Sheet ①

Omar atef
7619

Q.1

9- calculat distance from all 8 samples

1 → $\sqrt{(3-2)^2+(5-2)^2+(3.1-3)^2}$ = 3.1639 → ③

2 → $\sqrt{(3-2)^2+(5-2)^2+(3-2)^2}$ = 3.3166

3 → ——— = 5.0249

4 → ——— = 7.2973

5 → ——— = 9.1

6 → ——— = 1.005 → ①

7 → ——— = 1.7916 → ②

8 → ——— = 7.2829

The 3 nearest samples are 1, 6, 7

Their votes are Yes, NO, NO

so The prediction will be NO

10-

1 → 2.1932 → ③
2 → 2.8284
3 → 4.1533
4 → 6.2650
5 → 8.0006
6 → 1.4866 → ①
7 → 2.1000 → ②
8 → 6.1351

Vote
Yes, NO, No

Prediction is
NO

11-

1 → 1.7916 → ②
2 → 1.4142 → ①
3 → 2.6926 → ③
4 → 3.0414
5 → 5.4415
6 → 3.6277
7 → 3.1639
8 → 3.2311

Vote
NO, Yess, yes

Pred
Yes

12-

1 → 1.0770 → ①
2 → 1.8028 → ②
3 → 2 → ③
4 → 4.5826
5 → 6.0959
6 → 2.0880
7 → 2.7495
8 → 4.4821

Vote
NO, yes, Yes

Pred
Yes

Data after transforming

$$\frac{X - X_{min}}{X_{max} - X_{min}}$$

ID	PLY	DA	GPA	PA
1	0.5	0.25	0.6	1
2	0.5	0.25	0.05	0
3	0	0.5	0.8	1
4	1	0.75	0.3	1
5	0.5	1	1	0
6	0	0	0.5	0
7	0.5	0	0	0
8	1	0.75	0.65	1
9	0	-0.125	0.55	??
10	0.5	0	1.05	??
11	1	0.375	0.05	??
12	0	0.25	0.8	??

Ply:

$$x = \frac{X-2}{4-2} = \frac{X-2}{2}$$

DA:

$$x = \frac{X-3}{11-3} = \frac{X-3}{8}$$

GPA:

$$x = \frac{X-1.9}{3.9-1.9} = \frac{X-1.9}{2}$$

9)
1 → 0.6329
2 → 0.7701
3 → 0.6932
4 → 1.3437
5 → 1.3988
6 → 0.1950
7 → 0.7181
8 → 1.3372

Votes are
yes, yes, NO

Pred is
Yes

10)
1 → 0.5148
2 → 1.0308
3 → 0.7500
4 → 1.1726
5 → 1.0012
6 → 0.7433
7 → 1.0500
8 → 0.9861

Votes
yes, yes, NO

Pred: Yes

11)
1 → 0.7537
2 → 0.5154
3 → 1.2562
4 → 0.4507
5 → 1.2422
6 → 1.1589
7 → 0.6269
8 → 0.7075

Votes:
NO, yes, NO

Pred: NO

12)
1 → 0.5385
2 → 0.9014
3 → 0.2500
4 → 1.2247
5 → 0.9233
6 → 0.3905
7 → 0.9760
8 → 1.1281

Vote: yes, yes, NO

Pred: Yes

Conclusion:
3 of the 4 predictions changed after normalization as now all features contribute equally to the cost (dist)

$$R = \begin{bmatrix} -1, 0 \\ 0, 2 \\ 1, 4 \\ 2, 5 \end{bmatrix}$$

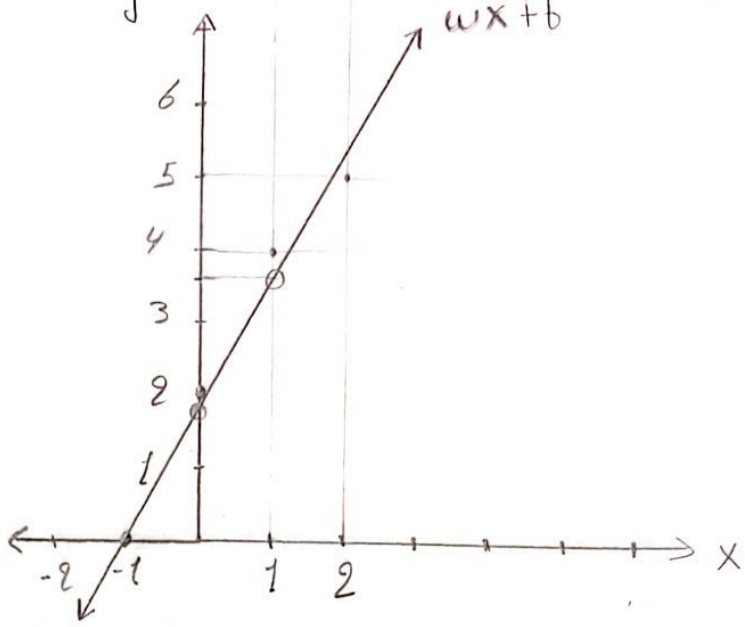
$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad t = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 5 \end{bmatrix}$$

$$w = (X^T \cdot X)^{-1} \cdot X^T \cdot t$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \end{bmatrix}$$

$$y = 1.7x + 1.9$$

y	0	1
x	1.9	3.6



$$\underset{2\times4}{X^T} \cdot \underset{4\times2}{X} = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} = A$$

$$A^{-1} = \frac{1}{24-4} \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}$$

$$\underset{2\times2}{A^{-1}} \cdot \underset{2\times4}{X^t} = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ -0.3 & -0.1 & 0.1 & 0.3 \end{bmatrix} = B$$

$$\underset{2\times4}{B} \cdot \underset{4\times1}{t} = \begin{bmatrix} 1.9 \\ 1.7 \end{bmatrix} = w$$

$$X=\begin{bmatrix}1&0\\1&1\\1&2\\1&3\\1&4\end{bmatrix} \quad t=\begin{bmatrix}2\\3\\5\\4\\6\end{bmatrix} \quad w=(X^T.X)^{-1}.X^t.t$$

$$\underset{2\times5}{(X^T.X)}\ \underset{5\times2}{=}\begin{bmatrix}5&10\\10&30\end{bmatrix}=A$$

$$A^{-1}=\frac{1}{150-100}\begin{bmatrix}30&-10\\-10&5\end{bmatrix}=\begin{bmatrix}0.6&-0.2\\-0.2&0.1\end{bmatrix}$$

$$X^T=\begin{bmatrix}1&1&1&1&1\\0&1&2&3&4\end{bmatrix} \quad B=\underset{2\times2}{A^{-1}}.\underset{2\times5}{X^t}=\begin{bmatrix}0.6&0.4&0.2&0&-0.2\\-0.2&-0.1&0&0.1&0.2\end{bmatrix}$$

$$w=\underset{2\times5}{B}.\underset{5\times1}{t}=\begin{bmatrix}2.2\\0.9\end{bmatrix}=b\\=w$$

$$\boxed{y=0.9x+2.2}$$

b) $y=0.9\times10+2.2=11.2$

a) square error $=\frac{1}{2\times5}\sum_{i=0}^{i=4}(0.9x^i+2.2-y^i)^2=\frac{1}{10}\left[(0.2)^2+(0.1)^2+(1)^2+(0.9)^2+(0.2\right.$

$$=\frac{1}{10}\times1.9=\boxed{0.19}$$

$$X=\begin{bmatrix}1&-2\\1&-1\\1&0\\1&1\\1&2\\1&2\end{bmatrix}\quad t=\begin{bmatrix}0\\1\\0\\-1\\0\\2\end{bmatrix}\quad w=(X^T.X)^{-1}.X^T.t$$

$$X^T.X=\underset{2\times6\ 6\times2}{\begin{bmatrix}6&2\\2&14\end{bmatrix}}=A$$

$$A^{-1}=\frac{1}{84-4}\begin{bmatrix}14&-2\\-2&6\end{bmatrix}=\begin{bmatrix}0.175&-0.025\\-0.025&0.075\end{bmatrix}$$

$$X^T=\begin{bmatrix}1&1&1&1&1&1\\-2&-1&0&1&2&2\end{bmatrix}$$

$$\underset{2\times2\ 2\times6}{A^{-1}.X^t}=B=\begin{bmatrix}0.225&0.2&0.175&0.15&\frac{1}{8}&\frac{1}{8}\\0.125&0.05&-0.025&-.1&\frac{-7}{40}&\frac{-}{4}\end{bmatrix}$$

$$\underset{2\times6\ 6\times1}{B.t}=\begin{bmatrix}0.3\\-0.025\end{bmatrix}$$

- after ignoring (2,2)
- original Line

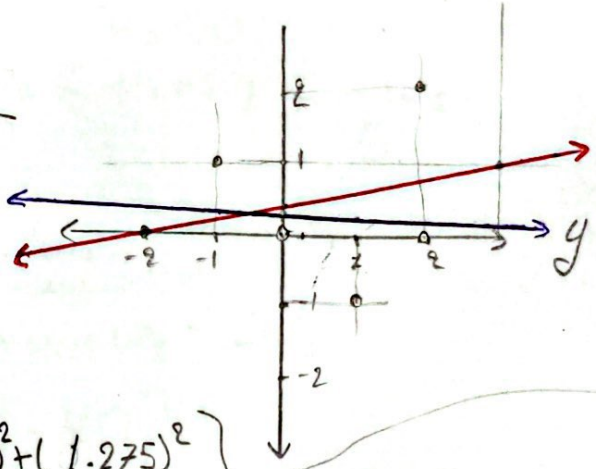
a- $w=-0.025$

$b=0.3$

$y=-0.025X+0.3$

B)

x	0	1
y	0.3	



c- squared loss:

$$L=\frac{1}{2\times6}\sum_{0}^{5}(y^i-t^i)^2=$$

$$=\frac{1}{12}\left[(0.35)^2+(0.675)^2+(0.3)^2+(1.275)^2+(0.25)^2+(1.75)^2\right]$$

$$=\frac{1}{12}\times5.42=\boxed{0.45}$$

D- $X^T.X=\begin{bmatrix}5&0\\0&10\end{bmatrix}=A\quad \underset{2\times5\ 5\times1}{B.t}=\begin{bmatrix}0.4\\0.2\end{bmatrix}$

$$A^{-1}=\frac{1}{50}\begin{bmatrix}10&0\\0&5\end{bmatrix}=\begin{bmatrix}0.2&0\\0&0.1\end{bmatrix}\qquad y=0.2X+0.4$$

$$\underset{2\times2\ 2\times5}{A^{-1}.X^t}=\begin{bmatrix}0.2&.2&.2&.2&.2\\-0.2&-.1&0&.1&.2\end{bmatrix}=B$$

x	0	3
y	0.4	1

Conclusion:
The model is very sensitive to change in data due to The very small dataset

$y = 0.2x + 0.4$

$x = -0.5, y = 0.3$
$x = 0.5, y = 0.5$
$x = 1.8, y = 0.7$

Q.3

1) a- true b- false c- true

XOR

1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$w_0 < 0$
$w_0 + w_2 \geq 0$
$w_0 + w_1 \geq 0$
$w_0 + w_1 + w_2 < 0$

$w_0 < 0$
$w_2 \geq -w_0$ $\Rightarrow$ $w_1 + w_2 \geq -2w_0$
$w_1 \geq -w_0$
$w_1 + w_2 < -w_0$

$w_0 = -5$

$w_0 < 0$
$-2w_0 \leq w_1 + w_2 < -w_0$
10 5

2) a- AND

1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$w_0 < 0$ ✓
$w_0 + w_2 < 0 \rightarrow w_2 < -w_0$ ✓
$w_0 + w_1 < 0 \rightarrow w_1 < -w_0$ ✓
$w_0 + w_1 + w_2 > 0 \rightarrow w_1 + w_2 > -w_0$ ✓

$w_0 = -5$
$w_1 = 3$
$w_2 = 3$

C- NOT $\overline{1}$

1	0	1	$w_0 \geq 0$ ✓
1	1	0	$w_0 + w_1 < 0$ ✓

$w_0 = -5$
$w_1 = 6$

B- OR

1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$w_0 < 0$ ✓
$w_0 + w_2 > 0$ ✓
$w_0 + w_1 > 0$ ✓
$w_0 + w_1 + w_2 > 0$ ✓

$w_0 = -5$
$w_1 = 6$
$w_2 = 6$

$y = 3.1 \times 1.2 - 1.9 \times 9.3 = -0.65$

$y < 0.4$ ∴ output is 0

4) a) $w = [6, 0, -1]^T$ → $w_0 + w_1 x_1 + w_2 x_2$

decision boundary is at $g(z) = 0.5$

which is at $z = 0$, $\frac{1}{1+e^0} = 0.5$

$6 - x_2 = 0 \quad x_2 = 6$

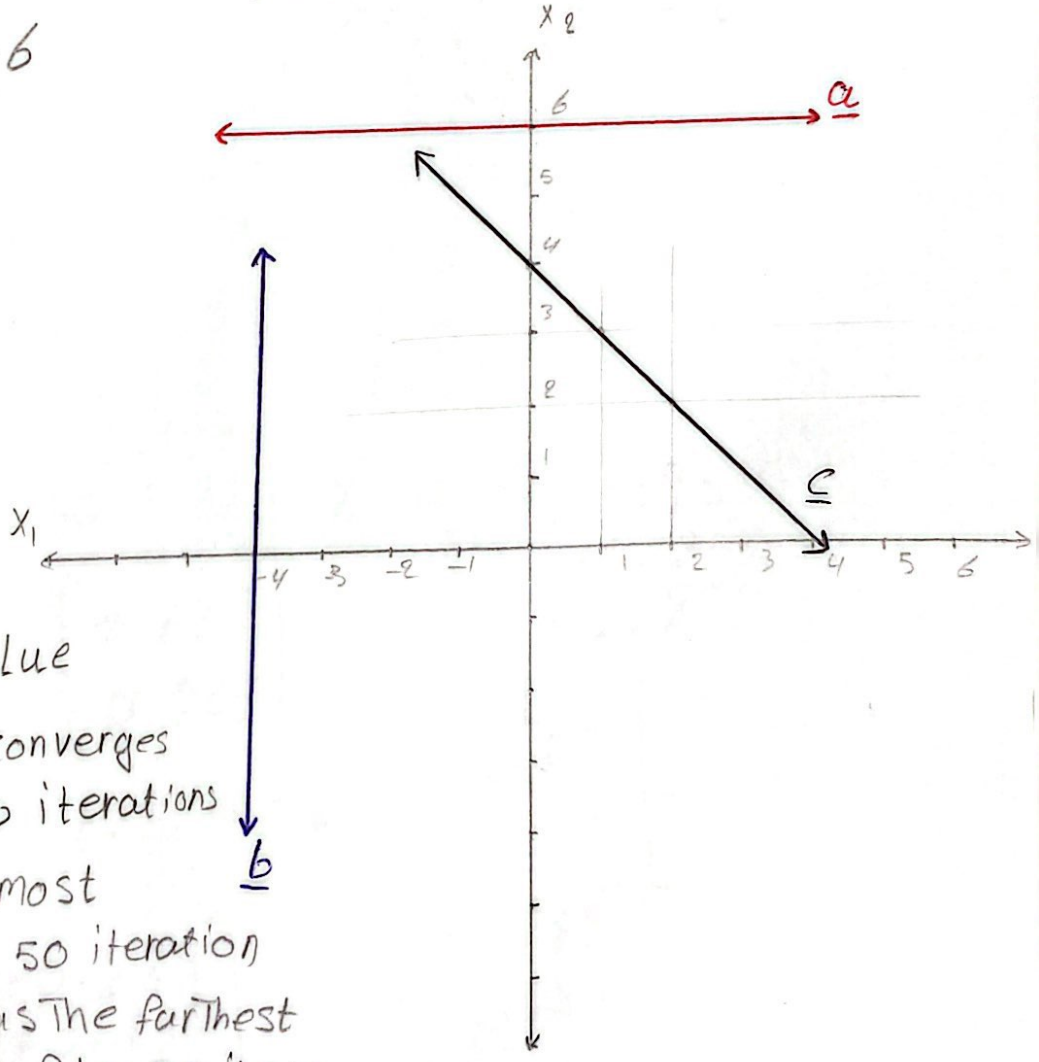
b) $8 + 2x_1 = 0$

$x_1 = -4$

c) $-4 + x_1 + x_2 = 0$

$x_1 + x_2 = 4$

x_1	0	2	1
x_2	4	2	3



5) green-red-blue

The green line converges afteronly 25-30 iterations where the red almost converged after 50 iteration and the blue was the farthest from converging after 50 iters

Accuracy: Sample 1: $-2-2\times-1+1\times0=0\rightarrow+$ $|y-t|$: 0

Sample 2: $=-0.5\rightarrow-$ 1

3: $=-4\rightarrow-$ 0

4: $=-2\rightarrow-$ 0

5: $=-4\rightarrow-$ 1

$=-5\rightarrow-$ 1

$=1\rightarrow+$ 1

$=-7\rightarrow-$ 0

$$acc=\frac{\Sigma|y-t|}{8}=0.5$$

iter 1:

$$\frac{\partial \ell}{\partial w}=\frac{1}{8}\left[X^T_{3\times8}\cdot(y-t)_{8\times1}\right]$$

$$X^T=\begin{bmatrix}1&1&1&1&1&1&1&1\\-1&-.5&2&.5&1&1&0&3\\0&.5&2&1&0&-1&3&1\end{bmatrix}\quad t=\begin{bmatrix}1\\1\\0\\0\\1\\1\\1\\0\\0\end{bmatrix}\quad y=\text{sigmoid}(z)$$

$$y=\begin{bmatrix}0.5\\0.378\\0.018\\0.119\\0.018\\0.007\\0.731\\0.001\end{bmatrix}\cdot(y-t)=\begin{bmatrix}-0.5\\-0.622\\0.18\\0.119\\-0.982\\-0.993\\0.731\\0.001\end{bmatrix}$$

$$X^T_{3\times8}\cdot(t-y)_{8\times1}=\begin{bmatrix}1.934\\-0.242\\2.855\end{bmatrix}$$

$$\frac{\partial \ell}{\partial w}=\begin{bmatrix}-0.279\\-0.133\\0.379\end{bmatrix}$$

$$w=w-\alpha\frac{\partial \ell}{\partial w}\quad \text{let } \alpha=1$$

$$w=\begin{bmatrix}-2\\-2\\1\end{bmatrix}-\begin{bmatrix}-0.279\\-0.133\\0.379\end{bmatrix}=\begin{bmatrix}-1.722\\-1.867\\0.621\end{bmatrix}$$

racy after iter①:

$X_{8\times3} \cdot w_{3\times1} = \begin{bmatrix} 0.145 \\ -0.477 \\ -4.913 \\ -9.034 \\ -3.589 \\ -4.909 \\ 0.142 \\ -6.701 \end{bmatrix}$ $y = \frac{1}{1+e^{-z}} = \begin{bmatrix} 0.536 \\ 0.383 \\ 0.015 \\ 0.116 \\ 0.027 \\ 0.015 \\ 0.535 \\ 0.001 \end{bmatrix}$ ✓ × ✓ ✓ × × ✓ ×

acc = 0.5

iter ②

$w -= 1 \times \frac{1}{8}(X^T \cdot (y-t))$ $w = \begin{bmatrix} -1.425 \\ -1.73 \\ 0.318 \end{bmatrix}$

acc:

$y = \frac{1}{1+e^{-z}} = \begin{bmatrix} 0.576 \\ 0.401 \\ 0.014 \\ 0.122 \\ 0.041 \\ 0.030 \\ 0.384 \\ 0.002 \end{bmatrix}$ ✓ × ✓ ✓ × × ✓ ✓

$acc = \frac{5}{8} = 0.625$

iter ③

$w = \begin{bmatrix} -1.121 \\ -1.591 \\ 0.071 \end{bmatrix}$

acc:

$y = \begin{bmatrix} 0.615 \\ 0.428 \\ 0.015 \\ 0.136 \\ 0.062 \\ 0.058 \\ 0.287 \\ 0.003 \end{bmatrix}$ ✓ × ✓ ✓ × × ✓ ✓ $acc = 0.625$