Flippin' Flingers Trebuchet Final Report

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1 Abstract

This final report provides an overview of the finalization of the project focused on the sketching, designing, and modeling of a trebuchet.

The report outlines the key milestones achieved since the inception of the project, including build process, changes from the initial design, detailed component specifications, analytical-numerical-physical test results, and dicussion on test results.

The report utilizes CAD, first principles, numerical simulations, and physical tests to achieve this goal.

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2 Introduction

This report's mission statement is to summarize the team's final results with:

- Detailed CAD drawings.
- Analytical Calculations.
- Numerical modeling.
- Real-life data.

The problem outline is to design and analyze the trebuchet to maximize projectile distance and accuracy.

The trebuchet is designed for maximum distance through general plane motion.[1]It is also designed for maximum accuracy through string measure.[2]

To maximize distance, the velocity of the projectile is considered, as distance and velocity are proportional. Increasing the counterweight fall distance can boost the projectile's velocity.[3] Increasing the counterweight-to-launch distance increases the velocity of the projectile.[4]

Lastly, adjusting the trebuchet firing angle to 45 degrees achieves the maximum projectile distance.[5]

[1]Hibbeler, 2015
[2]Rhoten, 2021
[3]Siano, 2001
[4]Denny, 2005
[5]Connel, 2001

3 Methodology

The team uses CATIA, a CAD software, to design components and present them in detailed technical drawings.

Next, the team applies the principles of Kinematics and Kinetics to analyze the motion of the trebuchet. Through this analysis, they are able to make accurate estimations regarding the distance the projectiles will travel.

The team also utilizes Working Model 2D, a CAE software, to numerically simulate the trebuchet's motion and provide numerical data for analyzing the impact of various factors on its performance.

Finally, the team conducts real-life tests of the trebuchet to compare and contrast the actual results with their analytical and numerical predictions. They carefully analyze the variations between the expected and observed outcomes, discussing the factors that contribute to these differences.

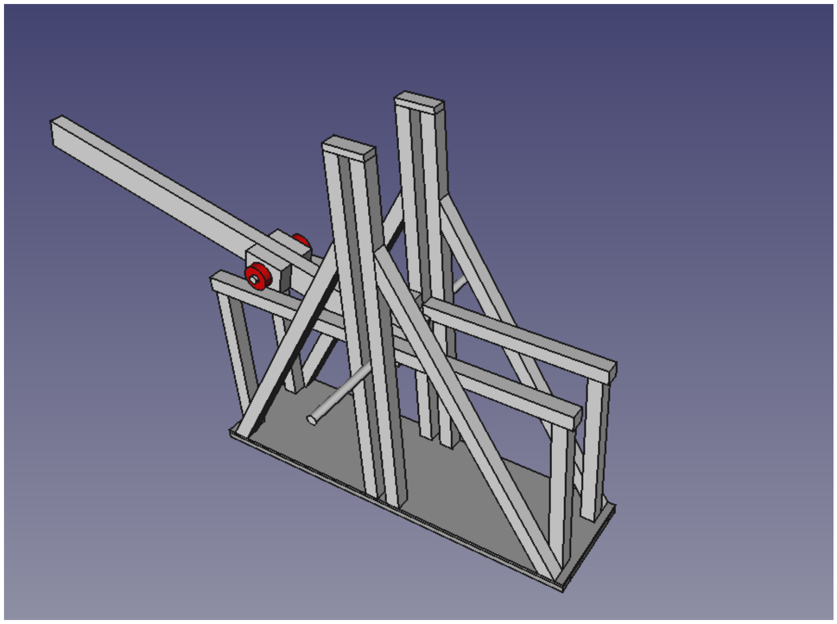

Figure 1: Floating Arm Trebuchet in CAD

4 Design

4.1 CAD

Figure 1 showcases the CAD model of the Floating Arm Trebuchet. The design features a robust physical body. The counterweight is attachable to the middle axle, and the arm holds the sling and guide chute at the end. To enhance portability, wheels are attachable to the corners of the base, facilitating easy transportation of the trebuchet.

4.2 Technical Drawings

5 Results

5.1 Analytical Calculations

Before the team can begin to analyze the motion of the trebuchet, they must first take some measurements. The team measures the length of the arm to be approximately 0.6 m. The team uses two 355mL drink cans. Therefore, the mass of the counterweight is $\frac{2*355}{1000} \approx 0.7$ kg. The mass of the arm with the supporting wheels is 0.3 kg. Hence, the combined mass of the counterweight with the arm is $0.7 + 0.3 = 1$ kg. The team measured the radius and height of the cans which were 0.066 m and 0.12 m respectively. See Figure 2.

Given: $m_a = 0.3$ kg, $m_w = 0.7$ kg, $m = 1$ kg, $l = h = 0.6$ m, $\theta = 45°$
$r = 0.066$ m and $h_c = 0.12$ m
RTF: Max velocity of point C, v_c
Assumptions: Rigid bodies, no energy loss, starts from rest.

To solve this problem, the team uses the General Work-Energy Equation:

$$T_1 + \Sigma U_{1-2} = T_2$$

Since, the trebuchet starts from rest, $T_1 = 0$. Now, to find ΣU_{1-2},

$$\Sigma U_{1-2} = mgh = (1)(9.8)(0.6) = 5.88 \approx 5.9\text{J}$$

To find T_2, the team needs to find the equivalent mass moment of inertia I_e,

$$I_e = I_a + I_w$$

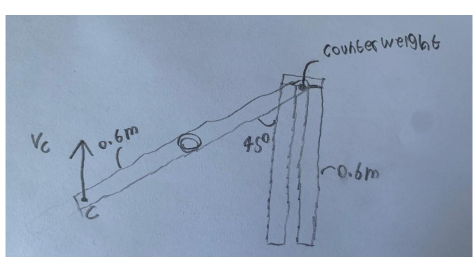

Figure 2: Diagram

The team assumes the arm to be a rod, and the counterweight to be a cylinder. Therefore,

$$\begin{aligned} I_e &= \frac{1}{12}m_a l^2 + \frac{1}{12}m_w(3r^2 + h_c^2) \\ &= \frac{1}{12}(0.3)(0.6)^2 + \frac{1}{12}(0.7)\left(3(0.066)^2 + (0.12)^2\right) = 0.0106\,\mathrm{kg}\cdot\mathrm{m}^2 \end{aligned}$$

To find T_2 we need to relate ω with v, the team uses the kinematic equation for rotational motion,

$$v = \omega l;\ \omega = \frac{v}{l}$$

Substituting this into the kinetic energy equation,

$$\begin{aligned} T_2 &= \frac{1}{2}mv^2 + \frac{1}{2}I_e\left(\frac{v}{l}\right)^2 \\ &= \frac{1}{2}(1)v^2 + \frac{1}{2}(0.0106)\left(\frac{v}{0.6}\right)^2 \\ &= 0.5v^2 + 0.0147v^2 = 0.5147v^2 \end{aligned}$$

Now, substituting ΣU_{1-2} and T_2 into the General Work-Energy Equation,

$$5.9 = 0.5147v^2$$

$$v = v_e = 3.38\,\mathrm{m/s};\ \omega = \frac{3.38}{0.6} = 5.63\,\mathrm{rad/s}$$

Now, to find the velocity of point C, the team uses the kinematic equation for rotational motion assuming the distance from the equivalent center to point C is 0.5 m,

$$v_c = v_e + \omega l_{c/e} = 3.38 + (5.63)(0.5) = 6.2\,\mathrm{m/s}$$

Now, to find the range of the projectile, the team uses the parabolic motion equation assuming that the time right before the ball lands is 4 seconds,

$$v_c = \frac{ds}{dt};\ \int_0^s ds = \int_0^4 6.2dt$$

$$\boxed{s = 24.8\,\mathrm{m}}$$

Therefore, the team predicts that the trebuchet will launch the projectile 24.8 meters.

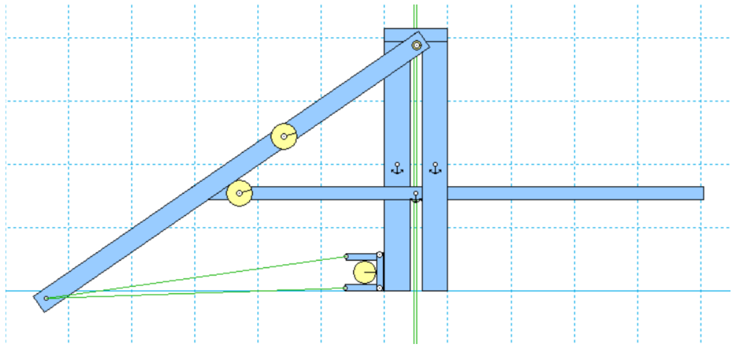

Figure 3: Floating Arm Trebuchet model in Working Model 2D

5.2 Numerical Model

The Floating Arm Trebuchet implemented in Working Model 2D can be found in Figure 3. Additionally, a tracing of the trebuchet's motion can be found in Figure 5. Tracing is a feature in Working Model 2D that shows each frame of the motion of an object.

Lastly, position-velocity-acceleration graphs for both vertical and horizontal axes of the projectile are found in Figure 4. The horizontal velocity of the ball increases linearly once the trebuchet fires then stays constant (ignoring air resistance). Hence, the team predicts with this model that they can achieve around 20 meters of projectile distance.

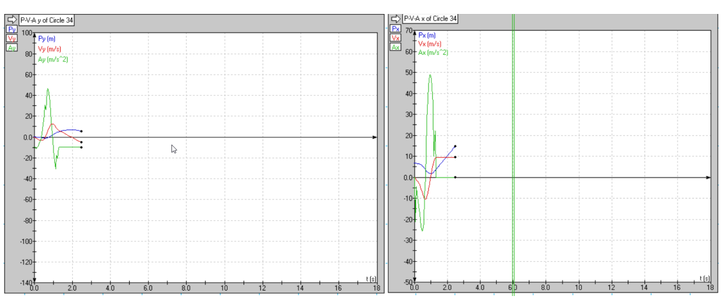

Figure 4: P-V-A graph of ball

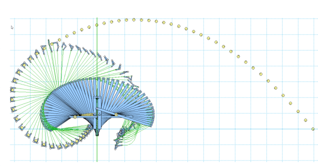

Figure 5: Motion of Trebuchet

5.3 Real-life Data

6 Comparison of Results

7 Discussion

8 Conclusions

The main objectives of this milestone were to create a blueprint for the construction of the final build. The key findings are summarized as follows:

1. In order to maximize the distance traveled by the projectile, it is essential to optimize the projectile's velocity.

2. General plane motion increases projectile velocity by harnessing the speed of the falling counterweight.

3. Floating Arm Trebuchets consist of five main components: frame, arm, counterweight, sling, and guide chute.

4. The release of the projectile at a 45 degree angle achieves the maximum projectile distance.

9 References

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