GMIT

INSTITIÚID TEICNEOLAÍOCHTA NA GAILLIMHE-MAIGH EO

GALWAY-MAYO INSTITUTE OF TECHNOLOGY

Galway-Mayo Institute of Technology

Semester 2 Examinations 2016/2017

MODULE: COMP08015 – Theory of Algorithms

PROGRAMME(S):

GA_KSOFG_H08 B.Sc. (Honours) Software Development

YEAR(S) OF STUDY: 4

EXAMINERS:

Dr. Ian McLoughlin	(Internal)
Dr. Des Chambers	(External)
Mr. Tom Davis	(External)

TIME ALLOWED: 2 hours

INSTRUCTIONS: **Answer 3 questions. All questions carry equal marks.**

Please do not turn this page until you are instructed to do so.

The use of programmable or text storing calculators is expressly forbidden. Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

There are no additional requirements for this paper.

Question 1

Consider the following Racket code and answer the questions below.

```
1 (define (twoper x)
2   (if (= 0 x)
3     1
4     (* 2 (twoper (- x 1)))))
5
6 (twoper 8)
7 (map twoper (range 8))
8 (foldl + 0 (map twoper (range 8)))
```

(a) Determine the output on line 6, and explain how the value is calculated. (40 marks)

(b) Explain what the `map` and `foldl` functions in Racket do with reference to lines 7 and 8 of the code above. (40 marks)

(c) Explain why the well-known programming concept of MapReduce is well-suited parallel programming environments. (20 marks)

Question 2

Consider the language $L = \{a^i b^i c^i | i \in \mathbb{N}, i > 0\}$ over the alphabet $A = \{a, b, c\}$.

(a) Give five examples of strings in L, each of different length, and give a general formula for the length of all strings in L in terms of i. (40 marks)

(b) Give the definition of the set A^*, where $*$ is the Kleene star, and determine and explain clearly the set of strings that are in A^* but not in L. (40 marks)

(c) State whether the empty string is in the intersection of L and A^*, and explain your reasoning. (20 marks)

Question 3

Consider the following state table for a Turing machine, where $\sqcup$ is the blank symbol and the usual conventions apply.

State	Input	Write	Move	Next
q_0	$\sqcup$	1	R	q_a
q_0	0	0	R	q_1
q_0	1	1	R	q_0
q_1	$\sqcup$	0	R	q_f
q_1	0	0	R	q_0
q_1	1	1	R	q_1

(a) Determine the final state and the output on the tape, where the initial input on the tape is 0010010010. Remember to clearly show your workings. (40 marks)

(b) Give the state table for a Turing machine that accepts inputs over the alphabet $\{0, 1\}$ and ends in the accept state if and only if the number of 1's in the input is a multiple of 4. You may assume 0 is a multiple of 4. (40 marks)

(c) Explain the difference between deterministic and non-deterministic Turing machines, with regard to state tables. (20 marks)

Question 4

Consider the well-known subset sum (SUBSETSUM) and Boolean satisfiability problems (SAT).

(a) Explain what a decision problem is, giving examples of both the decision and non-decision forms of the subset sum problem. (40 marks)

(b) Explain what it means for the subset sum problem to be NP-complete. (40 marks)

(c) Explain why a polynomial-time solution to SUBSETSUM would immediately lead to a polynomial-time solution of SAT. (20 marks)