 2.23386794])]
[array([1.16062009, 0.79901388]), array([1.8363361 , 2.17504166])]
[array([0.93329937, 0.97759638]), array([2.11606276, 1.78148375])]
[array([0.9986055 , 1.07649602]), array([2.24285238, 1.70709711])]
[array([0.86272636, 0.99789699]), array([2.16081022, 1.87144328])]
[array([0.98013833, 1.04247989]), array([2.15355015, 1.82210936])]
[array([1.04280776, 1.00771868]), array([2.02068867, 1.91002787])]
[array([1.02507636, 0.87423156]), array([2.01317073, 2.0864678 ])]
[array([0.80640918, 1.04962882]), array([2.12106105, 1.8754633 ])]
[array([1.05425878, 0.90262161]), array([1.96071607, 2.10267842])]
[array([0.78469714, 1.1424283 ]), array([1.9980087 , 2.19465726])]
```

```
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[array([0.94920769, 1.09558346]), array([2.04181521, 1.96686198])]
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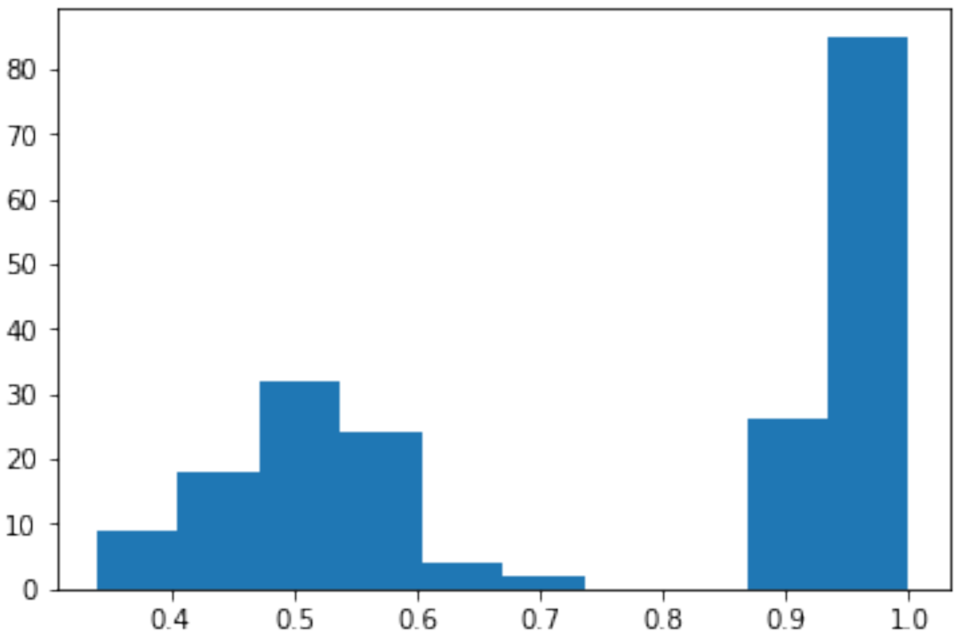
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