Dynamic Force Analysis

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Contents

List of Figures

List of Tables

1 Introduction

Mechanical systems involving the interaction of various components play a crucial role in understanding and optimizing the performance of machines. In this lab report, a dynamic force analysis will be conducted for an opposed two-cylinder crank/connecting rod/slider arrangement.

1.1 Objectives

The primary objective of this analysis is to delve into the kinematic and kinetic aspects of the system. Through numerical and symbolic calculations, we aim to determine key parameters, including angular velocities, angular accelerations, transmitted forces, input torque for constant angular velocity, and out-of-balance forces. These parameters will be crucial in comprehending the system's behavior and optimizing its design.

The investigation encompasses a time-dependent analysis covering two complete revolutions of the crank, and the obtained results will be graphically illustrated.

1.2 Approach

The analysis will be carried out utilizing computational software, namely MATLAB, and analytical methods. The analytical equations developed in class will be employed to conduct a comprehensive analysis of the system, particularly in determining the out-of-balance forces. The MATLAB software will be used to numerically solve the system's equations of motion.

This dual approach, combining computational tools and analytical methods, ensures a robust and comprehensive understanding of the mechanical system under consideration.

1.3 Literature Review

In the domain of dynamic force analysis for levitated planar actuators, Rovers[1] makes a substantial contribution. Their paper meticulously explores the dynamic forces and torques exerted within a moving planar actuator, shedding light on crucial aspects of its behavior.

The work by Korayem[2] stands out for its notable significance in the dynamic analysis of tapping-mode Atomic Force Microscopy (AFM). The paper focuses specifically on capillary force interactions, enriching our understanding of the intricacies involved.

Similarly, Williams[3] contributes significantly to the field with a study centered on the dynamic force analysis of planar mechanisms. The insights provided in this work are valuable for comprehending the nuanced behavior of such systems.

Cheng-ge[4] adds to the discourse with noteworthy research on the dynamic force analysis of power capacitors within a frame context. The detailed exploration carried out in this paper makes a substantial contribution to the relevant body of knowledge.

Lastly, the work by Schütte[5] holds considerable importance, delving into the discussion of ConDroid, a tool designed for targeted dynamic analysis of Android applications. This contribution extends the scope of analysis beyond mechanical systems, showcasing the interdisciplinary nature of dynamic force examination.

Collectively, these papers form a robust foundation for the comprehensive analysis of the mechanical system under consideration.

[1] Rovers, (2012)
[2] Korayem, (2011)
[3] Williams, (1981)
[4] Cheng-ge, (2010)
[5] Schütte, (2015)

2 Methodology

The methodology section will outline the approach taken to conduct the analysis and the tools utilized.

2.1 Analytical Approach

The analytical approach will be employed to determine the out-of-balance forces and the input torque for constant angular velocity. The equations of motion will be derived using the Newton-Euler method, and the out-of-balance forces will be determined using the method of dynamically equivalent masses and force balancing.

The Newton-Euler method is a powerful tool for deriving the equations of motion for a mechanical system. It involves the application of Newton's second law of motion and Euler's equations of motion. The method is particularly useful for systems with multiple degrees of freedom.

The method of dynamically equivalent masses and force balancing is a straightforward approach for determining the out-of-balance forces. It involves the application of the principle of virtual work, and it is particularly useful for systems with multiple degrees of freedom.

2.2 Computational Approach

The computational approach will be utilized to determine the angular velocities, angular accelerations, and transmitted forces. The equations of motion will be solved numerically using MATLAB.

The MATLAB software is a powerful tool for solving complex equations. It provides a robust platform for numerical analysis, and it is particularly useful for solving systems of equations.

3 Analysis

Assumptions made in the analysis include treating each linkage as a slender rod, neglecting the effects of gravity and friction, and confining all motion to a common plane. It is imperative to document and articulate any additional assumptions deemed necessary for the analysis.

The Required To Find (RTF) statements are as follows:

1. Determine the angular velocities and angular accelerations of the crank, connecting rod, and slider.

2. Determine the transmitted forces in the connecting rod and slider.

3. Determine the input torque for constant angular velocity.

4. Determine the out-of-balance forces.

3.1 Equations of Motion for Point A

A diagram of the system is shown in Figure 1. The diagram shows the crank as well as point A rotating at ω radians per second with angles θ and ϕ respectively.

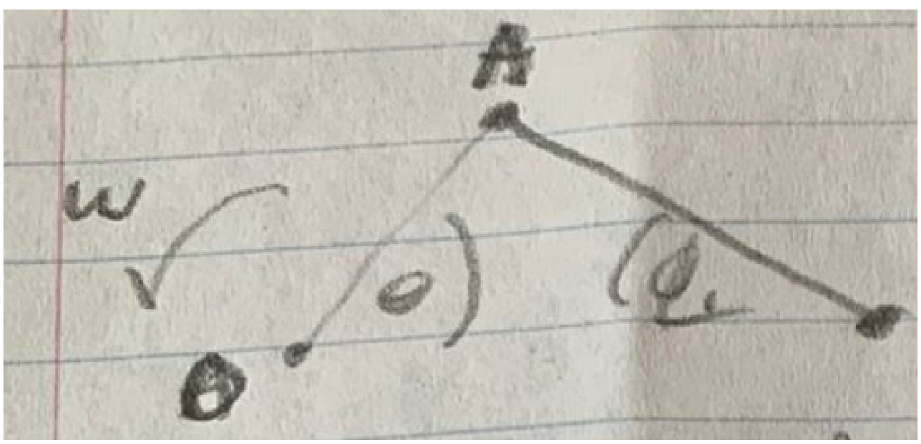

Figure 1: System Diagram of Point A

First, we will derive the equation of velocity for point A. The equation of velocity for point A is given by:

$$\vec{v}_A = \omega \times \vec{r}_A \angle R \cos\theta + R \sin\theta \tag{1}$$

Finding the x and y components of the equation of velocity for point A yields:

$$\begin{aligned} v_{Ax} &= -R\omega \sin\theta \\ v_{Ay} &= R\omega \cos\theta \end{aligned} \tag{2}$$

Next, we will derive the equation of acceleration for point A. The equation of acceleration for point A is given by:

$$\vec{a}_A = \omega \times (\omega \times \vec{r}_A) \angle R\cos\theta\omega^2, R\sin\theta\omega^2 \tag{3}$$

Finding the x and y components of the equation of acceleration for point A yields:

$$\begin{aligned} a_{Ax} &= -R\omega^2 \cos\theta \\ a_{Ay} &= -R\omega^2 \sin\theta \end{aligned} \tag{4}$$

3.2 Equations of Motion for Point B

A diagram of the system is shown in Figure 2. The diagram shows the connecting rod as well as point B with angles θ and ϕ respectively.

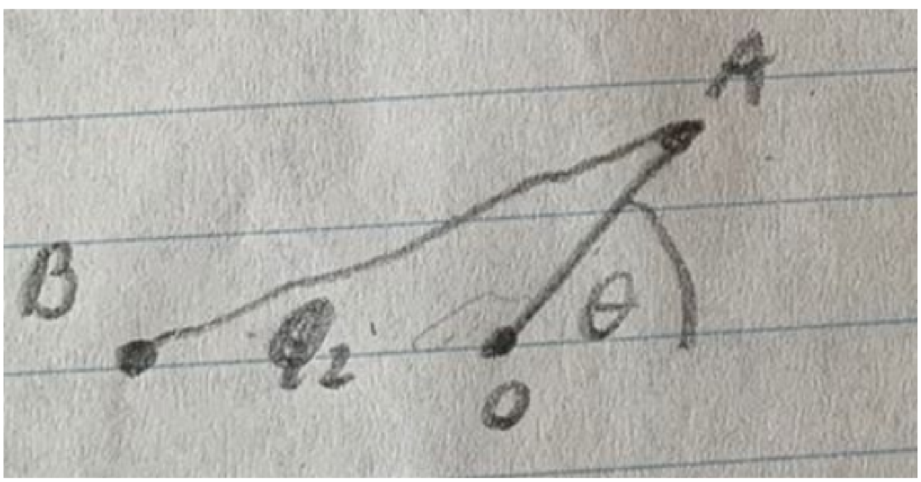

Figure 2: System Diagram of Point B

Next, we will derive the equation of velocity for point B. The equation of velocity for point B is given by:

$$\vec{v}_B = \vec{v}_A + \omega \times \vec{r}_{AB} \tag{5}$$

Finding the x and y components of the equation of velocity for point B yields:

$$\begin{aligned} v_{Bx} &= \vec{v}_{Ax} + L\omega \sin\phi \\ v_{By} &= \vec{v}_{Ay} - L\omega \cos\phi \end{aligned} \tag{6}$$

Next, we will derive the equation of acceleration for point B. The equation of acceleration for point B is given by:

$$\vec{a}_B = \vec{a}_A + \alpha \times \vec{r}_{AB} + \omega \times (\omega \times \vec{r}_{AB}) \tag{7}$$

Finding the x and y components of the equation of acceleration for point B yields:

$$\begin{aligned} a_{Bx} &= \vec{a}_{Ax} + L\alpha \sin\phi + L\omega^2 \cos\phi \\ a_{By} &= \vec{a}_{Ay} - L\alpha \cos\phi + L\omega^2 \sin\phi \end{aligned} \tag{8}$$

To find the acceleration of mass center B, we will use the following equation:

$$\vec{a}_{gB} = \vec{a}_B + \alpha \times \vec{r}_{Bc}/2 + \omega \times (\omega \times \vec{r}_{Bc}) \tag{9}$$

Finding the x and y components of the equation of acceleration for mass center B yields:

$$\begin{aligned} a_{gBx} &= \vec{a}_{Bx} + L/2\alpha \sin\phi + L\omega^2 \cos\phi \\ a_{gBy} &= \vec{a}_{By} - L/2\alpha \cos\phi + L\omega^2 \sin\phi \end{aligned} \tag{10}$$

3.3 Equations of Motion for Point C

A diagram of the system is shown in Figure 3. The diagram shows the slider as well as point C with angles θ and ϕ respectively.

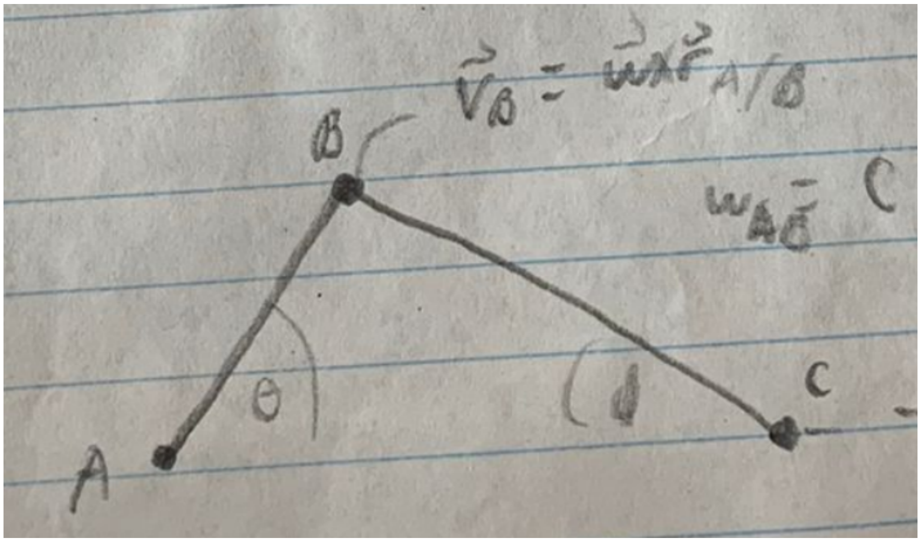

Figure 3: System Diagram of Point C

To derive the equations of motion for point C, we will use the following equation:

$$\vec{v}_C = \vec{v}_B + \omega \times \vec{r}_{BC} \tag{11}$$

Finding the x and y components of the equation of velocity for point C yields:

$$\begin{aligned} v_{Cx} &= \vec{v}_{Bx} + L\omega \sin\phi \\ v_{Cy} &= \vec{v}_{By} - L\omega \cos\phi \end{aligned} \tag{12}$$

Next, we will derive the equation of acceleration for point C. The equation of acceleration for point C is given by:

$$\vec{a}_C = \vec{a}_B + \alpha \times \vec{r}_{BC} + \omega \times (\omega \times \vec{r}_{BC}) \tag{13}$$

Finding the x and y components of the equation of acceleration for point C yields:

$$\begin{aligned} a_{Cx} &= \vec{a}_{Bx} + L\alpha \sin\phi + L\omega^2 \cos\phi \\ a_{Cy} &= \vec{a}_{By} - L\alpha \cos\phi + L\omega^2 \sin\phi \end{aligned} \tag{14}$$

To find the acceleration of mass center C, we will use the following equation:

$$\vec{a}_{gC} = \vec{a}_C + \alpha \times \vec{r}_{Cc}/2 + \omega \times (\omega \times \vec{r}_{Cc}) \tag{15}$$

Finding the x and y components of the equation of acceleration for mass center C yields:

$$\begin{aligned} a_{gCx} &= \vec{a}_{Cx} + L/2\alpha \sin\phi + L\omega^2 \cos\phi \\ a_{gCy} &= \vec{a}_{Cy} - L/2\alpha \cos\phi + L\omega^2 \sin\phi \end{aligned} \tag{16}$$

3.4 Calculating the Forces of the Crank

To calculate the out-of-balance forces of the crank, we will use the following equation:

$$F_{crank} = m\vec{a}_{gC} \tag{17}$$

Finding the x and y components of the out-of-balance force of the crank yields:

$$\begin{aligned} F_{crankx} &= m\vec{a}_{gCx} = m\frac{rc}{2}\omega^2 \cos\theta \\ F_{cranky} &= m\vec{a}_{gCy} = m\frac{rc}{2}\omega^2 \sin\theta \end{aligned} \tag{18}$$

The primary out-of-balance force of the crank is zero in this case. While the secondary out-of-balance force of the crank is given by:

$$F_{II} = \frac{2m_r r_c \omega^2}{n} \cos 2\theta \tag{19}$$

Where n can be calculated using the following equation:

$$n = \frac{r_c}{l} \approx 4 \tag{20}$$

3.5 Calculating the Forces of the Connecting Rod B

To calculate the out-of-balance forces of the connecting rod B, we will use the following equation:

$$F_B = m\vec{a}_{gB} \tag{21}$$

Finding the x and y components of the out-of-balance force of the connecting rod B yields:

$$\begin{aligned} F_{Bx} &= m\vec{a}_{gBx} \\ F_{By} &= m\vec{a}_{gBy} \end{aligned} \tag{22}$$

3.6 Calculating the Torque of the Crank

To calculate the torque of the crank, we will use the following equation:

$$T_{crank} = R\cos\theta(F_{cranky} + F_{By}) - R\sin\theta(F_{crankx} + F_{Bx}) \tag{23}$$

4 Discussion

5 Conclusions

6 References

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