DB Design Theory

Quick Recap

- Data Anomalies
- Functional Dependencies
  - *Defining and validating FD's*
  - *"Good" vs. "Bad" FDs: Intuition*
  - *Armstrong Rules*
  - *Closures*
  - *SuperKeys & Keys*

What's Next?

Minimal Cover

Equivalence of FDs

Quick RECAP

Equivalent to asking:

Given a set of FDs, $F = \{f_1, \ldots f_n\}$, does an FD **g** hold?

Inference problem: How do we decide?

Three simple rules called **Armstrong's Rules.**

1. **Reduction (Reflexive)**
   - If $Y \subseteq X$, then X -> Y
2. **Augmentation**
   - If X -> Y, then XZ -> YZ
3. **Transitivity**
   - If X -> Y and Y -> Z, then X -> Z

Two further rules that can be derived from Armstrong's Rules

- **Union**
- **Decomposition**

Quick RECAP - Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$, FDs F.
Repeat until X doesn't change;
do:
if $\{B_1, \dots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \dots, B_n\} \subseteq X$:
then add C to X.
Return X as X^+

$\{name\} \rightarrow \{color\}$

$\{category\} \rightarrow \{dept\}$

$\{color, category\} \rightarrow \{price\}$

$\{name, category\}^+ =$
$\{name, category\}$

$\{name, category\}^+ =$
$\{name, category, color\}$

$\{name, category\}^+ =$
$\{name, category, color, dept\}$

$\{name, category\}^+ = \{name, category, color, dept, price\}$

MINIMAL COVER

Minimal Cover of FDs

- A set of FDs is **minimal** if it satisfies the following conditions:

1. *Every dependency in **F** has a single attribute for its right side.*
2. *If for any FD in **F** we remove one or more attributes from the left side of **F**, the result is no longer a basis (cover).*
3. If any FD is removed from **F**, the result is no longer a basis(cover).

- Every set of FDs has an equivalent minimal set
- There can be several equivalent minimal sets

Example 1: Minimal Cover

- Consider a relation *R{A, B, C, D) with set of FDs*

$$\{AB \rightarrow D,\ B \rightarrow A,\ D \rightarrow A\}$$

- Step1: Canonical Form (Each dependency has a single attribute on right side)
- Step 2: We need to look for redundant attributes on the LHS
- Step3: We need to look for FDs that are redundant

Example 1: Minimal Cover

- Consider a relation *R{A, B, C, D) with set of FDs*

  $$\{AB \rightarrow D, B \rightarrow A, D \rightarrow A\}$$

- Step 2: We need to look for redundant attributes on the LHS
  - *Determine if AB → D has redundant attribute?*

Using RULES

Consider B → A
BB → AB (augment B)
B → AB
AB → D (given)
B → D (transitivity)

AB → D may be replaced by B → D.

Using CLOSURE

Compute
$A^+ = \{A\}$
$B^+ = \{B, A, D\}$

As B^+ contains D so A is redundant

Example 1: Minimal Cover

- Consider a relation *R{A, B, C, D) with set of FDs*

$$\{B \longrightarrow D,\ B \longrightarrow A,\ D \longrightarrow A\}$$

- Step3: We need to look for FDs that are redundant

Using RULES

$B \longrightarrow D$
$D \longrightarrow A$
$B \longrightarrow A$ (transitive rule)

Eliminate $B \longrightarrow A$

Using CLOSURE

- ***Hide $B \longrightarrow D$*** *and compute B^+* in $\{B \longrightarrow A,\ D \longrightarrow A\}$

$B^+ = \{B, A\}$ as it does not contain D so we cannot eliminate $B \longrightarrow D$

- ***Hide $B \longrightarrow A$*** *and compute B^+* in $\{B \longrightarrow D,\ D \longrightarrow A\}$

$B^+ = \{B, A, D\}$ as it contains A so eliminate $B \longrightarrow A$

- ***Hide $D \longrightarrow A$*** *and compute D^+* in $\{B \longrightarrow D\}$

$D^+ = \{D\}$, so we cannot eliminate $D \longrightarrow A$

Repeat till no more FD's can be removed

Example 2: Minimal Cover

- Consider a relation *R{A, B, C, D,E,G) with set of FDs*

{ AB -> C, C -> A, BC -> D, ACD -> B, D -> EG, BE -> C, CG -> BD, CE -> AG}

- Step1: Canonical Form
- Step 2: We need to look for redundant attributes on the LHS
- Step3: We need to look for FDs that are redundant

Example: Minimal Cover

- Consider a relation *R{A, B, C, D,E,G) with set of FDs*

{ AB -> C, C -> A, BC -> D, ACD -> B, D -> EG, BE -> C, CG -> BD, CE -> AG}

- Step1: Canonical Form

AB -> C
C -> A
BC -> D
ACD -> B
D -> E
D-> G
BE -> C
CG -> B
CG -> D
CE -> A
CE -> G

Example: Minimal Cover

- Step 2: Remove redundant attributes on the LHS

AB -> C
C -> A
BC -> D
ACD -> B
D -> E
D-> G
BE -> C
CG -> B
CG -> D
CE -> A
CE -> G

Using Closure

ACD -> B **Compute CD$^+$ ={ACDEGB}**

- The closure contains B, which tells us that **CD -> B** holds.

CE -> A **C$^+$ ={AC}**

- The closure contains A, which tells us that **C -> A** holds.

Using RULES

Prove that F logically implies CD -> B in place of ACD -> B.

C -> A (given in F)
CD -> ACD (augment with CD)
ACD -> B (given in F)
CD -> B (transitivity)

Example: Minimal Cover

- **Step3: Look for redundant FDs in set F**

AB -> C
C -> A
BC -> D
CD -> B
D -> E
D -> G
BE -> C
CG -> B
CG -> D
~~CE -> A~~
CE -> G

Using RULES remove Redundant FDs

- CG -> B

  CG -> D (given)
  CG ->CD (augment C)
  CD -> B (given)
  Thus CG -> B

No more redundant FDs in F.

Using Closure
Hide CG->B, compute CG^+={C,G,D,A,B,E

Check for rest of the FDs

Example: Minimal Cover

- **Step3: Look for redundant FDs in set F**

AB -> C
C -> A
BC -> D
CD -> B
D -> EG
BE -> C
~~CG -> B~~
CG -> D
~~CE -> A~~
CE -> G

We can have more than one cover

Example: Minimal Cover

- **Step3: Look for redundant FDs in set F**

AB -> C
C -> A
BC -> D
~~CD -> B~~
D -> EG
BE -> C
CG -> B
~~CG -> D~~
~~CE -> A~~
CE -> G

Using RULES remove Redundant FDs

- **CG -> D**
  - CG->B (given)
  - CG->BC (Augment C)
  - BC->D (given)
- **CD -> B**
  - D->G (given)
  - CD-> CG (Augment)
  - CG->B (given) …

No more redundant FDs in F.

Using Closure

…

Activity1: Minimal Cover

- Given a relation R (A, B, C, D, E, F) and
- a set of FDs F = {$A \rightarrow BCE$, $CD \rightarrow EF$, $E \rightarrow F$, $B \rightarrow E$, $AB \rightarrow CF$}.
- Compute the minimal cover for F

Equivalence of Sets of FDs

Equivalence of Sets of FDs

Minimal Cover 1

AB -> C
C -> A
BC -> D
CD -> B
D -> EG
BE -> C
~~CG -> B~~
CG -> D
~~CE -> A~~
CE -> G

ARE THEY EQUIVALENT ?

Minimal Cover 2

AB -> C
C -> A
BC -> D
~~**CD -> B**~~
D -> EG
BE -> C
CG -> B
~~CG -> D~~
~~CE -> A~~
CE -> G

Equivalence of Sets of FDs

- Two sets of FDs F and G are **equivalent** if:
  - *Every FD in F can be inferred from G, and*
  - *Every FD in G can be inferred from F*
  - *Hence, F and G are equivalent if* $\mathbf{F^+ = G^+}$

- **Covers:**
  - ***F covers G*** *if every FD in G can be inferred from F*
    - i.e. if $\mathbf{G^+ \subseteq F^+}$

- F and G are equivalent if F covers G and G covers F

Example 1: Equivalence of FDs

- Consider two sets of FDs, F and G,
  - *F = {A -> B, B -> C, AC -> D} and*
  - *G = {A -> B, B -> C, A -> D}*
- Are F and G equivalent?

- We need to prove that $F^+ = G^+$.
  - *This is computationally expensive we take a short cut.*
  - *We can conclude that F and G are equivalent,*
    - if we can prove that all FDs in F can be inferred from the set of FDs in G and vice versa.

Example 1: Equivalence of FDs

- Use attribute closure to infer all FDs in F using G
  - *F = {A -> B, B -> C, AC -> D}*
  - *G = {A -> B, B -> C, A -> D}*

- Take the attributes from the LHS of FDs in F and compute attribute closure for each using FDs in G:
  - $A^+ = ABCD$
  - $B^+ = BC$
  - $AC^+ = ABCD$

G = {A -> B,
B -> C,
A -> D}

- All FDs in F are inferred using FDs in G.

Example 1: Equivalence of FDs

- Use attribute closure to infer all FDs in F using G
  - $F = \{A \to B, B \to C, AC \to D\}$
  - $G = \{A \to B, B \to C, A \to D\}$

- Now we see if all FDs in G are inferred by F
  - $A^+ = ABCD$
  - $B^+ = BC$

F = {A -> B,
B -> C,
AC -> D}

- All FDs in F can be obtained from G and vice versa, so we conclude that F and G are equivalent.

Example 2: Equivalence of FDs

- Are F and G equivalent?
  - ***F = {A-> C, B -> A, BD ->C}***
  - ***G = {A -> C , B -> A, B->D}***

- A^+ using F = AC
- A^+ using G = AC
- B^+ using G = ABC
- B^+ using F = ABCD

- Thus, F and G are not equivalent

Activity

Minimal Cover 1

AB -> C

C -> A

BC -> D

CD -> B

D -> EG

BE -> C

CG -> D

CE -> G

ARE THEY EQUIVALENT ?

Minimal Cover 2

AB -> C

C -> A

BC -> D

D -> EG

BE -> C

CG -> B

CE -> G