VECTORS

1. Scalar and vector quantities

Scalar quantities : Physical quantities that have only magnitude and no direction are called scalar quantities or simply scalars.

Example : man, time, speed, temperature, potential etc.

Vector quantities : Physical quantities that have both magnitude and direction are called vector quantities or vectors.

Example : Displacement, velocity, force etc.

Geometrical representation of a vector

Generally, a vector is represented by a straight line with an arrow head. The length of line is equal to or proportional to the magnitude and arrow head shows the direction

In figure. 1 $\overrightarrow{OA}$ is a vector and are represented by the line OA. Arrow mark shows the direction.

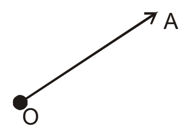


fig.1

2. Some definitions regarding vector

(i) Equal vectors - Two vectors are said to be equal if they have the same magnitude and same direction.

In figure. 2 $\vec{A}$ and $\vec{B}$ are two equal vectors

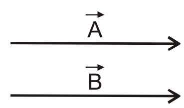


fig. 2

Note : If a vector is displaced parallel to itself, it remains equal to itself.

(ii) Negative of a vector - The negative of a vector is defined as another vector having the same magnitude but having an opposite direction. In fig. 3 vector $\vec{A}$ is the negative of vector $\vec{B}$ or vice-versa.

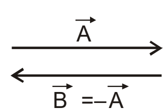


fig. 3

(iii) Modulus of a vector - The modulus of a vector means the length or the magnitude of that vector. It is a scalar quantity. Modulus of vector $\vec{A} = |\vec{A}| = A$

(iv) Unit vector - A unit vector is a vector of unit magnitude drawn in the direction of a given vector.

A unit vector in the direction of a given vector is found by dividing the given vector by its modulus. Thus a unit vector in the direction of vector $\vec{A}$ is given by

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

A unit vector in the direction of a given vector $\vec{A}$ is written as $\hat{A}$ and is pronounced as 'A carat' a 'A hat' or 'A cap'

Note : - The magnitude of a unit vector is unity. It just gives the direction of a vector. A unit vector has no units or dimensions.

(v) Collinear vectors - Collinear vector are vectors acting along the same line. They act in the same direction or opposite direction.

Parallel vector - Two collinear vectors acting along the same direction are called parallel vectors. The angle between the two vector must be zero. The magnitudes of two vectors need not be equal. They are also called like vectors.

In figure 4, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are parallel vectors.

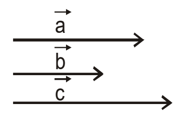


fig. 4

Antiparallel vectors - Two collinear vectors acting in the opposite directions are called antiparallel the angle between them is 180º. They are also called unlike vector. In figure 5. $\vec{a}$ and $\vec{b}$ are antiparallel vectors

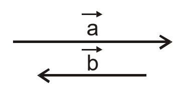


fig. 5

3. Addition of vectors

Triangle law of vector addition :

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is given both in magnitude and direction by the sides of the triangle taken in opposite order. Consider two vectors $\overline{OA} = \vec{a}$ are $\overline{OB} = \vec{b}$ (see in figure 6)

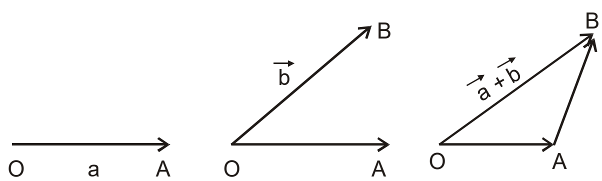


fig.6

To add $\vec{a}$ to $\vec{b}$, first draw vector $\vec{a}$, Next draw vector $\vec{b}$ with its tail at the head of $\vec{a}$ and draw a line from the tail of $\vec{a}$ to the head of $\vec{b}$. Then $\vec{c} = \vec{a}+\vec{b}$. $\vec{a}$ and $\vec{b}$ are in the same order, but $\vec{c}$ is in opposite order. We can add any number of vectors by this method.

Parallelogram law of vector addition :

If two vectors acting at a point be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

Let $\vec{a}$ and $\vec{b}$ are represented by adjacent sides $\overline{AB}$ and $\overline{AE}$ of a parallelogram ABCE as shown in figure. 7 given below.

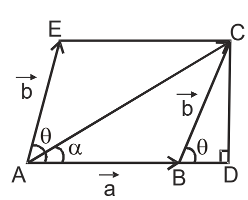


fig. 7

The resultant $\vec{c} = \vec{a} + \vec{b}$ will be represented by diagonal $\overrightarrow{AC}$. Let $\angle EAB = \theta$. Draw perpendicular CD on the extended portion of $\overrightarrow{AB}$.

from figure CD = $|\vec{b}|\sin\theta$

BD = $|\vec{b}|\cos\theta$

$\therefore\ |\vec{c}| =$ AC = $\sqrt{AD^2 + CD^2}$

$= \sqrt{(a + b\cos\theta)^2 + (b\sin\theta)^2}$

$= \sqrt{a^2 + b^2\cos\theta + 2ab\cos\theta + b^2\sin^2\theta}$

$= \sqrt{a^2 + b^2(\sin^2\theta + \cos^2\theta) + 2ab\cos\theta}$

$$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab\cos\theta} \qquad \text{...(i)}$$

Let $\overrightarrow{AC}$ makes an angle α with $\vec{a}$

$$\therefore \qquad \tan\alpha = \frac{CD}{AD} = \frac{b\sin\theta}{a + b\cos\theta} \qquad \text{....(ii)}$$

Special cases

(1) If $\theta = 0°$

from (i) $|\vec{c}| = a + b$

from (ii) $\alpha = 0$

(2) If $\theta = 180°$

from (i) $|\vec{c}| = a - b$

from (ii) $\alpha = 0$

(3) If $\theta = 90°$

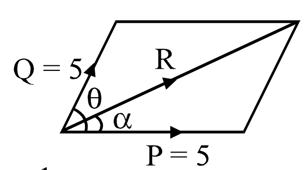
from (i) $|\vec{c}| = \sqrt{a^2 + b^2}$

from (ii) $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$

Illustration : 1

Two forces of equal magnitude 5 units have an angle 60° between them. Find the magnitude and direction of resultant force.

Solution



$R^2 = P^2 + Q^2 + 2PQ\cos\theta = 5^2 + 5^2 + 2[5][5]\cos 60°$

$= 25 + 25 + 25 = 75 \qquad R = 5\sqrt{3}$ unit

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{5\sin 60°}{5 + 5\cos 60°} = \frac{5\sqrt{3}/2}{5 + 5(1/2)} = \frac{5\sqrt{3}}{15} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \alpha = 30°$

Illustration : 2

The resultant of two forces 3p and 2p is R. If the first force is doubled keeping the same direction, then the resultant is also doubled. Find the angle between two forces.

Solution :

Let θ be the angle between two forces. We know resultant of two forces F_1 and F_2 inclined at an angle θ is given by

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

or $$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \text{. . . . . (i)}$$

Here $F_1 = 3p$ and $F_2 = 2p$

$\therefore \; R^2 = (3p)^2 + (2p)^2 + 2\times3p\times2p\cos\theta$

or $R^2 = 9p^2 + 4p^2 + 12p^2\cos\theta$

$= 13p^2 + 12p^2\cos\theta$ . . . . .(ii)

Now $F_1 = 2 \times 3p = 6p$ and $R = 2R$, but $F_2 = 2p$

$\therefore$ from equation (i)

$(2R)^2 = (6p)^2 + (2p)^2 + 2 \times 6p \times 2p\cos\theta$

$4R^2 = 36p^2 + 4p^2 + 24p^2\cos\theta$

$= 40p^2 + 24p^2\cos\theta$

or $R^2 = 10p^2 + 6p^2\cos\theta$ . . . . .(iii)

Equating equations (ii) and (iii), we get

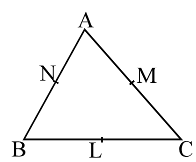
$13p^2 + 12p^2\cos\theta = 10p^2 + 6p^2\cos\theta$

or $3p^2 = -6p^2\cos\theta$ or $\cos\theta = -1/2$

$\therefore \; \theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^0$

Self effort

L, M and N are middle points of BC, CA and AB respectively. Prove that $\overrightarrow{AL} + \overrightarrow{BM} + \overrightarrow{CN} = 0$



4. Subraction of vectors

$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$ Consider the figure

$|\vec{a} - \vec{b}| = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\tan\phi = \dfrac{b\sin\theta}{a - b\cos\theta}$

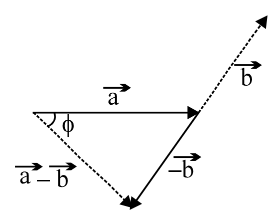


fig. 8

Illustration : 3

A particle moving with velocity $\vec{v}$ towards northward direction changes its direction and moves towards eastward with the same speed. Find the change in its velocity.

Solution :

$\vec{v}_1 = (\overrightarrow{ON})$ where $|v_1| = v$;

$\vec{v}_2 = (\overrightarrow{NE})$ where $|v_2| = v$;

$\therefore$ Change in velocity,

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$

$= (\overrightarrow{NE}) + (\overrightarrow{NO}) = (\overrightarrow{NA})$

and $|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos 90^\circ}$

$= \sqrt{v^2 + v^2} = v\sqrt{2}$

$\tan\beta = \dfrac{|\vec{v}_1|}{|\vec{v}_2|} = \dfrac{v}{v} = 1$ or $\beta = 45^\circ$

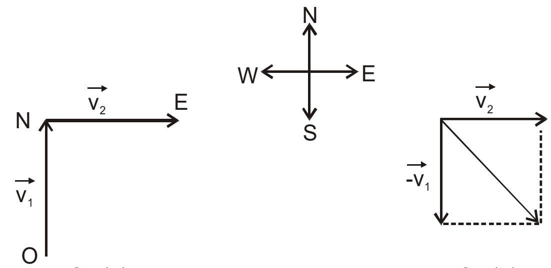


fig (a) fig (b)

Thus the direction of $\Delta\vec{v}$ wil be along south east direction.

fig. 9

5. Resolution of a vector

Let $\overrightarrow{OA}$ be a vector in the X-Y plane. Let OA = r. Let $\angle AOX = \alpha$, $\angle AOY = \beta$. Draw AN perpendicular on OX and AM perpendicular on OY. Then cosα = ON/OA and cosβ = OM/OA. Hence ON=rcosα and OM = rcosβ. By vector addition $\overrightarrow{OA} = \overrightarrow{ON} + \overrightarrow{NA} = \overrightarrow{ON} + \overrightarrow{OM}$. Thus, we have resolved(broken) the vector $\overrightarrow{OA}$ into two parts, one along OX and other along OY.

$\overrightarrow{OA} = r\cos\alpha\hat{i} + r\cos\beta\hat{j}$

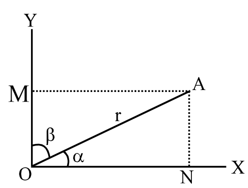


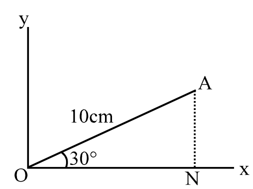
fig. 10

rcosα is called the component of vector $\overrightarrow{OA}$ along X-axis.

rcosβ is called the component of vector $\overrightarrow{OA}$ along Y-axis.

Illustration : 4

Let OA = 10 cm. Write vector $\overrightarrow{OA}$ with the help of $\hat{i}$ and $\hat{j}$



Solution :

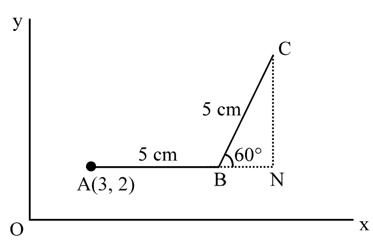
Let AN be perpendicular to OX. cos30° = ON/OA

$\Rightarrow$ ON = OA cos30° = (10)$\sqrt{3}$/2 = $5\sqrt{3}$, sin 30° = AN/10

$\Rightarrow$ $AN = 5$ $\therefore$ $A = (5\sqrt{3}, 5)$ $\therefore$ $\overrightarrow{OA} = 5\sqrt{3}\,\hat{i} + 5\hat{j}$

Illustration : 5

Write the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{OC}$ in terms of $\hat{i}$ and $\hat{j}$.



Solution :

$\overrightarrow{OA} = 3\hat{i} + 2\hat{j}$ B = (8, 2) $\therefore$ $\overrightarrow{OB} = 8\hat{i} + 2\hat{j}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA} = 8\hat{i} + 2\hat{j} - 3\hat{i} - 2\hat{j} = 5\hat{i}$

sin60° = C N/5, cos60° = BN/5

$\therefore$ BN = 5cos60°, CN = 5 sin 60°

$\therefore$ C = (3 + 5 + 5cos60°, 2 + 5sin 60°)

$\therefore$ $\overrightarrow{OC} = \hat{i}$ (3 + 5 + 5cos60°) + $\hat{j}$ (2 + 5sin60°)

6. Multiplication of a vector by a real number

Suppose a vector $\vec{a}$ is multipled by a real number, the vector increases in magnitude, the direction remains constant. For example, let $\vec{a}$ be multiplied by 2 then the vector $2\vec{a}$ has the same direction but magnitude becomes twice. When the vector $\vec{a}$ is multiplied by -2, the direction is opposite to that of $\vec{a}$

When the vector $\vec{a}$ is multiplied by the scalar m the new vector has the same direction and same unit but the magnitude becomes m times the magnitude of $\vec{a}$

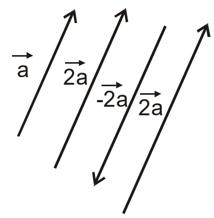


fig. 11

7. Dot product of two vectors

The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantitiy given by the product of magnitude of $\vec{A}$ and $\vec{B}$ and the cosin of the smaller angle between then

$$\vec{A}.\vec{B} = AB\cos\theta$$

where, θ is the angle between the two vectors

Geometrical meaning of scalar product

As shown in fig 12. suppose two vectors $\vec{A}$ and $\vec{B}$ are represented by $\overline{OP}$ and $\overline{OQ}$ and $\angle POQ = \theta$

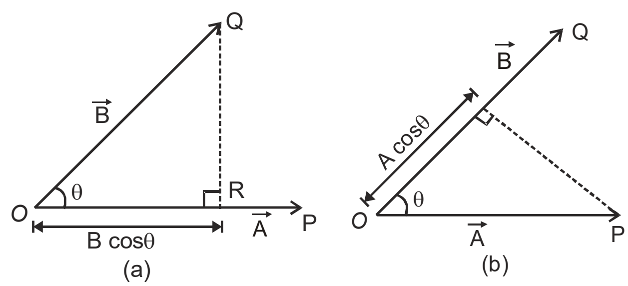


fig.12

From figure 12 (a)

$\vec{A}.\vec{B}$ = AB cosθ = A (B cosθ) = A (OR)

= A (magnitude of component of $\vec{B}$ in the direction of $\vec{A}$)

From figure 12 (b)

$\vec{A}.\vec{B}$ = AB cosθ = B (A cosθ) = B (OS)

= B (magnitude of component of $\vec{A}$ in the direction of $\vec{B}$)

Thus the scalar product of two vectors is equal to the product of magnitude of one vector and the magnitude of component of other vector in the direction of the first vector

PROPERTIES OF DOT PRODUCT

1. $\vec{A}.\vec{B}$ = AB cosθ = AB, where $\theta = 0$, 2π
   when the two vectors are parallel
2. $\vec{A}.\vec{B}$ = AB cosθ , $\vec{B}.\vec{A}$ = BA cosθ
   $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ i.e. dot product is commutative

3. Magnitude of $\hat{i}$, $\hat{j}$ and $\hat{k}$ is one. The angle between any two of them is 90º

So, $\hat{i}\,.\,\hat{i} = 1\times 1\cos\theta = 1$, $\cos\theta = 1$ as $\theta = 0$

$\hat{j}\,.\,\hat{j} = 1$

$\hat{k}\,.\,\hat{k} = 1$

$\hat{i}\,.\,\hat{j} = 1\times 1\times\cos 90° = 0$

$\hat{k}\,.\,\hat{i} = 0$

$\hat{k}\,.\,\hat{j} = 0$

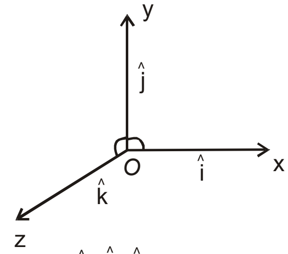


fig. 13

Illustration : 6

Given two vectors $\vec{A} = -2i + 2j$, $\vec{B} = 3\hat{i}$

(a) Find $\vec{A}\,.\,\vec{B}$

(b) Find angle between the two vectors

(c) Find projection of $\vec{A}$ on $\vec{B}$

(d) Find projection of $\vec{B}$ on $\vec{A}$

Solution :

(a) $\vec{A}\,.\,\vec{B} = (-2\hat{i} + 2\hat{j}).\,(3\hat{i})$

$= -6(\hat{i}\,.\,\hat{i}) + 6(\hat{j}\,.\,\hat{i})$

$= -6$

(b) $(-2\hat{i} + 2\hat{j}).(3\hat{i}) = |-2\hat{i} + 2\hat{j}||\,3\hat{i}\,|\cos\theta$

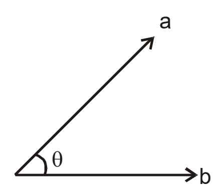
$\Rightarrow$ $-6(\hat{i}.\hat{i}) + 6(\hat{j}.\hat{i}) = (\sqrt{(-2)^2 + (2)^2})\;\; 3\cos\theta$

$\Rightarrow$ $\cos\theta = -1/2$ $\Rightarrow$ $\theta = 120°$

(c) Projection of vector a on vector b = a cosθ

Projection here $\vec{A}$ on $\vec{B}$ = $A\cos\theta$

$$= \frac{\vec{A}\,.\,\vec{B}}{|\vec{B}|}$$



$$= \frac{-6}{|3\hat{i}|} = \frac{-6}{3} = -2$$

(d) Projection of $\vec{B}$ on $\vec{A}$ = B cosθ

$$= \frac{\vec{B}.\vec{A}}{|\vec{A}|}$$

$$= \frac{-6}{\sqrt{(-2)^2 + (2)^2}} = \frac{-6}{\sqrt{4+4}}$$

$$= \frac{-6}{\sqrt{8}} = \frac{-6}{2\sqrt{2}}$$

$$= \frac{-3}{\sqrt{2}} = -1.5\sqrt{2}$$

8. Cross Product of two vectors

Before defining cross product we want to find the area of the parallelogram formed by two vectors $\vec{a}$ and $\vec{b}$.

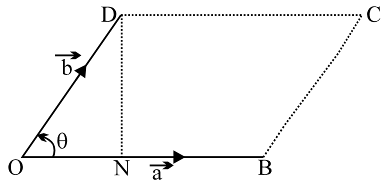


fig. 14

Draw DN perpendicular to OB.

$\sin\theta = DN/OD$

$\Rightarrow$ DN = (OD)sinθ = bsinθ

Area of the parallelogram = (base) (height)

= (OB) (DN) = absinθ

Cross product of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$.

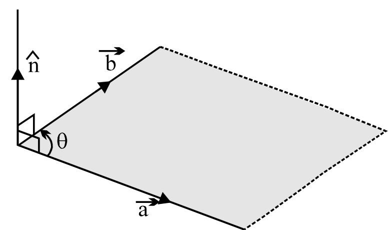


fig. 15

We define $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\,\mathbf{\sin\theta}\ \hat{\mathbf{n}} = \mathbf{ab\sin\theta}\ \hat{\mathbf{n}}$

where $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ both

and θ is angle $(0 \le \theta \le \pi)$ between two vectors $\vec{a}$ and $\vec{b}$ as shown in the figure.

Remark

(i) The direction of $\vec{a} \times \vec{b}$ (or $\hat{n}$) is the direction of rotation when $\vec{a}$ is rotated towards $\vec{b}$ through the angle θ.

(ii) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(iii) $|\vec{a} \times \vec{b}| = |ab\sin\theta\ \hat{n}| = |ab\sin\theta||\hat{n}| = ab\sin\theta$

= area of the parallelogram formed by the vectors $\vec{a}$ and $\vec{b}$.

The direction of the resultant vector can be represented by the following three rules.

Flemming left hand rule

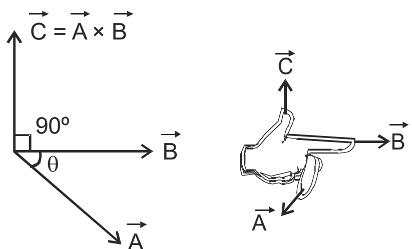


fig. 16

If the middle finger and the fore finger point in the direction of vector $\vec{A}$ & $\vec{B}$ respectively, the thumb points in the direction of the vector $\vec{A} \times \vec{B}$

Screw rule

If a screw rotates in the direction of corresponding to vector $\vec{B}$ from vector $\vec{A}$ the direction in which it moves (upwards or downwards) gives the direction of the vector $\vec{A} \times \vec{B}$

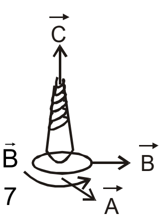


fig. 17

Right hand thumb rule

If the fingers of the right hand curl in the direction from $\vec{A}$ to $\vec{B}$, the thumb points in the direction of vector $\vec{A} \times \vec{B}$

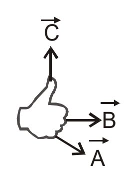


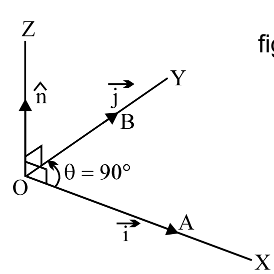
fig. 18

Cross Product of Two unit Vectors

In the diagram $\overrightarrow{OA} = \hat{i}$, $\overrightarrow{OB} = \hat{j}$

$\hat{i} \times \hat{j} = |\hat{i}||\hat{j}|\sin 90°\ \hat{n}$

where direction of $\hat{n}$ is along Z-axis but a unit vector along Z-axis is denoted by $\hat{k}$ so $\hat{n} = \hat{k}$



Hence $\hat{i} \times \hat{j} = \hat{k}$

fig. 19

Similarly $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

also $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$

As angle between $\hat{i}$ and $\hat{i}$ is zero so

$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0° \ \hat{n} = (1)(1)(0) \ \hat{n} = 0$

Similarly $\hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

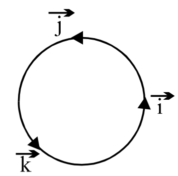


fig. 20

Illustration : 7

$\vec{a} = \hat{i} + \hat{j}, \ \vec{b} = \hat{j} + \hat{k}$. Find

(i) $\vec{a} \times \vec{b}$

(ii) area of the parallelogram formed by the vectors $\vec{a}$ and $\vec{b}$

(iii) Using cross product find the angle between vectors $\vec{a}$ and $\vec{b}$

(iv) a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ both

(v) a vector of length 5 unit and perpendicular to $\vec{a}$ and $\vec{b}$ both

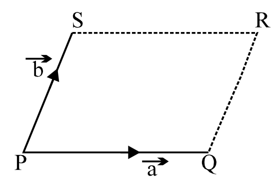
(vi) area of the ΔPQS formed by vectors $\vec{a}$ and $\vec{b}$

Solution :

(i) $\vec{a} \times \vec{b} = (\hat{i} + \hat{j}) \times (\hat{j} + \hat{k})$

$= \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{j} + \hat{j} \times \hat{k}$

$= \hat{k} - \hat{j} + 0 + \hat{i} = \hat{i} - \hat{j} + \hat{k}$



(ii) Area of parallelogram formed by vectors $\vec{a}$ and $\vec{b}$

$= |\vec{a} \times \vec{b}| = |\hat{i} + \hat{j} + \hat{k}| = \sqrt{3}$

(iii) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \ \hat{n}$

$(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k}) = |\hat{i} + \hat{j}||\hat{j} + \hat{k}| \sin\theta \ \hat{n}$

$\hat{i} - \hat{j} + \hat{k} = \sqrt{2} \ \sqrt{2} \ \sin\theta \ \hat{n}$

$|\hat{i} - \hat{j} + \hat{k}| = |\sqrt{2} \ \sqrt{2} \ \sin\theta \ \hat{n}|$

$\sqrt{3} = |2||\sin\theta||\hat{n}| = 2\sin\theta$

$\Rightarrow \ \theta = 60°$

(iv) $\vec{a}$ and $\vec{b}$ is a vector perpendicular to $\vec{a}$ and $\vec{b}$ both

so $\hat{i} - \hat{j} + \hat{k}$ is a vector perpendicular to $\vec{a}$ and $\vec{b}$ both

But $|\hat{i} - \hat{j} + \hat{k}| = \sqrt{3}$

So $\dfrac{\hat{i} - \hat{j} + \hat{k}}{3}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ both

(v) $5\left[\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}\right]$ is vector of length 5 unit and perpendicular to $\vec{a}$ and $\vec{b}$ both

(vi) Area of ΔPQS = (1/2) area of parallelogram PQRS

$= \left(\dfrac{1}{2}\right)|\vec{a} \times \vec{b}| = \sqrt{3}/2$

EXERCISE 1

INTRODUCTION

1. A vector may change if :

   (A) frame of reference is translated (B) frame of reference is rotated

   (C) vector is translated parallel to itself (D) vector is rotated

2. One of the following is not a vector :

   (A) displacement (B) work (C) force (D) gravitational field.

3. Which one of the following is not a scalar :

   (A) time (B) Length (C) mass (D) weight.

4. Direction of zero vector

   (A) does not exist (B) towards origin (C) indeterminate (D) away the origin.

ADDITION

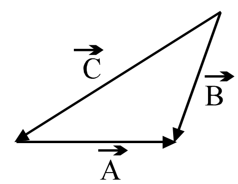
5. The magnitudes of four pairs of displacement vectors are given. Which pairs of displacement vectors cannot be added to give a resultant vector of magnitude 4 cm ?

   (A) 2 cm, 3 cm (B) 1 cm, 3 cm (C) 1 cm, 5 cm (D) 1 cm, 7 cm

6. For the fig.

   (A) $\vec{A}+\vec{B}=\vec{C}$ (B) $\vec{B}+\vec{C}=\vec{A}$

   (C) $\vec{C}+\vec{A}=\vec{B}$ (D) $\vec{A}+\vec{B}+\vec{C}=0$



7. When two vectors $\vec{A}$ and $\vec{B}$ of magnitude a and b are added, the magnitude of the resultant vector is always :

   (A) equal to (a + b) (B) less than (a + b) (C) greater than (a + b) (D) not greater than (a + b)

8. If the magnitudes of vectors $\vec{A},\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively and $\vec{A}+\vec{B}=\vec{C}$, the angle between vectors $\vec{A}$ and $\vec{B}$ is :

   (A) 0 (B) π (C) $\pi/2$ (D) $\pi/4$

9. If $\vec{A}=\vec{B}+\vec{C}$ and the magnitudes of $\vec{A},\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is :

   (A) $\cos^{-1}(3/5)$ (B) $\cos^{-1}(4/5)$ (C) $\pi/2$ (D) $\sin^{-1}(3/4)$

10. Two forces of 4 dynes and 3 dynes act upon a body. The resultant force on the body can only be

    (A) More than 3 dynes (B) More than 4 dynes

    (C) Between 3 and 4 dynes (D)Between 1 and 7 dynes

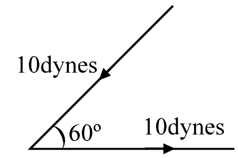
11. In case of three vector quantities of same type, in which case the resultant cannot be zero?

    (A) 10,10,10 (B) 10,10,20 (C) 10, 20,20 (D) 10,20,40

12. Three forces of magnitudes 30,60 and P Newton acting at a point are in equilibrium. If the angle between the first two is 60º, the value of P is :

    (A) $25\sqrt{2}$ (B) $30\sqrt{3}$ (C) $30\sqrt{6}$ (D) $30\sqrt{7}$

13. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is :



(A) 10 dynes (B) 20 dynes (C) $10\sqrt{3}$ dynes (D) 5 dynes

SUBTRACTION

14. Two vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}|$ = $|\vec{A}-\vec{B}|$. The angle between the vectors $\vec{A}$ and $\vec{B}$ is

(A) 0 (B) $\pi/3$ (C) $\pi/2$ (D) π

15. A particle moves through angular displacement θ on a circular path of radius r. The linear displacement will be

(A) 2r sin (θ/2) (B) 2r cos (θ/2) (C) 2r tan (θ/2) (D) 2r cot (θ/2)

RESOLUTION

16. The vector $\vec{P}$ makes 120° with the x-axis and the vector $\vec{Q}$ makes 30° with the y-axis. What is their resultant ?

(A) P + Q (B) P – Q (C) $\sqrt{P^2+Q^2}$ (D) $\sqrt{P^2-Q^2}$

17. A man travels 1 mile due east, then 5 miles due south, then 2 miles due east and finally 9 miles due north, how far is he from the starting point :

(A) 3 miles (B) 5 miles (C) 4 miles (D) between 5 and 9 miles

18. The angle that the vector $\vec{A}=2\hat{i}+3\hat{j}$ makes with y-axis is :

(A) $\tan^{-1}(3/2)$ (B) $\tan^{-1}(2/3)$ (C) $\sin^{-1}(2/3)$ (D) $\cos^{-1}(3/2)$

19. A man moves towards 3 m north then 4m towards east and finally 5 m towards 37° south of west. His displacement from origin is:

(A) $5\sqrt{2}$ m (B) 0 m (C) 12 m (D) 5 m

DOT PRODUCT

20. If $3\hat{i}+2\hat{j}+8\hat{k}$ and $2\hat{i}+x\hat{j}+\hat{k}$ are at right angles then x =

(A) 7 (B) – 7 (C) 5 (D) – 4.

21. $a_1\hat{i}+a_2\hat{j}$ is a unit vector perpendicular to $4\hat{i}-3\hat{j}$ if

(A) a_1 = .6, a_2 = .8 (B) a_1 = 3, a_2 = 4 (C) a_1 = .8, a_2 = .6 (D) a_1 = 4, a_2 = 3.

22. If $\vec{a}$ is a vector and x is a non-zero scalar, then

(A) x$\vec{a}$ is a vector in the direction of $\vec{a}$ (B) x$\vec{a}$ is a vector collinear to $\vec{a}$

(C) x$\vec{a}$ and $\vec{a}$ have independent directions (D) none of these.

CROSS PRODUCT

23. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then

(A) $\vec{a}\times(\vec{b}\times\vec{c})=1$ (B) $\vec{a}\times(\vec{b}\times\vec{c})=0$ (C) $\vec{a}\times(\vec{b}\times\vec{c})=-1$ (D) None of these.

EXERCISE 2

INTRODUCTION

1. Pick out the two scalar quantities in the following list:

   Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.

2. Pick out the only vector quantity in the following list:

   Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

ADDITION

3. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90º with the force of smaller magnitude, what are the magnitudes of forces ?

4. The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to minimum force, then the forces are :

5. When two forces of magnitude P and Q are perpendicular to each other, their resultant is of magnitude R. When they are at an angle of 180° to each other, their resultant is of magnitude $\frac{R}{\sqrt{2}}$. Find the ratio of P and Q.

6. The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

7. Two vectors have magnitudes 3 units and 4 units respectively. What should be the angle between them if the magnitude of the resultant is

   (a) 1 unit, (b) 5 units and (c) 7 units.

SUBTRACTION

8. A vector $\vec{A}$ of length 10 units makes an angle of 60° with a vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A}-\vec{B}$ & the angle it makes with vector $\vec{A}$.

9. A body is moving with uniform speed v in a horizontal circle in anticlockwise direction. What is the change in velocity in (a) half revolution (b) first quarter revolution.

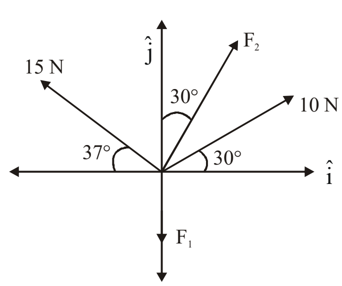
10. If the sum of two unit vectors is also a unit vector. Find the magnitude of their difference ?

RESOLUTION

11. The x and y components of vector $\vec{A}$ are 4 and 6 m respectively. The x and y components of vector $\vec{A}+\vec{B}$ are 10 and 9 m respectively. Calculate for the vector $\vec{B}$ the following :

    (a) its x and y components (b) its length, and (c) the angle it makes with x-axis.

12. If the four forces as shown are in equilibrium, Express $\vec{F}_1$ & $\vec{F}_2$ in unit vector form.



13. A plane body has perpendicular axes OX and OY marked on it and is acted on by following forces:
5 P in the direction OY

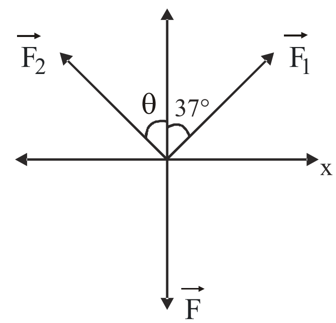
4 P in the direction OX

10 P in the direction OA where A is the point (3a, 4a)

15 P in the direction AB where B is the point (–a, a)

Express each force in the unit vector form & calculate the magnitude & direction of sum of the vectors of these forces.

14. A body acted upon by three given forces is under equilibrium.



(a) If $|\vec{F}_1|$ = 10 Nt, $|\vec{F}_2|$ = 6 Nt, find the values of $|\vec{F}_3|$ & angle (θ).

(b) Express $\vec{F}_2$ in unit vector form.

15. A particle is acted upon by the forces $\vec{F}_1 = 2\hat{i} + a\hat{j} - 3\hat{k}$, $\vec{F}_2 = 5\hat{i} + c\hat{j} - b\hat{k}$, $\vec{F}_3 = b\hat{i} + 5\hat{j} - 7\hat{k}$, $\vec{F}_4 = c\hat{i} + 6\hat{j} - a\hat{k}$. Find the values of the constants a, b, c in order that the particle will be in equilibrium.

16. A vector makes an angle of 30° with the horizontal. If horizontal component of the vector is 250, find the magnitude of vector and its vertical component?

17. $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$, when a vector $\vec{B}$ is added to $\vec{A}$, we get a unit vector along x-axis. Find the value of $\vec{B}$? Also find its magnitude.

18. In the above question, find a unit vector along $\vec{B}$?

19. Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ have magnitude 5, $5\sqrt{2}$ and 5 respectively, directions of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are towards east, North-East and North respectively. If $\hat{i}$ and $\hat{j}$ are unit vectors along East and North respectively, express the sum $\vec{A} + \vec{B} + \vec{C}$ in terms of $\hat{i}, \hat{j}$. Also find magnitude and direction of the resultant.

20. You walk 3 Km west and then 4 Km headed 60° north of east. Find your resultant displacement:

(a) graphically, and (b) using vector components.

21. A car travels due east on a level road for 30 km. It then turns due north at an intersection and travels 4 0 km before stopping. Find the resultant displacement of the car.

22. A particle is given a displacement of 5.0 cm in the east direction and then a displacement of 4.0 cm 60° north of east. Find the magnitude and the direction of the resultant displacement.

23. A particle is given a displacement of 4.0 m in x-y plane. If the x-component of the displacement vector is 2.0 m, find the y-component. Also find the angle subtended by the displacement vector with the x-axis.

24. An aeroplane takes off at an angle of 30° to the horizontal runway. The component of its velocity along the runway is 200 km h^{-1}. What is the actual velocity of the plane ? What is the vertical component of its velocity ?

25. A 50 kg block is placed on an inclined plane with an angle of 30°. Then find the components of the weight (i) perpendicular (ii) parallel to the inclined plane.

26. Find the magnitude of resultant of following three forces acting on a particle.

$\vec{F}_1 = 20N$ in eastward direction,

$\vec{F}_2 = 20N$ due north east and

$\vec{F}_3 = 20N$ in southward direction

DOT PRODUCT

27. Prove that the three vectors $6\hat{i}+2\hat{j}+4\hat{k}$, $\hat{i}+5\hat{j}-4\hat{k}$ and $2\hat{i}-2\hat{j}-2\hat{k}$ are at right angles to one another.

28. If the Vectors $\vec{P}=a\hat{i}+a\hat{j}+3\hat{k}$ and $\vec{Q}=a\hat{i}-2\hat{j}-\hat{k}$ are perpendicular to each other, find the value of a ?

29. Find the component of $3\hat{i}+4\hat{j}$ along $\hat{i}+\hat{j}$?

30. Find the angle between $\vec{A}=3\hat{i}+4\hat{j}$ and $\vec{B}=12\hat{i}+5\hat{j}$?

31. Find the components of vector $\vec{A}=2\hat{i}+3\hat{j}$ along the direction of $\hat{i}+\hat{j}$?

32. Given that $\vec{a}=\hat{i}+\hat{j}+\hat{k};\ -\vec{b}=-\hat{i}+\hat{j}-\hat{k};\ \vec{c}=\hat{i}+\hat{j}-\hat{k}$, evaluate

(i) $(\vec{a}.\vec{b})+(\vec{b}.\vec{c})+(\vec{c}.\vec{a})$ (ii) $(\vec{a}.\vec{c})\vec{c}+(\vec{c}.\vec{b})\vec{a}$.

CROSS PRODUCT

33. Find the area of a parallelogram formed from the vectors $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{B}=3\hat{i}-2\hat{j}+\hat{k}$ as adjacent sides.

34. Verify that $\vec{b}\times\vec{a}=-(\vec{a}\times\vec{b})$ where:

(i) $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=3\hat{i}-\hat{j}+\hat{k}$ (ii) $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\hat{k}$

35. If $\vec{A}$ is eastwards and $\vec{B}$ is downwards. find the direction of $\vec{A}\times\vec{B}$?

36. If $\vec{A}$. $\vec{B}$ = $|\vec{A}\times\vec{B}|$, find angle between $\vec{A}$ and $\vec{B}$

37. Find $\vec{A}\times\vec{B}$ if $\vec{A}=\hat{i}-2\hat{j}+4\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+2\hat{k}$.

38. If $\vec{A}$ is along North-East and $\vec{B}$ is down wards, find the direction of $\vec{A}\times\vec{B}$.

39. Find $\vec{B}$ x $\vec{A}$ if $\vec{A}=3\hat{i}-2\hat{j}+6\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+\hat{k}$

EXERCISE 3

INTRODUCTION

1. The resultant of two forces F_1 and F_2 is P. If F_2 is reversed, then resultant is Q. Then the value of $(P^2 + Q^2)$ in terms of F_1 is:

   (A) $2\left(F_1^2 + F_2^2\right)$ (B) $F_1^2 + F_2^2$ (C) $\left(F_1 + F_2\right)^2$ (D) None of these

2. Which of the following is a vector quantity?

   (A) Temperature (B) Surface tension (C) Heat (D) Force

ADDITION

3. The resultant of two forces acting at an angle of 150º is 10 kg wt, and is perpendicular to one of the forces. The smaller force is :

   (A) $10\sqrt{3}$ kg wt(B) $20\sqrt{3}$ kg wt (C) 20 kg wt (D) $(20/\sqrt{3})$ kg wt

4. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is:

   (A) 150° (B) 90° (C) 60° (D) 120°

5. The two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point and $\vec{C} = \vec{A} + \vec{B},$ then angle between $\vec{A}$ and $\vec{B}$ is

   (A) 90^0 if $C^2 \neq A^2 + B^2$ (B) greater than 90^0 if $C^2 < A^2 + B^2$

   (C) greater than 90^0 if $C^2 > A^2 + B^2$ (D) None of these

6. If $\vec{e}_1$ & $\vec{e}_2$ are two unit vectors and θ is the angle between them , then sin $\left(\frac{\theta}{2}\right)$ is :

   (A) $\frac{1}{2}\left|\vec{e}_1 + \vec{e}_2\right|$ (B) $\frac{1}{2}\left|\vec{e}_1 - \vec{e}_2\right|$ (C) $\frac{\vec{e}_1 \cdot \vec{e}_2}{2}$ (D) $\frac{\left|\vec{e}_1 \text{ x } \vec{e}_2\right|}{2\left|\vec{e}_1\right|\left|\vec{e}_2\right|}$

SUBTRACTION

7. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, which of the following options in not true

   (A) $\vec{A}$ is a null vector (B) $\vec{B}$ is a null vector (C) $\vec{A} \perp$ is to $\vec{B}$ (D) $\vec{A}$ is || to $\vec{B}$

8. If $\vec{C} = \vec{A} + \vec{B}$, then which of the following relations is necessarily valid.

   (A) A < B (B) A > B (C) A = B (D) none of these

RESOLUTION

9. If $\vec{A} + \vec{B}$ is a unit vector along x-axis and $\vec{A} = \hat{i} - \hat{j} + \hat{k}$, then what is $\vec{B}$?

   (A) $\hat{j} + \hat{k}$ (B) $\hat{j} - \hat{k}$ (C) $\hat{i} + \hat{j} + \hat{k}$ (D) $\hat{i} + \hat{j} - \hat{k}$

10. With respect to a rectangular cartesian co-ordinate system three vectors are expressed as $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$ where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along the x,y,z axes respectively. The unit vector along the direction of sum of these vectors is :

(A) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (B) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j}-\hat{k})$

(C) $\hat{r} = \frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$ (D) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

11. Three forces P, Q & R are acting at a point in the plane. The angle between P & Q and Q & R are 150° & 120° respectively. Then for equilibrium, forces P, Q & R are in the ratio :

(A) 1 : 2 : 3 (B) 1 : 2 : $\sqrt{3}$ (C) 3 : 2 : 1 (D) $\sqrt{3}$: 2 : 1

12. A bird moves from point (1, –2, 3) to (4, 2, 3). If the speed of the bird is 10 m/sec, then the velocity vector of the bird is:

(A) $5\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ (B) $5\left(4\hat{i}+2\hat{j}+3\hat{k}\right)$ (C) $0.6\,\hat{i}+0.8\,\hat{j}$ (D) $6\,\hat{i}+8\,\hat{j}$

DOT PRODUCT

13. If $\vec{a} = 2\hat{i}-3\hat{j}$, $\vec{b} = 2\hat{j}+3\hat{k}$, then $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) =$

(A) 0 (B) -8 (C) 9 (D) -10.

14. A unit vector in xy-plane that makes an angle of 45^0 with the vector $\hat{i}+\hat{j}$ and an angle of 60^0 with the vector $3\hat{i}-4\hat{j}$ is

(A) $\hat{i}$ (B) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (C) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ (D) None of these.

15. The force determined by the vector $\vec{r}$ = (1, –8, –7) is resolved along three mutually perpendicular directions , one of which is the direction of the vector $\vec{a} = 2\hat{i}+2\hat{j}+\hat{k}$. Then the vector component of the force $\vec{r}$ in the direction of the vector $\vec{a}$ is :

(A) $-14\hat{i}-14\hat{j}-7\hat{k}$ (B) $-\frac{14}{3}\hat{i}-\frac{14}{3}\hat{j}-\frac{7}{3}\hat{k}$ (C) $-\frac{2}{3}\hat{i}-\frac{2}{3}\hat{j}-\frac{1}{3}\hat{k}$ (D) none of these

CROSS PRODUCT

16. If $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$ then the value of $|\vec{A}+\vec{B}|$ is :

(A) $(A^2+B^2+\sqrt{3}AB)^{1/2}$ (B) $(A^2+B^2+AB)^{1/2}$

(C) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$ (D) A + B

17. Let $\vec{a} = 2\vec{i}+\vec{j}-2\hat{k}$ and $\vec{b} = \vec{i}+\vec{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|,\ |\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30^0, then $|(\vec{a}\times\vec{b})\times\vec{c}| =$

(A) 2/3 (B) 3/2 (C) 2 (D) 3

18. If the vectors $\vec{a},\ \vec{b},\ \vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then

(A) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = 0$ (B) $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

(C) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ (D) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} = 0$

19. The value of $\hat{i}\ x\left(\vec{r}\ x\ \hat{i}\right) + \hat{j}\ x\left(\vec{r}\ x\ \hat{j}\right) + \hat{k}\ x\left(\vec{r}\ x\ \hat{k}\right)$ is :

(A) $\vec{r}$ (B) $2\vec{r}$ (C) $3\vec{r}$ (D) $4\vec{r}$

INTRODUCTION

1. What is the maximum number of rectangular components into which a vector can be split in space ?

2. What is the maximum number of rectangular components into which a vector can be split in its own plane

ADDITION

3. Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}+\vec{B}=\vec{C}$ and $A^2 + B^2 = C^2$. Find the angle θ between the two vectors.

SUBTRACTION

4. If $\vec{A}+\vec{B}$ is a unit vector along x-axis and $\vec{A}=\hat{i}-\hat{j}+\hat{k}$, then what is $\vec{B}$.

5. The direction of a vector $\vec{A}$ is reversed. Find the values of $\Delta\vec{A}$ and $\Delta|\vec{A}|$?

RESOLUTION

6. A body is moving uniformly on a circle with speed v. Find the magnitude of change in its velocity when it has turned an angle θ.

DOT PRODUCT

7. $\hat{i}$ and $\hat{j}$ are unit vectors along x-axis and y- axis respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$, and $\hat{i}-\hat{j}$? What are the components of a vector $\vec{A}=2\hat{i}+3\hat{j}$ along the directions of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

8. Given that A = B. Find the angle between $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

CROSS PRODUCT

9. Angle between $\vec{P}$ and $\vec{Q}$ is θ . Find the value of $\vec{P}.\vec{Q}\times\vec{P}$?

10. Find the value of p for which the vectors $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+p\hat{j}+3\hat{k}$ are (i) perpendicular (ii) parallel.

ANSWER SHEET

EXERCISE # 1

1.	(D)	2.	(B)	3.	(D)	4.	(C)
5.	(D)	6.	(C)	7.	(D)	8.	(C)
9.	(A)	10.	(D)	11.	(D)	12.	(D)
13.	(A)	14.	(C)	15.	(A)	16.	(A)
17.	(B)	18.	(B)	19.	(B)	20.	(B)
21.	(A), (B)	22.	(B)	23.	(B)		

EXERCISE # 2

1. work & current
2. Impulse
3. 5 & 13
4. 6 N & 10 N
5. $2 \pm \sqrt{3}$
6. 150º
7. (a) 180º (b) 90º (c) 0º
8. Ans. $\sqrt{76}$ Units, $\tan^{-1}\frac{3\sqrt{3}}{7}$
9. (a) 2 v directed south (b) $\sqrt{2}$ v south-west
10. Unity
11. (a) $B_x = 6m$; $B_y = 3m$ (b) $3\sqrt{5}$ m (c) $\tan^{-1}(1/2) = 26.6º$
12. $F_2 = (12 - 5\sqrt{3})\hat{i} + (12\sqrt{3} - 15)\hat{j}$, $F_1 = -(12\sqrt{3} - 6)\hat{j}$
13. $5P\hat{j}, 4P\hat{i}, 6P\hat{i} + 8P\hat{j}, -12P\hat{i} - 9P\hat{j}, \sqrt{20}, \tan^{-1}[-2]$ with the +ve x-axis.
14. (a) $\theta = 90º$, $F_3 = 8$ (b) – 6i
15. a = –7, b = – 3, c = –4
16. $\frac{500}{\sqrt{3}}, \frac{250}{\sqrt{3}}$
17. $-2\hat{j} + 3\hat{k}$, $\sqrt{13}$
18. $\frac{-2\hat{j} + 3\hat{k}}{\sqrt{13}}$
19. $10\sqrt{2}$, 45º N of E
20. $1\hat{i} + 2\sqrt{3}\,\hat{j}$
21. 50 km, 53º N of E
22. $\sqrt{61}$ 26º3 N of E
23. $2\sqrt{3}$, 60º
24. $\frac{400}{\sqrt{3}}$ $\frac{200}{\sqrt{3}}$
25. (i) $250\sqrt{3}$ (ii) 250

26.	$20\sqrt{3}$	28.	3, –1
29.	$7\sqrt{2}$	30.	$\cos^{-}\left(\frac{56}{65}\right)$
31.	$5/\sqrt{2}$	32.	(i) 1 (ii) $2\hat{i}+2\hat{j}$
33.	$4\sqrt{6}$	35.	North
36.	$\pi/4$	37.	$6\hat{j}+3\hat{k}$
38.	Nort		

EXERCISE # 3

1.	(A)	2.	(D)	3.	(A)	4.	(C)
5.	(B)	6.	(B)	7.	(B)	8.	(D)
9.	(B)	10.	(A)	11.	(D)	12.	(D)
13.	(A)	14.	(D)	15.	(B)	16.	(B)
17.	(B)	18.	(B)	19.	(B)		

EXERCISE # 4

1.	3	2.	2	3.	$\pi/2$	4.	$\hat{j}-\hat{k}$
5.	$-2\vec{A},0$	6.	2V sin (θ/2)	7.	$\sqrt{2}$, 45º N of E, $\sqrt{2}$, 135º N of W, $\frac{5}{\sqrt{2}},\frac{1}{\sqrt{2}}$		
8.	90º	9.	zero	10.	(i) –15 (ii) $\frac{2}{3}$		