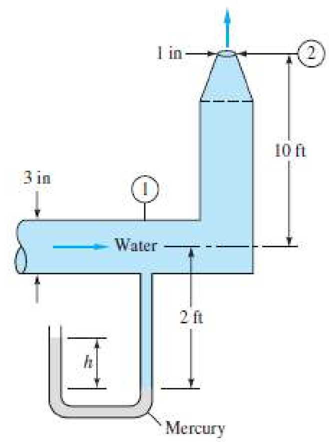
Assignment#6-F20+Solutions

Fluid Mechanics (University of Windsor)

Mechanical, Automotive and Materials Engineering
Fluid Mechanics I
MECH-3233-F20

Assignment Problems Set #6

Problem 1: (P3.131 White): In Fig. P3.131 both fluids (ρ_{water} = 1.94 slug/ft^3 and $\gamma_{mercury}$ = 846 lb/ft^3, γ_{water} = 62.4 lb/ft^3) are at 20°C. If V_1 = 1.7 ft/s and losses are neglected, what should the manometer reading h ft be?

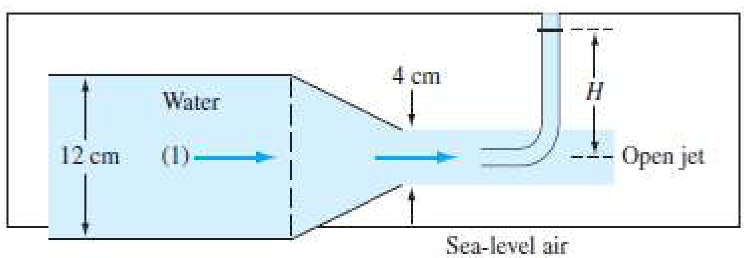


P3.131

Problem 2: (P3.127 White) In Fig. P3.127 the open jet of water at 20°C (ρ_w= 998 kg/m^3) exits a nozzle into sea-level air (p_a =101325 Pa) and strikes a stagnation tube as shown. If the pressure at the centerline at section 1 is 110 kPa (abs), and losses are neglected, estimate

(*a*) the mass flow in kg/s and

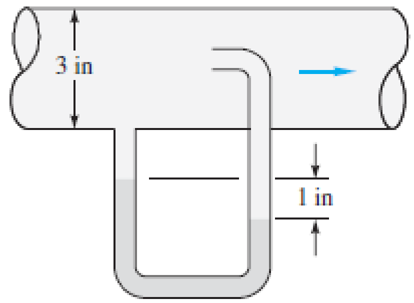
(*b*) the height *H* of the fluid in the stagnation tube.



P3.127

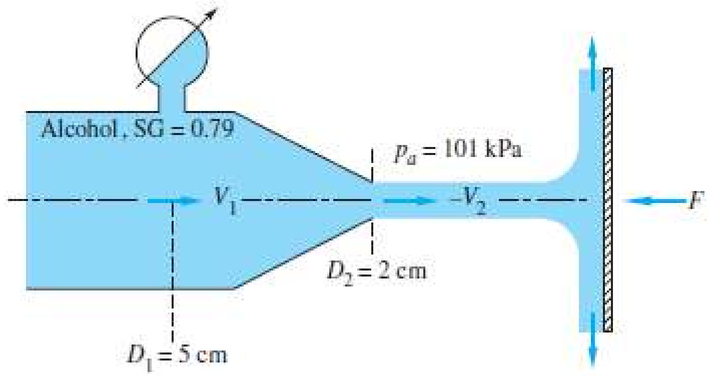
Problem 3: (P3.120 White) The manometer fluid in Fig. P3.120 is mercury ($\rho_{Hg} = 26.34$ slug/ft^3). Estimate the volume flow in the tube if the flowing fluid is (a) gasoline (ρ = 1.32 slug/ft^3) and (b) nitrogen, at 20°C and 1 atm.

Note: For nitrogen, use ideal gas law and $R_{nitrogen}$ = 297 J/kg·°C to find the density.



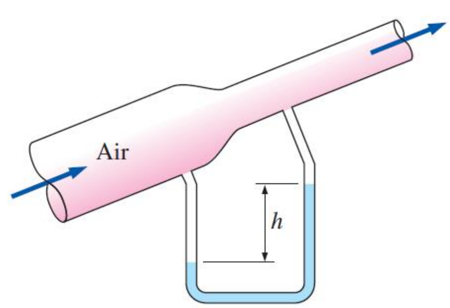
P3.120

Problem 4 (P3.112 White): A jet of alcohol strikes the vertical plate in Fig. P3.112. A force $F \approx$ 425 N is required to hold the plate stationary. Assuming there are no losses in the nozzle, estimate (a) the mass flow rate of alcohol and (b) the **absolute** pressure at section 1.
Note: Take the density of water to be ρ = 998kg/m^3



P3.112

Problem 5: Air at 105 kPa and 37°C flows upward through a 6-cm-diameter inclined pipe at a rate of 65 L/s. The pipe diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water (ρ = 1000 kg/m^3) manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height, h between the fluid levels of the two arms of the manometer. Take that the gas constant of air is R = 0.287 kPa·m^3/kg·K.



November 5, 2020

Solutions:

Problem 1: By continuity, establish V_2:

$$V_2 = V_1(D_1/D_2)^2 = 1.7(3/1)^2 = 15.3\ \frac{\text{ft}}{\text{s}}$$

Now apply Bernoulli between 1 and 2 to establish the pressure at section 2:

$$p_1 + \frac{\rho}{2}V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}V_2^2 + \rho g z_2,$$

or: $p_1 + (1.94/2)(1.7)^2 + 0 \approx 0 + (1.94/2)(15.3)^2 + (62.4)(10),\qquad p_1 = 848\ \text{lb/ft}^2$

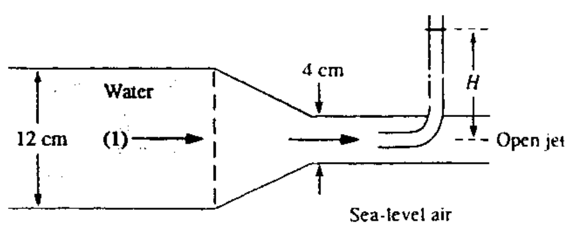
This is gage pressure. Now the manometer *reads* gage pressure, so

$$p_1 + \rho_{\text{water}} g(2\text{ft}) - \rho_{\text{merc}} gh = 0,$$
$$p_1 + \rho_{\text{water}} g(2\text{ft}) = \rho_{\text{merc}} gh,$$

Solve for h

$$h = \frac{\rho_{\text{water}} g(2\text{ft}) + p_1}{\rho_{\text{merc}} g} = \frac{62.4(2\text{ft}) + 848\ \text{lb/ft}^2}{846} = 1.14 ft$$

Problem 2:



Writing Bernoulli and continuity between pipe and jet yields jet velocity, using absolute pressures

$$p_1 - p_a = \frac{\rho}{2}V_{jet}^2\left[1 - \left(\frac{D_{jet}}{D_1}\right)^4\right] = 110000 - 101350 = \frac{998}{2}V_{jet}^2\left[1 - \left(\frac{4}{12}\right)^4\right],$$

$$V_{jet} = \sqrt{\frac{2(p_1 - p_a)}{\rho\left[1 - \left(\frac{D_{jet}}{D_1}\right)^4\right]}} = \sqrt{\frac{2 \times (110000 - 101350)}{998 \times \left(1 - \left(\frac{4}{12}\right)^4\right)}} = 4.19 m/s$$

$$solve\quad V_{jet} = \mathbf{4.19}\ \frac{\text{m}}{\text{s}}$$

$$\textit{Then the mass flow is}\quad \dot{m} = \rho A_{jet}V_{jet} = 998\frac{\pi}{4}(0.04)^2(4.19) = \mathbf{5.25}\ \frac{\mathbf{kg}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(a)}$$

(b) The water in the stagnation tube will rise above the jet surface by an amount equal to the stagnation pressure head of the jet:

$$H = \frac{V_{jet}^2}{2g} = \frac{(4.19)^2}{2(9.81)} = 0.89\ m \quad Ans.(b)$$

Problem 3:

For gasoline (a) take $\rho = 1.32$ slug/ft^3.

For nitrogen (b), R ≈ 297 J/kg·°C and

$\rho = \text{p/RT} = (101350)/[(297)(293)] \approx 1.165\ \text{kg/m}^3 = 0.00226$ slug/ft^3.

For mercury, take

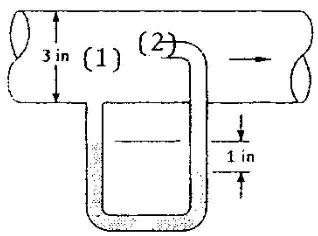


Fig. P3.120

The pitot tube (2) reads stagnation pressure, and the wall hole (1) reads static pressure. Thus Bernoulli's relation becomes, with Δz = 0,

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2, \quad \text{or} \quad V_1 = \sqrt{2(p_2 - p_1)/\rho}$$

The pressure difference is found from the manometer reading, for each fluid in turn:

(a) Gasoline: $\Delta p = (\rho_{\text{Hg}} - \rho)gh = (26.34 - 1.32)(32.2)(1/12\ \text{ft}) \approx 67.1\ \text{lbf/ft}^2$

$$V_1 = [2(67.1)/1.32]^{1/2} = 10.1\ \frac{\text{ft}}{\text{s}},\quad Q = V_1A_1 = (10.1)\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2 = \mathbf{0.495}\ \frac{\mathbf{ft^3}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(a)}$$

(b) N_2: $\Delta p = (\rho_{\text{Hg}} - \rho)gh = (26.34 - 0.00226)(32.2)(1/12) \approx 70.7\ \text{lbf/ft}^2$

$$V_1 = [2(70.7)/0.00226]^{1/2} = 250\ \frac{\text{ft}}{\text{s}},\quad Q = V_1A_1 = (250)\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2 \approx \mathbf{12.3}\ \frac{\mathbf{ft^3}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(b)}$$

November 5, 2020

Problem 4: A momentum analysis of the plate will give

$$F = \dot{m}V_2 = \rho A_2 V_2^2 = 0.79(998)\frac{\pi}{4}(0.02)^2 V_2^2 = 425 \text{ N},$$

solve for $V_2 \approx 41.4$ m/s

$$and\ \dot{m} = 0.79(998)(\pi/4)(0.02)^2(41.4) \approx 10.3\frac{kg}{s} \qquad Ans.(a)$$

We find V_1 from mass conservation and then find p_1 from Bernoulli:

Incompressible mass conservation: $V_1 = V_2(D_2/D_1)^2 = (41.4)(2/5)^2 = 6.62$ m/s

Applying Bernoulli, $z_1 = z_2$:

$$p_1 = p_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) = 101000 + \frac{0.79(998)}{2}[(41.4)^2 - (6.62)^2] \approx 759.4\ kPa\ Ans.(b)$$

Problem 5:

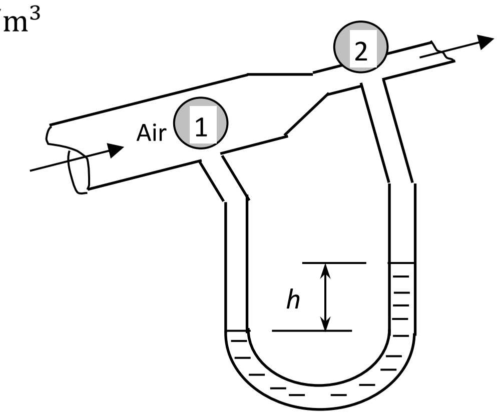
Assumptions **1** The flow through the duct is steady, incompressible and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** Air is an ideal gas. **3** The effect of air column on the pressure change is negligible because of its low density. **4** The air flow is parallel to the entrance of each arm of the manometer, and thus no dynamic effects are involved.

Properties We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m^3/kg·K.

Analysis We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \rho_{\text{air}}\frac{V_2^2 - V_1^2}{2} + \rho g(z_2 - z_1)$$

where $\rho_{air} = \frac{P}{RT} = \frac{105 \text{ kPa}}{(0.287 \text{ kPa}\cdot m^3/\text{kg}\cdot K)(37+273 \text{ K})} = 1.180 \text{ kg/m}^3$



$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.065 \text{ m}^3/\text{s}}{\pi(0.06 \text{ m})^2/4} = 22.99 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.065 \text{ m}^3/\text{s}}{\pi(0.04 \text{ m})^2/4} = 51.73 \text{ m/s}$$

Substituting,

$$P_1 - P_2 = (1.180\ \mathrm{kg/m^3})\frac{(51.73\ \mathrm{m/s})^2 - (22.99\ \mathrm{m/s})^2}{2} + (1.180\ \mathrm{kg/m^3}) \times 9.81 \times 0.2$$
$$= 1269.31\ \mathrm{Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P = \rho_{water} g h$ to be

$$h = \frac{P_1 - P_2}{\rho_{water} g} = \frac{1269.31\ \mathrm{N/m^2}}{(1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})}\left(\frac{1\ \mathrm{kg \cdot m/s^2}}{1\ \mathrm{N}}\right) = 0.1293\ \mathrm{m} = 12.9\ \mathrm{cm}$$