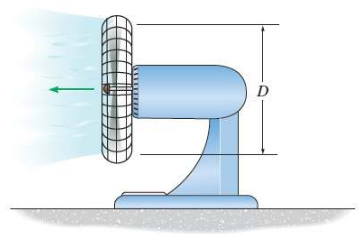
Assignment#8-F20+Solutions

Fluid Mechanics (University of Windsor)

Mechanical, Automotive and Materials Engineering
Fluid Mechanics I
MECH-3233-F20

Assignment Problems Set #8

Problem 1: The pressure difference Δp of air that flows through a fan is a function of the diameter D of the blade, its angular rotation ω, the density ρ of the air, and the flow Q. Using D, ρ and ω as repeating variables, determine the relation between Δp and these parameters.



Problem 2: (P5.37 White) The volume flow Q through an orifice plate is a function of pipe diameter D, pressure drop Δp across the orifice, fluid density ρ and viscosity μ, and orifice diameter d. Using D, ρ, and Δp as repeating variables, express this relationship in dimensionless form.

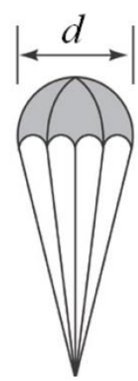
Problem 3: (P5.39 White) The volume flow Q over a certain dam is a function of dam width b, gravity g, and the upstream water depth H above the dam crest. It is known that Q is proportional to b $(Q \propto b)$. If $b = 120$ ft and $H = 15$ inches, the flow rate is 600 ft^3/s. What will be the flow rate if $H = 3$ ft?

Problem 4: (P5.73 White) The power P generated by a certain windmill design depends upon its diameter D, the air density ρ, the wind velocity V, the rotation rate Ω, and the number of blades n. Use (***D*, $\boldsymbol{\rho}$, *V***) as repeating variables.

(a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when $V = 40$ m/s and when rotating at 4800 rev/min.

(b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude ($\rho_{air} = 1.0067$ kg/m^3)?

(c) What is the appropriate rotation rate of the prototype? Take the density of air in see level to be $\rho_{air} = 1.2255$ kg/m^3.



Problem 5: Flow characteristics for a 30-ft-diameter prototype parachute are to be determined by tests of a 1-ft-diameter model parachute in a water tunnel. Some data collected with the model parachute indicate a drag of 17 lb when the water velocity is 4 ft/s.

(a) Use the dimensional analysis and find a suitable pi parameter for this problem.

(b) Use the model data to predict the drag on the prototype parachute falling through the air at 10 ft/s. Assume the drag to be a function of the velocity, V, the fluid density, ρ, and the parachute diameter, d. ($\rho_{water} = 1.94$ slugs/ft^3 and $\rho_{air} = 2.38 \times 10^{-3}$ slugs/ft^3)

Solutions:
Problem 1:

Physical Variables. There are $n = 6$ variables and the unknown function is $f(\Delta p, D, \omega, \rho, Q) = 0$. Using the $F - L - T$ system,

Δp	FL^{-2}
D	L
ω	T^{-1}
ρ	FT^2L^{-4}
Q	L^3T^{-1}

Here, all three base dimensions are used, so that $m = 3$. Thus, there are

$$n - m = 5 - 3 = 2\Pi \text{ terms}$$

Dimensional Analysis. Here, D, ω, and ρ are chosen as $m = 3$ repeating variables. Thus, the q variables are Δp for Π_1 and Q for Π_2.

$$\Pi_1 = D^a\omega^b\rho^c\Delta p = (L^a)(T^{-b})(F^cT^{2c}L^{-4c})(FL^{-2}) = F^{c+1}L^{a-4c-2}T^{-b+2c}$$

F: $\quad 0 = c + 1$

L: $\quad 0 = a - 4c - 2$

T: $\quad 0 = -b + 2c$

Solving, $a = -2$, $b = -2$, and $c = -1$. Thus,

$$\Pi_1 = D^{-2}\omega^{-2}\rho^{-1}\Delta p = \frac{\Delta p}{\rho\omega^2D^2}$$

$$\Pi_2 = D^d\omega^e\rho^fQ = (L^d)(T^{-e})(F^fT^{2f}L^{-4f})(L^3T^{-1}) = F^fL^{d-4f+3}T^{-e+2f-1}$$

F: $\quad 0 = f$

L: $\quad 0 = d - 4f + 3$

T: $\quad 0 = -e + 2f - 1$

Solving, $d = -3$, $e = -1$, and $f = 0$. Thus,

$$\Pi_2 = D^{-3}\omega^{-1}\rho^0Q = \frac{Q}{\omega D^3}$$

Therefore, the function can be written as

$$f_1\left(\frac{\Delta p}{\rho\omega^2D^2}, \frac{Q}{\omega D^3}\right) = 0$$

Solving for Δp,

$$\frac{\Delta p}{\rho\omega^2D^2} = f\left(\frac{Q}{\omega D^3}\right)$$

$$\Delta p = \rho\omega^2D^2f\left(\frac{Q}{\omega D^3}\right) \qquad \textbf{Ans.}$$

Problem 2: There are 6 variables and 3 primary dimensions (MLT), and we already know that $j = 3$, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$\Pi_1 = D^a\rho^b\Delta p^cQ$; **Solve for** $a = -2,\ b = 1/2,\ c = -1/2.$ **Thus** $\quad \Pi_1 = \dfrac{Q\,\rho^{1/2}}{D^2\,\Delta p^{1/2}}$

$\Pi_2 = D^a\rho^b\Delta p^cd$; **Solve for** $a = -1\ \ b = 0\ \ c = 0.$ **Thus** $\quad \Pi_1 = \dfrac{d}{D}$

$\Pi_3 = D^a\rho^b\Delta p^c\mu$; **Solve for** $a = -1,\ b = -1/2,\ c = -1/2.$ **Thus** $\quad \Pi_1 = \dfrac{\mu}{D\,\rho^{1/2}\Delta p^{1/2}}$

The final requested orifice-flow function is:

$$\frac{Q\rho^{1/2}}{D^2\,\Delta p^{1/2}} \;=\; fcn\left(\frac{d}{D},\; \frac{\mu}{D\rho^{1/2}\Delta p^{1/2}}\right) \qquad\qquad Ans.$$

Problem 3: Work this problem in BG units. Given that $Q \propto b$, use dimensional analysis:

$$\begin{array}{ccc} Q/b & = & fcn(\;g\;,\quad H\;) \\ \{L^3/T/L\} & & \{L/T^2\}\quad\{L\} \end{array}$$

Assembling the dimensional matrix

	Q/b	g	H
M	0	0	0
L	2	1	1
T	-1	-2	0

The rank of the matrix is m = 2 since the largest matrix with non zero determinant is 2 × 2.

Then $n = 3$ and $m = 2$ (L and T), so we expect $n - m = 1$, or only *one* Pi group, which can be found by:

$$\Pi_1 = g^{x_1}H^{y_1}\frac{Q}{b}$$

To find the exponents substitute with the basic dimensions and solve

$$\Pi_1 = (L^1T^2)^{x_1}L^{y_1}(L^2T^{-1}) = M^0L^0T^0$$

$$\text{L:}\quad x_1 + y_1 + 2 = 0$$

$$\text{T:}\quad -2x_1 - 1 = 0$$

Solving for x_1 = -1/2 and y_1 = -3/2 and the dimensionless group is

$$\Pi_1 \;=\; \frac{Q}{b\,g^{1/2}H^{3/2}} \;=\; constant \quad (dimensionless)$$

Introduce the given data to find the dimensionless constant:

$$\Pi_1 = \frac{(600\,ft^3/s)/(120\,ft)}{(32.2\,ft/s^2)^{1/2}(1.25\,ft)^{1.5}} \;=\; \mathbf{0.63}$$

Then, for the new water depth H = 3 ft, we obtain, by scaling,

$$Q = 0.63(120ft)(32.2ft/s^2)^{\frac{1}{2}}(3ft)^{\frac{3}{2}} = 2229.1\;ft^3/s \qquad Ans.$$

Problem 4: a) For the function $P = f(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using $(\boldsymbol{D, \rho, V})$ as repeating variables, we obtain the desired dimensionless function:

$$\frac{P}{\rho D^2 V^3} = f\left(\frac{\Omega D}{V},\ n\right) \qquad Ans.\ (a)$$

(c) “Geometrically similar” requires that *n* is the same for both windmills. For “dynamic similarity,” the advance ratio ($\Omega D/V$) must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{\left(\frac{4800}{60}\right) rev/s(0.5m)}{40\ m/s} = 1.0$$

$$\left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ \frac{m}{s}} = 1$$

$$\Omega_{proto} = 2.4\ rev/s \times 60 = 144\ rev/\min \qquad Ans.(c)$$

(b) At 2000 m altitude, $\rho = 1.0067$ kg/m³. At sea level, $\rho = 1.2255$ kg/m³. Since $\Omega D/V$ and *n* are the same, it follows that the power coefficients equal for model and prototype:

$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = 0.138 = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$solve \quad \mathbf{P_{proto} = 5990\ W \approx 6\ kW} \quad Ans.\ \text{(b)}$$

Problem 5:

(a) Pi parameter

$$D = f(V, \rho, d)$$

Where, $D \doteq F, V \doteq LT^{-1}, \rho \doteq FL^{-4}T^2, d = L$, and 4 – 3 = 1 pi parameter.

Thus,

$$\Pi = DV^a\rho^b d^c = (F)(LT^{-1})^a(FL^{-4}T^2)^b(L)^c = F^0L^0T^0$$

or

$$\Pi = \frac{D}{\rho V^2 d^2}$$

(b) For similarity between model and prototype,

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}$$

So that

$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2 \left(\frac{d}{d_m}\right)^2 D_m$$

$$= \left(\frac{2.38 \times 10^{-3}\, \frac{slugs}{ft^3}}{1.94\, \frac{slugs}{ft^3}}\right)\left(\frac{10\, \frac{ft}{s}}{4\, \frac{ft}{s}}\right)^2 \left(\frac{30\, ft}{1\, ft}\right)^2 (17\, lb) = 117\, lb$$