1) A Sinction is lonvex if its sciond dinvidive & Should be positive.

The los likelihood is given by.

Here g (67 xi) is given by 1

Using thin vule, we got.

$$g(z) = \frac{1}{1+e^{-2}}; g'(z) = \frac{1}{(1+e^{-2})^{2}} \cdot e^{-2}$$

$$= \frac{1}{1+e^{-2}} \left(1 - \frac{1}{1+e^{-2}}\right)$$

$$= g(z) g(1-z).$$

$$\frac{\partial}{\partial 0} \left(-\log g(z) \right) = \frac{-g'(z)}{g(z)} \cdot \frac{\partial}{\partial 0} \cdot z \cdot = \left(g(z) - 1 \right) \chi.$$

$$\frac{\partial^2}{\partial \sigma^2} = \left(-\log g\left(\sigma^{(x)}\right)\right) = \frac{\partial}{\partial \sigma} \left(g(z) - 1\right) x.$$

$$= \frac{\partial}{\partial \sigma} \left(g(z) \cdot x\right) = g'(z) \cdot x.$$

=
$$g(z)(1-g(z))\frac{dz}{do} \cdot x = g(i)(1-g(i))$$

$$\frac{\partial}{\partial o} \left(-\log \left(1 - g(0 | x) \right) \right) = -\frac{1}{1 - g(z)} \frac{\partial}{\partial o} \left(1 - g(z) \right).$$

$$- \left(-g(z) \frac{\partial}{\partial z} \left(1 - g(z) \right) \times \frac{1 - g(z)}{1 - g(z)}.$$

$$= g(z) \pi.$$

$$\frac{\partial^{2} \left(-\log \left(1-g\left(0^{\frac{1}{2}}x\right)\right)\right)}{\partial 0^{2}} = \frac{\partial \left(g(z)x\right)}{\partial 0} = \chi g(z)\left(1-g(z)\right) x^{\frac{1}{2}} 20$$

The xxi term in buth are squared term implyed it can never be negative.

Since both the terms second dorivative have XXT and are thantre greater than O, that the function as a whole is convex.

a). X, ord X2 ore continuous.

For three Classes and J=3, each class will have.

3x (J=1). parameters # making shot. 6

b) Assumed X, is contagorical, with 5 contegories and Xz is continous

There are 5 brown one hot variables, needed to represent the

Bosides that. Xz which is continous making an total of 6.

```
In [1]:
```

```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import random
from sklearn import linear_model
import scipy.stats
from sklearn.metrics import accuracy_score
np.set_printoptions(suppress=True)
```

In [2]:

```
def genData(numPoints):
    x = np.zeros(shape=(numPoints, 3))
    y = np.zeros(shape=numPoints)

x[:,0] = 1
    #x[:,1] = [i for i in range(numPoints)]
    x[:,1] = np.random.uniform(0, 3, size = numPoints)
    x[:,2] = np.random.uniform(0, 3, size = numPoints)

for i in range(0, numPoints): y[i] = 1/(1 + np.exp( - (-3 + (x[i][1]) + (x[i][2]))))
    #for i in range(0, numPoints): y[i] = 1/(1 + np.exp( - (-3 + (x[i][1]))))

y = np.array([1 if i > 0.5 else 0 for i in y])
    return x, y
```

In [3]:

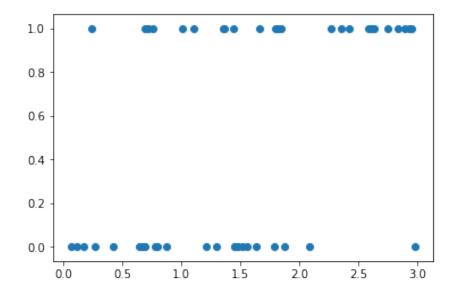
```
x,y = (genData(50))
#y_temp = np.array([1 if i > 0.5 else 0 for i in y])
```

```
In [4]:
```

```
plt.scatter(x[:,1], y)
```

Out[4]:

<matplotlib.collections.PathCollection at 0x7ff2ba97cd30>



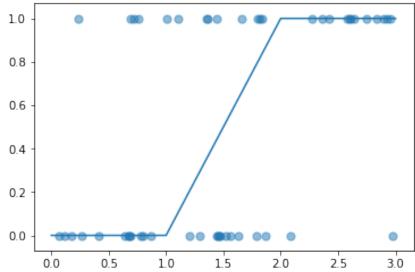
In [5]:

```
def gradientDescent(x, y, theta, alpha, m, numIterations):
    xTrans = x.transpose()
    for i in range(0, numIterations):
        loss = (1 / (1 + np.exp(-(1 + (np.dot(x, theta)))))) - y
        #print (np.mean(loss))
        #gradient = np.dot(xTrans, np.dot(x, theta) - y) / m #Partial derivativ
        gradient = np.dot(xTrans, loss) / m
        theta = theta - alpha * gradient
```

In [6]:

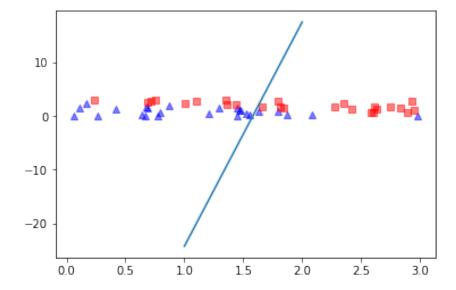
```
m, n = np.shape(x)
theta = gradientDescent(x,y, np.ones(n), 1, m, 70000)
```

```
In [7]:
theta
Out[7]:
array([-66.12729013, 20.52518735, 21.32420216])
In [8]:
hypothesis = lambda x: 1 / (1 + (np.exp(-(x * theta[1] + x * theta[2]+ theta[0]))))
plt.plot([i for i in range(0,4)],[hypothesis(i) for i in range(0,4)])
plt.scatter([i[1] for i in x], y, s=50, alpha = 0.5)
plt.show()
```

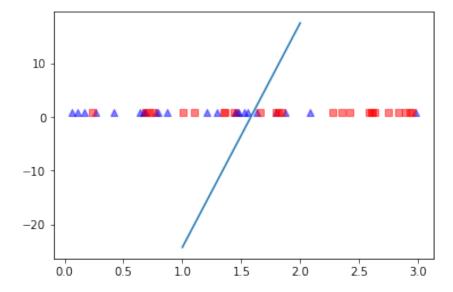


Decision boundaries for X[1] vs X[2] and X[1] vs X[0] respectively for Batch GD

In [9]:



In [10]:



```
In [11]:
def gradientDescentSto(x, y, theta, alpha, m, numIterations, delta=2, conv=0.000
00001):
   xTrans = x.transpose()
   count = 0
   theta prev = theta + delta + 1
   while (count < numIterations):</pre>
          count += 1
           for i in range(m):
              [2]
              loss = 1/(1 + np.exp(- hypothesis)) - y[i]
              \#loss = hypothesis - y[i]
              gradient1 = x[i][0] * loss
              gradient0 = x[i][1] * loss
              theta prev = theta
              theta[1] = theta[1] - alpha * gradient0
              theta[0] = theta[0] - alpha * gradient1
   return theta
```

```
In [12]:
theta = gradientDescentSto(x, y, theta, 0.08, m, 10000)
```

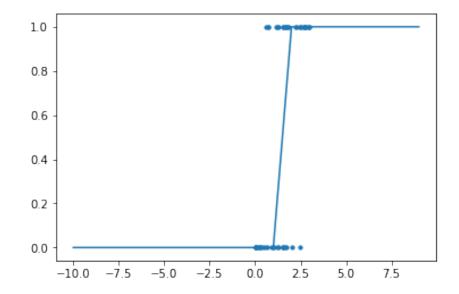
```
In [13]:
theta
```

```
Out[13]:
array([-65.34984189, 20.63300455, 21.32420216])
```

```
In [14]:
```

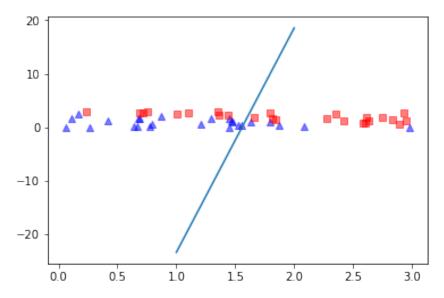
```
hypothesis = lambda x: 1 / (1 + (np.exp(-(x * theta[1] + x * theta[2] + theta[0]
))))

plt.plot([i for i in range(-10,10)],[hypothesis(i) for i in range(-10,10)])
plt.scatter([i[2] for i in x], y, s=10)
plt.show()
```

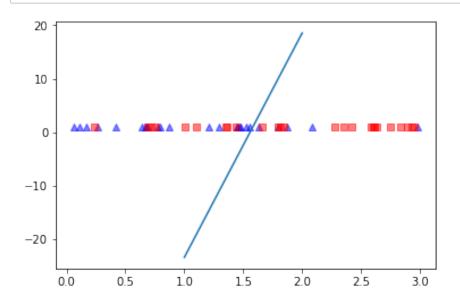


Decision boundaries for X[1] vs X[2] and X[1] vs X[0] respectively for Stochastic GD

```
In [15]:
```



```
In [16]:
d_boundary_plot = lambda x: x*theta[1] + x*theta[2] + theta[0]
plt.plot([i for i in range(1,3)],[d_boundary_plot(i) for i in range(1,3)])
#plt.scatter([i[1] for i in x], [i[2] for i in x], s=50, alpha = 0.5)
for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')):
    plt.scatter(x=x[:,1].real[y == label],
```



marker=marker,
color=color,
alpha=0.5

y=x[:,0].real[y == label],

```
In [17]:
d_boundary = lambda x: x[1]*theta[1] + x[2]*theta[2] + theta[0] * x[0]
y_hat = [1 if i > 0 else 0 for i in [d_boundary(i) for i in x]]
```

```
In [18]:
accuracy_score(y, y_hat)
Out[18]:
1.0
```

Linear discriminant analysis

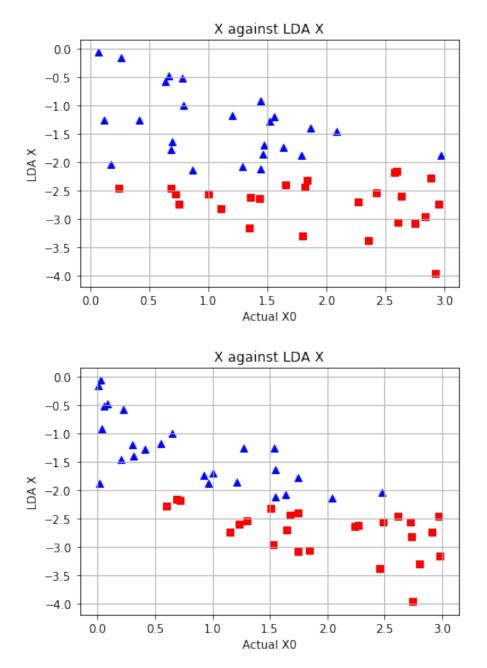
```
In [19]:

x_old = x
x = x[:, 1:]
```

```
In [20]:
mean vs = []
y temp = np.array([1 if i > 0.5 else 0 for i in y])
for i in range(0,2):mean_vs.append(np.mean(x[y_temp==i], axis = 0))
print(mean vs)
[array([1.11687638, 0.83202265]), array([1.90450382, 1.97216123])]
In [21]:
within class = np.zeros((2,2))
for cl,mv in zip(range(0,2), mean vs):
    class scatter matrix = np.zeros((2,2))
    for row in x[y_temp == cl]:
        row, mv = row.reshape(2,1), mv.reshape(2,1)
        class scatter matrix += (row-mv).dot((row-mv).T)
    within class += class scatter matrix
within class
Out[21]:
array([[ 28.46191249, -14.49320429],
       [-14.49320429, 26.81805413]])
In [22]:
mean = np.mean(x, axis=0)
between class = np.zeros((2,2))
for i,mean vec in enumerate(mean vs):
    n = x[y_{temp}=i+1,:].shape[0]
    mean vec = mean vec.reshape(2,1)
    overall mean = mean.reshape(2,1)
    between_class += n * (mean_vec - mean).dot((mean_vec - mean).T)
between class
Out[22]:
array([[ 5.91092904, 10.74652962],
```

[10.74652962, 19.63921255]])

```
In [23]:
eig vals, eig vecs = np.linalg.eig(np.linalg.inv(within class).dot(between class
))
for i in range(len(eig_vals)): eigvec_sc = eig_vecs[:,i].reshape(2,1)
print(eig vals, eig vecs)
[0.00058142 1.85935676] [[-0.87693265 -0.6255713 ]
 [ 0.48061328 -0.780167 ]]
In [29]:
eig pairs = sorted([(np.abs(eig vals[i]), eig vecs[:,i]) for i in range(len(eig
vals))]
                   , key=lambda k: k[0], reverse=True)
eig pairs
Out[29]:
[(1.859356764932039, array([-0.6255713, -0.780167])),
 (0.0005814232087433258, array([-0.87693265, 0.48061328]))]
In [33]:
W = eig pairs[0][1].reshape(2,1)
x lda = x.dot(W)
In [41]:
from matplotlib import pyplot as plt
def plot step lda(feature):
    ax = plt.subplot(111)
    for label,marker,color in zip(range(0,2),('^', 's', 'o'),('blue', 'red', 'gr
een')):
        plt.scatter(x=x[:,feature].real[y == label],
                y=x lda[:,0].real[y == label],
                marker=marker, color=color)
    plt.xlabel('Actual X0')
    plt.ylabel('LDA X')
    plt.title('X against LDA X')
    plt.grid()
    plt.tight layout
    plt.show()
plot_step_lda(0)
plot step lda(1)
```



The above plot shows the relationship between actual X and LDA transformed X. Both X0 and X1 are shown to be linearly seperated at X \sim 2.0

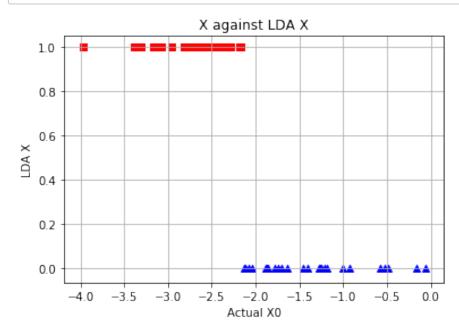
```
In [42]:
```

0.96

```
ax = plt.subplot(111)
for label,marker,color in zip(range(0,2),('^', 's'),('blue', 'red')): plt.scatte
r(x=x_lda[:,0].real[y == label],y=y.real[y == label],marker=marker,color=color)

plt.xlabel('Actual X0')
plt.ylabel('LDA X')
plt.title('X against LDA X')

plt.grid()
plt.tight_layout
plt.show()
```



There is a clean seperation once LDA is applied to X

```
In [584]:

y_hat = [1 if i>=2 else 0 for i in x_lda]
accuracy_score(y, y_hat)

Out[584]:
```

Histograms of the three methods

```
In [609]:
```

```
accuracies_gd = []

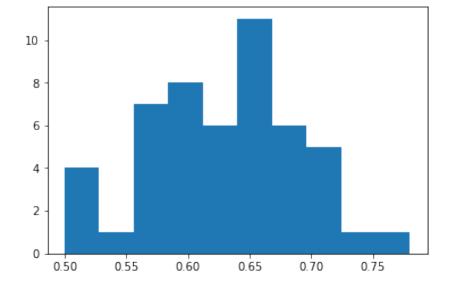
for run in range(50):
    x,y = (genData(50))
    m, n = np.shape(x)
    theta = gradientDescent(x,y, np.ones(n), 1, m, 70000)

    d_boundary = lambda x: x[0]*theta[1] + x[1]*theta[2] + x[2]*theta[0]

    y_hat = [0 if i > 0 else 1 for i in [d_boundary(i) for i in x]]
    accuracies_gd.append(accuracy_score(y, y_hat))

plt.hist(np.array(accuracies_gd), label = 'GD')
```

Out[609]:



```
In [620]:
```

```
accuracies_sto = []

for run in range(20):

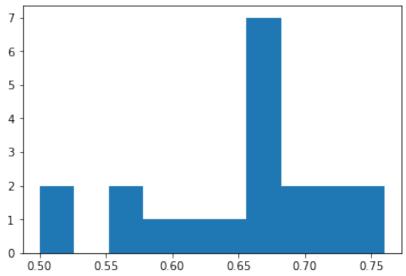
    print (run)
    x,y = (genData(50))
    m, n = np.shape(x)
    theta = gradientDescentSto(x,y, np.ones(n), 1, m, 10000)

d_boundary = lambda x: x[0]*theta[1] + x[1]*theta[2] + x[2]*theta[0]

y_hat = [0 if i > 0 else 1 for i in [d_boundary(i) for i in x]]
    accuracies_sto.append(accuracy_score(y, y_hat))

plt.hist(np.array(accuracies_sto), label = 'GD')
```

```
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Out[620]:
(array([2., 0., 2., 1., 1., 1., 7., 2., 2., 2.]),
array([0.5 , 0.526, 0.552, 0.578, 0.604, 0.63 , 0.656, 0.682, 0.70
8,
        0.734, 0.76 ]),
<a list of 10 Patch objects>)
```



```
In [45]:
```

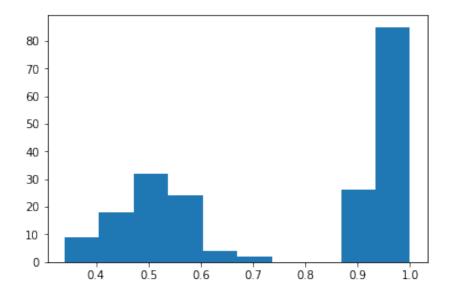
```
accuracies lda = []
for run in range(200):
    x,y = (genData(50))
    x = x[:, 1:]
   mean vs = []
    y temp = np.array([1 if i > 0.5 else 0 for i in y])
    for i in range(0,2):mean_vs.append(np.mean(x[y_temp==i], axis = 0))
    print(mean vs)
    within class = np.zeros((2,2))
    for cl,mv in zip(range(0,2), mean vs):
        class scatter matrix = np.zeros((2,2))
        for row in x[y_temp == cl]:
            row, mv = row.reshape(2,1), mv.reshape(2,1)
            class_scatter_matrix += (row-mv).dot((row-mv).T)
        within_class += class_scatter_matrix
    overall mean = np.mean(x, axis=0)
    mean = np.mean(x, axis=0)
    between class = np.zeros((2,2))
    for i,mean_vec in enumerate(mean_vs):
        n = x[y temp==i+1,:].shape[0]
        mean_vec = mean_vec.reshape(2,1)
        overall mean = mean.reshape(2,1)
        between_class += n * (mean_vec - mean).dot((mean_vec - mean).T)
    eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(within_class).dot(between_c
lass))
    for i in range(len(eig_vals)): eigvec_sc = eig_vecs[:,i].reshape(2,1)
    eig_pairs = sorted([(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len())
eig_vals))]
                   , key=lambda k: k[0], reverse=True)
    W = eig_pairs[0][1].reshape(2,1)
    x_{da} = x.dot(W)
    y_hat = [1 if i>=2 else 0 for i in x_lda]
    accuracies_lda.append(accuracy_score(y, y_hat))
plt.hist(np.array(accuracies_lda), label = 'GD')
```

```
[array([0.83929/04, 1.00614434]), array([2.26066898, 1.98306645])]
[array([1.06232932, 1.05398051]), array([1.69221586, 2.20116009])]
[array([0.8595049 , 1.12125005]), array([1.90860151, 2.04333948])]
[array([0.66273389, 1.12640724]), array([2.0469637, 1.79945824])]
[array([1.39749888, 0.72031956]), array([1.97557825, 1.95054578])]
[array([1.16347381, 1.01042746]), array([2.07469372, 1.86183587])]
[array([0.98200943, 0.92036327]), array([1.80268477, 2.2628307 ])]
[array([1.13439717, 1.09243617]), array([2.17336398, 2.28375346])]
[array([1.25256252, 0.97733217]), array([1.93141848, 2.28168542])]
[array([0.84419823, 0.91675769]), array([2.26498206, 1.89756332])]
[array([0.73760576, 1.41559017]), array([2.01185166, 1.8043372])]
[array([1.19567031, 0.6815785 ]), array([2.25383607, 1.77951421])]
[array([1.13134132, 1.07579967]), array([1.98603291, 1.97414516])]
[array([1.08014185, 0.9773905 ]), array([2.1687166 , 2.05693904])]
[array([0.92437414, 1.27888806]), array([1.77244258, 2.20451583])]
[array([0.94577157, 0.82670936]), array([2.10106905, 1.89203975])]
[array([1.04056687, 0.86694862]), array([1.84393481, 2.1233452 ])]
[array([1.1376197 , 1.01674285]), array([1.95185591, 1.80288235])]
[array([0.79833992, 1.04388146]), array([1.99347243, 2.13186886])]
[array([1.11348563, 0.92840595]), array([2.16059205, 1.77703197])]
[array([1.17397058, 0.7901681 ]), array([2.05593733, 1.98202127])]
[array([1.02757846, 1.06142999]), array([2.05920861, 1.95480949])]
[array([1.09639654, 0.80886487]), array([1.92884076, 1.97198275])]
[array([1.03332626, 0.88547872]), array([2.0162863, 2.060518])]
[array([1.01642599, 0.94183089]), array([1.96869499, 2.05641464])]
[array([0.8334427 , 0.79823615]), array([2.21331414, 1.83948053])]
[array([1.0974606 , 0.92377971]), array([2.01187447, 2.04512918])]
[array([0.90334048, 1.13076672]), array([2.11443585, 1.97782092])]
[array([0.94844166, 0.98771563]), array([2.23343228, 1.69019859])]
[array([1.02636027, 1.28305872]), array([1.97338957, 2.02634463])]
[array([1.00490103, 0.99833925]), array([1.96769705, 1.75880355])]
[array([1.04004681, 0.98802631]), array([1.90451316, 2.00244706])]
[array([0.8371501 , 1.10444209]), array([1.84189095, 2.16167476])]
[array([1.14778684, 0.95436929]), array([2.13162302, 1.83861561])]
[array([0.76344194, 1.33282895]), array([1.96857004, 1.95690778])]
[array([0.93710826, 1.15980332]), array([1.88983318, 2.02440109])]
[array([1.07358488, 1.07455568]), array([1.95049053, 2.13105952])]
[array([0.99597244, 1.20381687]), array([1.91977011, 2.21975561])]
[array([0.98242316, 0.89770331]), array([2.11831111, 2.1170304])]
[array([0.81203613, 1.10178435]), array([1.94710593, 1.90929428])]
[array([0.89721168, 1.0803754]), array([1.9731049, 1.9711887])]
[array([0.95514714, 0.96270942]), array([2.05381239, 1.7455297 ])]
[array([1.01984562, 0.9033315 ]), array([1.91527404, 2.12371993])]
[array([0.94577072, 1.02794188]), array([1.72712178, 2.23386794])]
[array([1.16062009, 0.79901388]), array([1.8363361 , 2.17504166])]
[array([0.93329937, 0.97759638]), array([2.11606276, 1.78148375])]
[array([0.9986055 , 1.07649602]), array([2.24285238, 1.70709711])]
[array([0.86272636, 0.99789699]), array([2.16081022, 1.87144328])]
[array([0.98013833, 1.04247989]), array([2.15355015, 1.82210936])]
[array([1.04280776, 1.00771868]), array([2.02068867, 1.91002787])]
[array([1.02507636, 0.87423156]), array([2.01317073, 2.0864678])]
[array([0.80640918, 1.04962882]), array([2.12106105, 1.8754633 ])]
[array([1.05425878, 0.90262161]), array([1.96071607, 2.10267842])]
[array([0.78469714, 1.1424283 ]), array([1.9980087 , 2.19465726])]
```

```
[array([1.0187896 , 0.95930482]), array([2.03487846, 2.00838616])]
[array([0.94920769, 1.09558346]), array([2.04181521, 1.96686198])]
[array([0.9850755 , 1.05782142]), array([2.05664234, 1.96258126])]
[array([0.97467426, 0.92362596]), array([1.95959361, 2.22828744])]
[array([1.01354359, 0.88067864]), array([1.7399228 , 2.15377644])]
[array([1.03096888, 0.95656048]), array([2.01971526, 2.27358889])]
[array([0.83884659, 1.00104054]), array([2.2737725 , 1.81754569])]
[array([0.93150606, 1.01448678]), array([1.87034135, 1.88970036])]
[array([0.98982547, 1.12657339]), array([1.75187528, 2.14249861])]
[array([1.08722262, 1.12176508]), array([1.76054122, 2.22835194])]
[array([0.89317817, 1.32315068]), array([2.14885953, 1.81801011])]
[array([0.96495706, 1.05547138]), array([2.06152634, 1.98093605])]
[array([1.06401855, 0.91694693]), array([1.94244596, 1.99419318])]
[array([1.1576754 , 0.77499116]), array([1.74686025, 2.20172313])]
[array([0.8644793 , 0.87279762]), array([1.89238983, 1.98834183])]
[array([0.77768114, 1.20253381]), array([1.99136713, 1.95884892])]
[array([1.13023102, 0.91269429]), array([2.17933597, 1.7709569 ])]
[array([0.98295646, 1.03465817]), array([2.04238763, 2.01288952])]
[array([0.94075902, 1.11511166]), array([2.01883025, 2.02861991])]
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```

```
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```

```
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Out[45]:
(array([ 9., 18., 32., 24., 4., 2., 0., 0., 26., 85.]),
array([0.34, 0.406, 0.472, 0.538, 0.604, 0.67, 0.736, 0.802, 0.86
8,
        0.934, 1.
<a list of 10 Patch objects>)
```



Problem 6

```
In [626]:
```

```
hw3 = pd.read_csv("hw3_dataset.txt", delim_whitespace=True, names = ['id', 'fund
ing', 'fv', 'shares', 'l_buyout'])
```

6 A

Trying out EDA

In [630]:

hw3.describe()

Out[630]:

	funding	fv	shares	l_buyout
count	482.000000	4.820000e+02	4.820000e+02	482.000000
mean	0.439834	2.651672e+07	2.227942e+06	0.093361
std	0.496883	2.632174e+07	1.413872e+06	0.291240
min	0.000000	1.200000e+06	3.000000e+05	0.000000
25%	0.000000	1.025000e+07	1.300000e+06	0.000000
50%	0.000000	1.950000e+07	2.000000e+06	0.000000
75%	1.000000	3.250000e+07	2.700000e+06	0.000000
max	1.000000	2.346000e+08	1.101862e+07	1.000000

The fv and shares features have really large values compared to I_buyout and funding columns.

```
In [627]:
```

hw3.head()

Out[627]:

	id	funding	fv	shares	I_buyout
0	1	0	1200000	3000000	0
1	2	0	1454000	1454000	1
2	3	0	1500000	300000	0
3	4	0	1530000	510000	0
4	5	0	2000000	800000	0

In [628]:

```
we know that id is not giving any useful information
also, we'll check for missing values here
"""
hw3 = hw3.drop('id', axis=1)
nas = hw3.isnull().sum()
print(nas)
```

funding 0 fv 0 shares 0 l_buyout 0 dtype: int64

There are no missing values in this dataset

```
In [632]:
```

```
features = hw3.drop('funding', axis=1)
target = hw3['funding']
```

In [633]:

```
X_train, X_test, y_train, y_test = train_test_split(features, target, test_size=
0.33, random_state=42)
```

```
In [634]:
```

X train.info()

<class 'pandas.core.frame.DataFrame'> Int64Index: 322 entries, 157 to 102 Data columns (total 3 columns): fv 322 non-null int64 shares 322 non-null int64 $1_{ extbf{buyout}}$ 322 non-null int64 dtypes: int64(3)

memory usage: 10.1 KB

In [635]:

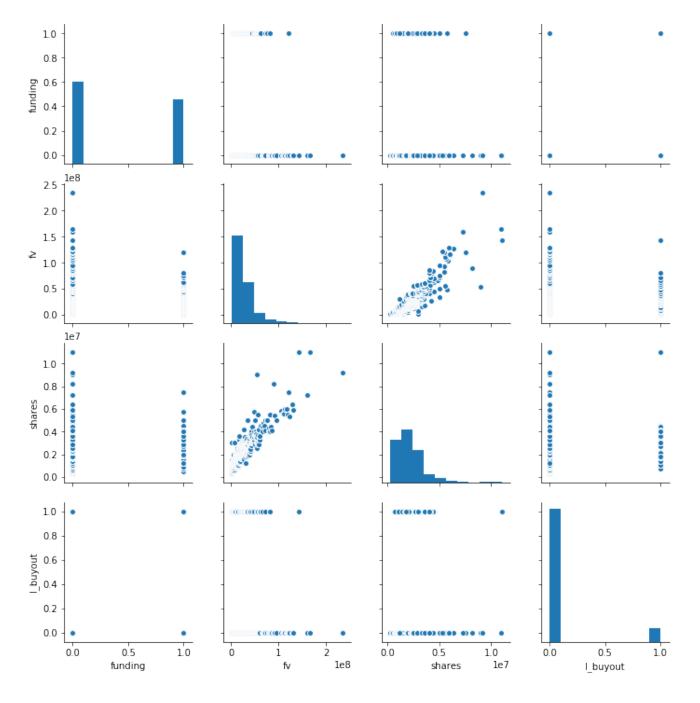
```
X train.head()
```

Out[635]:

	fv	shares	I_buyout	
157	13000000	1300000	0	
449	63000000	4500000	1	
118	10103125	1325000	0	
114	9625000	1375000	0	
439	55000000	2500000	0	

In [653]:

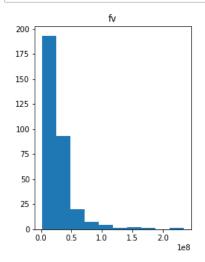
```
sns.pairplot(hw3, vars=hw3.columns)
plt.show()
```

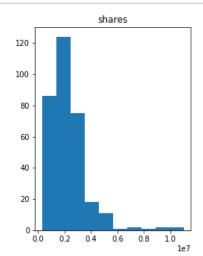


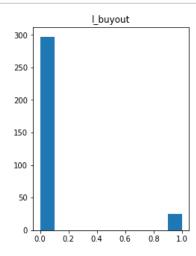
```
In [644]:
```

```
fig, axs = plt.subplots(1,3, figsize=(15, 5), facecolor='w', edgecolor='k')
fig.subplots_adjust(hspace = .5, wspace=.5)

for i,j in enumerate(X_train.columns):
    axs[i].hist(X_train[j])
    axs[i].set_title(j)
```



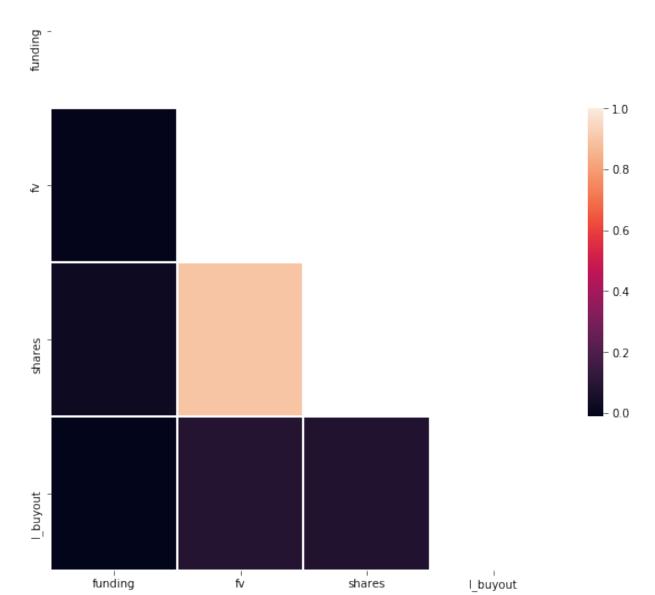




In [649]:

Out[649]:

<matplotlib.axes. subplots.AxesSubplot at 0x7f01443329b0>



Inference

```
In [654]:
```

```
X_train.head()
```

Out[654]:

	fv	shares	I_buyout
157	13000000	1300000	0
449	63000000	4500000	1
118	10103125	1325000	0
114	9625000	1375000	0
439	55000000	2500000	0

6 B

```
In [ ]:
```

```
X_train['capital'] = np.log(X_train['face_value'] * X_train['n_shares'])
X_validate['capital'] = np.log(X_validate['face_value'] * X_validate['n_shares'])
X_test['capital'] = np.log(X_test['face_value'] * X_test['n_shares'])

X_train['fvl'] = np.log(X_train.face_value)
X_train['nsl'] = np.log(X_train.n_shares)

X_validate['fvl'] = np.log(X_validate.face_value)
X_validate['nsl'] = np.log(X_validate.n_shares)

X_test['fvl'] = np.log(X_test.face_value)
X_test['nsl'] = np.log(X_test.n_shares)
```

The first thing to try is the log value of the two features which have a positively skewed distr.

```
In [660]:

X_train['fv_log'] = np.log(X_train['fv'])

X_test['fv_log'] = np.log(X_test['fv'])

X_train['shares_log'] = np.log(X_train['shares'])

X_test['shares_log'] = np.log(X_test['shares'])

/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:4: Sett ingWithCopyWarning:
```

```
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:4: Sett
ingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/panda
s-docs/stable/indexing.html#indexing-view-versus-copy
    after removing the cwd from sys.path.
/opt/conda/lib/python3.6/site-packages/ipykernel_launcher.py:5: Sett
ingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/panda
s-docs/stable/indexing.html#indexing-view-versus-copy
"""
```

In [710]:

```
# method to print basic metrics of a classifier

def print_metrics(y_test, y_hat):
    print("Accuracy of model is ", metrics.accuracy_score(y_test, y_hat))
    print("Precision of model is ", metrics.precision_score(y_test, y_hat))
    print("Recall of model is ", metrics.recall_score(y_test, y_hat))
    print("Confusion matrix\n", metrics.confusion_matrix(y_test, y_hat))

# method to plot AUC

def plot_roc_auc(model, X_test):
    y_hat_probabilities = model.predict_proba(X_test)[::,1]
    fpr, tpr, _ = metrics.roc_curve(y_test, y_hat_probabilities)
    auc = metrics.roc_auc_score(y_test, y_hat_probabilities)
    plt.plot(fpr,tpr,label="data 1, auc="+str(auc))
    plt.legend(loc=4)
    plt.show()
```

```
In [711]:
```

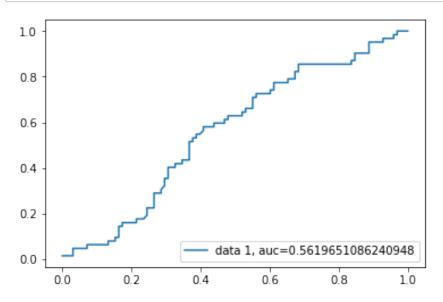
```
lr = LogisticRegression()
lr.fit(X_train[['fv_log', 'shares_log', 'l_buyout']], y_train)

y_hat = lr.predict(X_test[['fv_log', 'shares_log', 'l_buyout']])
print_metrics(y_test, y_hat)
```

So, that's an okayish accuracy. We just tried all variables we thought would work. Let's try plotting the Area under curve.

In [712]:

```
plot_roc_auc(lr, X_test[['fv_log', 'shares_log', 'l_buyout']])
```



Let's try more feature transformation. Since face_value and number of shares are highly correlated in stock market, it's worth trying. Moreover, there's no point in using two highly correlated features in building a model, the correlation between them is greater than 0.8. I will multiply them and then take a log over the multiplied quantity.

```
In [713]:
# All the values are greater than 0 so no need to do log(p+1) as shown in class.
X_train['stock_value'] = np.log((X_train['fv'] * X_train['shares']))
X test['stock value'] = np.log((X test['fv'] * X test['shares']))
/opt/conda/lib/python3.6/site-packages/ipykernel launcher.py:4: Sett
ingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row indexer,col indexer] = value instead
See the caveats in the documentation: http://pandas.pydata.org/panda
s-docs/stable/indexing.html#indexing-view-versus-copy
  after removing the cwd from sys.path.
In [714]:
lr = LogisticRegression()
lr.fit(X_train[['stock_value', 'l_buyout']], y_train)
y_hat = lr.predict(X_test[['stock_value', 'l_buyout']])
print metrics(y test, y hat)
Accuracy of model is 0.55625
Precision of model is 0.0
Recall of model is 0.0
Confusion matrix
 [[89 9]
 [62 0]]
In [715]:
plot_roc_auc(lr, X_test[['stock_value', 'l_buyout']])
1.0
0.8
0.6
0.4
0.2
                      data 1, auc=0.5354674127715602
```

Didn't give a significant improvement. Logistic regression has different solver so will try a different solver.

1.0

0.8

0.0

0.0

0.2

0.4

0.6

```
In [736]:

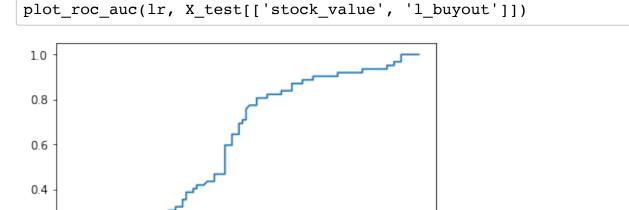
lr = LogisticRegression(solver='lbfgs')
lr.fit(X_train[['stock_value', 'l_buyout']], y_train)

y_hat = lr.predict(X_test[['stock_value', 'l_buyout']])
print_metrics(y_test, y_hat)

Accuracy of model is 0.55
Precision of model is 0.4074074074074
Recall of model is 0.3548387096774194

Confusion matrix
[[66 32]
[40 22]]

In [717]:
```



Though it didn't improve significantly, it is definitely better than the previous model.

0.6

data 1, auc=0.5800691244239631

0.8

6 C

0.2

0.0

0.0

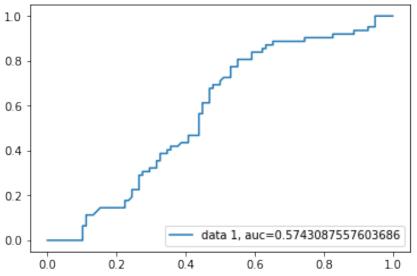
0.2

0.4

Now, we try another kind of classifier- LDA. Again following a similar approach, let's start with all features and slowly narrow down to the ones which we found useful from our logistic regression model.

1.0

```
In [718]:
lda = sklearn lda()
lda.fit(X_train[['fv_log', 'shares_log', 'l_buyout']], y_train)
y_hat = lda.predict(X_test[['fv_log', 'shares_log', 'l_buyout']])
print metrics(y test, y hat)
Accuracy of model is
                      0.575
Precision of model is 0.45
Recall of model is 0.43548387096774194
Confusion matrix
 [[65 33]
 [35 27]]
Now, trying the enhanced version:
In [719]:
lda = sklearn lda()
lda.fit(X_train[['stock_value', 'l_buyout']], y_train)
y hat = lda.predict(X test[['stock value', 'l buyout']])
print metrics(y test, y hat)
Accuracy of model is
                      0.55
Precision of model is 0.4074074074074
Recall of model is 0.3548387096774194
Confusion matrix
[[66 32]
 [40 22]]
In [720]:
plot_roc_auc(lda, X_test[['stock_value', 'l_buyout']])
1.0
0.8
0.6
```



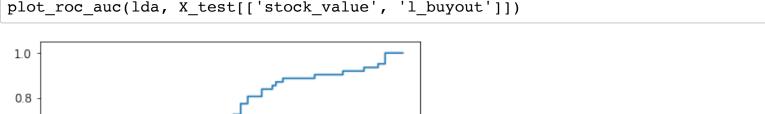
```
In [731]:

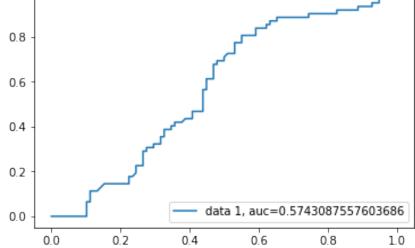
lda = sklearn_lda(solver='svd')
lda.fit(X_train[['stock_value', 'l_buyout']], y_train)

y_hat = lda.predict(X_test[['stock_value', 'l_buyout']])
print_metrics(y_test, y_hat)

Accuracy of model is 0.55
Precision of model is 0.4074074074074
Recall of model is 0.3548387096774194
Confusion matrix
[[66 32]
[40 22]]

In [732]:
plot_roc_auc(lda, X_test[['stock_value', 'l_buyout']])
```





Let's even test the training accuracy of our model

```
In [723]:

y_hat_train = lda.predict(X_train[['stock_value', 'l_buyout']])
print_metrics(y_train, y_hat_train)
```

Here, we observe again that feature transformation provides better AUC compared to fitting the model on the original features. However, changing solver here didn't make any difference.

6 D

Let's try finding training accuracies: a) Logistic regression

```
In [725]:
```

```
lr = LogisticRegression(solver='lbfgs')
lr.fit(X_train[['stock_value', 'l_buyout']], y_train)

y_hat_train = lr.predict(X_train[['stock_value', 'l_buyout']])
print_metrics(y_train, y_hat_train)

y_hat_probabilities = lr.predict_proba(X_train[['stock_value', 'l_buyout']])[::,

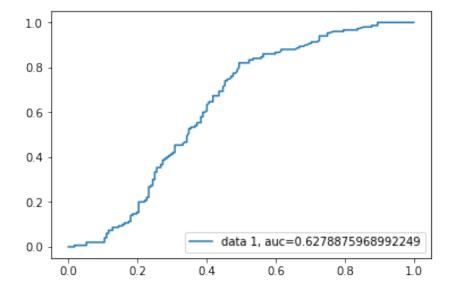
l]
fpr, tpr, _ = metrics.roc_curve(y_train, y_hat_probabilities)
auc = metrics.roc_auc_score(y_train, y_hat_probabilities)
plt.plot(fpr,tpr,label="data 1, auc="+str(auc))
plt.legend(loc=4)
plt.show()
```

```
Accuracy of model is 0.5745341614906833

Precision of model is 0.5546218487394958

Recall of model is 0.44

Confusion matrix
[[119 53]
[ 84 66]]
```

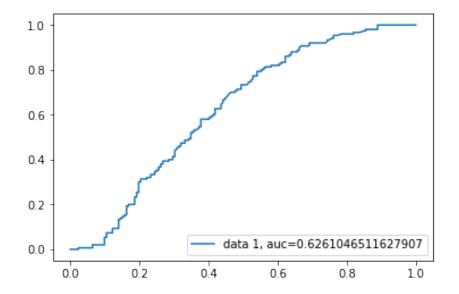


```
In [727]:
```

```
lr = LogisticRegression(solver='lbfgs')
lr.fit(X_train[['fv_log', 'l_buyout', 'shares_log']], y_train)

y_hat_train = lr.predict(X_train[['fv_log', 'l_buyout', 'shares_log']])
print_metrics(y_train, y_hat_train)

y_hat_probabilities = lr.predict_proba(X_train[['fv_log', 'l_buyout', 'shares_log']])[::,1]
fpr, tpr, _ = metrics.roc_curve(y_train, y_hat_probabilities)
auc = metrics.roc_auc_score(y_train, y_hat_probabilities)
plt.plot(fpr,tpr,label="data 1, auc="+str(auc))
plt.legend(loc=4)
plt.show()
```



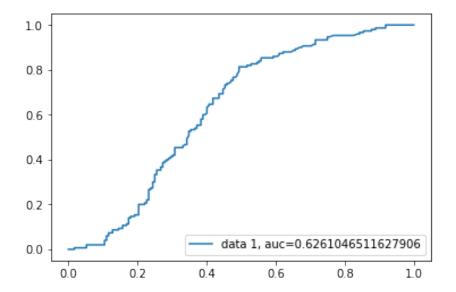
b) Linear Discriminant Analysis

```
In [737]:
```

```
lda = sklearn_lda(solver='svd')
lda.fit(X_train[['stock_value', 'l_buyout']], y_train)

y_hat_train = lda.predict(X_train[['stock_value', 'l_buyout']])
print_metrics(y_train, y_hat_train)

y_hat_probabilities = lda.predict_proba(X_train[['stock_value', 'l_buyout']])[::
,1]
fpr, tpr, _ = metrics.roc_curve(y_train, y_hat_probabilities)
auc = metrics.roc_auc_score(y_train, y_hat_probabilities)
plt.plot(fpr,tpr,label="data 1, auc="+str(auc))
plt.legend(loc=4)
plt.show()
```



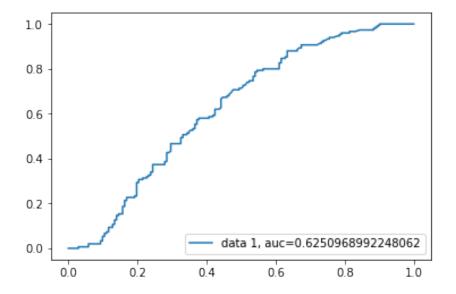
```
In [738]:
```

```
lda = sklearn_lda(solver='svd')
lda.fit(X_train[['fv_log', 'l_buyout', 'shares_log']], y_train)

y_hat_train = lda.predict(X_train[['fv_log', 'l_buyout', 'shares_log']])
print_metrics(y_train, y_hat_train)

y_hat_probabilities = lda.predict_proba(X_train[['fv_log', 'l_buyout', 'shares_log']])[::,1]
fpr, tpr, _ = metrics.roc_curve(y_train, y_hat_probabilities)
auc = metrics.roc_auc_score(y_train, y_hat_probabilities)
plt.plot(fpr,tpr,label="data 1, auc="+str(auc))
plt.legend(loc=4)
plt.show()
```

```
Accuracy of model is 0.5900621118012422
Precision of model is 0.5671641791044776
Recall of model is 0.50666666666667
Confusion matrix
[[114 58]
[ 74 76]]
```



Here I tested out traning accuracies in my model.