Vector

Vector is a physical quantity which magnitude and direction.

Vector = magnitude x dir

dir of a vector is given by its Unit vector.

Unit Vector

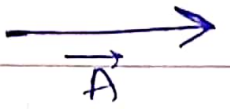
Vector which has magnitude of 1.

Vector Notation

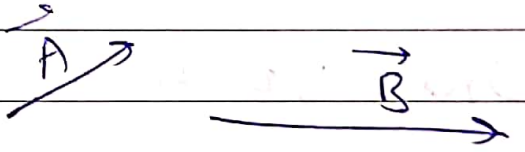
$\vec{A}$

magnitude of $\vec{A} = |\vec{A}|$
or simply A

Geometrically it is represented by arrow



Length of arrow will give an idea of magnitude of vector



Here $|\vec{B}| > |\vec{A}|$

Representation of Unit vector

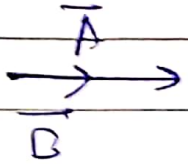
$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

(#) (3)

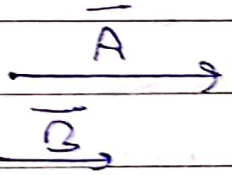
Types of vector

(1) Co-linear

Vectors acting along same line are known as collinear vector.



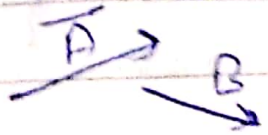
$\bar{A}$ and $\bar{B}$ are collinear vector



parallel or antiparallel vectors are collinear

(2) Co-planar

Vectors acting along same plane are known as coplanar vector

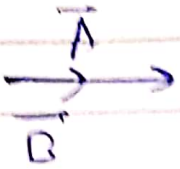


Vector $\bar{A}$ and $\bar{B}$ are on the plane of copy. Therefore they are coplanar.

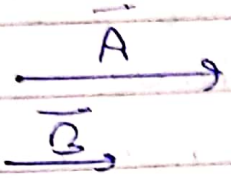
Types of Vectors

(1) Co-linear

Vectors acting along same line are known as collinear vector.



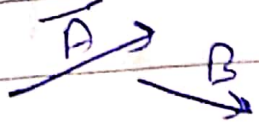
$\vec{A}$ and $\vec{B}$ are collinear vector



parallel or antiparallel vectors are collinear

(2) Co-planar

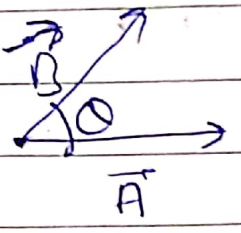
Vectors acting along same plane are known as coplanar vector



Vector $\vec{A}$ and $\vec{B}$ are on the plane of copy. Therefore they are coplanar.

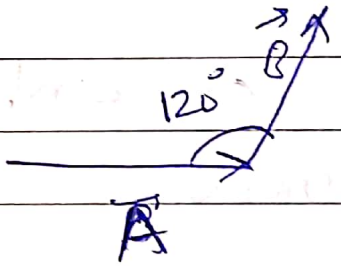
Co-initial vector

Vectors starting from same point are known as Co-initial vector



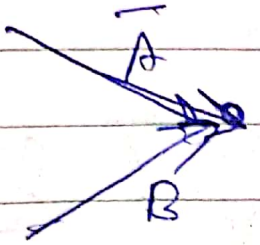
θ = angle between vectors

To find angle between vectors we have to make vector co-initial.



Angle between vectors = 60°

Coterminus Vectors terminating at same point is known as coterminus



Equal vectors

If two vectors have magnitude and directions are known as equal vectors.

Null vector

Vector which has magnitude zero and has a direction which is not fixed.

If $\vec{A} + \vec{B} = 0$

then 0 must be a null vector because sum of two vectors is a vector

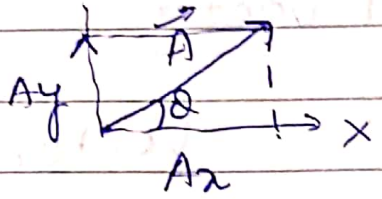
Therefore $\vec{A} + \vec{B} = \vec{0}$

(4) (5)

Component of a vector

Rectangular Component

Component along X, Y and Z axis.

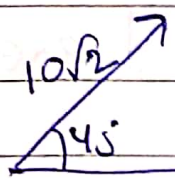


$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$$

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$$

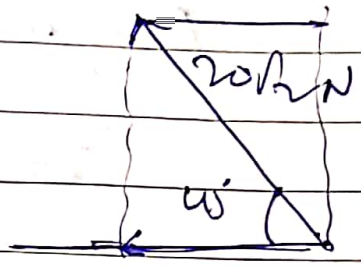
$$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$

Find force



$$\vec{F} = 10\sqrt{2}\cos 45^\circ\,\hat{i} + 10\sqrt{2}\sin 45^\circ\,\hat{j}$$

$$= 10\hat{i} + 10\hat{j}$$



$$\vec{F} = 20\sqrt{2}\cos 45^\circ(-\hat{i}) + 20\sqrt{2}\sin 45^\circ(\hat{j})$$

$$\vec{F} = -10\hat{i} + 10\hat{j}$$

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

$$A_x^2 + A_y^2 = A^2[\cos^2\theta + \sin^2\theta]$$

$$A^2 = A_x^2 + A_y^2$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Q 16 $\vec{A} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

Find magnitude A.

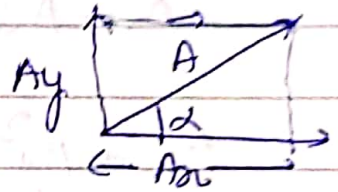
(6) (8)

$$A = \sqrt{3^2+4^2+(-2)^2}$$

$$= \sqrt{29}$$

Find Unit vector of A

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i}+4\hat{j}-2\hat{k}}{\sqrt{29}}$$



$A_x = A\cos\alpha$ $\quad \cos\alpha = \frac{A_x}{A}$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma = \frac{A_x^2+A_y^2+A_z^2}{A^2} = 1$$

$$\boxed{\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1}$$

$\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are directions cosine.

(9)

Q. if $\vec{A} = \hat{i} - \hat{j} + k$

Find angle made by vector $\vec{A}$ with x-axis.

$$\cos\alpha = \frac{A_x}{A}$$

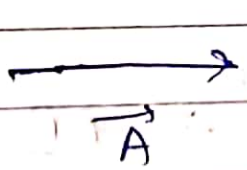
$$A = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

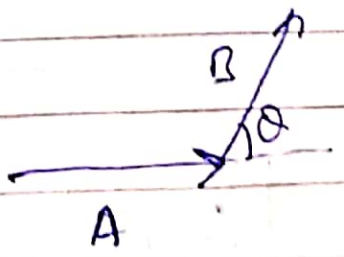
$$\cos\alpha = \frac{1}{\sqrt{3}}$$

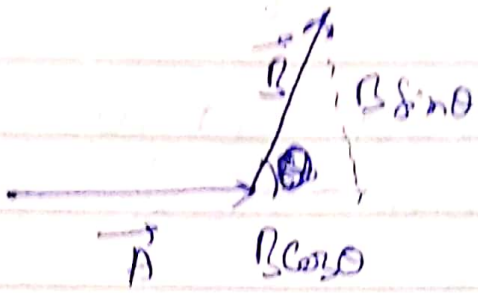
$$\boxed{\alpha = \cos^{-1}\frac{1}{\sqrt{3}}}$$

Vector addition

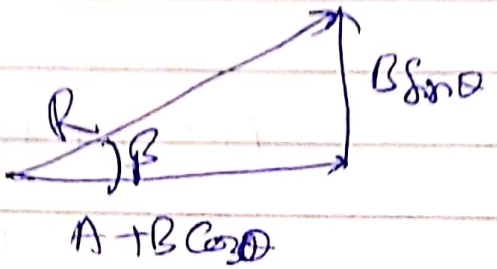
Triangle law of vector addition







$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$



$$R = \sqrt{(A+B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

θ = angle between vectors

Max value of R

$R = R_{max}$ when $\cos\theta = 1$

$\theta = 0^\circ$

$R_{max} = A + B$

Min value of R

$R = R_{min}$ when $\cos\theta = -1$

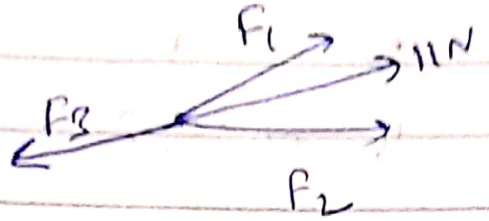
$\theta = 180$

$R_{min} = A - B$

$(A-B) \leq R \leq (A+B)$

Q If $F_1 = 5N$, $F_2 = 8N$ and $F_3 = 11N$ are acting on a body, can net force on be zero.

Ans



Net force = 0

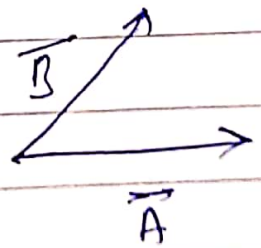
Q if $F_1 = 3N$, $F_2 = 5N$

$F_3 = 8N$ $F_4 = 14N$

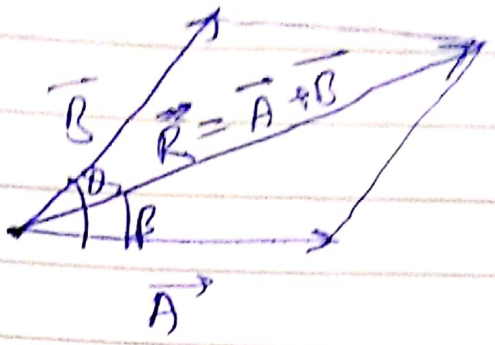
are acting on a body. Can net force on the body be zero?

Parallelogram law of vector Addition

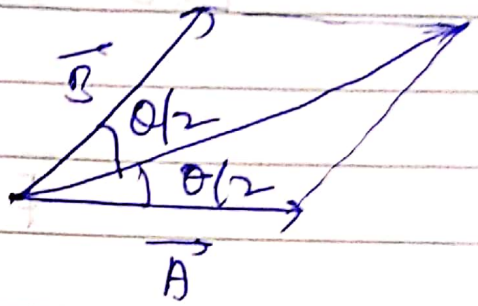
for Co-initial vector



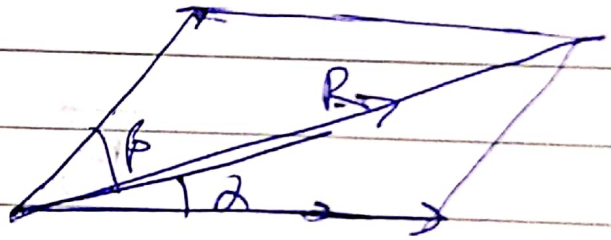
Complete parallelogram using these two sides



If $|\vec{A}| = |\vec{B}|$

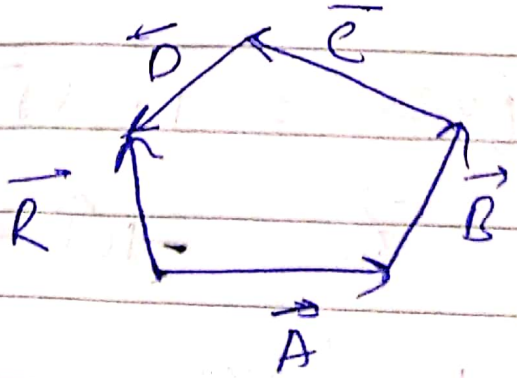


If $|\vec{A}| > |\vec{B}|$



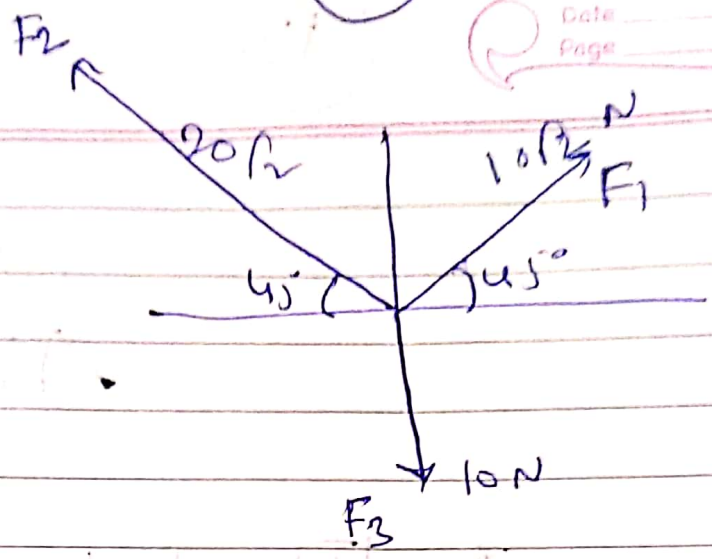
$\alpha < \beta$

Polygon law of vector addition



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

(14)



Find net force acting on body

$$\vec{F_1} = 10\sqrt{2}\cos 45^\circ \hat{i} + 10\sqrt{2}\sin 45^\circ \hat{j}$$

$$= 10\hat{i} + 10\hat{j}$$

$$\vec{F_2} = 20\sqrt{2}\cos 45^\circ(-\hat{i}) + 20\sqrt{2}\sin 45^\circ \hat{j}$$

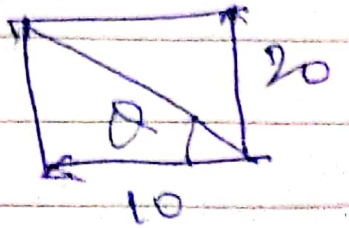
$$= -20\hat{i} + 20\hat{j}$$

$$\vec{F_3} = -10\hat{j}$$

Net force $= \vec{F_1} + \vec{F_2} + \vec{F_3}$

$$\vec{F} = -10\hat{i} + 20\hat{j}$$

$$|\vec{F}| = \sqrt{500}$$



$\tan\theta = \frac{2}{1}$