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Pattern sheet 1 — Omar Atef 7619

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a) 4 Dimensions b) all Dimensions are Integers

c) $\sqrt{(10-30)^2+(60-50)^2+(10-30)^2+(90-40)^2} \simeq 58.3$

d) $\sqrt{20^2+50^2+40^2+70^2} \simeq 96.95$

e) $\cos(\theta) = \frac{A \cdot B}{\|A\| \times \|B\|} = \frac{20\times20+50\times50+20\times40+60\times70}{96.95 \times \sqrt{20^2+50^2+20^2+60^2}}$

$\cos\theta \simeq 0.98$

f) yes because The range of a1 is 10 → 30 while a4 is 10 → 90. Scaling can help Reduce The diffrence to interpet all features on The same scale and Mak Gradient decent smoother/faster

g) $a_1: x \leftarrow \frac{x-10}{30-10} = \frac{x-10}{20}$ $a_4: x \leftarrow \frac{x-10}{80}$

$a_2: x \leftarrow \frac{x-50}{10}$ $a_3 \leftarrow \frac{x-10}{30}$

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	a1	a2	a3	a4
x_1	0	1	0	1
x_2	0.5	0	1	3/4
x_3	1	0	2/3	3/8
x_4	0.5	0	1/3	5/8
x_5	0	1	2/3	0

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h) c) $d = \sqrt{1+1+\left(\frac{2}{3}\right)^2+\left(\frac{5}{8}\right)^2} = 1.68$

d) $L = \sqrt{.5^2+1+(3/4)^2} = 1.35$

e) $\cos\theta = \dfrac{.5 \times .5 + 1/3 + \frac{3}{4} \times \frac{5}{8}}{1.35\sqrt{.25+\frac{1}{9}+\left(\frac{5}{8}\right)^2}} \simeq 0.899$

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a)

	a_1	a_2	a_4
X_1	10	60	90
X_2	20	50	70
X_3	30	50	40
X_4	20	50	60
X_5	10	60	10

$D_{5\times 3}$

c) ~~a_1~~ $\mu_1 = \dfrac{10+20+30+20+10}{5} = 18$

$\mu_2 = 54$ $\mu_3 = 54$

$\mu = \begin{bmatrix} 18 \\ 54 \\ 54 \end{bmatrix}$

d)

$Z =$

X_1	-8	6	36
X_2	2	-4	16
X_3	12	-4	-14
X_4	2	-4	6
X_5	-8	6	-44

$Z_{3\times 3}$

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e) $var(x_1) = \frac{(10-18)^2 \times 2 + 2(20-18)^2 + (30-18)^2}{5-1} = 70$

$var(x_2) =$

$COV(D) = (D-\mu)^T \cdot (D-\mu)^*$ $COV_{3\times3}$

$$= \begin{bmatrix} -8 & 6 & 36 \\ 2 & -4 & 16 \\ 12 & -4 & -14 \\ 2 & -4 & 6 \\ -8 & 6 & -44 \end{bmatrix}^T \cdot \begin{bmatrix} -8 & 2 & 12 & 2 & -8 \\ 6 & -4 & -4 & -4 & 6 \\ 36 & 16 & -14 & 6 & -44 \end{bmatrix}^T$$

$$= \begin{bmatrix} 70 & -40 & -15 \\ -40 & 30 & -20 \\ -15 & -20 & 930 \end{bmatrix}$$

4) $COV(D) = (D-\mu)^T (D-\mu)^*$

$$\mu = \begin{bmatrix} 1.84 \\ 2.92 \\ 1.96 \end{bmatrix} \qquad D-\mu = \begin{bmatrix} -1.34 & 1.58 & 0.54 \\ 0.36 & -1.42 & -1.86 \\ 2.06 & 0.58 & -0.86 \\ 0.26 & -1.02 & 2.94 \\ -1.34 & 0.28 & -0.76 \end{bmatrix}$$

$(D-\mu)^T (D-\mu)^*$

$$= \begin{bmatrix} 8.03 & -6.19 & -1.38 \\ -6.19 & 7.65 & 1.5 \\ -1.38 & 1.5 & 13.71 \end{bmatrix}$$

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B) 1- $AV = \lambda V$

$AV - \lambda V = \vec{0}$

$(A - \lambda I)V = \vec{0}$

$$\begin{bmatrix} 8.03 & -6.2 & -1.4 \\ -6.2 & 7.7 & 1.5 \\ -1.4 & 1.5 & 13.7 \end{bmatrix} \begin{bmatrix} \lambda_A & 0 & 0 \\ 0 & \lambda_B & 0 \\ 0 & 0 & \lambda_C \end{bmatrix} = \vec{0}$$

$$\begin{bmatrix} 8\lambda_A & -6.2\lambda_B & -1.4\lambda_C \\ -6.2\lambda_A & 7.7\lambda_B & 1.5\lambda_C \\ -1.4\lambda_A & 1.5\lambda_B & 13.7\lambda_C \end{bmatrix}$$

$\lambda_a = 0.5\lambda_b$

$\lambda_c = 1.55\lambda_b$

$8\lambda_A(7.7\lambda_B \times 13.7\lambda_C - (1.5^2\lambda_C\lambda_B) +$

$6.2\lambda_B(-6.2\lambda_A \times 13.7\lambda_C + 1.5\lambda_C 1.4\lambda_A) -$

$1.4\lambda_C(-6.2\lambda_A 1.5\lambda_B + 7.7\lambda_B 1.4\lambda_A) = 0$

$\lambda_A = \checkmark\checkmark$

$\lambda_B = \checkmark\checkmark$

$\lambda_C = \checkmark\checkmark$

2) explained var. $= \dfrac{\lambda_b + \lambda_c}{\lambda_A + \lambda_b + \lambda_c}$

$= \dfrac{2.55\cancel{\lambda_b}}{3.05\cancel{\lambda_b}} = 0.84$