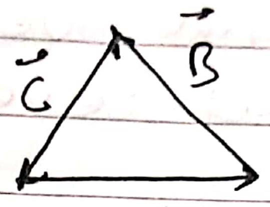
class notes (Physics)　　Date - 09/04/2020

Polygon law (continued)

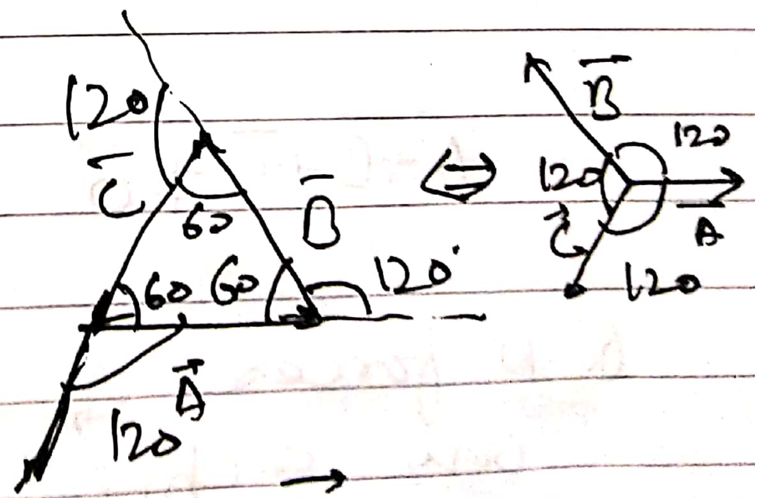


$$\vec{A} + \vec{B} + \vec{C} = \vec{0}$$

$\vec{A}$, $\vec{B}$ and $\vec{C}$ are forming closed polygon. Therefore resultant will be zero.

Now suppose

$$|\vec{A}| = |\vec{B}| = |\vec{C}|$$



Angle between $\vec{A}$ and $\vec{B}$ $= 120$

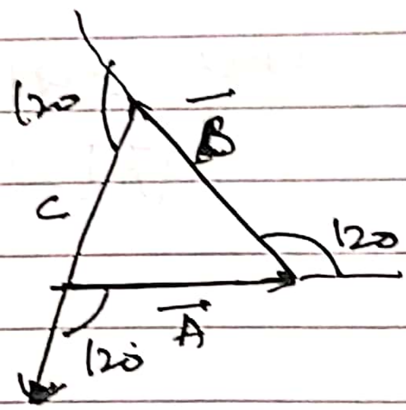
$\vec{B}$ and $\vec{C}$ $= 120$

$\vec{C}$ and $\vec{A}$ $= 120$

We can say if vectors are forming closed polygon then resultant will be zero. And sum of angles between consecutive vectors is equal to 360°. But reverse is not true.

If sum of angle between consecutive vectors is equal to 360° then they will not neccessarly form closed polygon

For example



$\vec{A}+\vec{B}+\vec{C} \neq \vec{0}$

Q N forces of magnitude F each are acting on a body. Angle between any two consecutive force is $\frac{2\pi}{N}$. Find resultant

Ans. Resultant = 0, because they will form closed polygon

Q (N-1) forces ~~are acting on a~~ ~~bo~~ of magnitude F each are acting on a body. Angle between any two consecutive force is $\frac{2\pi}{N}$. Find resultant

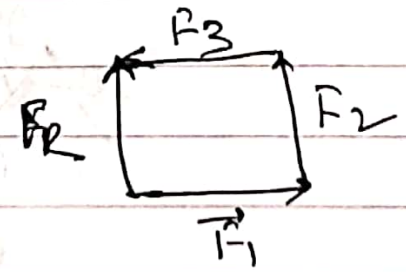
Ans → F

let $N = 4$

$N - 1 = 3$ forces

$\theta = \frac{2\pi}{4} = \frac{\pi}{2}$

angle between consecutive forces



F_R = Resultant force

magnitude of F_R = magnitude of $F_2 = F$

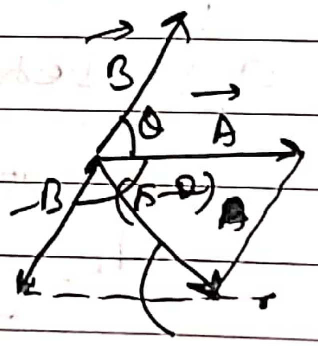
Substraction of two vectors

$$\vec{R} = \vec{A} - \vec{B}$$

We can write

$$\vec{R} = \vec{A} + (-\vec{B})$$

$\vec{R}$ is basically sum of two vectors $\vec{A}$ and $(-\vec{B})$



$\vec{R} = \vec{A} - \vec{B}$

θ = angle between vectors

Resultant $\vec{A}$ and $-\vec{B}$

$$R = \sqrt{|\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}||-\vec{B}|\cos(180-\theta)}$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

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$$|\bar{A}-\bar{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$$

Q If sum of two unit vectors is a unit vector. Find their difference.

Res

$$|\bar{A}+\bar{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$1 = \sqrt{1^2+1^2+2\times1\times1\cos\theta}$$

$$\boxed{\cos\theta = -\frac{1}{2}}$$

$$|\bar{A}-\bar{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$$

$$= \sqrt{1^2+1^2-2\times1\times1\times\left(-\frac{1}{2}\right)}$$

$$= \sqrt{3}$$

Examples of Subtraction of two Vectors

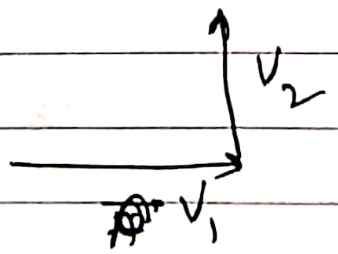
① Change in velocity

$$\Delta\vec{V} = \vec{V_2} - \vec{V_1}$$

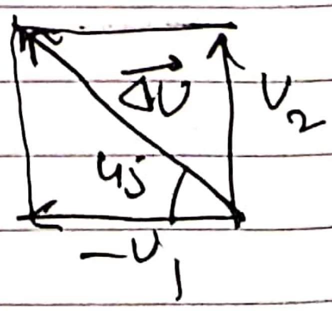
$\vec{V_1}$ = initial velocity

$\vec{V_2}$ = final velocity

Q A car is moving towards east with speed 10 m/s. So turns towards north without change in speed. Find change in velocity.



$$\Delta\vec{V} = \vec{V_2} - \vec{V_1}$$

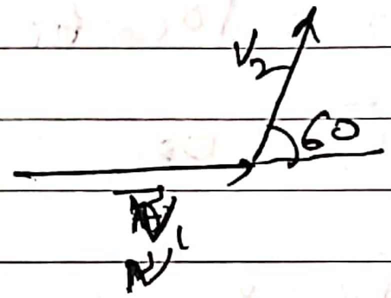


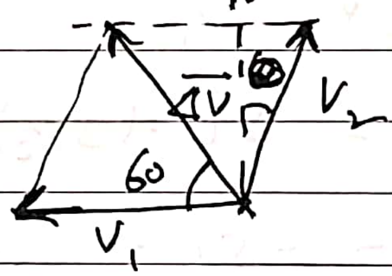
$$|\Delta\vec{V}| = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos 90^\circ}$$

$$= 10\sqrt{2}\ m/s$$

dir → N–W

Q. A Car is moving towards east with speed 10 m/s. It turns by angle of 60° towards north without change in speed. Find change in velocity.





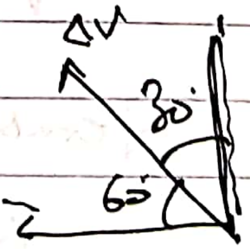
$$|\Delta \bar{v}| = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos 60}$$

$$= \sqrt{10^2 + 10^2 - 2\times 10\times 10\times \tfrac{1}{2}}$$

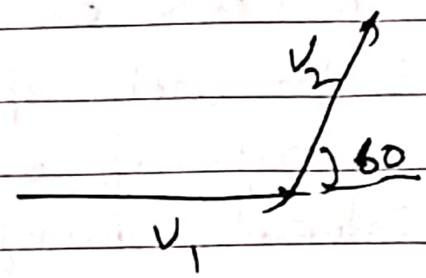
$$= 10 \text{ m/s}$$

Dir

60° N of W
or
30° W of N



Second method



$\bar{v}_1 = 10\hat{i}$

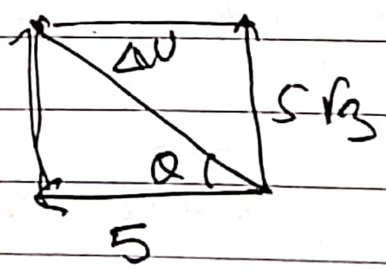
$\bar{v}_2 = 10\cos 60^\circ\,\hat{i} + 10\sin 60\,\hat{j}$

$\bar{v}_2 = 5\hat{i} + 5\sqrt{3}\hat{j}$

$\bar{\Delta v} = \bar{v}_2 - v_1$

$= 5\hat{i} + 5\sqrt{3}\hat{j} - 10\hat{i}$

$= -5\hat{i} + 5\sqrt{3}\hat{j}$



$|\Delta\bar{v}| = \sqrt{5^2 + (5\sqrt{3})^2} = 10$

$\tan\theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$

$\theta = 60^\circ$