Flippin' Flingers Trebuchet Final Report

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1 Abstract

This final report provides an overview of the finalization of the project.

The report outlines the key milestones achieved since the inception of the project, including build process, changes from the initial design, detailed component specifications, analytical-numerical-physical test results, and dicussion on test results.

The report utilizes CAD, first principles, numerical simulations, and physical tests to achieve this goal.

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2 Introduction

This report's mission statement is to summarize the team's final results with:

- Detailed CAD drawings.
- Analytical Calculations.
- Numerical modeling.
- Real-life data.

The problem outline is to design and analyze the trebuchet to maximize projectile distance and accuracy.

The trebuchet is designed for maximum distance through general plane motion.[1] It is also designed for maximum accuracy through string measure.[2]

To maximize distance, the velocity of the projectile is considered, as distance and velocity are proportional. Increasing the counterweight fall distance can boost the projectile's velocity.[3] Increasing the counterweight-to-launch distance increases the velocity of the projectile.[4]

Lastly, adjusting the trebuchet firing angle to 45 degrees achieves the maximum projectile distance.[5]

[1] Hibbeler, 2015
[2] Rhoten, 2021
[3] Siano, 2001
[4] Denny, 2005
[5] Connel, 2001

3 Methodology

The team uses CATIA, a CAD software, to design components and present them in detailed technical drawings.

Next, the team applies the principles of Kinematics and Kinetics to analyze the motion of the trebuchet. Through this analysis, they are able to make accurate estimations regarding the distance the projectiles will travel.

The team also utilizes Working Model 2D, a CAE software, to numerically simulate the trebuchet's motion and provide numerical data for analyzing the impact of various factors on its performance.

Finally, the team conducts real-life tests of the trebuchet to compare and contrast the actual results with their analytical and numerical predictions. They carefully analyze the variations between the expected and observed outcomes, discussing the factors that contribute to these differences.

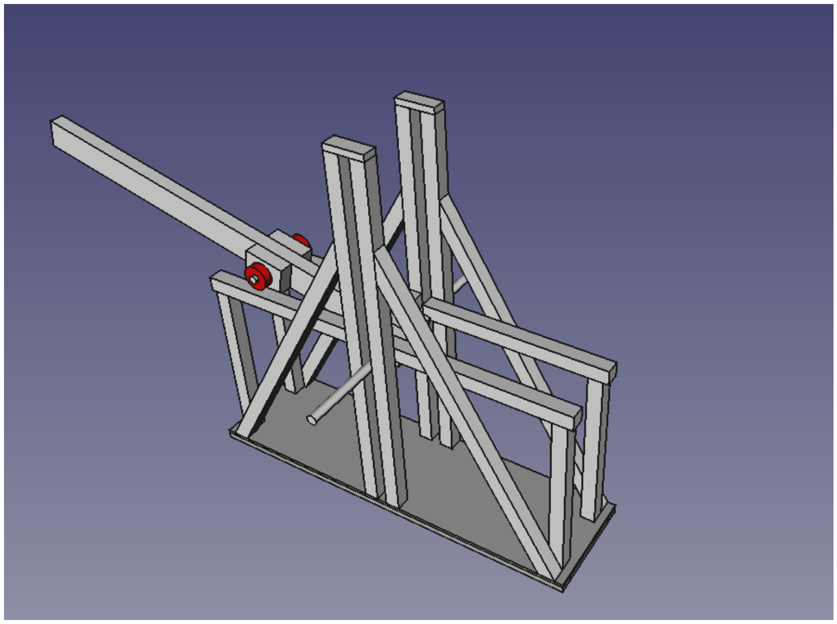

Figure 1: Floating Arm Trebuchet in CAD

4 Design

Figure 1 showcases the CAD model of the Floating Arm Trebuchet. The design features a robust physical body. The counterweight is attachable to the middle axle, and the arm holds the sling and guide chute at the end. To enhance portability, wheels are attachable to the corners of the base, facilitating easy transportation of the trebuchet.

4.1 Technical Drawings

The trebuchet's structural body is designed using two main components: 60 cm and 30 cm wooden rods. The 60 cm rods are used for the base length, arm, ramp, and uprights. While the 30 cm rods are used for the base width and frame.

The design is very simple yet effective. The team used CATIA to design and create drafts for these components. Technical drawings for these components are shown in Figures 2 and 3.

The technical drawings showcase the front, top, left, and isometric views of the components. The wooden rods are the same dimensions except their length, so the width and height of the rods are 3/4" 3/8" or 1.905 cm 0.9525 cm respectively.

The CAD, technical drawings, and working model 2D files can be found in the submission folder.

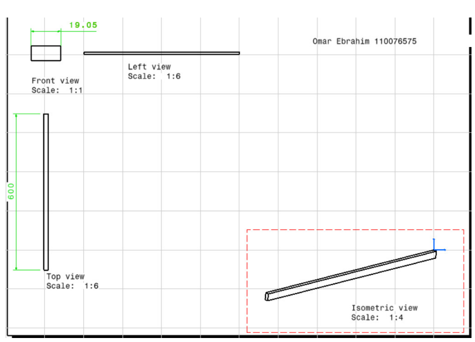

Figure 2: 60 cm rod

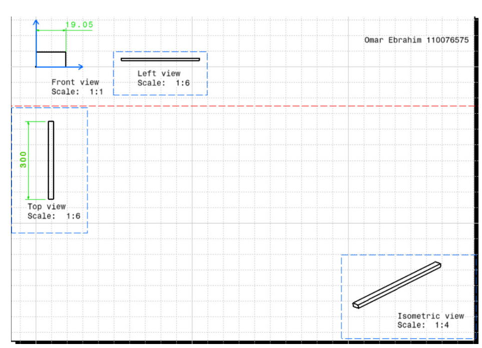

Figure 3: 30 cm rod

5 Results

5.1 Analytical Calculations

Before the team can begin to analyze the motion of the trebuchet, they must first take some measurements. The team measures the length of the arm to be approximately 0.6 m. The team uses two 355mL drink cans. Therefore, the mass of the counterweight is $\frac{2*355}{1000} \approx 0.7$ kg. The mass of the arm with the supporting wheels is 0.3 kg. Hence, the combined mass of the counterweight with the arm is $0.7 + 0.3 = 1$ kg. The team measured the radius and height of the cans which were 0.066 m and 0.12 m respectively. See Figure 4.

Given: $m_a = 0.3$ kg, $m_w = 0.7$ kg, $m = 1$ kg, $l = h = 0.6$ m, $\theta = 45°$
$r = 0.066$ m and $h_c = 0.12$ m
RTF: Range of projectile, s
Assumptions: Rigid bodies, no energy loss, starts from rest.

To solve this problem, the team uses the General Work-Energy Equation:

$$T_1 + \Sigma U_{1-2} = T_2$$

Since, the trebuchet starts from rest, $T_1 = 0$. Now, to find ΣU_{1-2},

$$\Sigma U_{1-2} = mgh = (1)(9.8)(0.6) = 5.88 \approx 5.9\text{J}$$

To find T_2, the team needs to find the equivalent mass moment of inertia I_e,

$$I_e = I_a + I_w$$

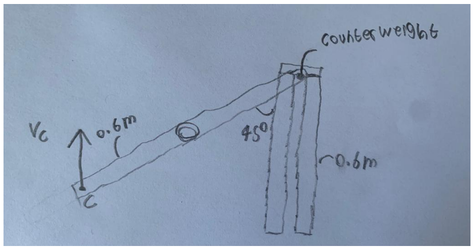

Figure 4: Diagram

The team assumes the arm to be a rod, and the counterweight to be a cylinder. Therefore,

$$\begin{aligned} I_e &= \frac{1}{12}m_a l^2 + \frac{1}{12}m_w(3r^2 + h_c^2) \\ &= \frac{1}{12}(0.3)(0.6)^2 + \frac{1}{12}(0.7)\left(3(0.066)^2 + (0.12)^2\right) = 0.0106\,\mathrm{kg}\cdot\mathrm{m}^2 \end{aligned}$$

To find T_2 we need to relate ω with v, the team uses the kinematic equation for rotational motion,

$$v = \omega l;\ \omega = \frac{v}{l}$$

Substituting this into the kinetic energy equation,

$$\begin{aligned} T_2 &= \frac{1}{2}mv^2 + \frac{1}{2}I_e\left(\frac{v}{l}\right)^2 \\ &= \frac{1}{2}(1)v^2 + \frac{1}{2}(0.0106)\left(\frac{v}{0.6}\right)^2 \\ &= 0.5v^2 + 0.0147v^2 = 0.5147v^2 \end{aligned}$$

Now, substituting ΣU_{1-2} and T_2 into the General Work-Energy Equation,

$$5.9 = 0.5147v^2$$

$$v = v_e = 3.38\,\mathrm{m/s};\ \omega = \frac{3.38}{0.6} = 5.63\,\mathrm{rad/s}$$

Now, to find the velocity of point C, the team uses the kinematic equation for rotational motion assuming the distance from the equivalent center to point C is 0.5 m,

$$v_c = v_e + \omega l_{c/e} = 3.38 + (5.63)(0.5) = 6.2\,\mathrm{m/s}$$

Now, to find the range of the projectile, the team uses the parabolic motion equation assuming that the time right before the ball lands is 4 seconds,

$$v_c = \frac{ds}{dt};\ \int_0^s ds = \int_0^4 6.2dt$$

$$\boxed{s = 24.8\,\mathrm{m}}$$

From this analysis, the team predicts that the trebuchet will launch the projectile 24.8 meters.

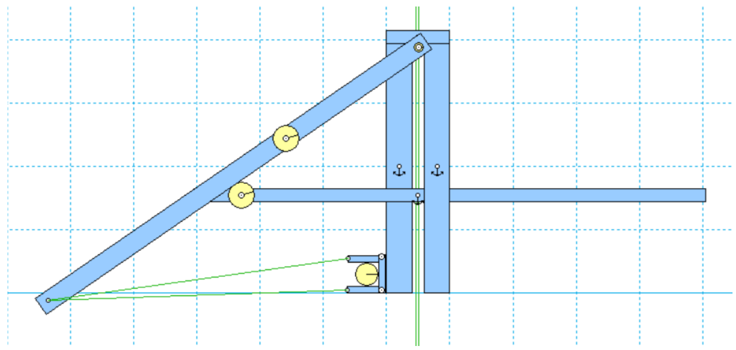

Figure 5: Floating Arm Trebuchet model in Working Model 2D

5.2 Numerical Model

The Floating Arm Trebuchet implemented in Working Model 2D can be found in Figure 5. Additionally, a tracing of the trebuchet's motion can be found in Figure 7. Tracing is a feature in Working Model 2D that shows each frame of the motion of an object.

The simulation creates a P-V-A graph as shown in Figure 6. The parameters used in the simulation are as follows: $m_w = 0.7$ kg, $l_{arm} = h = 0.6$ m, $\theta = 45°$, and $r_{circle} = 0.066$ m. As shown in the graph, the ball's velocity is initially zero, decreases then increases to its maximum velocity at time $t = 1.5$ s. The ball's position increases as it is launched until it reaches the ground after 4 seconds. By looking at the x direction graph at t = 4 s, the team predicts that the range of the projectile is approximately 24 meters.

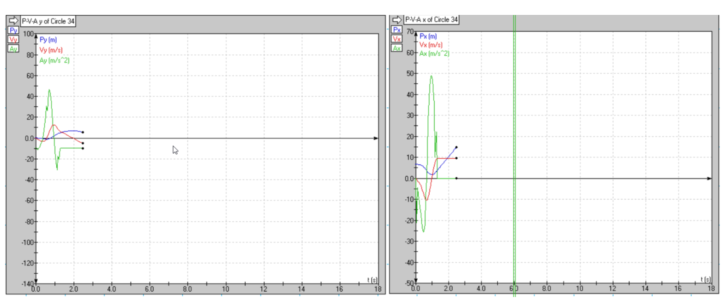

Figure 6: P-V-A graph of ball

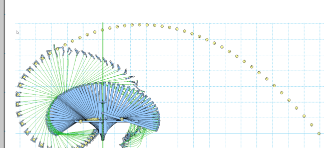

Figure 7: Motion of Trebuchet

5.3 Real-life Data

The team conducted a real-life experiment to test the range of the trebuchet. The results can be found in Figure 8.

The team uses the mean of the data to find the average range of the that the projectile traveled. The mean is calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{2*5+5*6+7+8+9}{10} = 6.4\,\mathrm{m}$$

Hence, to find the efficiency of the trebuchet, the team uses the following equation:

$$\eta = \frac{\text{Actual Range}}{\text{Predicted Range}} = \frac{6.4}{24.8} = 0.258$$

Therefore, the team concludes that the trebuchet is 25.8% efficient.

Throw	Range (m)
1	8
2	6
3	9
4	5
5	6
6	6
7	7
8	5
9	6
10	6

Figure 8: Data of 10 throws

6 Comparison of Results

To compare the results of the analytical calculations, numerical model, and the real-life data the team uses this equation:

$$\text{Percent Error} = \frac{\text{Predicted Range} - \text{Actual Range}}{\text{Predicted Range}} * 100\%$$

6.1 Analytical/Numerical

The team starts by comparing the analytical calculations and the numerical model. The team uses the following equation to find the percent error:

$$\text{Percent Error} = \frac{24.8 - 24}{24.8} * 100\% = 3.226\%$$

6.2 Analytical/Real-life

Now to find the percent error between the analytical calculations and the real-life data, the team uses the following equation:

$$\text{Percent Error} = \frac{24.8 - 6.4}{24.8} * 100\% = 74.2\%$$

6.3 Numerical/Real-life

Lastly to find the percent error between the numerical model and the real-life data, the team uses the following equation:

$$\text{Percent Error} = \frac{24 - 6.4}{24} * 100\% = 73.3\%$$

7 Discussion

From the Comparison of Results section, the team concludes that the analytical calculations and numerical model are very similar because they have a percent error of 3.226%. This is because both the analytical calculations and numerical model use the same principles.

7.1 Limitations

The real-life data is very different from both the analytical calculations and numerical model. This is because the real-life data is affected by many factors such as wind, friction, and the release angle. These factors are not taken into account in the analytical calculations and numerical model.

These results don't discredit the analytical calculations and numerical model because these calculations and models are based on the ideal conditions.

The analytical calculations and numerical model help the team to find the best case scenario for the trebuchet. The team can use this to make meaningful predictions about how the trebuchet will perform.

7.2 Improvements

The team can improve the design of the trebuchet by approaching the ideal conditions. The team can do this by reducing the friction between the projectile and the sling as well as increase the time of release.

To deal with friction, the team can use a lubricant on key parts that are affected by friction. To increase the time of release, the team can use a longer sling and tweak the trebuchet to release the projectile at 45 degrees.

8 Conclusions

The main objectives of this milestone were to provide updated detailed technical drawings, calculate the range of the projectile using analytical calculations and numerical models, compare the results, and discuss the limitations and improvements of the design.

The key findings are summarized as follows:

1. To maximize distance, the velocity of the projectile should be considered because the range of the projectile is proportional to the velocity.

2. The analytical calculations and numerical model predictions are very similar. The analytical calculations predict that the projectile will travel 24.8 meters while the numerical model predicts that the projectile will travel 24 meters.

3. The average range of the projectile in the real-life experiment is 6.4 meters. This is very different from the analytical calculations and numerical model. This is because the real-life experiment is affected by many factors such as wind, friction, and the release angle.

4. To improve the design of the trebuchet, the trebuchet should be tweaked to approach the ideal conditions. This can be done by reducing friction and increasing the time of release.

9 References

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