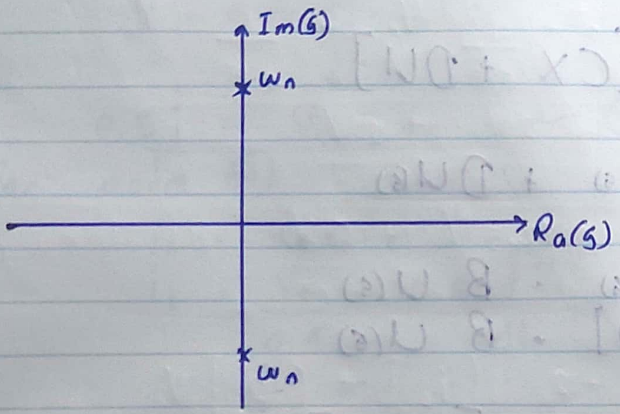
$\frac{6}{5} \times 3$ $\quad 5\overline{)15}$ 3

Control Systems Design January 2024

(i) undompmped

$\zeta = 0$

$s = \pm j\omega_n$



(ii) $\dot{X} = AX + BU$

$Y = CX + DU$

$\dot{X}$ = Derivative of the state of the system
A = System matrix
B = Input matrix
C = Output matrix
D = Direct transmission matrix
U = Input to the system
Y = Output to the system

(iii) $\dot{X} = AX + BU$
$Y = CX + DU$

$L[\dot{X}] = L[AX + BU]$

$sX(s) = AX(s) + BU(s)$

$L[Y] = L[CX + DU]$

$Y(s) = CX(s) + DU(s)$

$sX(s) - AX(s) = BU(s)$

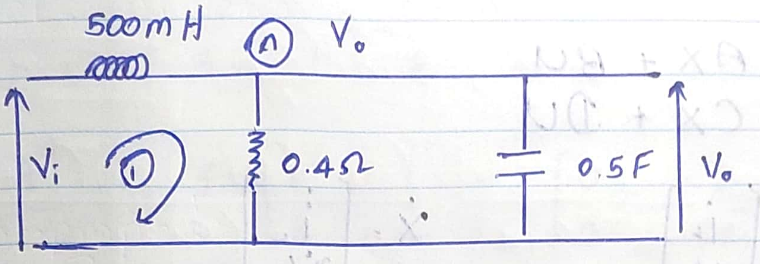
$X(s)[s - A] = BU(s)$

$X(s) = [sI - A]^{-1} BU(s)$

$Y(s) = C[sI - A]^{-1} BU(s) + DU(s)$

$= C[sI - A]^{-1} B + D$

(b)



① KVL

$$V_i = 0.5\frac{di}{dt} + V_o$$

$$0.5\dot{i} = V_i - V_o \sim ①$$

For node Ⓐ

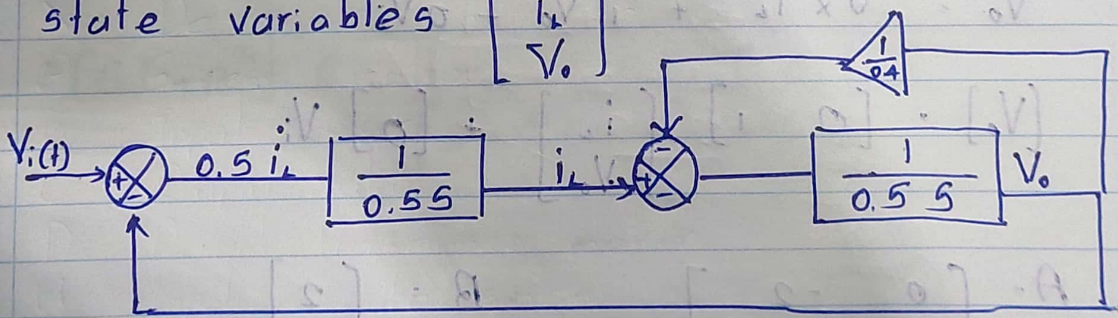
$$i_L = i_R + i_C$$

$$i_L = \frac{V_o}{0.4} + 0.5\frac{dV_o}{dt}$$

$$0.5\dot{V}_o = i_L - \frac{V_o}{0.4} \sim ②$$

state variables are V_o and i_L

state variables $\begin{bmatrix} i_L \\ V_o \end{bmatrix}$



$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

$$X = \begin{bmatrix} i_L \\ V_o \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{i}_L \\ \dot{V}_o \end{bmatrix}$$

(1) $\Longrightarrow$ $\dot{i}_L = 0 \times i_L - \frac{1}{0.5} V_o + \frac{1}{0.5} V_i$

$\dot{i}_L = 0 \times i_L - 2 V_o + 2 V_i$

(2) $\Longrightarrow$ $\dot{V}_o = \frac{1}{0.5} i_L - 2.5 V_o + 0 V_i$

$$\begin{bmatrix} \dot{i}_L \\ \dot{V}_o \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & -2.5 \end{bmatrix} \begin{bmatrix} i_L \\ V_o \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} V_i$$

$V_o = V_o$

$V_o = 0 \times i_L + 1 V_o + 0 V_i$

$$[V_o] = [0 \quad 1] \begin{bmatrix} i_L \\ V_o \end{bmatrix} + [0] V_i$$

$$A = \begin{bmatrix} 0 & -2 \\ 2 & -2.5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 1] \qquad D = [0]$$

$$G(s) = C[sI - A]^{-1}B + D$$

$$[sI - A] = s\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}0 & -2\\2 & -12.5\end{bmatrix}$$

$$= \begin{bmatrix}s & 2\\-2 & s+12.5\end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{s(s+12.5)+4}\begin{bmatrix}s+12.5 & -2\\2 & s\end{bmatrix}$$

$$= \frac{1}{s^2+12.5s+4}\begin{bmatrix}s+12.5 & -2\\2 & s\end{bmatrix}$$

$$C[sI-A]^{-1} = \frac{\begin{bmatrix}0 & 1\end{bmatrix}}{s^2+12.5s+4}\begin{bmatrix}s+12.5 & -2\\2 & s\end{bmatrix}$$

$$= \frac{2 \cdot 1}{s^2+12.5s+4}\begin{bmatrix}2 & s\end{bmatrix}$$

$$C[sI-A]^{-1}B = \frac{\begin{bmatrix}2 & s\end{bmatrix}}{[s^2+12.5s+4]}\begin{bmatrix}2\\0\end{bmatrix}$$

$$= \frac{4}{s^2+12.5s+4}$$

$$= \frac{4}{(s+1)(s+4)}$$

(iv) Transfer function from graph:

$$G_1(s) = \frac{(s+1)(s+1-2i)(s+1+2i)}{(s+2)}$$

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(iv) overall transfer function

$$G(s) = \frac{4}{\cancel{(s+1)}(s+4)} \cdot \frac{\cancel{(s+1)}(s+1-2i)(s+1+2i)}{(s+2)}$$

$$= \frac{4\,(s+1-2i)(s+1+2i)}{(s+2)(s+4)}$$

(v) $L(3) = \frac{3}{s}$

all poles are left half of the plane
using final value theorem

$$y_{ss} = \lim_{s\to 0} s\,Y(s)$$

$$= \lim_{s\to 0} \cancel{s} \times \frac{3}{\cancel{s}} \times 4\,\frac{(s+1-2i)(s+1+2i)}{(s+2)(s+4)}$$

$$= [12\,(1-2i)(1+2i)]\,/6$$

$$= [12\,[1+2i-2i+4]]\,/6$$

$$= \frac{12\times 5}{6}$$

$$= \underline{\underline{10}}$$

(vi) all poles are left side of the plane. $\therefore$ the system is stable.

Q2

a) (i) P $= \dfrac{K_p G(s)}{1 + K_p G(s) H(s)}$

PI $= \dfrac{(K_i/s) G(s)}{1 + (K_i/s) H(s) G(s)}$

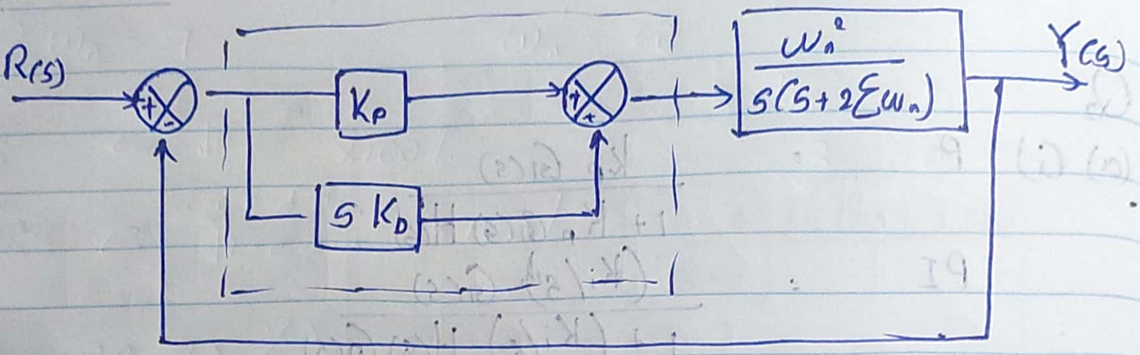
PD $= \dfrac{(K_p + s K_D) G(s)}{1 + (K_p + s K_D) H(s) G(s)}$

PID $= \dfrac{[K_p + K_i/s + K_D s] G(s)}{1 + [K_p + K_i/s + K_D s] H(s) G(s)}$

(ii) Remove the steady state error
Reduse the steady state error to zero

(iii) Gradually increase K_p until the system begins to oscillate around the setpoint, Find the highest K_p value that gives a stable response with minimal oscillation, but don't aim to eliminate the error completely yet.
Now slowly increase K_i to reduse the steady-state error
as increasing K_i too much can introduce oscillations and overshoot.
Graducly increase K_d to reduce overshoot and dampen oscillations caused by K_p and K_i

(b)



$$G_1(s) = K_p + sK_D$$

$$G_{new}(s) = \frac{[K_p + K_D s]\,\omega_n^2}{s(s+2\zeta\omega_n) + (K_p + K_D s)\omega_n^2}$$

$$= \frac{(K_p + K_D s)\,\omega_n^2}{s^2 + 2\zeta\omega_n s + K_D\omega_n^2 s + K_p\omega_n^2}$$

characteristic eq.

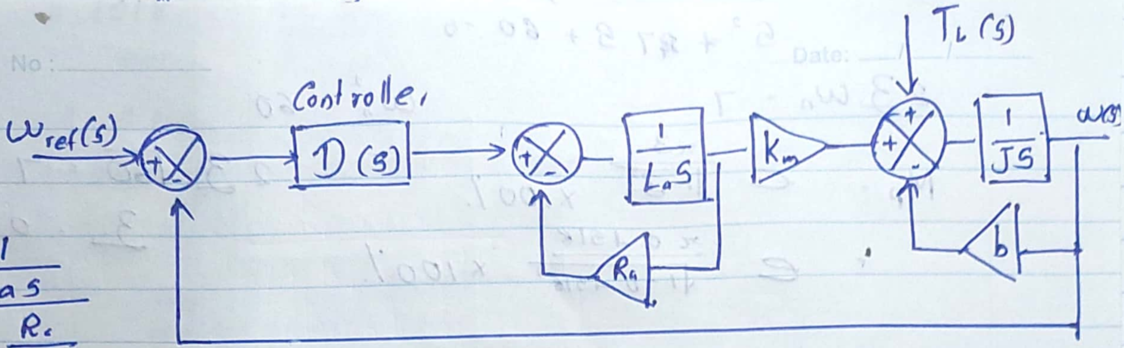
$$s^2 + 2\underbrace{\left(\zeta + \frac{K_D\omega_n}{2}\right)}_{\zeta_{new}}\omega_n s + K_p\omega_n^2 = 0$$

$$\zeta_{new} = \zeta + \frac{K_D\omega_n}{2}$$

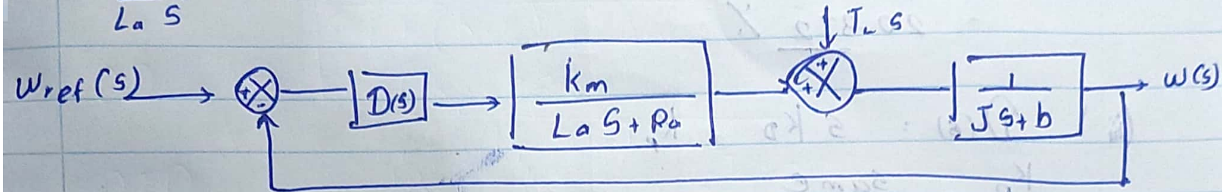
$$\zeta < \zeta_{new}$$

So the dampin of the system can be improved by adding PD type controller

(c)



$$\frac{\frac{1}{L_a s}}{1+\frac{R_a}{L_a s}}$$



Consider the $T_L s = 0$

$$\frac{\omega(s)}{\omega_{ref}(s)} = \frac{\left[\frac{D(s) K_m}{L_a s + R_a}\right]\frac{1}{(Js+b)}}{1+\left[\frac{D(s) K_m}{L_a s + R_a}\right]\frac{1}{(Js+b)}}$$

$$= \frac{[D(s) \; K_m]}{[Js+b][L_a s + R_a] + D(s) K_m} \quad (1)$$

for propotional controler $D(s) = K_p$

$$\frac{\omega(s)}{\omega_{ref}(s)} = \frac{K_p K_m}{(Js+b)[L_a s + R_a] + K_p K_m}$$

$$(Js+b)(L_a s + R_a) + K_p \times K_m = 0$$

$$J L_a s^2 + R_a J s + b L_a s + R_a b + K_p K_m = 0$$

$$J L_a s^2 + s(R_a J + b L_a) + R_a b + K_p K_m = 0$$

$\Delta = 0$

$$(R_a J + b L_a)^2 - 4 J L_a (R_a b + K_p K_m) = 0$$

$$\frac{(R_a J + b L_a)^2}{K_m 4 J L_a} - \frac{4 J L_a R_a b}{K_m 4 J L_a} = K_p$$

$$K_p = \frac{(b L_a + R_a J)^2}{K_m 4 J L_a} - \frac{b R_a}{K_m}$$

characteristic eqn

$$J L_a s^2 + (R_a J + b L_a) s + R_a b + K_p K_m = 0$$

$$0.2 \times 0.5 s^2 + (1 \times 0.2 + 1 \times 0.5) s + 1 \times 1 + K_p \times 5 = 0$$

$$0.1 s^2 + 0.7 s + 6 = 0$$

$$s^2 + 7s + 60 = 0$$

$2\zeta\omega_n = 7$ $\qquad \omega_n^2 = 60$

$2\zeta\sqrt{60} = 7$

$\zeta = 0.4518$

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$$

$$= e^{\frac{-\pi\, 0.4518}{\sqrt{1-0.4518^2}}} \times 100\%$$

$$= 20.372\ \%$$

(iii) $D(s) = sK_D + K_p$

K_p same

$$K_p = \frac{(bL_a + JR_a)^2}{4JK_mL_a} - \frac{bR_a}{K_m}$$

$$= \left[\frac{1 \times 0.5 + 0.2 \times 1}{4 \times 0.2 \times 5 \times 0.5}\right] - \frac{1 \times 1}{5}$$

$$= 0.045$$

$D(s) = sK_D + 0.045$

using (1)

$$\frac{W(s)}{W_{ref}(s)} = \frac{[sK_D + 0.045]}{(Js+b)[L_as + R_a] + [sK_D + 0.045]K_m}$$

$0 = JL_as^2 + s(R_aJ + bL_a) + R_ab + [sK_D + 0.045]K_m$

$0 = 0.1s^2 + 0.7s + 1 + [sK_D + 0.045]5$

$= 0.1s^2 + s(0.7 + 5K_D) + 1.225$

$0 = s^2 + s(7 + 50K_D) + 12.25$

$\cancel{2\zeta\omega_n} =$

$2\zeta\omega_n = 7 + 50K_D$ $\qquad 12.25 = \omega_n^2$

$\omega_n = 3.5$

$2\zeta(3.5) = 7 + 50K_D$

$$K_D = \frac{7\zeta_{new} - 7}{50}$$

$$\text{new over shoot} = \left[1 - \frac{40}{100}\right] \times 20.372 = 12.2232$$

$$12.2232\% = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$$

$$\ln\left(\frac{12.2232}{100}\right) = \ln e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$-2.1018 = \frac{-\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$2.1018 = \frac{\pi \zeta}{\sqrt{1-\zeta^2}}$$

$$4.41 = \frac{\pi^2 \zeta^2}{1-\zeta^2}$$

$$4.41 - 4.41\zeta^2 = 9.8696\,\zeta^2$$

$$\frac{4.41}{4.41+9.86} = \zeta^2$$

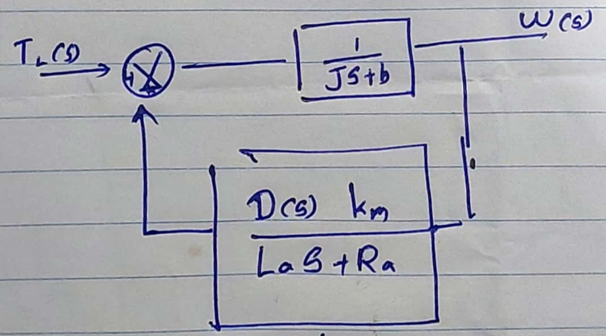
$$\zeta = 0.5559$$

$$K_D = \frac{7 \times 0.5559 - 7}{50}$$

$$= -0.062$$

(IV) $T_L(s) = 3\ \text{Nm}$

$$T_L(s) = \frac{3}{s}$$



$$\frac{\omega'(s)}{T_L(s)} = \frac{\frac{1}{Js+b}}{1 + \left[\frac{D(s)K_m}{L_a s+R_a}\right]\left[\frac{1}{Js+b}\right]} \qquad D = K_p +$$

$$\omega'(s) = \frac{T_L(s)}{[L_a s+R_a][Js+b] + k_m[K_p + sK_D]}$$

$$s\,\omega'(s) = \frac{3}{s} \times s \quad \frac{1}{[L_a s+R_a][Js+b] + k_m[k_p + sk_D]}$$

$$\omega'_{ss} = \lim_{s\to 0} \frac{3}{[L_a s+R_a][Js+b] + k_m[k_p + sk_D]}$$

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$s^2 + 27s + 60 = 0$

$$\omega_{ss}' = \frac{3}{R_a + K_p} < 0.01$$

$$\frac{3}{1 + K_p} < 0.01$$

$$3 < \cancel{1 + K_p}\ 0.01 + 0.01 K_p$$

$$\cancel{2} < \cancel{K_p}$$

$$3 - 0.01 < 0.01 K_p$$

$$\underline{299 < K_p}$$

Q3
(a) (i) $1 + k\,L(s) = 0$ → open loop transfer function

(ii) $L(s) = \left|\frac{b(s)}{a(s)}\right| = \left|\frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)(s-p_3)}\right|$

$= \frac{|s-z_1|\,|s-z_2|}{|s-p_1|\,|s-p_2|\,|s-p_3|}$

$\angle L(s) = \angle \frac{b(s)}{a(s)}$ phase condition

$= \angle b(s) - \angle a(s)$

$= (\psi_1 + \psi_2) - (\phi_1 + \phi_2 + \phi_3)$

$= 180 + 360(l-1)$

$|L(s)| = \frac{1}{k}$ magnitude condition

(b) $G(s) = \frac{s+3}{s(s^2+6s+8)}$

$$G'(s) = \frac{K_p \frac{(s+3)}{s(s^2+6s+8)}}{1 + K_p \frac{(s+3)}{s(s^2+6s+8)}}$$

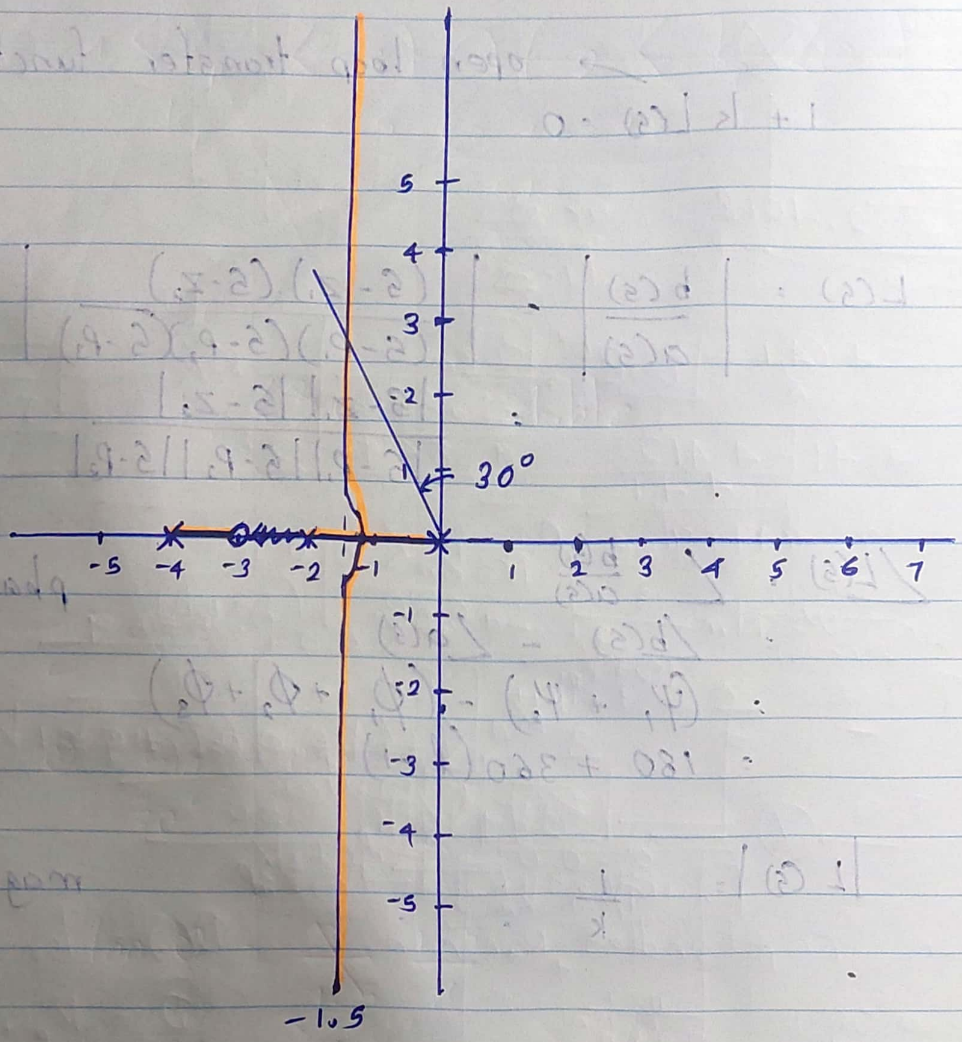
$$1 + K_p\left[\frac{s+3}{s(s^2+6s+8)}\right] = 0$$

Poles of $L(s)$

$s = 0$ $s = -2$ $s = -4$

zero

$s = -3$



Rule 03

$$\alpha = \frac{\sum P - \sum Z}{n-m}$$

$$= \frac{(0-2-4)-(-3)}{3-1}$$

$$= \frac{-6+3}{2}$$

$$= -1.5$$

$$\theta = \frac{180 + 360(l-1)}{n-m}$$

$$\theta_1 = \frac{180 + 360(1-1)}{3-1}$$

$$= 90$$

$$\theta_2 = \frac{180 + 360(2-1)}{2}$$

$$= 270$$

$$\theta_3 = \frac{180 + 360(3-1)}{2}$$

Rule 04

$$\phi_{dep\,2} = (\psi_1 + \psi_2) - (\phi_1 + \phi_2) + 180$$

$$\phi_{dep(0)} = \cancel{180}\,0 - (\cancel{180}\,0 + \cancel{180}\,0) + 180$$

$$= 180$$

$$\phi_{dep(-2)} = \cancel{180}\,0 - (0 + 180) + 180$$

$$= \cancel{180}\,0$$

$$\phi_{dep(-4)} = 180 - \cancel{180}\,(\cancel{0}\,180 + \cancel{0}\,180) + 180$$

$$= \cancel{180}\,0$$

$$\psi_{arr(-3)} = (0 + \cancel{0}\,180 + 180) + 180$$

$$= \cancel{360}\,\cancel{180}\;540$$

$$= 360 + 180 = 180$$

Break away point

$$K_p = -\frac{s(s^2+6s+8)}{s+3}$$

$$\frac{dK_p}{ds} = -\frac{d}{ds}\left[\frac{s(s^2+6s+8)}{s+3}\right]$$

$$\frac{dK_p}{ds} = -\left[\frac{2s^3+15s^2+36s+24}{(s+3)^2}\right]$$

$$2s^3 + 15s^2 + 36s + 24 = 0$$

$s = -1.08$ other are imaginary

(i) ~~$(-4 < K_p < -3) \cap (-2 < K_p < 0)$~~

$$K_p = -\frac{s(s^2+6s+8)}{s+3}$$

$s = 0$

$K_p = 0$

$K_p < 30$ stable

(ii) $\xi = 0.5$

$\theta = \sin^{-1}(0.5)$

$= 30^\circ$

$\tan\theta = \cancel{\frac{x}{1.5}} \quad \frac{1.5}{x}$

$1.5 \tan 30 = x$

$x = \cancel{0.8660} + 2.6$

$P\ K_p = \cancel{-1.5 + 0.8660}i - 2.6 + 1.5i$

~~$L(s) \quad \left|\frac{b(s)}{a(s)}\right| = \frac{\sqrt{(-4+1.5)^2 + (-2+1.5)^2 + (0+1.5)^2 + 3\times 0.866^2}}{\sqrt{(-3+1.5)^2 + 0.866^2}}$~~

use

c) $G(s) = \frac{1}{s(s+2)}$

$$G_1(s) = \frac{K_p}{s(s+2) + K_p}$$

$$= \frac{K_p}{s^2 + 2s + K_p}$$

characteristic eq

$$s^2 + 2s + k_p = 0$$

$$s = \frac{-2 \pm \sqrt{4 - 4k}}{2}$$

$$= -1 \pm \sqrt{1 - k_p}$$

$K_p = 0$ $\quad$ $s_1 = 0$ or $s_2 = -2$

$k_p = 1$ $\quad$ $s_1 = -1$ or $s_2 = -1$

so when $0 \leq k \leq 1$

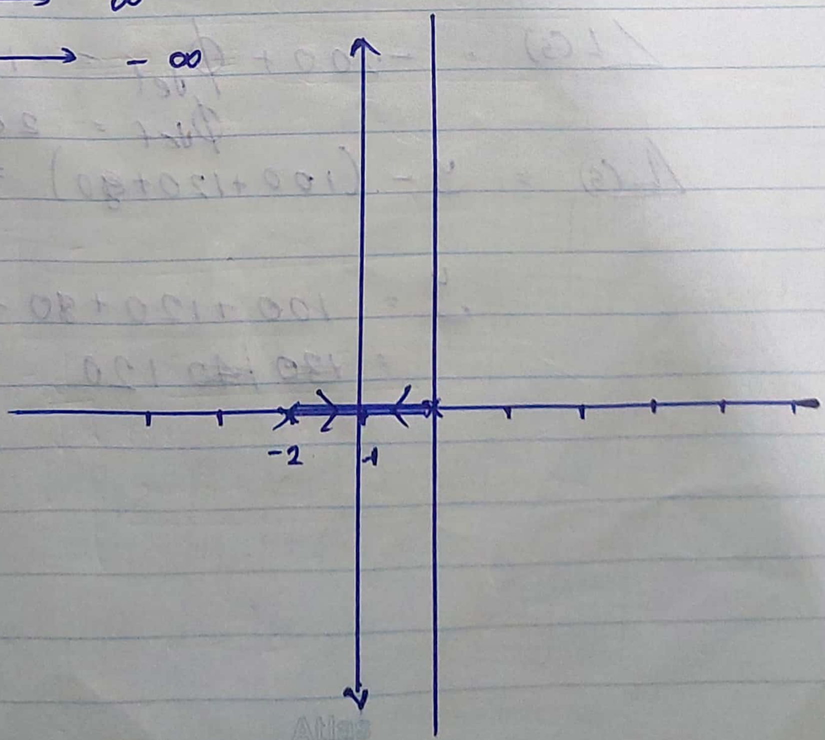
s_1 : $0 \rightarrow -1$

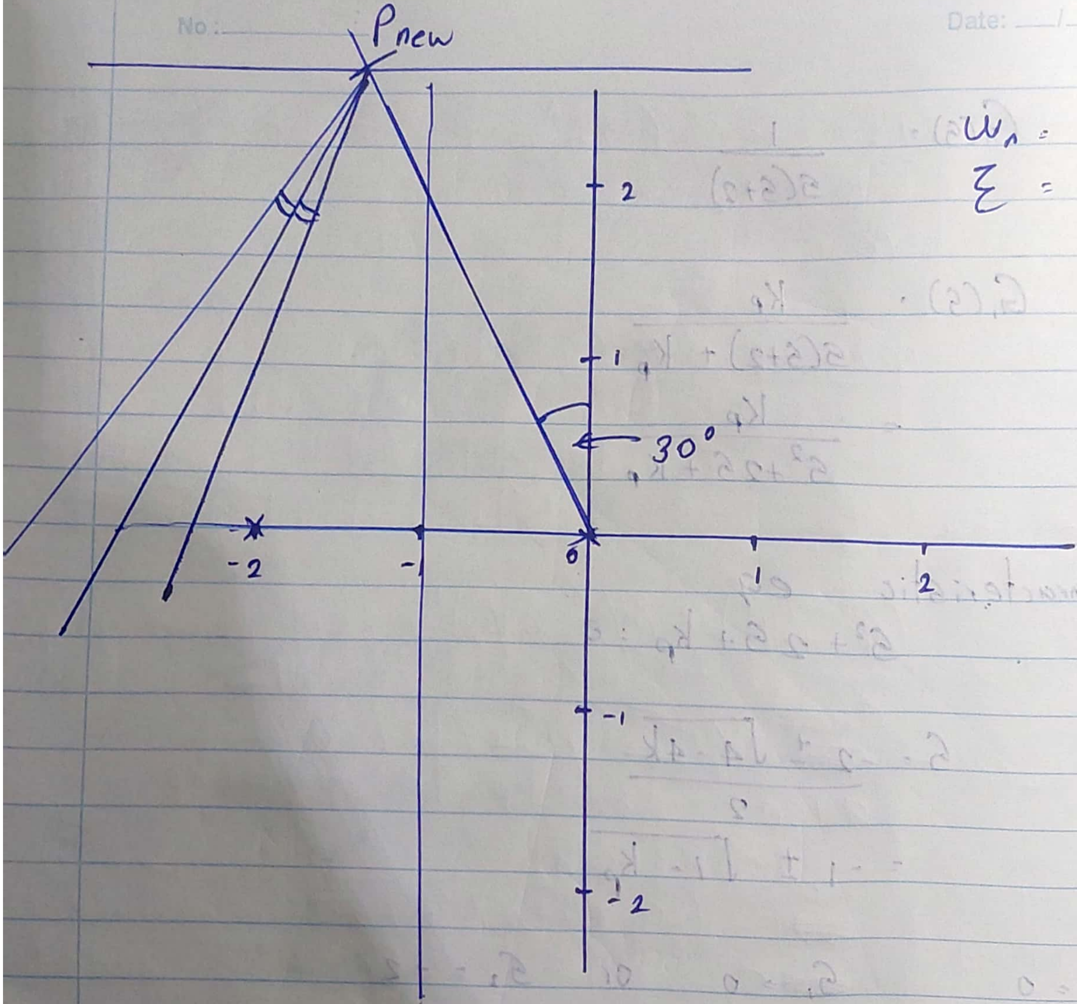
s_2 : $-2 \rightarrow -1$

$1 \leq k < \infty$

$Im(s_1) \longrightarrow \infty$

$Im(s_2) \longrightarrow -\infty$





$\omega_n = 3 \text{ rad } s^{-1}$

$\xi = 0.5$

$$\angle \frac{b(s)}{a(s)} = \sum \psi_i - \sum \phi_i$$

$$= 0 - (120 + \cancel{100}\,80)$$

$$= \cancel{-220}\ -200$$

zero & pole

$$\angle L(s) = -200 + \phi_{def} = -180^\circ$$

$$\phi_{def} = 20$$

$$\angle L(s) = y - (100 + 120 + 80) = -180^\circ$$

$$y = 100 + 120 + 80 - 180$$

$$= \cancel{130}\ \cancel{140}\ 120$$

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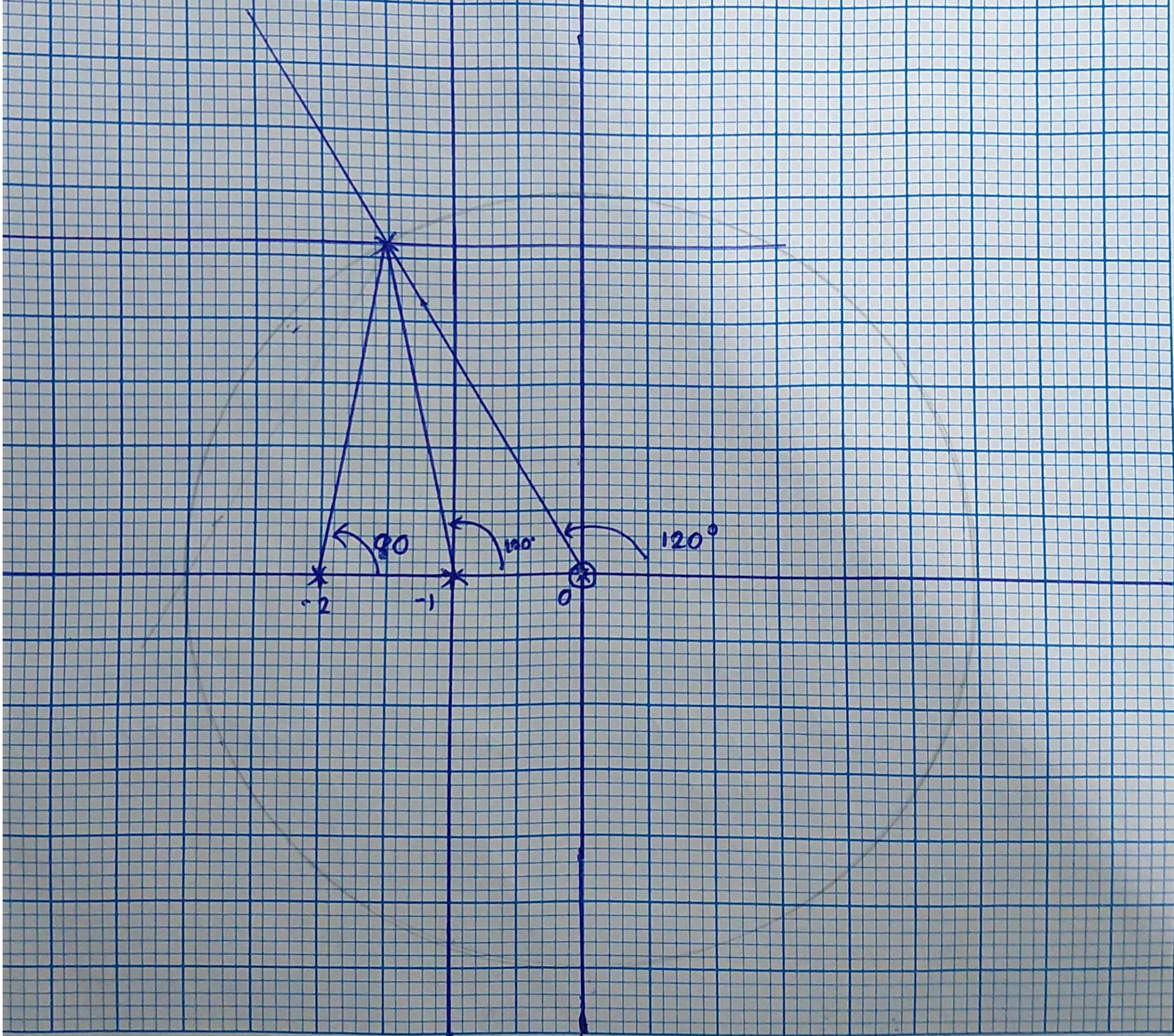

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$s = -3\cos 30 + 3\sin 30\, i$

$= -2.6 + 1.5i$

$L(s) G(s) = \dfrac{k}{(s+2)(s+1)}$

$L(s) = \left| \dfrac{b(s)}{a(s)} \right| = \left| \dfrac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)} \right|$

$\dfrac{1}{k} = \dfrac{1}{|-2.6+1.5i-2|\,|-2.6+1.5i+1|}$

$\dfrac{1}{k} = \dfrac{1}{|-0.6+1.5i|\,|-1.6+1.5i|}$

$\dfrac{1}{k} = \dfrac{10 \times 10}{3\sqrt{29} \times \sqrt{481}}$

$\dfrac{1}{k} = 0.282$

$k = \underline{3.54}$

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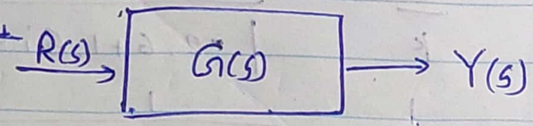
Q_4

(a) (i) A linear system's response to a sinusoidal input holds the system frequency response.

(ii) $r(t) = A \sin \omega t$

$y_{ss}(t) = AM \sin(\omega t + \phi)$

~~G(s) = S +~~



$$M = G(\omega j)$$

$$\phi = \angle G(\omega j)$$

(iii) $$M_{dB} = 20 \log_{10} |G(j\omega)|$$

$$\phi = \angle G(j\omega)$$

(iv) phase Margin (γ) and Gain margin should be positive

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(b) $G(s) = \frac{\theta_L(s)}{V_r(s)} = \frac{K_M}{s(T_f s+1)(T_M s+1)}$

$K_M = 1$
$T_f = 1$
$T_M = 0.1$

(i) $G(s) = \frac{1}{s(s+1)(0.1s+1)}$

$= \frac{10}{s(s+1)(s+10)}$

(ii) $V_r = 2\sin(4t - 30^\circ)$

$V_r(s) = \left[\frac{s\sin(30) + \cos(-30^\circ)}{s^2+4^2}\right]2$

$= \frac{s + \sqrt{3}}{s^2+4^2}$

$Y(s) = \frac{10(s+\sqrt{3})}{s(s+1)(s+10)(s^2+4^2)}$

Atlas

$$G(s) = \frac{1}{s(s+1)(0.1s+1)}$$

$$20\log|G(j\omega)| = 20\log\frac{1}{j\omega} + 20\log\frac{1}{1+j\omega} + 20\log\frac{1}{1+0.1j\omega}$$