<u>EE 5351 : CONTROL SYSTEMS DESIGN</u>

LABORATORY 01

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GROUP No	: CE 07
DATE	: 03/04/2025

Table 1: Summative Laboratory Form

Semester	05
Module Code	EE 5351
Module Name	Control System Design
Lab Number	01
Lab Name	Laboratory Session-1
Lab Conduction date	05/11/2024
Report Submission date	04/03/2024

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01.Observation

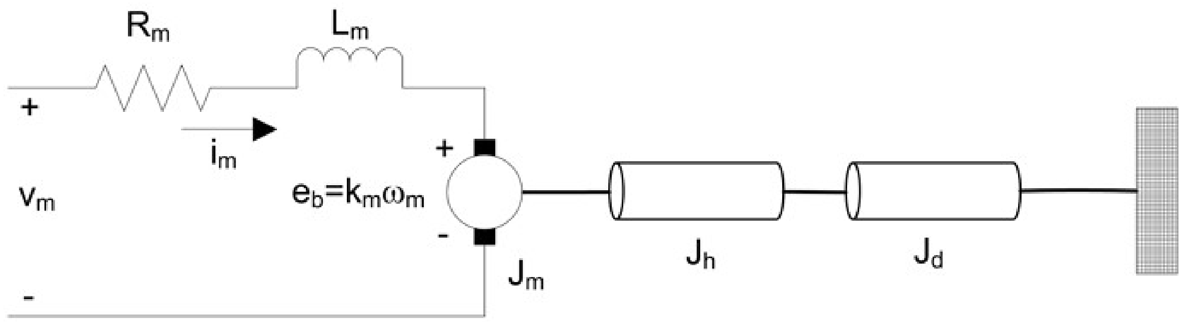

Figure 1: QUBEServo3 DC motor and load

Table 2 : QUBEServo3 parameter

Terminal Resistance (R_m)	8.4Ω
Rotor inductance(L_m)	1.16 mH
Equivalent rotor inertia(J_{eq})	2.09×10^{-5} kgm^2
Torque constant(k_t)	0.042Nm/A
Voltage constant (k_m)	0.042 Nm/A

02.Calculation

Q1)

i) Dynamic Equation for DC motor and load

$$V_m \quad = i_m R_m + L_m \frac{dim}{dt} + e_b$$

$$e_b \quad = k_m \omega_m$$

$$T_m \quad = J_{eq} \frac{d\omega m}{dt}$$

$$T_m \quad = k_t i_m$$

ii) Transfer function

$$\frac{\omega(s)}{Vm(s)} = \frac{kt}{\{JeqS[Rm + LmS] + kmkt\}}$$

$$\frac{\omega(s)}{Vm(s)} = \frac{0.042}{\{2.09 \times 10^{-5} S[8.4 + 1.16 \times 10^{-3} S] + 0.042 \times 0.042}$$

$$\frac{\omega(s)}{Vm(s)} = \frac{0.042}{2.424 \times 10^{-8} \times S^2 + +17.556 \times 10^{-3} S + 1.764 \times 10^{-3}}$$

$$\frac{\Theta m(s)}{Vm(s)} = \frac{kt}{S\{JeqS[Rm + LmS] + kmkt\}}$$

$$\frac{\Theta m(s)}{Vm(s)} = \frac{0.042}{2.424 \times 10^{-8} S^3 + 1.7556 \times 10^{-4} S^2 + 1.764 \times 10^{-3} S}$$

iii) Obtain the domain speed response

MATLAB code

```
% Parameters
 Rm = 8.4; % Terminal resistance (Ohms)
Lm = 1.16e-3; % Rotor inductance (H)

Jeq = 2.09e-5; % Equivalent inertia (kg*m^2)
kt = 0.042; % Torque constant (Nm/A)
km = 0.042; % Voltage constant (V/rad/s)
% Transfer function for speed control
num = kt;
den = [Jeq*Lm, Jeq*Rm, kt*km];
sys = tf(num, den);
% Simulate step response for 3V input
input_voltage = 3; % Applied voltage
t = 0:0.001:1; % Time vector
[u, t] = step(input_voltage * sys, t);
figure;
plot(t, u, 'LineWidth', 1.5, 'Color', 'b'); % Improved aesthetics
title('Time domain speed response', 'FontWeight', 'bold');
xlabel('Time (s)', 'FontSize', 12);
ylabel('Speed (rad/s)', 'FontSize', 12);
grid on;
xlim([0, 1]); % Ensure the time axis is within range
ylim([0, max(u) * 1.1]); % Adjust y-axis for better visualization
legend('time domain speed response');
```

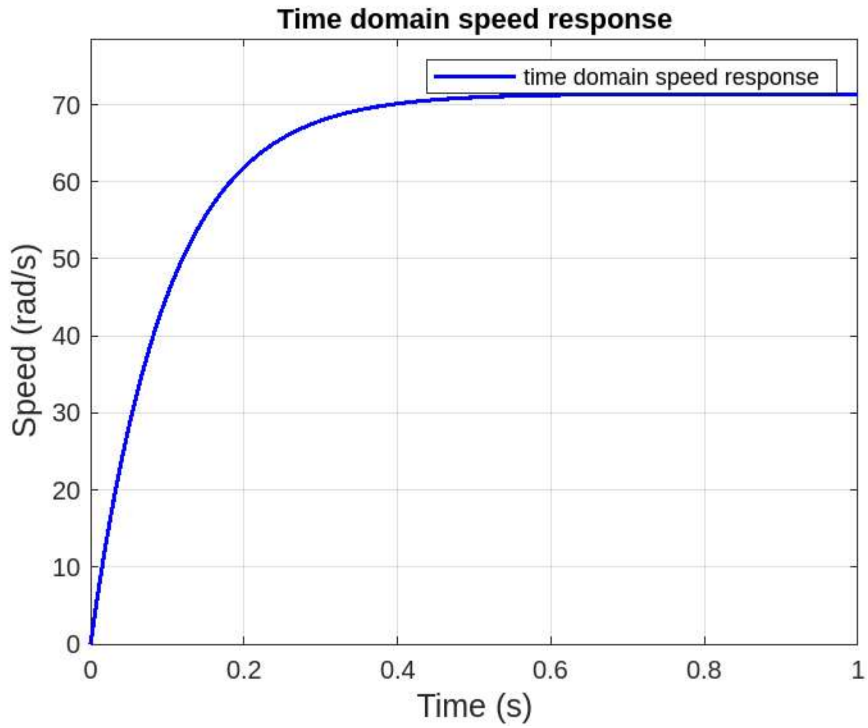

Figure 2: Time Domain Response of ω(t)(MATLAB)

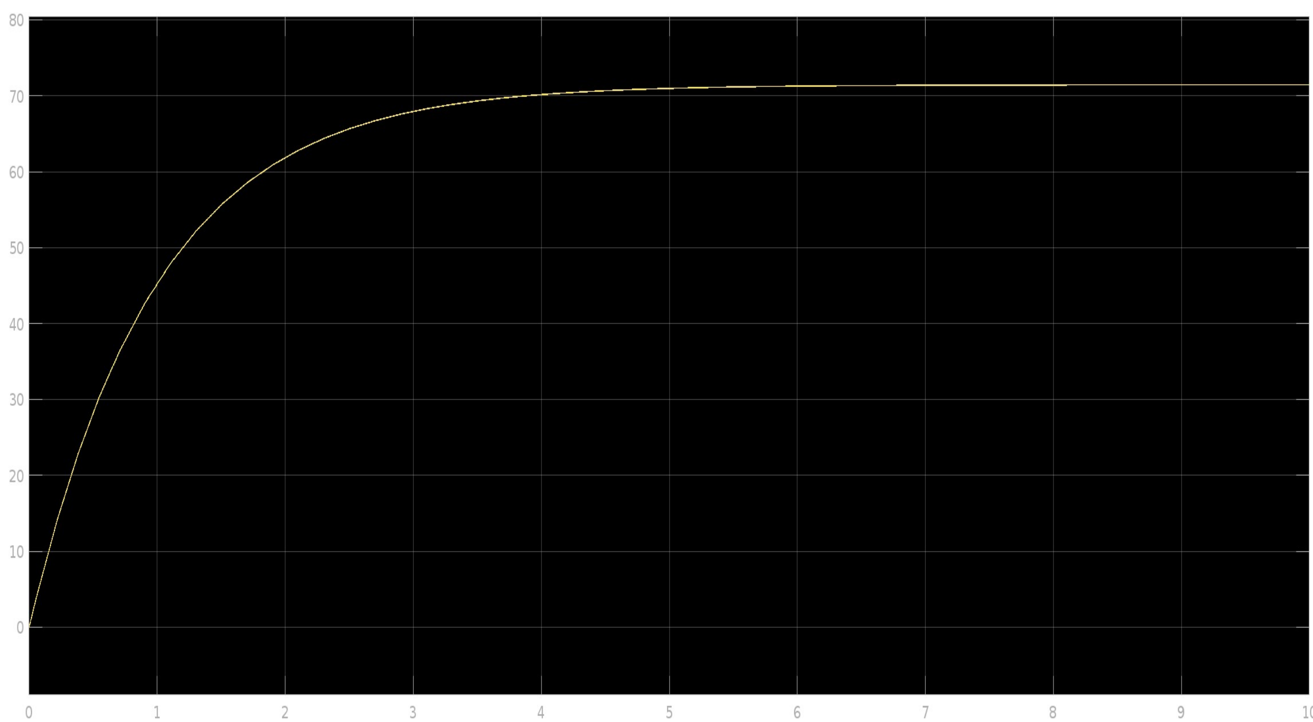

Figure 3: Time Domain Response of ω(t)(Simulink)

iv) Transfer function (negligible rotor inductance)

$$\frac{\omega(s)}{Vm(s)} = \frac{k_t}{\{J_{eq}R_mS + k_mk_t\}}$$

$$\frac{\omega(s)}{Vm(s)} = \frac{0.042}{2.09 \times 10^{-5} \times 8.4S + 0.042 \times 0.042}$$

$$\frac{\Theta m(s)}{Vm(s)} = \frac{k_t}{S\{J_{eq}R_mS + k_mk_t\}}$$

$$\frac{\Theta m(s)}{Vm(s)} = \frac{0.042}{1.7556 \times 10^{-4}S^2 + 1.764 \times 10^{-3}S}$$

v) Simulink

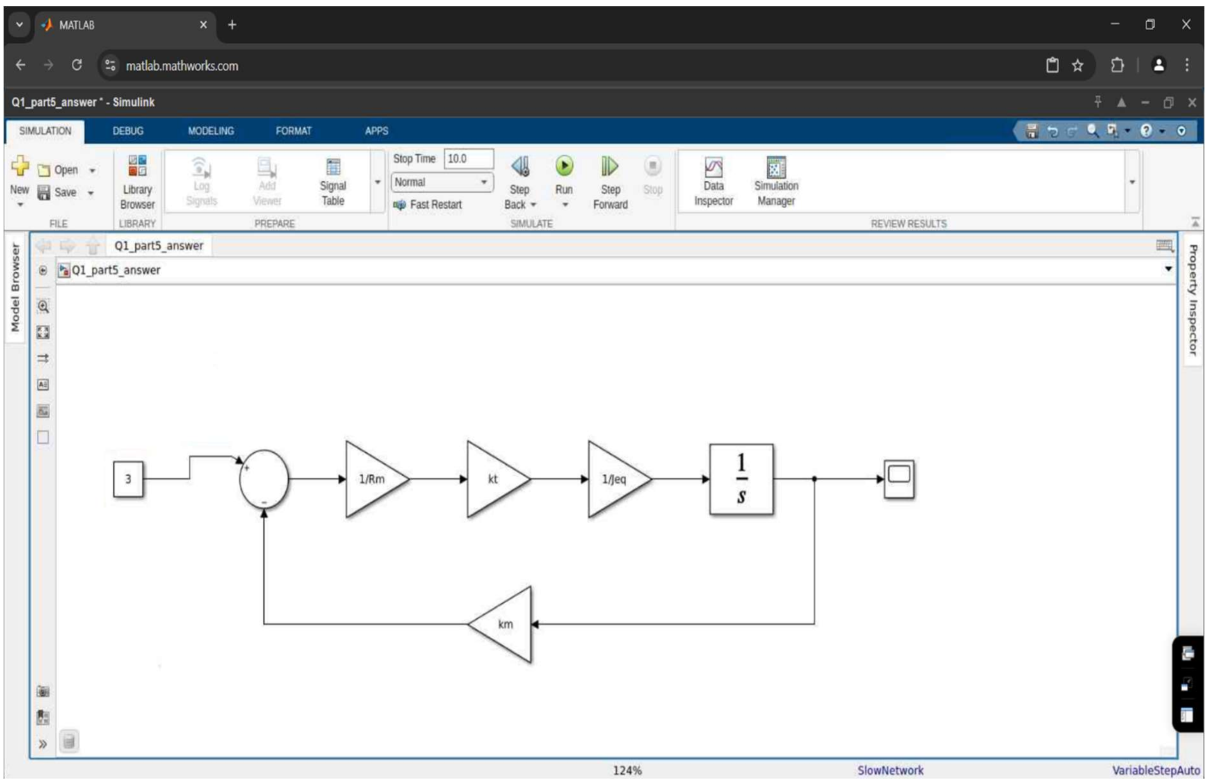

Figure 4: Simulink Q1(v)

vi) State Space Model (armature current and rotor speed)

$$\dot{Im} = -\left(\frac{R_m}{L_m}\right) i_m - \left(\frac{k_m}{L_m}\right) \omega_m + \frac{V_m}{L_m}$$

$$\dot{\omega_m} = (\frac{k_t}{J_{eq}})\, i_m + 0 \times \omega_m + 0 \times V_m$$

$$\begin{bmatrix} i_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} \frac{-R_m}{L_m} & \frac{-k_m}{L_m} \\ \frac{k_t}{j_{eq}} & 0 \end{bmatrix} \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_m} \\ 0 \end{bmatrix} Vm$$

$$\begin{bmatrix} i_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} -7241.38 & -36.21 \\ 2009.57 & 0 \end{bmatrix} \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 862.07 \\ 0 \end{bmatrix} Vm$$

$$[\omega_m] = [0 \quad 1] \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} + [0]\, Vm$$

vii) State Space Model (rotor position and rotor speed)

$$\Theta = 0 \times \dot{\theta_m} + \omega_m + 0 \times V_m$$

$$\dot{\omega} = 0 \times \theta_m - (\frac{k_t k_m}{R_m J_{eq}}) + (\frac{k_t}{J_{eq} R_m})$$

$$\begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-k_t k_m}{R_m J_{eq}} \end{bmatrix} \begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_t}{J_{eq} R_m} \end{bmatrix} V_m$$

$$\begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -10.05 \end{bmatrix} \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ 293.23 \end{bmatrix} V_m$$

$$[\omega_m] = [1 \quad 0] \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} + [0]\, V_m$$

viii) Plot the time domain speed responses

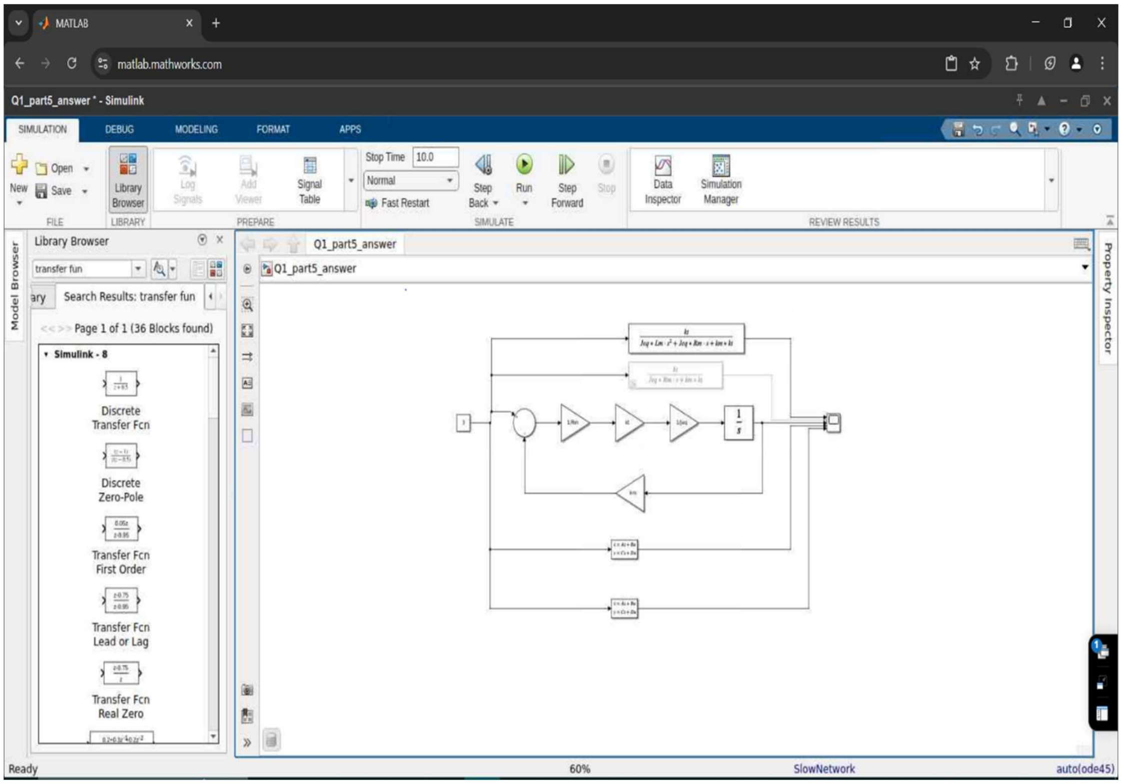

Figure 5: Simulink Q1(VIII)

Q2)

i) Obtain time response

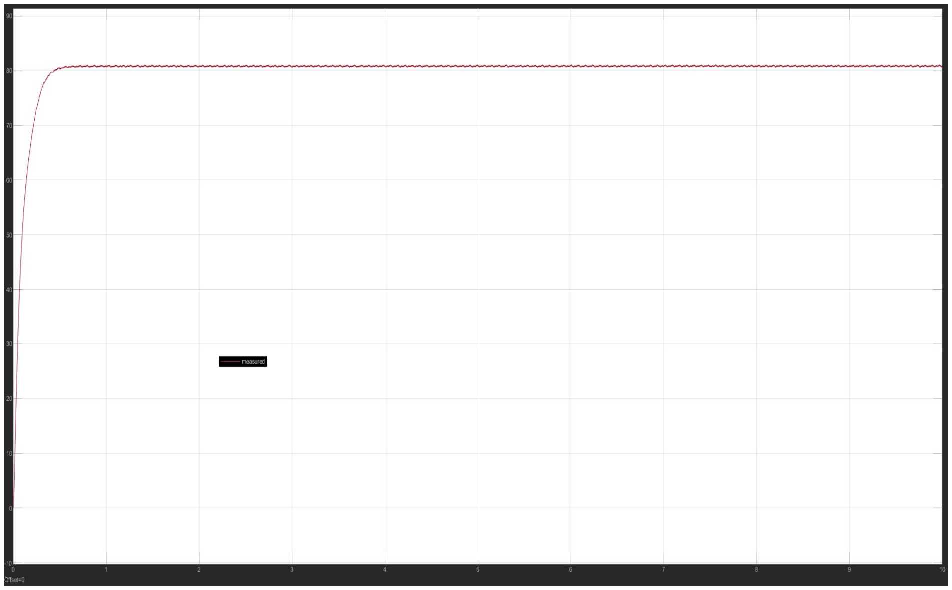

Figure 6 : Time domain speed response when input voltage 3V

ii) Compare Results

Figure 7 : Graph of Comparison negligible rotor inductance and applied 3V

Table 3 : Result Comparison

Steady state speed	Based on simplified transfer function	Match with real behavior for 3v input.
Rise time	Determine simplified dynamics(Jeq, Rm, kt)to be optimistic.	Reflect actual damping and delay present In QUBEServo3.
Settling time	Simplified model response faster without external disturbances.	Simulink model for actual motor inertia and damping. It potentially showing longer settling time

3)

i) Kp=1

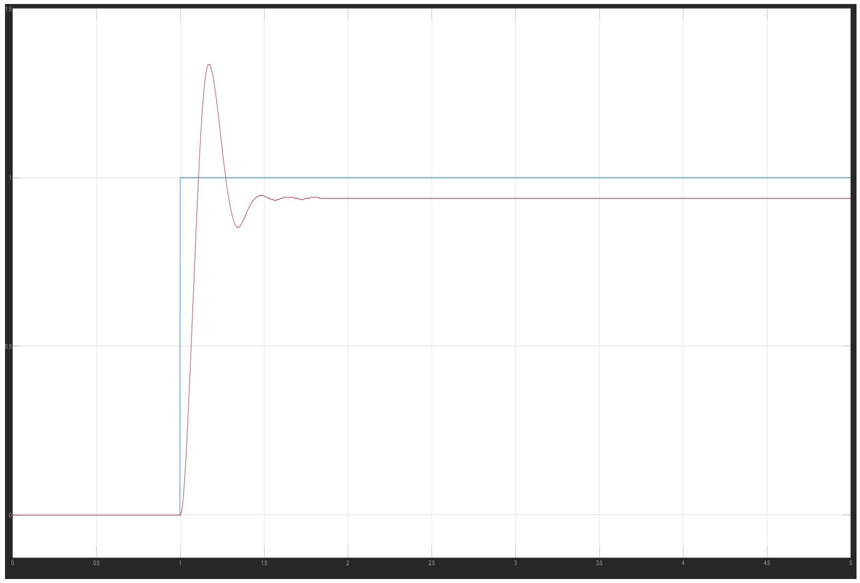

Figure 8 : Graph of steady state error(Kp=1)

ii) According to the Figure 8,

Overshoot $= \frac{(\max value-stead \quad state)}{steady\ state\ value} \times 100$

$= \frac{1.335-1}{1} \times 100 = 33.5\%$

Steady state error = 1 – 0.938 = 0.062

iii) Kp=1.25

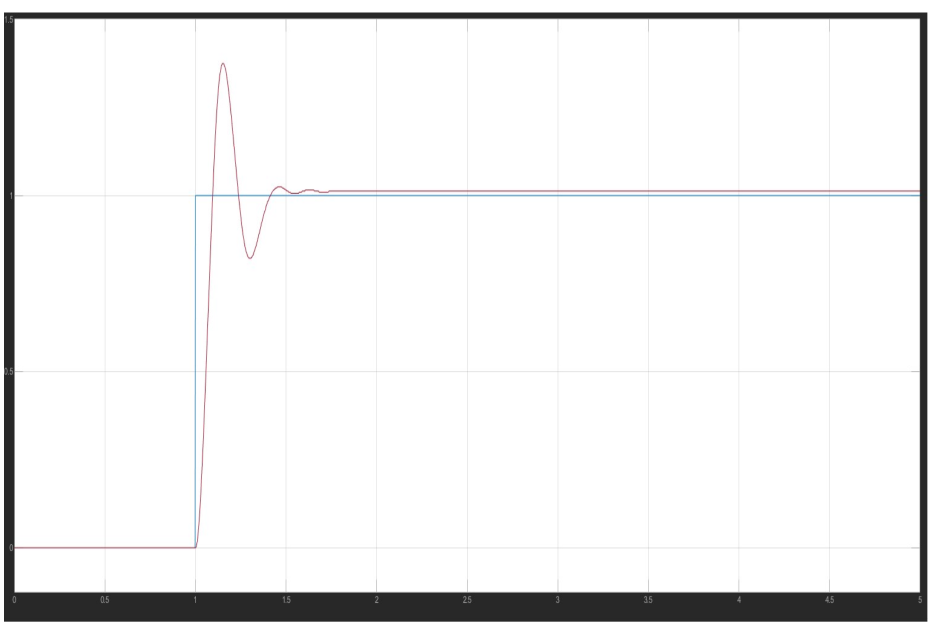

Figure 9 : Graph of steady state error(Kp=1.25)

According to the Figure 9

Steady state error = $1 - 1.012$

= 0.012

Overshoot = $\frac{1.374-1}{1} \times 100\%$

= 37.4%

Kp=1.5,

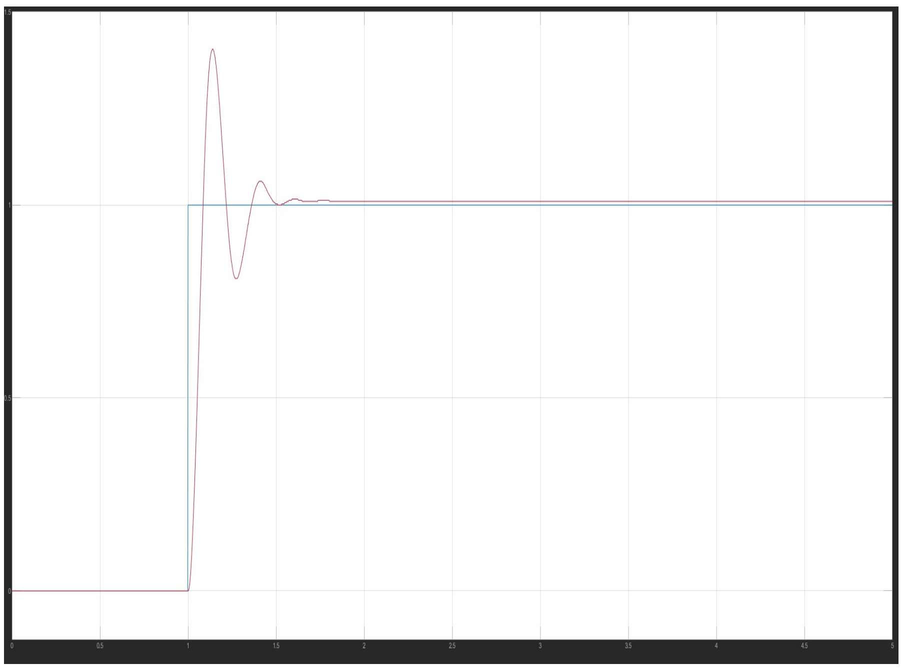

Figure 10 : Graph of steady state error(Kp=1.5)

According to the Figure 10

Overshoot = $\frac{1.405-1}{1} \times 100$

= 40.5 %

Steady state error = $1 - 1.009$

= 0.009

Kp=1.75

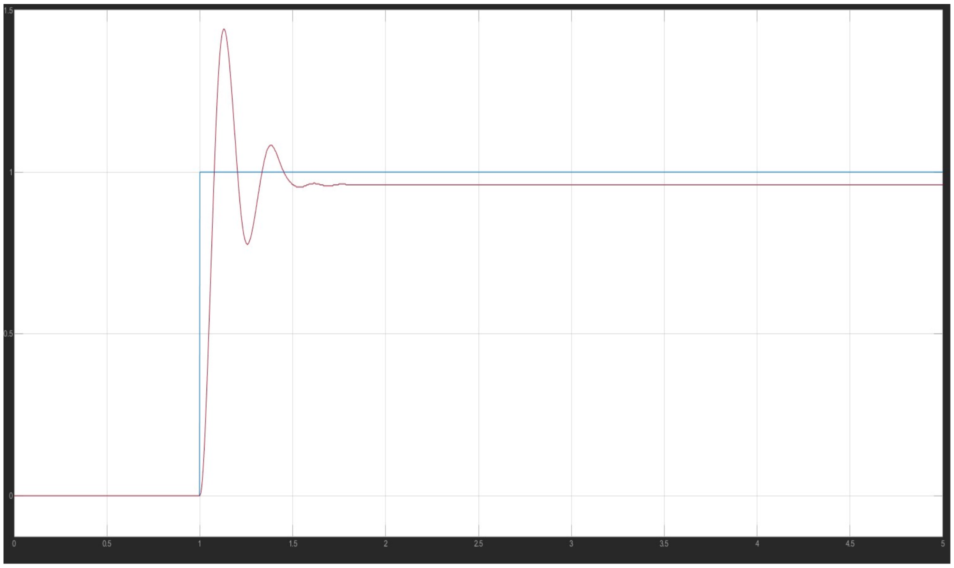

Figure 11 : Graph of steady state error(Kp=1.75)

According to the Figure 12

Steady state error $= 1- 0.9603$

$= 0.0397$

Overshoot $=\frac{1.442-1}{1} \times 100$

$=44.2\%$

Kp=2

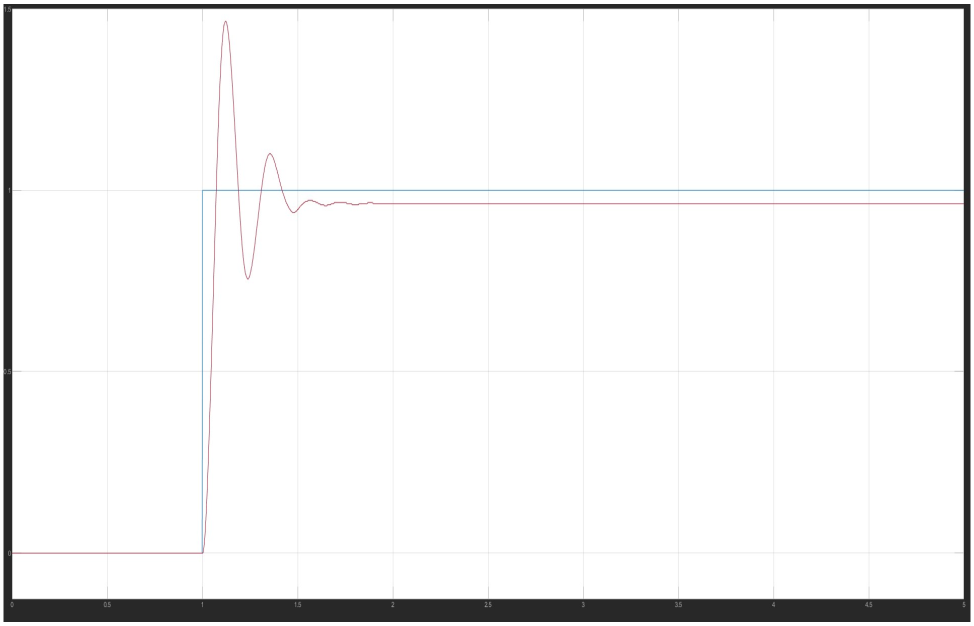

Figure 12 : Graph of steady state error(Kp=2)

According to the figure

Steady state error = 1- 0.9633

= 0.0367

Overshoot = $\frac{1.466-1}{1} \times 100$

= 46.6%

03.Reference

[1] MATLAB. [Online]. Available: https://www.mathworks.com/matlabcentral/answers/2000762-how-to-convert-state-space-to-transfer-function.

[2] "Science Direct," [Online]. Available: https://www.sciencedirect.com/topics/engineering/steady-state-error.

[3] "Quanser," [Online]. Available: https://docs.quanser.com/quarc/documentation/qube_servo3_usb.html.