Assignment#9-F20+Solutions

Fluid Mechanics (University of Windsor)

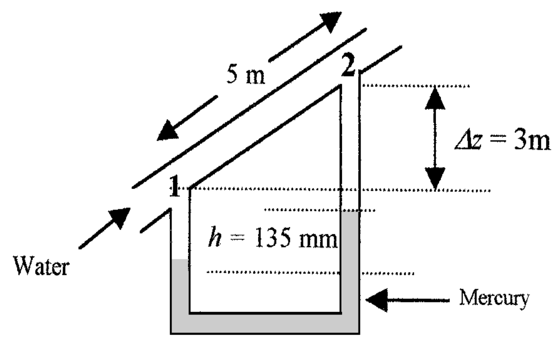
27 November 2020

Mechanical, Automotive and Materials Engineering
Fluid Mechanics I
MECH-3233-F20

Assignment Problems Set #9

Problem 1: (White 6.11) Water at 20°C (γ_w = 9790 N/m³) flows upward at 4 m/s in a 6-cm-diameter pipe. The pipe length between points 1 and 2 is 5m, and point 2 is 3m higher as shown in the Figure to the right. A mercury manometer, connected between 1 and 2, has a reading h = 135 mm, with p_1 higher.

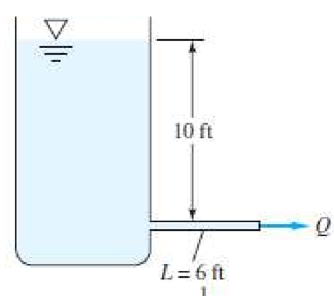
(a) What is the pressure change ($p_1 - p_2$)?
(b) What is the head loss, in meters?
(c) Is the manometer reading proportional to head loss? Explain.
(d) What is the friction factor of the flow? Is the flow laminar or turbulent?



Problem 2: (White 6.13) A soda straw is 20 cm long and 2 mm in diameter. It delivers cold cola, approximated as water at 10°C, at a rate of 3 cm³/s.

(a) What is the head loss through the straw?
What is the axial pressure gradient $\partial p/\partial x$ if the flow is (b) vertically up or (c) horizontal?
Note: For water at 10°C, take ρ = 1000 kg/m³ and μ = 1.307×10⁻³ kg/m·s.

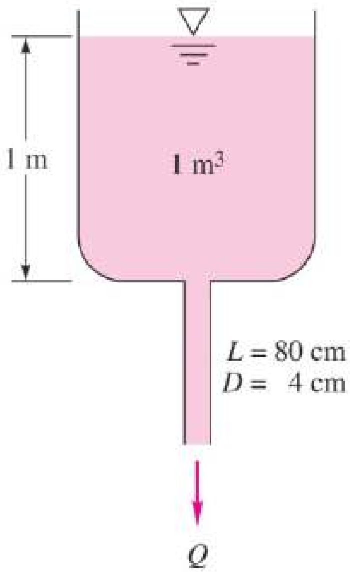
Problem 3: (White 6.19) An oil (SG = 0.9) issues from the pipe in Fig. P6.19 at Q = 35 ft³/h. What is the kinematic viscosity of the oil in ft³/s? Is the flow laminar?
Note: Neglect minor losses.



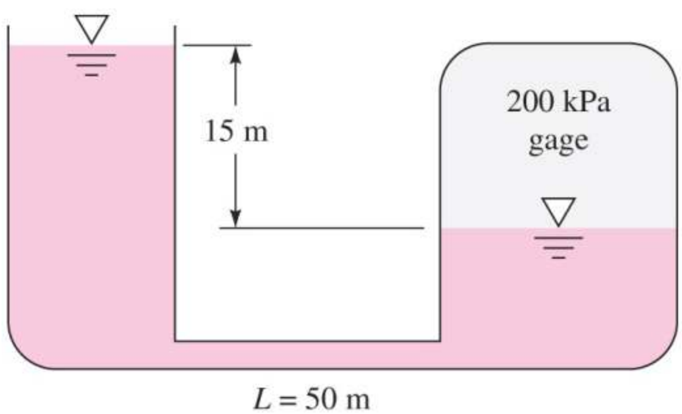
P6.19

Problem 4: A tank contains 1 m^3 of water at 20°C (ρ=998 kg/m^3, μ=0.001 Pa.s) and has a drawn outlet tube with D = 4 cm and L=80 cm at the bottom, as shown in the Figure below. Take ε = 0.0015 mm and find the outlet flow rate Q in m^3/s.

Note: Use trial and error procedure



Problem 5: In the Figure below the connecting pipe is commercial steel (ε = 0.046 mm) 6 cm in diameter. Estimate the flow rate, in m^3/s, if the fluid is water at 20°C (ρ = 998 kg/m^3, μ = 0.001 Pa.s).



Solutions:

Problem 1: By moving through the manometer, we obtain the pressure change between points 1 and 2 as follows:

$$p_1 + \gamma_w h - \gamma_m h - \gamma_w \Delta z = p_2$$

Substituting

$$p_1 - p_2 = \left(133100 - 9790\ \frac{N}{m^3}\right)(0.135\ m) + \left(9790\ \frac{N}{m^3}\right)(3\ m) = \mathbf{46.01\ kPa} \qquad Ans.(a)$$

Using the energy equation between point 1 and 2 the, head loss h_L is given by

$$h_L = \frac{\Delta p}{\gamma_w} - \Delta z = \frac{46010\ Pa}{9790 N/m^3} - 3\ m = \mathbf{1.7\ m} \quad Ans.(b)$$

By comparing the manometer relation to the head-loss relation above, we find that:

$$h_L = \frac{(\gamma_m - \gamma_w)}{\gamma_w} h \text{ and thus head loss is proportional to manometer reading.} \qquad Ans.(c)$$

From the Darcy–Weisbach equation the friction factor is

$$f = h_L \frac{d}{L} \frac{2g}{V^2} = (1.7\ m)\left(\frac{0.06\ m}{5\ m}\right)\frac{2(9.81 m/s^2)}{(4 m/s)^2} = \mathbf{0.025} \quad \boldsymbol{Ans.(d)}$$

Problem 2: For water at 10°C, take $\rho = 1000$ kg/m^3 and $\mu = 1.307\times10^{-3}$ kg/m·s.

Check Re:

$$\text{Re} = \frac{4\rho Q}{\pi \mu d} = \frac{4(1000)\left(\frac{3\times10^{-6}\ \text{m}^3}{s}\right)}{\pi(1.307\times10^{-3})(0.002)} = 1461 < 2300 \text{ (laminar flow)}$$

The head loss is, $h_L = \frac{128 \mu L Q}{\pi \rho g D^4} = \frac{128(1.307\times10^{-3})(0.2)(3\times10^{-6})}{\pi(1000)(9.81)(0.002)^4} = 0.204\ m \qquad Ans.(a)$

If the straw is *horizontal*, then the pressure gradient is simply due to the head loss:

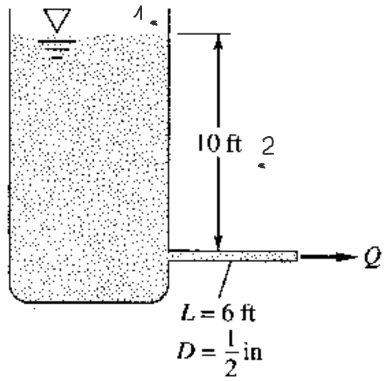
$$\frac{\Delta p}{L}\Big|_{\text{horiz}} = \frac{\rho g h_L}{L} = \frac{1000(9.81)(0.204\ \text{m})}{0.2\ \text{m}} = 10006\ \frac{Pa}{m} \qquad Ans.(c)$$

If the straw is *vertical*, with flow *up*, the head loss and elevation change add together:

$$\frac{\Delta p}{L}\Big|_{\text{vertical}} = \frac{\rho g (h_L + \Delta z)}{L} = \frac{1000(9.81)(0.204 + 0.2)}{0.2} = \mathbf{19816}\ \frac{Pa}{m} \quad Ans.(b)$$

Problem 3: Apply steady-flow energy:

$$\frac{p_{\text{atm}}}{\rho g} + 0 + z_1 = \frac{p_{\text{atm}}}{\rho g} + \frac{V_2^2}{2\text{g}} + z_2 + h_L,$$



where $V_2 = \frac{Q}{A} = \frac{35/3600}{\pi(0.25/12)^2} \approx 7.13\ \frac{\text{ft}}{\text{s}}$

$$\text{Solve} \quad h_L = z_1 - z_2 - \frac{V_2^2}{2\text{g}} = 10 - \frac{(7.13)^2}{2(32.2)} = 9.21\ \text{ft}$$

Assuming laminar pipe flow, use to relate head loss to viscosity:

$$h_L = 9.21\ \text{ft} = \frac{128\nu \text{LQ}}{\pi \text{gd}^4} = \frac{128(6)(35/3600)\nu}{\pi(32.2)(0.5/12)^4}, \qquad solve\ \ \nu = \frac{\mu}{\rho} \approx \mathbf{3.76 \times 10^{-4}} \frac{\boldsymbol{ft^2}}{\boldsymbol{s}} \quad Ans.$$

Check Re = 4Q/($\pi\nu$d) = 4(35/3600)/[π(3.76E–4)(0.5/12)] ≈ 790 (OK, laminar)

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Problem 4:

Analysis

$$\varepsilon/d = 0.0015/40 \approx 0.0000375$$

The steady-flow energy equation, with p1 = p2 and V1 ≈ 0, gives

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \Delta z - \frac{V^2}{2g}$$

$$\frac{V^2}{2g}\left(1+\frac{0.8}{0.04}f\right) \approx 1.8 \text{ m}$$

$$V^2 \approx \frac{35.32}{1+20f}$$

$$\text{Guess} \quad f \approx 0.015, \quad V = \left[\frac{35.32}{1+20(0.015)}\right]^{1/2} \approx 5.21 \ \frac{m}{s}, \quad Re = \frac{998(5.21)(0.04)}{0.001} \approx 208000$$

$$f_{better} \approx 0.0158, \quad V_{better} \approx 5.18 \text{ m/s}, \quad Re \approx 207000 \text{ (converged)}$$

Thus $V \approx 5.18$ m/s, $Q = (\pi/4)(0.04)^2(5.18) = 0.00651\ m^3/s \approx \mathbf{23.4\ m^3/h}$.

Problem 5:

Energy equation with $p_1 = V_1 = V_2 = 0$

$$z_1 = \frac{p_2}{\rho g} + z_2 + h_f$$

$$h_f = (z_1 - z_2) - \frac{p_2}{\rho g} = 15 - \frac{200000}{998 * 9.81} = -5.43\ m\ (flow\ to\ left)$$

Major loss

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{50}{0.06}\frac{V^2}{2 * 9.81} = 5.43$$

$$fV^2 = 0.1278\ (1)$$

$$f = f\left(\frac{\varepsilon}{D}, Re\right) = f\left(\frac{0.046}{60}, Re\right) = f(0.000767, Re)\ (2)$$

$$Re = \frac{\rho D V}{\mu} = \frac{998 * 0.06 * V}{0.001} = 59880V\ (3)$$

Solving equation 1-3 iteratively

$$f_{guess} = 0.0184 \rightarrow V = 2.64\frac{m}{s} \rightarrow Re = 158000 \rightarrow f = 0.0204$$

$$f_{guess} = 0.0204 \rightarrow V = 2.5\frac{m}{s} \rightarrow Re = 149700 \rightarrow f = 0.0205$$

Flow rate calculation

$$Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}0.06^2 * 2.5 = 0.007\frac{m^3}{s}$$