1). a)

The basis functions can be defined by.

$h_1(x) = 1$ $\qquad$ $h_5(x) = (x - \xi_1)^3$

$h_2(x) = x$ $\qquad$ $h_6(x) = (x - \xi_2)^3$

$h_3(x) = x^2$ $\qquad$ $h_7(x) = (x - \xi_3)^3$

$h_4(x) = x^3$

The model is specified as. ~~other~~ ~~regress~~

$$y = \sum_{m=1}^{m} \theta_m h_m(x)$$

as in any regression model.

where $m = [1 \ldots 7]$

In vector form,

$y = \theta^T X.$ ~~where~~

where, $X = \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ (x-\xi_1)^3 \\ (x-\xi_2)^3 \\ (x-\xi_3)^3 \end{bmatrix}$ $\quad \theta = \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_6 \end{bmatrix}$ $\quad y = \begin{bmatrix} y_0 \\ \vdots \\ y_{12} \end{bmatrix}$

b) To estimate the parameters of the model, we could use

~~gradient descent or anat~~

gradient descent or analytical solution. We shall describe the analytical solution below.

$$Y = X\theta + e.$$

$$J(\theta) = \| Y - X\theta \|_2^2.$$

$$J(\theta) = (Y - \theta X)^T (Y - X\theta)$$

$$-X^T Y - X^T Y + 2X^T X\theta = 0$$

finally,

$$X^T X \theta = X^T Y$$

$$\hat{\theta} = (X^T X)^{-1} X^T Y$$