# <u>EE 5351 : CONTROL SYSTEMS DESIGN</u>

LABORATORY 02


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|---|---|
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| DATE | : 04/03/2025 |

Table 1: Summative Laboratory Form

| | |
|---|---|
| Semester | 05 |
| Module Code | EE 5351 |
| Module Name | Control System Design |
| Lab Number | 02 |
| Lab Name | Laboratory Session-2 |
| Lab Conduction date | 05/11/2024 |
| Report Submission date | 04/03/2024 |

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## 01.Observation

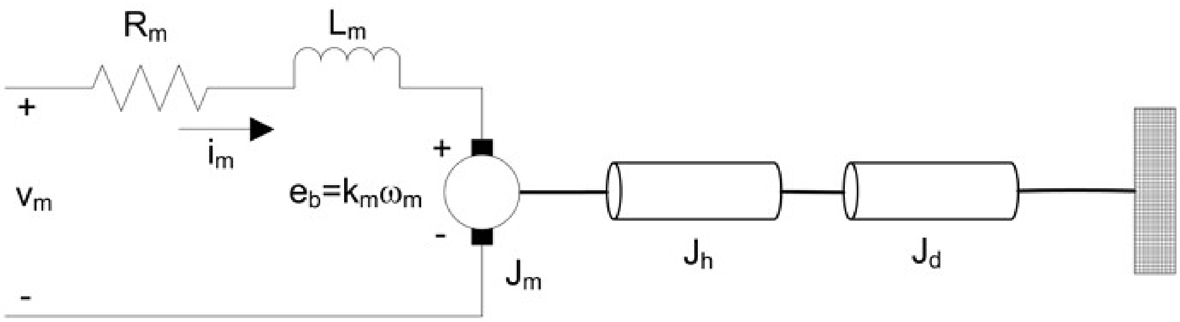


Figure 1: QUBEServo3  DC motor and load

Table 2 : QUBEServo3 parameter

| | |
|---|---|
| Terminal Resistance ($R_m$) | 8.4Ω |
| Rotor inductance($L_m$) | 1.16 mH |
| Equivalent rotor inertia($J_{eq}$) | $2.09\times10^{-5}$ kgm$^2$ |
| Torque constant($k_t$) | 0.042Nm/A |
| Voltage constant ($k_m$) | 0.042 Nm/A |

## 02.Calculation

Q1)

i) Dynamic Equation for DC motor and load

$$V_m = i_m R_m + L_m \frac{dim}{dt} + e_b$$

$$e_b = k_m \omega_m$$

$$T_m = J_{eq} \frac{d\omega m}{dt}$$

$$T_m = k_t i_m$$

ii) Transfer function

$$Vm(t) = R_m i_m(t) + L_m \frac{\mathrm{d}i_m(t)}{dt} + k_m \omega_m$$

*transform to laplace domain*

$$Vm(s) = R_m i_m(s) + SL_m i_m(s) + k_m \omega_m$$

$$i_m(t) = \frac{J_{eq}}{k_t} \frac{\mathrm{d}\omega_m}{dt}$$

*transform to laplace domain*

$$i_m(s) = \frac{SJ_{eq}}{k_t} \frac{\mathrm{d}\omega_m(s)}{dt}$$

$$\frac{\omega_m(s)}{Vm(s)} = \frac{kt}{(SJ_{eq} + km)[Rm + LmS] + kmkt}$$

$$\omega_m(t) = \frac{\mathrm{d}\theta_m(t)}{dt}$$

*transform to laplace domain*

$$\omega_m(s) = S\theta_m(s)$$

$$\frac{\Theta\mathrm{m(s)}}{Vm(s)} = \frac{kt}{S\{JeqS[Rm + LmS] + kmkt\}}$$

iii) DC motor Simulink

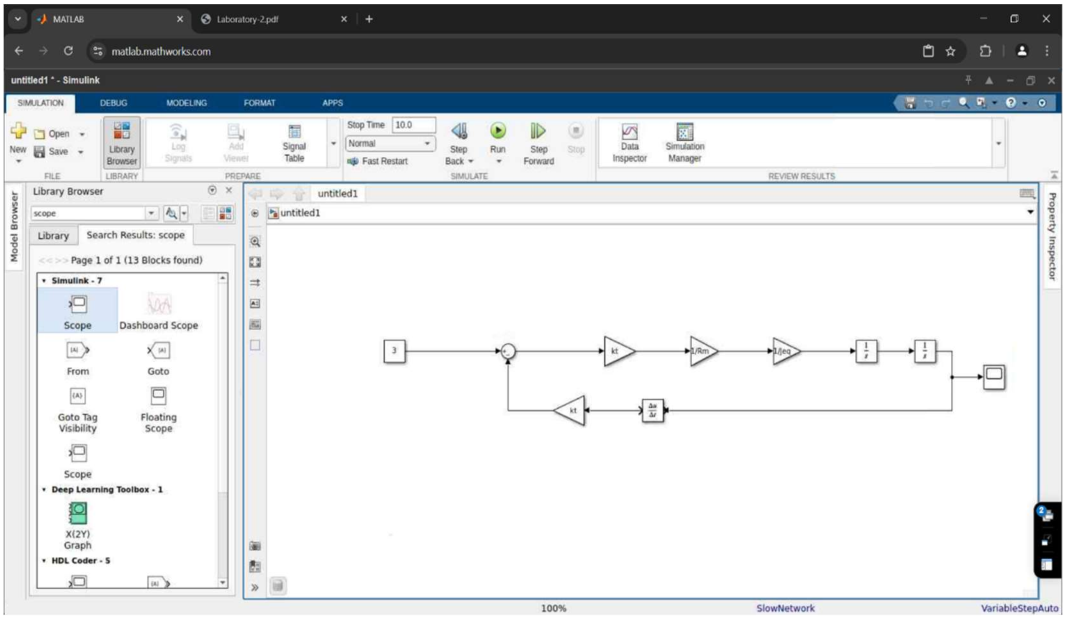


Figure 2:Simulink Q1)iii

iv) Closed loop transfer function of DC motor

$$G(s) = \frac{\Theta m(s)}{Vm(s)} = \frac{0.042}{2.424 \times 10^{-8} S^3 + 17.556 \times 10^{-5} S^2 + 1.764 \times 10^{-3} S}$$

$$\frac{\theta_m(s)}{\theta_{ref}(s)} = \frac{K_P G(s)}{1 + K_p G(s)}$$

$$\frac{\theta_m(s)}{\theta_{ref}(s)} = \frac{0.042}{2.424 \times 10^{-8} S^3 + 17.556 \times 10^{-5} S^2 + 1.764 \times 10^{-3} S + 0.042 K_p}$$

Negligible rotor speed;

$$\frac{\theta_m(s)}{\theta_{ref}(s)} = \frac{0.042}{1.7556 \times 10^{-4} S^2 + 1.764 \times 10^{-3} S + 0.042}$$

v) Plot domain response

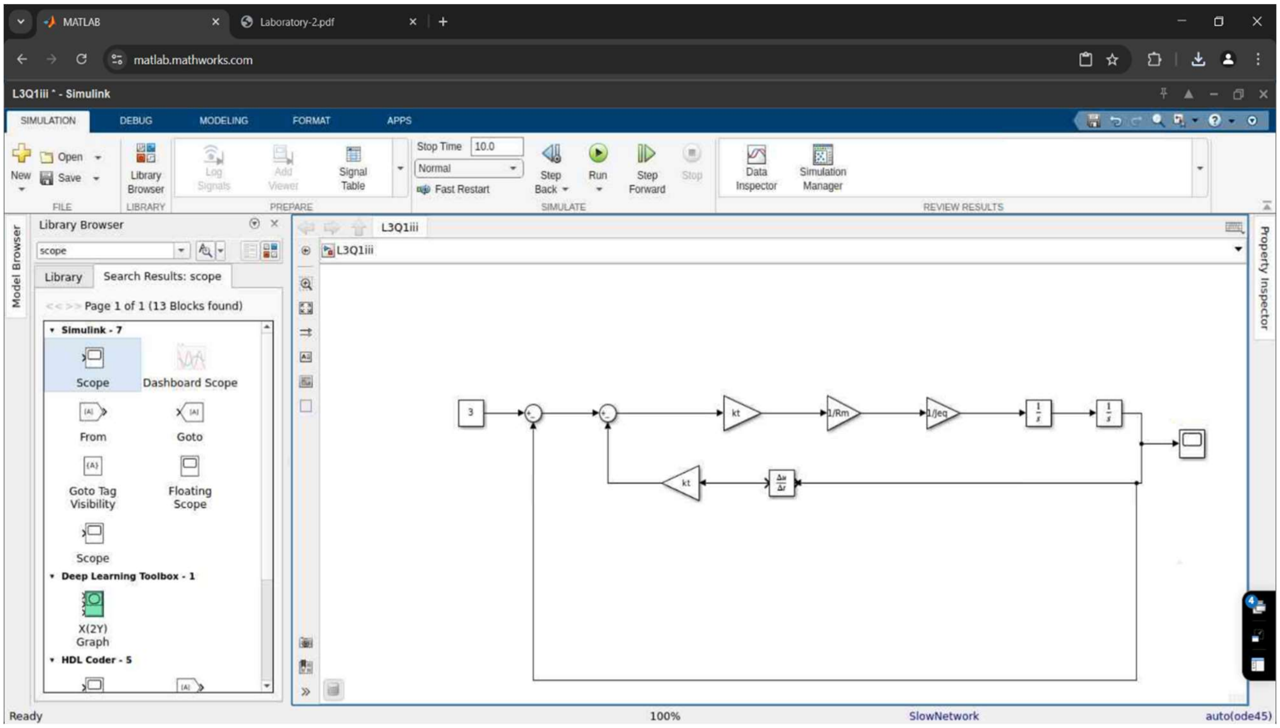


Figure 3 : Simulink

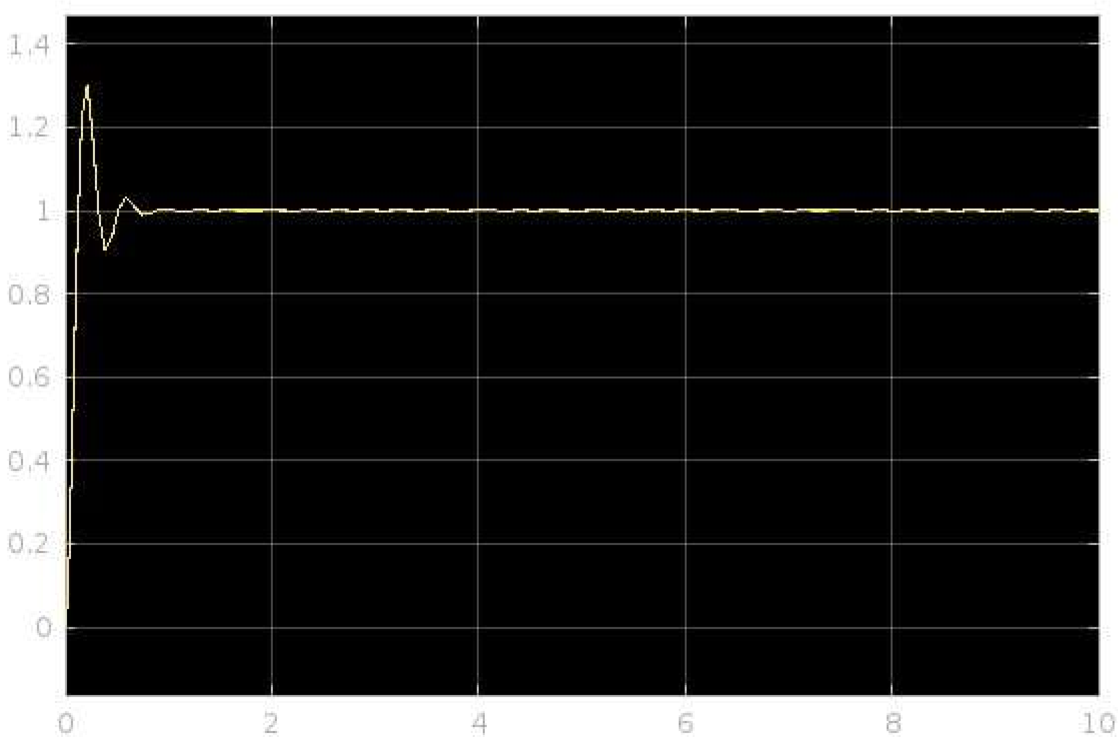


Figure 4 : Step response of Closed-loop system

vi) Calculate percentage overshoot

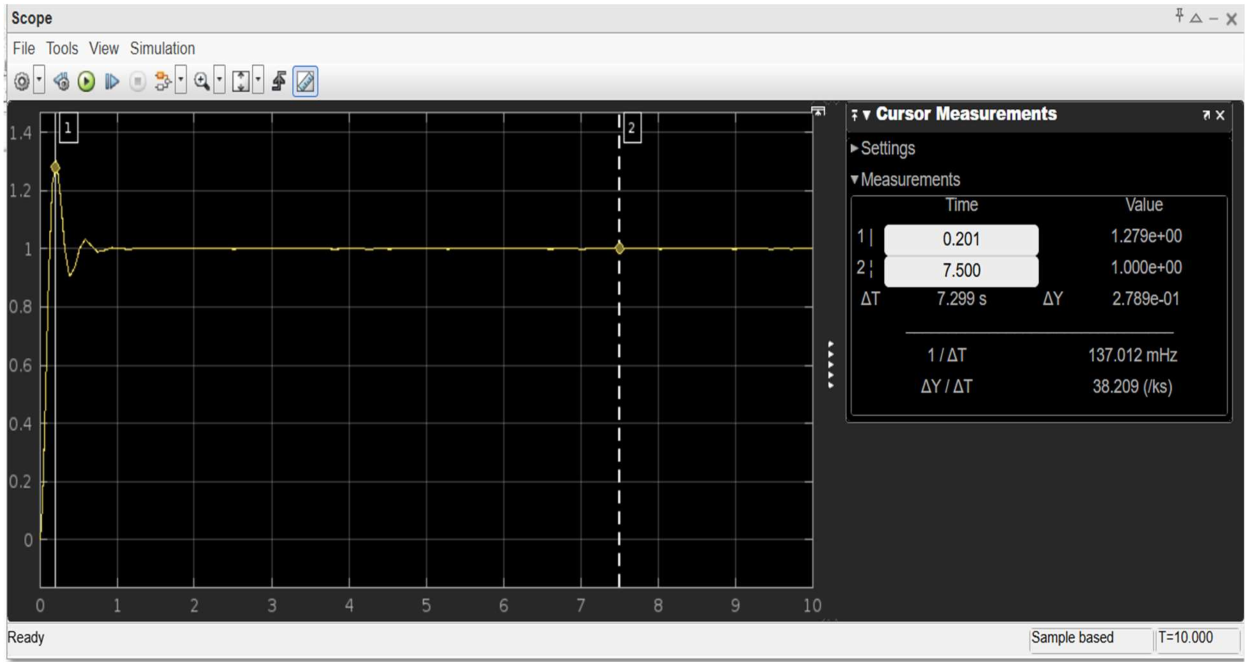


Figure 5: Cursor Measurement

$$overshoot = \frac{Maximum\ Overshoot - Steady\ state\ value}{steady\ state\ value} \times 100\%$$

$$= \frac{1.287-1}{1} \times 100\% = 28.7\%$$

Q2)

i) Characteristic equation

$$\frac{\theta_m(s)}{\theta_{ref}(s)} = \frac{0.042}{17.556 \times 10^{-5}S^2 + 1.764 \times 10^{-3}S + 0.042}$$

By considering characteristic equation,

$$1.7556 \times 10^{-4}S^2 + 1.764 \times 10^{-3}S + 0.042 = 0$$

$$S^2 + 10.047S + 239.234 = 0$$

ii) Percentage of overshoot

$$S^2 + 2\zeta\omega_n S + 0.042 = 0$$

$$S^2 + 10.047S + 239.234 = 0$$

$$\omega_n^2 = 239.234$$

$$2\zeta\omega_n = 10.047$$

$$\omega_n = 15.467 rad/s$$

$$\zeta = 0.325$$

$$\text{overshoot} = e^{\frac{0.325\pi}{\sqrt{1-0.325^2}}} \times 100\%$$

$$= 33.97\%$$

iii) Calculate k $_p$ and k $_d$ parameter

$M_P \quad = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$

From 30% of overshoot

$M_P(new) \quad = 23.79$

$23.79 \quad = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$

ζ (new) = 0.416

By considering the PD controller;

$$S^2 + 2\left(\zeta(\text{new}) + \frac{k_d\,\omega_{n(new)}}{2}\right)\omega_{n\,(new)}S + k_{p\,new}\omega_n{}^2 = 0$$

when $t_p < 2s$;

$$t_p = \frac{\pi}{\omega_n(new)\sqrt{1-\zeta\,(\text{new})^2}}$$

$$t_p = \frac{\pi}{\sqrt{k_p\,\omega}_n\sqrt{1-\zeta\,(\text{new})^2}}$$

$$2 > \frac{\pi}{\omega_n\sqrt{k_p}}$$

$$k_p > \left(\frac{\pi}{2 \times 15.467}\right)^2$$

$$k_p > 0.0103$$

$$0.325 + \frac{k_d\,\omega_{n\,new}}{2} = 0.416$$

$$0.325 + \frac{k_d \times 15.467}{2} = 0.416$$

$$k_d = 0.0117$$

Q3)

i) Plot the domain response

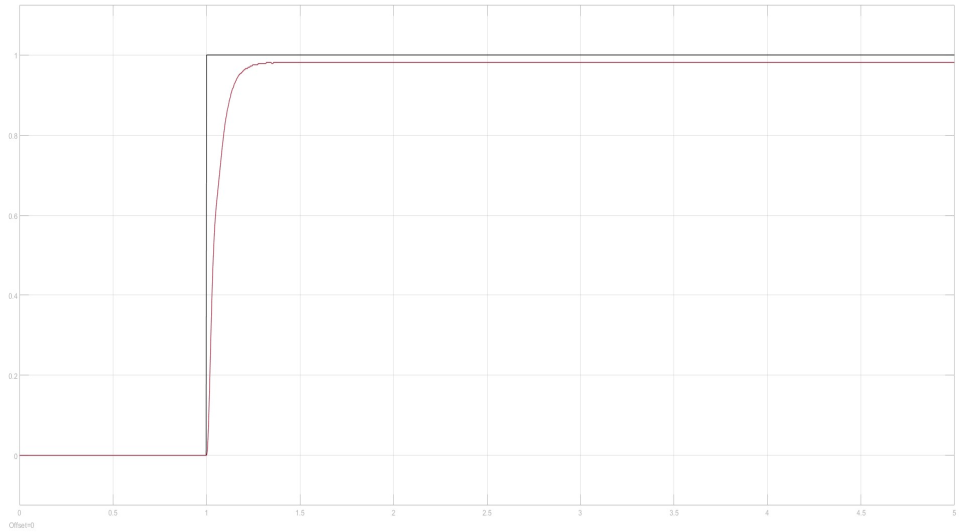

Figure 6: Time Response

ii) Percentage of overshoot

$$\text{Overshoot} = \frac{1.36217-0.972544}{0.972544} \times 100\%$$

$$= 40.06\%$$

iii) Design PD controller

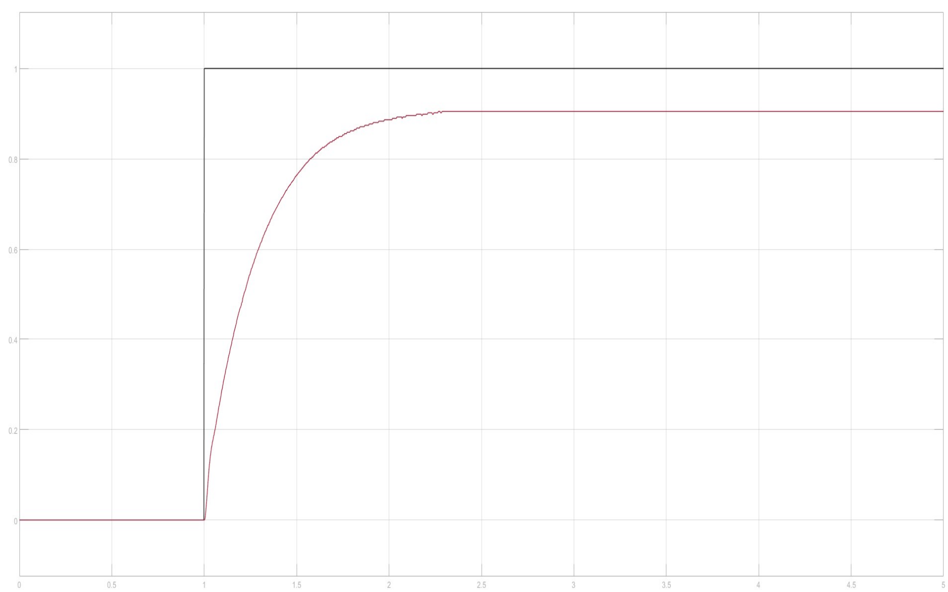

Figure 7: PD controller design

## 03.Reference


[1] "LibreTexts," [Online]. Available: https://eng.libretexts.org/Bookshelves/Industrial_and_Systems_Engineering/Introduction_to_Control_Systems_(Iqbal)/03%3A_Feedback_Control_System_Models/3.3%3A_PI_PD_and_PID_Controllers.

[2] "PID Explained," [Online]. Available: https://pidexplained.com/how-to-tune-a-pid-controller/.

[3] "Medium," [Online]. Available: https://medium.com/@svm161265/when-and-why-to-use-p-pi-pd-and-pid-controller-73729a708bb5.