Assignment#7-F20+Solutions

Fluid Mechanics (University of Windsor)

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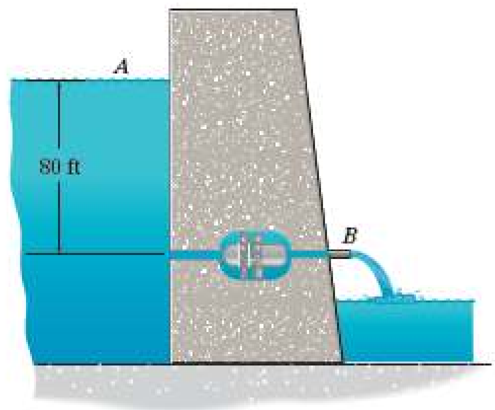
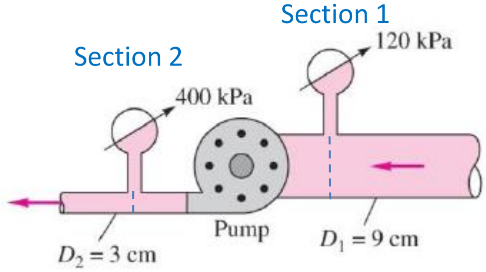
Mechanical, Automotive and Materials Engineering
Fluid Mechanics I
MECH-3233-F20

Assignment Problems Set #7

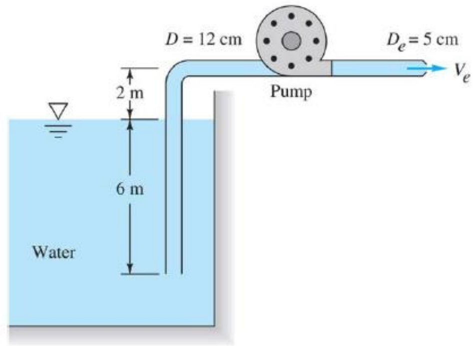
Problem 1: Water ($\rho = 1.94$ slug/ft^3) from the reservoir passes through a turbine at the rate of 18 ft^3/s. If it is discharged at B with a velocity of 15 ft/s, and the turbine withdraws 100 hp, determine the head loss in the system.



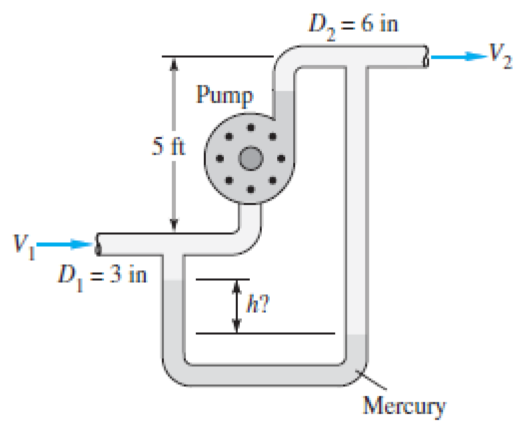
Problem 2: The horizontal pump in the Figure below discharges water at 57 m^3/h. The losses between 1 and 2 are given by $h_L = K\frac{V_1^2}{2g}$, where $K \approx 7.5$ is a dimensionless loss coefficient. Take the kinetic energy correction factor $\alpha \approx 1.06$ for both sections 1 and 2 and find the power delivered to the water by the pump (water density is 1000 kg/m^3).



Problem 3: When the pump in the Figure below draws 220 m^3/hr of water at 20°C (ρ = 998 kg/m^3) from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.
Note: Take turbulent pipe flow kinetic correction factor $\alpha = 1.11$.

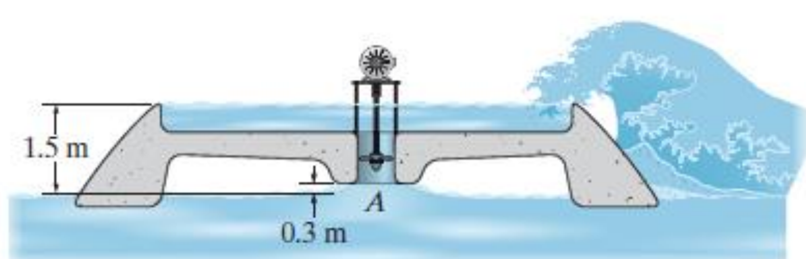


Problem 4 (P3.185 White) Kerosene at 20°C ($\rho = 804$ kg/m^3) flows through the pump in Fig. P3.185 at 2.3 ft^3/s. Head losses between 1 and 2 are 8 ft, and the pump delivers 8 hp to the flow. What should the mercury ($\gamma_{Hg} = 846$ lbf/ft^3) manometer reading h ft be?



P3.185

Problem 5: The wave overtopping device consists of a floating reservoir that is continuously filled by waves, so that the water level in the reservoir is always higher than that of the surrounding ocean. As the water drains out at A, the energy is drawn by the low-head hydro turbine, which then generates electricity. Determine the power that can be produced by this system if the water level in the reservoir is always 1.5 m above that in the ocean. The waves add 0.3 m^3/s to the reservoir, and the diameter of the tunnel containing the turbine is 600 mm. The head loss through the turbine is 0.2 m. Take $\rho_w = 1050$ kg/m^3.



Solutions:

Problem 1:

$$\dot{W}_s = Q\gamma h_s$$

$$(100\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = (18\text{ ft}^3/\text{s})(62.4\text{ lb/ft}^3)h_s$$

$$h_{turb} = \quad h_s = 48.97\text{ ft}$$

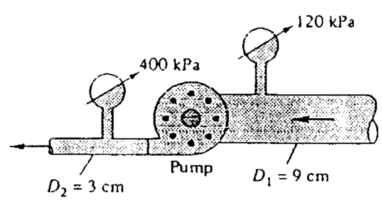
Energy Equation. Take the water from A to B to be the control volume. Since A and B are both free surfaces, $p_A = p_B = 0$. Also, due to the large source at the reservoir, $V_A = 0$. If the datum passes through B, $z_A = 80$ ft and $z_B = 0$.

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$0 + 0 + 80\text{ ft} + 0 = 0 + \frac{(15\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 + 48.97\text{ ft} + h_L$$

$$h_L = 27.5\text{ ft} \qquad \textbf{Ans.}$$

Problem 2: First we need to compute the velocities at sections (1) and (2):



$$\bar{V}_1 = \frac{Q}{A_1} = \frac{57/3600}{\pi(0.045)^2} = 2.49\ \frac{m}{s}; \quad \bar{V}_2 = \frac{Q}{A_2} = \frac{57/3600}{\pi(0.015)^2} = 22.4\ \frac{m}{s}$$

$$\frac{p_1}{\rho g} + \frac{\alpha\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha\bar{V}_2^2}{2g} + z_2 + h_L - h_p,$$

$$h_L = 7.5\frac{V_1^2}{2g} = 7.5\frac{(2.49\frac{m}{s})^2}{2\times 9.81 m/s^2} = 2.37m$$

or: $\frac{120000}{9810} + \frac{1.06(2.49)^2}{2(9.81)} + 0 = \frac{400000}{9810} + \frac{1.06(22.4)^2}{2(9.81)} + 0 + 2.37 - h_p$, solve for $h_p = 57.69$ m

Then the pump power is $P_p = \gamma Q h_p = 9810\left(\frac{57}{3600}\right)(57.69) = 8960.7\text{ W} = 8.96\ kW \quad Ans.$

Problem 3:

Continuity:

$$Q = 220\ \frac{m^3}{hr} = \frac{(220)}{(3600)}\ \frac{m^3}{s} = 0.0611\ \frac{m^3}{s}$$

$$Q = A_e V_e$$

$$V_e = \frac{Q}{A_e} = \frac{Q}{\pi \frac{D_e{}^2}{4}} = \frac{(0.0611)}{\pi \frac{(0.05)^2}{4}} = 31.12\ \frac{m}{s}$$

Take point 1 at the free surface and 2 at the pipe exit:

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_{turbine} - h_{pump} + h_{friction}$$

Pressure is zero at the free surface and at the exit to atmosphere. Also assume that velocity at the free surface is almost zero:

$$0 + 0 + z_1 = 0 + \frac{\alpha_e V_e^2}{2g} + z_2 + 0 - h_{pump} + h_{friction}$$

$$h_{pump} = \frac{\alpha_e V_e^2}{2g} + (z_2 - z_1) + h_{friction}$$

$$h_{pump} = \frac{(1.11)(31.12)^2}{2(9.81)} + (2\ m) + (5\ m) = 61.79\ m$$

The pump power is:

$$P_{pump} = \rho g Q h_{pump} = (998)(9.81)(0.0611)(61.79) = 36{,}962.3\ W = 36.96\ kW$$

Problem 4: First establish the two velocities:

$$V_1 = \frac{Q}{A_1} = \frac{\frac{2.3\text{ ft}^3}{s}}{(\pi/4)(3/12\text{ ft})^2} = 46.9\ \frac{\text{ft}}{s};$$

$$V_2 = \frac{1}{4} V_1 = 11.7\ \frac{\text{ft}}{s}$$

For kerosene take $\rho = 804$ kg/m^3 = 1.56 slug/ft^3, or γk = 1.56(32.2) = 50.2 lbf/ft^3. For mercury take γm = 846 lbf/ft^3. Then apply a manometer analysis to determine the pressure difference between points 1 and 2:

$$p_2 - p_1 = (\gamma_m - \gamma_k)h - \gamma_k \Delta z = (846 - 50.2)h - \left(50.2 \frac{lbf}{ft^3}\right)(5\,ft) = 796h - 251\ \frac{lbf}{ft^2}$$

Now apply the steady flow energy equation between points 1 and 2:

$$\frac{p_1}{\gamma_k} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_k} + \frac{V_2^2}{2g} + z_2 + h_f - h_p, \quad \text{where } h_p = \frac{P}{\gamma_k Q} = \frac{8(550)\,ft{\cdot}lbf/s}{(50.2)(2.3\,ft^3/s)} = 38.1\,ft$$

$$\text{Thus:} \quad \frac{p_1}{50.2} + \frac{(46.9)^2}{2(32.2)} + 0 = \frac{p_2}{50.2} + \frac{(11.7)^2}{2(32.2)} + 5 + 8 - 38.1\,ft \quad \text{Solve } p_2 - p_1 = 2866\ \frac{lbf}{ft^2}$$

Now, with the pressure difference known, apply the manometer result to find h:

$$p_2 - p_1 = 2866 = 796h - 251, \quad \text{or:} \quad h = \frac{2866 + 251\ lbf/ft^2}{796\ lbf/ft^3} = \mathbf{3.92\ ft} \quad \textit{Ans.}$$

Problem 5:

From the discharge,

$$Q = V_{out} A_{out}; \qquad 0.3\ \text{m}^3/\text{s} = V_{out}\left[\pi(0.3\ \text{m})^2\right]$$

$$V_{out} = 1.061\ \text{m/s}$$

Take the water within the turbine to be the control volume. We will apply the energy equation between the inlet and the outlet.

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{pump} = \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + h_{turb} + h_L$$

Here, $V_{in} = 0$ since the inlet is the surface of a large reservoir. $p_{in} = p_{out} = p_{atm} = 0$ since the inlet and outlet are exposed to the atmosphere. Here, the datum is set at the ocean water level. Then, $z_{in} = 1.5$ m and $z_{out} = 0.3$ m.

$$0 + 0 + 1.5\ \text{m} + 0 = 0 + \frac{(1.061\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.3\ \text{m} + h_{turb} + 0.2\ \text{m}$$

$$h_{turb} = 0.9426\ \text{m}$$

The turbine is

$$\begin{aligned}\dot{W}_s &= Q\gamma_{sw}h_{turb}\\ &= (0.3\ \text{m}^3/\text{s})\left[(1050\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right](0.9426\ \text{m})\\ &= 2912.83\ \text{W}\\ &= 2.91\ \text{kW} \qquad \textbf{Ans.}\end{aligned}$$