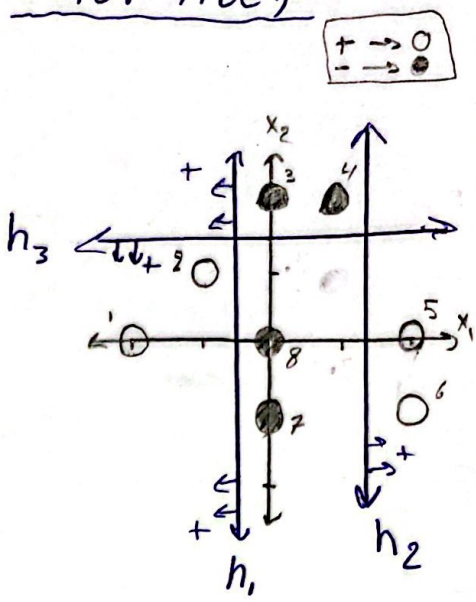
Sheet 3

7619

omar Atef

Q.1

$\frac{1}{2}$	$1-w$
$\frac{1}{2}$	$1+w$



iter: one

$w_1 = [1/8, 1/8 \text{ ---}]$

$h_1 \rightarrow$ split at $x_1 = 0.25$

$err = \frac{2}{8} = 0.25$

$\alpha_1 = \frac{1}{2} \ln\left(\frac{1-0.25}{0.25}\right) \approx 0.55$ $\quad I \in (0,1)$

$w_2 = w_1 e^{2 \times 0.55 \times \frac{1}{2}(1-I)} = w_1 e^{\overset{x}{A}\ \overset{\alpha_1}{\beta}(1-I)}$

$w_2 = [1/8, 1/8, 1/8, 1/8, 0.38, 0.38, 1/8, 1/8]$

iter: Two

$h_2 \rightarrow$ split at $x_1 = 0.75$

$err_2 = \frac{1/8 + 1/8}{6/8 + 2 \times 0.38} \approx 0.167$

$\alpha_2 = \frac{1}{2} \ln\left(\frac{1-err}{err}\right) = 0.8$

$w_3 = [0.62, 0.62, 1/8, 1/8, 0.38, 0.38, 1/8, 1/8]$

iter: Three $\quad h_3 \rightarrow$ at $x_2 = 0.75$

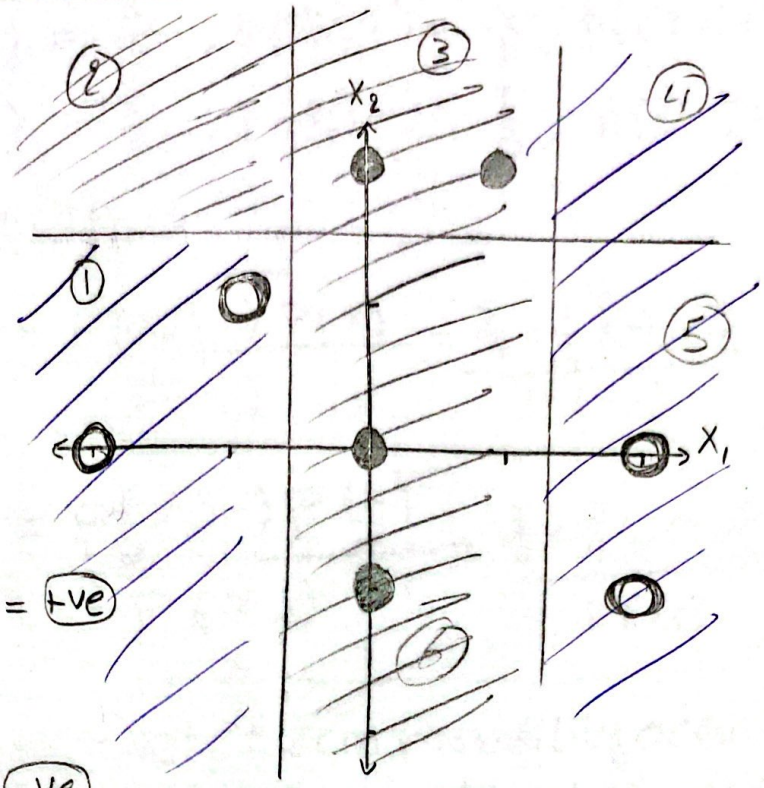
$err_3 = \frac{2/8}{4/8 + 2 \times 0.38 + 2 \times 0.62} = 0.1$, $\alpha_3 = 1.098$

$w_4 = [0.62, 0.62, 1/8, 1/8, 0.38, 0.38, 1.12, 1.12]$

Final classifier:

/// → -ve

/// → +ve



region (1):

$\alpha_1 + \alpha_3 - \alpha_2 =$ (+ve)

reg (2)

$\alpha_1 - \alpha_2 - \alpha_3 =$ (-ve)

reg (3): $-\alpha_1 - \alpha_2 - \alpha_3 =$ (-ve), reg (4): $-\alpha_1 + \alpha_2 + \alpha_3 =$ (+ve)

reg (5): $-\alpha_1 + \alpha_2 + \alpha_3 =$ (+ve), reg (6): $-\alpha_1 - \alpha_2 + \alpha_3 =$ (-ve)

$$err_4 = \frac{0}{\Sigma w} = 0$$

Q.2

$X_{11} = (red, suv, Dom)$

$$P(X_{11} \mid stolen=yes) = \frac{P(red \mid yes)\, P(suv \mid yes)\, P(Dom \mid yes)\, P(C)}{P(red)\, P(suv)\, P(Dom) = P(X_{11})}$$

$$P(X_{11} \mid yes) = \frac{3/5 \times 1/5 \times 2/5 \times 5/10}{P(X_{11})} = \frac{0.024}{P(X_{11})}$$

$$P(X_{11} \mid No) = \frac{2/5 \times 3/5 \times 3/5 \times 5/10}{P(X_{11})} = \frac{0.072}{P(X_{11})}$$

Pred = NO

Q.3

a) $P(A|+) = \frac{3}{5}$, $P(B|+) = \frac{1}{5}$, $P(C|+) = \frac{4}{5}$

$P(A|-) = \frac{2}{5}$, $P(B|-) = \frac{2}{5}$, $P(C|-) = \frac{5}{5}$

b) $X_{11} = 0, 1, 0$

$$P(+|X_{11}) = \frac{P(X_{11}|+)\,P(+)}{P(X_{11})} = \frac{\frac{2}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{5}{10}}{P(X_{11})} = \frac{0.008}{P(X_{11})}$$

$$P(-|X_{11}) = \frac{P(X_{11}|-)\,P(-)}{P(X_{11})} = \frac{\frac{3}{5} \times \frac{2}{5} \times 0 \times \frac{5}{10}}{P(X_{11})} = 0$$

Pred = +

Note: zero probability occured which is not accurate and should be avoided by using MAP

Q.4

a) 2 Dim b) all integers

c) $= \sqrt{20^2+10^2+20^2+50^2} = 10\sqrt{34}$

d) $= \sqrt{20^2+50^2+40^2+70^2} = 10\sqrt{94}$

e) $\cos\theta = \frac{X_2 \cdot X_4}{\|X_2\|\|X_4\|} = \frac{20^2+50^2+40\times20+70\times60}{\sqrt{20^2+50^2+20^2+60^2} \times 10\sqrt{94}} = 0.98$

Q.5

ID	a_1	a_2	a_4
x_1	10	60	96
x_2	20	50	70
x_3	30	50	40
x_4	20	50	60
x_5	10	60	10

c) $[18, 54, 54]$

d) $\begin{bmatrix} -8 & 6 & 36 \\ 2 & -4 & 16 \\ 12 & -4 & -14 \\ 2 & -4 & 6 \\ -8 & 6 & -44 \end{bmatrix}$

e) $= \begin{bmatrix} 56 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 744 \end{bmatrix}$

$$V(x) = \frac{\sum_1^n (x_i - \mu)^2}{n}$$

$$V(a_1) = \frac{8^2 + 2^2 + 12^2 + 2^2 + 8^2}{5}$$

$$= \frac{56}{5} = 11.2$$

$V(a_2) = 24$

$V(a_3) =$

$$\begin{bmatrix} V(a_1) & cov(a_1, a_2) & cov(a_2, a_3) \\ cov(a_1, a_2) & V(a_2) & cov(a_2, a_3) \\ cov(a_1, a_3) & cov(a_2, a_3) & V(a_3) \end{bmatrix}$$

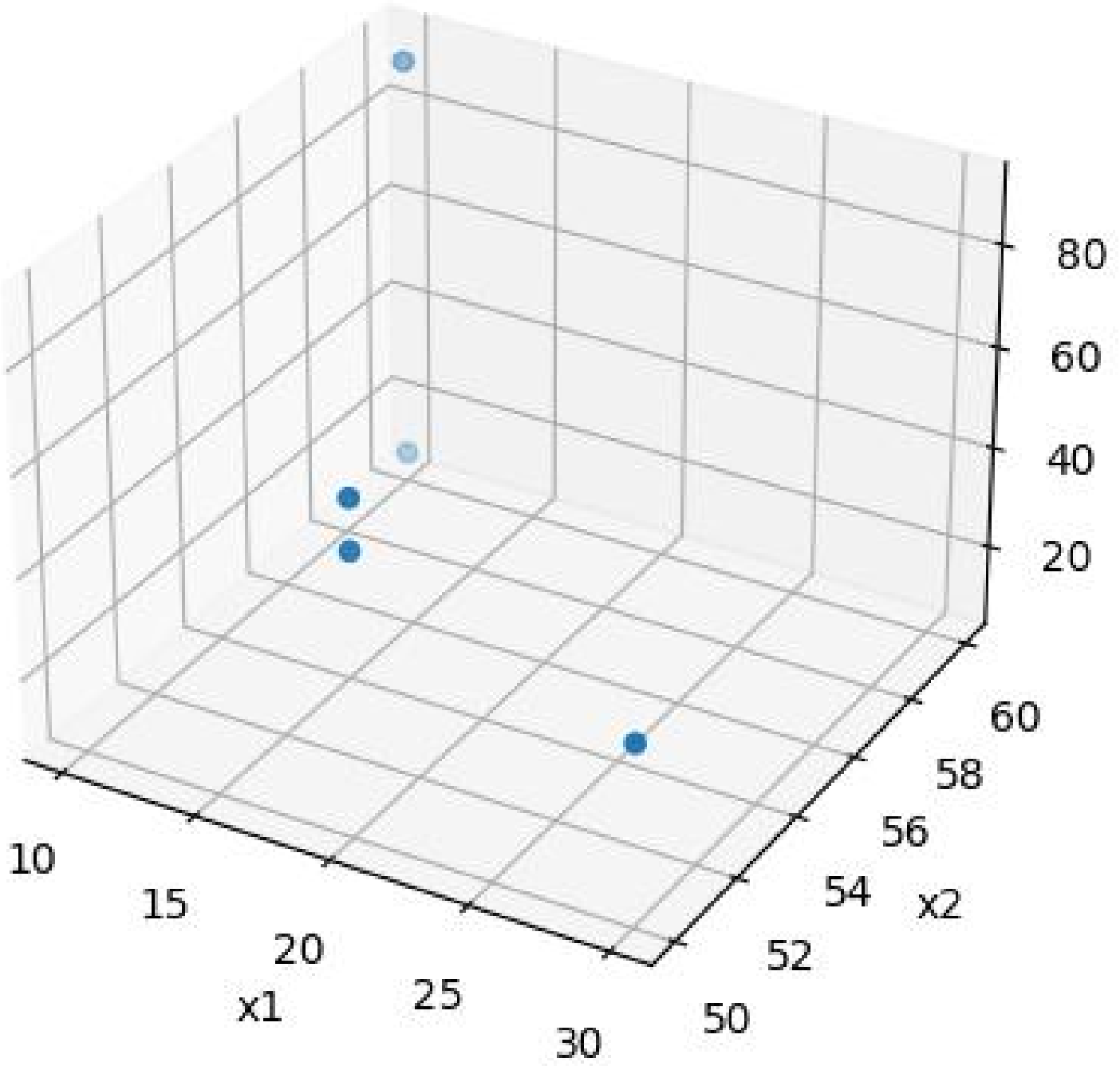
$$cov(a_1, a_2) = \frac{-8 + 2 + 12 + 2 - 8 + 6 - 4 - 4 - 4 + 6}{5}$$

$$= 0$$

$$cov(a_1, a_3) = \frac{0 + 36 + 16 - 14 + 6 - 44}{5}$$

$$= 0$$

$$cov(a_2, a_3) = \frac{0}{5}$$


80
60
40
20
60
58
56
54
x2
52
50
10
15
20
25
30
x1