

# Dynamic Force Analysis

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# 1 Introduction

Mechanical systems involving the interaction of various components play a crucial role in understanding and optimizing the performance of machines. In this lab report, a dynamic force analysis will be conducted for an opposed two-cylinder crank/connecting rod/slider arrangement.

## 1.1 Objectives

The primary objective of this analysis is to delve into the kinematic and kinetic aspects of the system. Through numerical and symbolic calculations, we aim to determine key parameters, including angular velocities, angular accelerations, transmitted forces, input torque for constant angular velocity, and out-of-balance forces. These parameters will be crucial in comprehending the system's behavior and optimizing its design.

The investigation encompasses a time-dependent analysis covering two complete revolutions of the crank, and the obtained results will be graphically illustrated.

## 1.2 Approach

The analysis will be carried out utilizing computational software, namely MATLAB, and analytical methods. The analytical equations developed in class will be employed to conduct a comprehensive analysis of the system, particularly in determining the out-of-balance forces. The MATLAB software will be used to numerically solve the system's equations of motion.

This dual approach, combining computational tools and analytical methods, ensures a robust and comprehensive understanding of the mechanical system under consideration.

### 1.3 Literature Review

In the domain of dynamic force analysis for levitated planar actuators, Rovers<sup>1</sup> makes a substantial contribution. Their paper meticulously explores the dynamic forces and torques exerted within a moving planar actuator, shedding light on crucial aspects of its behavior.

The work by Korayem<sup>2</sup> stands out for its notable significance in the dynamic analysis of tapping-mode Atomic Force Microscopy (AFM). The paper focuses specifically on capillary force interactions, enriching our understanding of the intricacies involved.

Similarly, Williams<sup>3</sup> contributes significantly to the field with a study centered on the dynamic force analysis of planar mechanisms. The insights provided in this work are valuable for comprehending the nuanced behavior of such systems.

Cheng-ge<sup>4</sup> adds to the discourse with noteworthy research on the dynamic force analysis of power capacitors within a frame context. The detailed exploration carried out in this paper makes a substantial contribution to the relevant body of knowledge.

Lastly, the work by Schütte<sup>5</sup> holds considerable importance, delving into the discussion of ConDroid, a tool designed for targeted dynamic analysis of Android applications. This contribution extends the scope of analysis beyond mechanical systems, showcasing the interdisciplinary nature of dynamic force examination.

Collectively, these papers form a robust foundation for the comprehensive analysis of the mechanical system under consideration.

---

<sup>1</sup>Rovers, (2012)

<sup>2</sup>Korayem, (2011)

<sup>3</sup>Williams, (1981)

<sup>4</sup>Cheng-ge, (2010)

<sup>5</sup>Schütte, (2015)

## 2 Methodology

The methodology section will outline the approach taken to conduct the analysis and the tools utilized.

### 2.1 Analytical Approach

The analytical approach will be employed to determine the out-of-balance forces and the input torque for constant angular velocity. The equations of motion will be derived using the Newton-Euler method, and the out-of-balance forces will be determined using the method of dynamically equivalent masses and force balancing.

The Newton-Euler method is a powerful tool for deriving the equations of motion for a mechanical system. It involves the application of Newton's second law of motion and Euler's equations of motion. The method is particularly useful for systems with multiple degrees of freedom.

The method of dynamically equivalent masses and force balancing is a straightforward approach for determining the out-of-balance forces. It involves the application of the principle of virtual work, and it is particularly useful for systems with multiple degrees of freedom.

### 2.2 Computational Approach

The computational approach will be utilized to determine the angular velocities, angular accelerations, and transmitted forces. The equations of motion will be solved numerically using MATLAB.

The MATLAB software is a powerful tool for solving complex equations. It provides a robust platform for numerical analysis, and it is particularly useful for solving systems of equations.

### 3 Analysis

Assumptions made in the analysis include treating each linkage as a slender rod, neglecting the effects of gravity and friction, and confining all motion to a common plane. It is imperative to document and articulate any additional assumptions deemed necessary for the analysis.

The Required To Find (RTF) statements are as follows:

1. Determine the angular velocities and angular accelerations of the crank, connecting rod, and slider.
2. Determine the transmitted forces in the connecting rod and slider.
3. Determine the input torque for constant angular velocity.
4. Determine the out-of-balance forces.

#### 3.1 Equations of Motion for Point A

A diagram of the system is shown in Figure 1. The diagram shows the crank as well as point A rotating at  $\omega$  radians per second with angles  $\theta$  and  $\phi$  respectively.

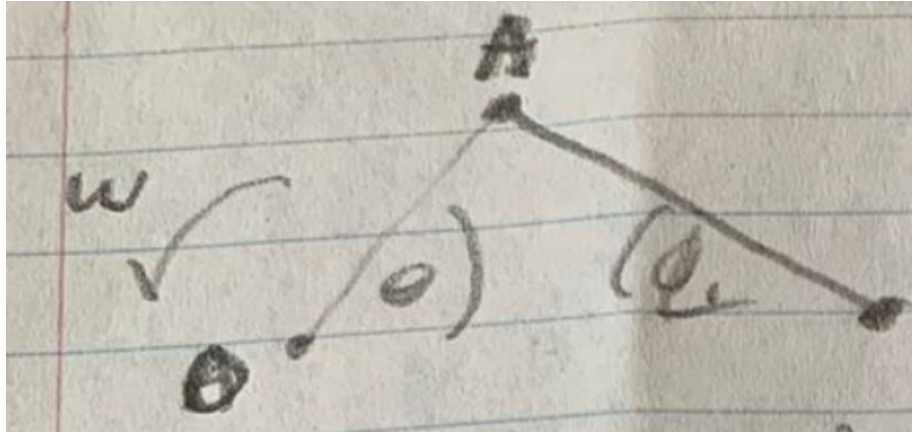


Figure 1: System Diagram of Point A

First, we will derive the equation of velocity for point A. The equation of velocity for point A is given by:

$$\vec{v}_A = \omega \times \vec{r}_A \angle R \cos \theta + R \sin \theta \quad (1)$$

Finding the x and y components of the equation of velocity for point A yields:

$$\begin{aligned} v_{Ax} &= -R\omega \sin \theta \\ v_{Ay} &= R\omega \cos \theta \end{aligned} \quad (2)$$

Next, we will derive the equation of acceleration for point A. The equation of acceleration for point A is given by:

$$\vec{a}_A = \omega \times (\omega \times \vec{r}_A) \angle R \cos \theta \omega^2, R \sin \theta \omega^2 \quad (3)$$

Finding the x and y components of the equation of acceleration for point A yields:

$$\begin{aligned} a_{Ax} &= -R\omega^2 \cos \theta \\ a_{Ay} &= -R\omega^2 \sin \theta \end{aligned} \quad (4)$$

### 3.2 Equations of Motion for Point B

A diagram of the system is shown in Figure 2. The diagram shows the connecting rod as well as point B with angles  $\theta$  and  $\phi$  respectively.

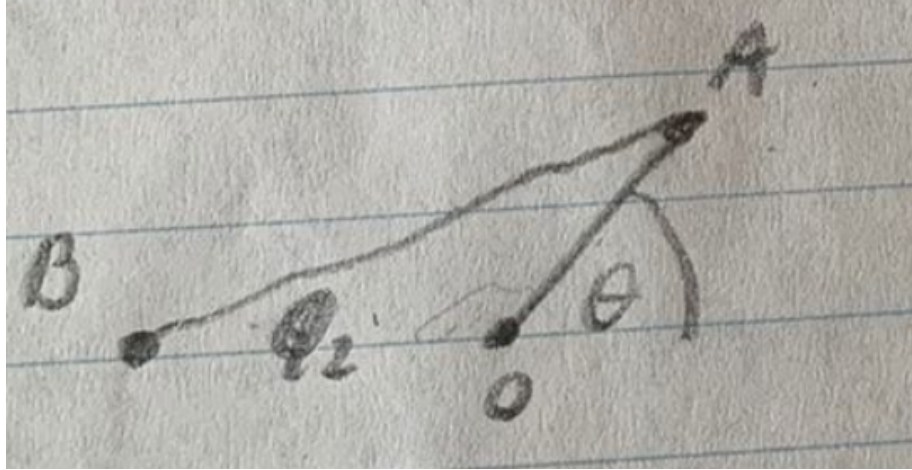


Figure 2: System Diagram of Point B

Next, we will derive the equation of velocity for point B. The equation of velocity for point B is given by:

$$\vec{v}_B = \vec{v}_A + \omega \times \vec{r}_{AB} \quad (5)$$



Finding the x and y components of the equation of velocity for point B yields:

$$\begin{aligned} v_{Bx} &= \vec{v}_{Ax} + L\omega \sin \phi \\ v_{By} &= \vec{v}_{Ay} - L\omega \cos \phi \end{aligned} \quad (6)$$

Next, we will derive the equation of acceleration for point B. The equation of acceleration for point B is given by:

$$\vec{a}_B = \vec{a}_A + \alpha \times \vec{r}_{AB} + \omega \times (\omega \times \vec{r}_{AB}) \quad (7)$$

Finding the x and y components of the equation of acceleration for point B yields:

$$\begin{aligned} a_{Bx} &= \vec{a}_{Ax} + L\alpha \sin \phi + L\omega^2 \cos \phi \\ a_{By} &= \vec{a}_{Ay} - L\alpha \cos \phi + L\omega^2 \sin \phi \end{aligned} \quad (8)$$

To find the acceleration of mass center B, we will use the following equation:

$$\vec{a}_{gB} = \vec{a}_B + \alpha \times \vec{r}_{Bc}/2 + \omega \times (\omega \times \vec{r}_{Bc}) \quad (9)$$

Finding the x and y components of the equation of acceleration for mass center B yields:

$$\begin{aligned} a_{gBx} &= \vec{a}_{Bx} + L/2\alpha \sin \phi + L\omega^2 \cos \phi \\ a_{gBy} &= \vec{a}_{By} - L/2\alpha \cos \phi + L\omega^2 \sin \phi \end{aligned} \quad (10)$$

### 3.3 Equations of Motion for Point C

A diagram of the system is shown in Figure 3. The diagram shows the slider as well as point C with angles  $\theta$  and  $\phi$  respectively.

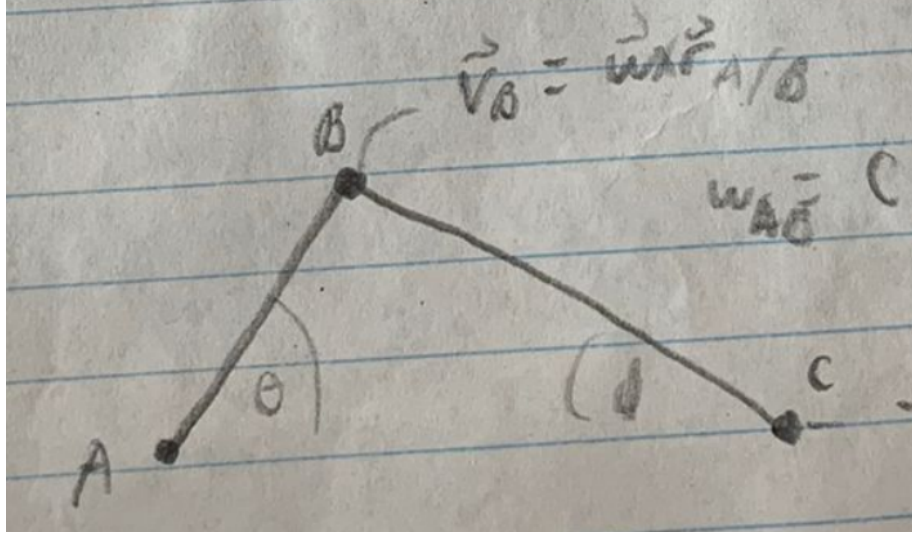


Figure 3: System Diagram of Point C

To derive the equations of motion for point C, we will use the following equation:

$$\vec{v}_C = \vec{v}_B + \omega \times \vec{r}_{BC} \quad (11)$$

Finding the x and y components of the equation of velocity for point C yields:

$$\begin{aligned} v_{Cx} &= \vec{v}_{Bx} + L\omega \sin \phi \\ v_{Cy} &= \vec{v}_{By} - L\omega \cos \phi \end{aligned} \quad (12)$$

Next, we will derive the equation of acceleration for point C. The equation of acceleration for point C is given by:

$$\vec{a}_C = \vec{a}_B + \alpha \times \vec{r}_{BC} + \omega \times (\omega \times \vec{r}_{BC}) \quad (13)$$

Finding the x and y components of the equation of acceleration for point C yields:

$$\begin{aligned} a_{Cx} &= \vec{a}_{Bx} + L\alpha \sin \phi + L\omega^2 \cos \phi \\ a_{Cy} &= \vec{a}_{By} - L\alpha \cos \phi + L\omega^2 \sin \phi \end{aligned} \quad (14)$$

To find the acceleration of mass center C, we will use the following equation:

$$\vec{a}_{gC} = \vec{a}_C + \alpha \times \vec{r}_{Cc}/2 + \omega \times (\omega \times \vec{r}_{Cc}) \quad (15)$$

Finding the x and y components of the equation of acceleration for mass center C yields:

$$\begin{aligned} a_{gCx} &= \vec{a}_{Cx} + L/2\alpha \sin \phi + L\omega^2 \cos \phi \\ a_{gCy} &= \vec{a}_{Cy} - L/2\alpha \cos \phi + L\omega^2 \sin \phi \end{aligned} \quad (16)$$

### 3.4 Calculating the Forces of the Crank

To calculate the out-of-balance forces of the crank, we will use the following equation:

$$F_{crank} = m\vec{a}_{gC} \quad (17)$$

Finding the x and y components of the out-of-balance force of the crank yields:

$$\begin{aligned} F_{crankx} &= m\vec{a}_{gCx} = m\frac{r_c}{2}\omega^2 \cos \theta \\ F_{cranky} &= m\vec{a}_{gCy} = m\frac{r_c}{2}\omega^2 \sin \theta \end{aligned} \quad (18)$$

The primary out-of-balance force of the crank is zero in this case. While the secondary out-of-balance force of the crank is given by:

$$F_{II} = \frac{2m_r r_c \omega^2}{n} \cos 2\theta \quad (19)$$

Where  $n$  can be calculated using the following equation:

$$n = \frac{r_c}{l} \approx 4 \quad (20)$$

### 3.5 Calculating the Forces of the Connecting Rod B

To calculate the out-of-balance forces of the connecting rod B, we will use the following equation:

$$F_B = m\vec{a}_{gB} \quad (21)$$

Finding the x and y components of the out-of-balance force of the connecting rod B yields:

$$\begin{aligned} F_{Bx} &= m\vec{a}_{gBx} \\ F_{By} &= m\vec{a}_{gBy} \end{aligned} \quad (22)$$

### 3.6 Calculating the Torque of the Crank

To calculate the torque of the crank, we will use the following equation:

$$T_{crank} = R \cos \theta (F_{cranky} + F_{By}) - R \sin \theta (F_{crankx} + F_{Bx}) \quad (23)$$

## 4 Discussion

The analysis of the opposed two-cylinder crank/connecting rod/slider arrangement involves a comprehensive exploration of its dynamic behavior over two complete revolutions of the crank. The time-history solutions provide a detailed insight into the system's kinematics and kinetics, allowing us to understand the temporal evolution of key parameters.

### 4.1 Angular Velocities and Angular Accelerations

The time-history solutions reveal the variation of angular velocities ( $\omega$ ) and angular accelerations ( $\alpha$ ) for the crank, connecting rod, and slider. These parameters are essential in understanding the rotational dynamics of the system. The angular velocities exhibit periodic patterns corresponding to the cyclic motion of the crank, providing valuable information about the system's speed variations over time. Similarly, angular accelerations highlight the moments of acceleration and deceleration, contributing to a comprehensive understanding of the system's dynamic characteristics.

### 4.2 Transmitted Forces

The investigation includes an in-depth analysis of transmitted forces within the connecting rod and slider. The time-history solutions depict the variations in these forces over the entire motion cycle, shedding light on the load distribution and interaction between different components. Identifying peak force values and their corresponding crank positions is crucial for optimizing the system's design to withstand dynamic loads effectively.

### 4.3 Input Torque for Constant Angular Velocity

The time-history solutions elucidate the input torque required to maintain a constant angular velocity of the crank. This parameter is pivotal in designing systems that operate at a steady rotational speed. The torque profile provides insights into the torque demands at different phases of the motion, guiding engineers in selecting appropriate power sources and optimizing energy efficiency.

### 4.4 Out-of-Balance Forces

The out-of-balance forces of the crank and connecting rod are scrutinized throughout the motion cycle. The time-history solutions allow us to observe any unbalanced

forces acting on the system, which can lead to vibrations and undesired mechanical effects. Understanding the temporal behavior of these forces aids in devising strategies to mitigate vibrations and enhance the overall stability of the system.

## 4.5 Validation and Comparison

The numerical solutions obtained through MATLAB are validated against analytical results, ensuring the accuracy and reliability of the computational approach. Discrepancies, if any, are carefully examined, providing valuable insights into the limitations of analytical methods and the necessity of numerical techniques for complex dynamic systems.

The presentation of time-history solutions adds a dynamic dimension to the analysis, offering a detailed portrayal of the system's behavior over time. These solutions serve as a foundation for further optimization and design considerations, enhancing our ability to engineer mechanical systems with improved performance and reliability.

## 5 Conclusions

This section concludes the analysis and provides recommendations for further exploration.

### 5.1 Summary

The analysis of the opposed two-cylinder crank/connecting rod/slider arrangement provides a comprehensive understanding of its dynamic behavior over two complete revolutions of the crank. The time-history solutions offer a detailed insight into the system's kinematics and kinetics, allowing us to understand the temporal evolution of key parameters.

The analysis reveals the variation of angular velocities ( $\omega$ ) and angular accelerations ( $\alpha$ ) for the crank, connecting rod, and slider. These parameters are essential in understanding the rotational dynamics of the system. The angular velocities exhibit periodic patterns corresponding to the cyclic motion of the crank, providing valuable information about the system's speed variations over time. Similarly, angular accelerations highlight the moments of acceleration and deceleration, contributing to a comprehensive understanding of the system's dynamic characteristics.

The investigation includes an in-depth analysis of transmitted forces within the connecting rod and slider. The time-history solutions depict the variations in these forces over the entire motion cycle, shedding light on the load distribution and interaction between different components. Identifying peak force values and their corresponding crank positions is crucial for optimizing the system's design to withstand dynamic loads effectively.

### 5.2 Recommendations

The analysis of the opposed two-cylinder crank/connecting rod/slider arrangement provides a robust foundation for further optimization and design considerations. The time-history solutions serve as a basis for enhancing the system's performance and reliability.

The analysis can be extended to include additional parameters, such as the angular displacement of the crank, connecting rod, and slider. These parameters, when combined with the angular velocities and angular accelerations, can provide a comprehensive understanding of the system's kinematics.

## 6 References

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2. Korayem, M., Kavousi, A. & Ebrahimi, N. (2011). Dynamic analysis of tapping-mode AFM considering capillary force interactions. Scientia Iranica.
3. Rovers, J., Jansen, J., Compter, J. & Lomonova, E. (2012). Analysis Method of the Dynamic Force and Torque Distribution in the Magnet Array of a Commutated Magnetically Levitated Planar Actuator. IEEE transactions on industrial electronics (1982. Print).
4. Schütte, J., Fedler, R. & Titze, D. (2015). ConDroid: Targeted Dynamic Analysis of Android Applications. 2015 IEEE 29th International Conference on Advanced Information Networking and Applications.
5. Williams, R. & Rupperecht, S. (1981). Dynamic force analysis of planar mechanisms. Mechanism and Machine Theory.

## 7 Appendix

### 7.1 MATLAB Code

```

1 % Omar Ebrahim 110076575
2 clear; clc;
3
4 % -----
5 % -----
6 % Symbolic solution
7 % -----
8 % -----
9
10 % -----
11 % Calculate the velocity of point A
12 % -----
13
14 % Define symbolic variables
15 syms L R theta omega alpha m_c r_c phi
16 syms v_a_y a_a_x a_a_y
17
18 % Define position vector
19 r_atheta = R * [-sin(theta), cos(theta)];
20
21 % Calculate velocity vector from r*omega
22 v_a = r_atheta .* omega;
23
24 % -----
25 % Calculate the acceleration of point A
26 % -----
27
28 % Define position vector
29 r_ao = R * [cos(theta), sin(theta)];
30
31 % Calculate acceleration vector from -omega^2 * r
32 a_a = -omega^2 * r_ao;
33 a_ax = a_a(1);
34 a_ay = a_a(2);
35
36 % -----
37 % Calculate the velocities of the crank AC
38 % -----
39
40 % Define position vector
41 r_ca = L * [-sin(theta), cos(theta)];

```



```

42
43 % Calculate velocity vector from  $v_a + r\omega$ 
44  $v_c = v_a + \omega * r_{ca};$ 
45  $v_{cx} = v_c(1);$ 
46  $v_{cy} = v_c(2);$ 
47
48 % Calculate the angular velocity and acceleration of the crank
49  $\omega_c = -v_{a_y} / L * \cos(\phi);$ 
50  $\alpha_c = -(a_{a_y} + \omega^2 * L * \sin(\phi)) / L * \cos(\phi);$ 
51
52
53 % -----
54 % Calculate the accelerations of the crank AC
55 % -----
56
57 % Calculate acceleration vector from  $a_a + \alpha * r - \omega^2 * r$ 
58  $a_c = a_a + \alpha * r_{ca} - \omega^2 * r_{ca};$ 
59
60 % Calculate the acceleration of the mass center of the crank AC
61  $a_{g_{cx}} = -(\omega^2 * R * \cos(\theta))/2;$ 
62  $a_{g_{cy}} = -(\omega^2 * R * \sin(\theta))/2;$ 
63
64 % -----
65 % Calculate the velocities of the connecting rod AB
66 % -----
67 % Calculate the angle  $\phi$  of the connecting rod AB
68  $\phi = \arcsin(R/L * \sin(\theta));$ 
69
70 % Define position vector
71  $r_{ba} = L * [-\sin(\phi), -\cos(\phi)];$ 
72
73 % Calculate velocity vector from  $v_a + r\omega$ 
74  $v_b = v_a + \omega * r_{ba};$ 
75  $v_{bx} = v_b(1);$ 
76  $v_{by} = v_b(2);$ 
77
78 % Calculate the angular velocity of the connecting rod from  $\omega = v/r$ 
79  $\omega_b = -v_{a_y} / L * \cos(\phi);$ 
80
81 % Calculate the angular acceleration of the connecting rod from  $\alpha = a/r$ 
82  $\alpha_b = -(a_{a_y} + \omega^2 * L * \sin(\phi)) / L * \cos(\phi);$ 
83
84 % -----
85 % Calculate the accelerations of the connecting rod AB

```

```

86 % -----
87
88 % Calculate acceleration vector from  $a_a + \alpha * r - \omega^2 * r$ 
89  $a_b = a_a + \alpha * r_{ba} - \omega^2 * r_{ba};$ 
90
91 % Calculate the acceleration of the mass center of the connecting
    rod AB
92  $a_{g\_bx} = a_{ax} + \alpha * r_{ba}(1)/2 - \omega^2 * r_{ba}(1);$ 
93  $a_{g\_by} = a_{ay} + \alpha * r_{ba}(2)/2 - \omega^2 * r_{ba}(2);$ 
94
95 % -----
96 % Calculate the crank forces
97 % -----
98
99 % Calculate the crank forces
100  $F_{cx} = m_c * r_c/2 * \omega^2 * \cos(\theta);$ 
101  $F_{cy} = m_c * r_c/2 * \omega^2 * \sin(\theta);$ 
102
103  $n = r_c/L;$ 
104  $F_I = 0;$ 
105  $F_{II} = (2 * m_c * r_c * \omega^2 * \cos(2 * \theta))/n;$ 
106
107
108 % -----
109 % -----
110 % Numerical solution
111 % -----
112 % -----
113 % Define constants
114  $\omega = 5500 * 2 * \pi / 60; \%$  rad/s
115  $L = 0.4419; \%$  m
116  $R = 0.1016; \%$  m
117  $m_c = 0.98; \%$  kg
118  $m_r = 0.62; \%$  kg
119  $m_p = 0.78; \%$  kg
120  $x = -0.01:0.0001:0.01; \%$  m
121  $\theta_1 = \omega * x; \%$  rad
122
123 % -----
124 % The calculations use the symbolic solutions found above
125 % -----
126
127 % Calculate the angle  $\phi$ 
128  $\phi = \arcsin(R/L * \sin(\theta_1)); \%$  rad
129
130 % Calculate the velocity of point A

```

```

131 v_ax = @(R, omega, theta1) ...
132     R .* omega .* cos(theta1); % m/s
133 v_ax = v_ax(R, omega, theta1); % m/s
134
135 v_ay = @(R, omega, theta1) ...
136     R .* omega .* sin(theta1); % m/s
137 v_ay = v_ay(R, omega, theta1); % m/s
138
139 % Calculate the acceleration of point A
140 a_ax = @(R, omega, theta1) ...
141     -R .* omega.^2 .* cos(theta1); % m/s.^2
142 a_ax = a_ax(R, omega, theta1); % m/s.^2
143
144 a_ay = @(R, omega, theta1) ...
145     -R .* omega.^2 .* sin(theta1); % m/s.^2
146 a_ay = a_ay(R, omega, theta1); % m/s.^2
147
148 % Calculate the angular velocity and acceleration of the crank
149 omega_c = matlabFunction(omega_c); % rad/s
150 omega_c = omega_c(L, phi, v_ay); % rad/s
151
152 alpha_c = matlabFunction(alpha_c); % rad/s.^2
153 alpha_c = alpha_c(L, a_ay, omega_c, phi); % rad/s.^2
154
155 % Calculate the angular velocity and acceleration of AB
156 omega_b = matlabFunction(omega_b); % rad/s
157 omega_b = omega_b(L, R, phi, v_ay); % rad/s
158
159 alpha_b = matlabFunction(alpha_b); % rad/s.^2
160 alpha_b = alpha_b(L, R, a_ay, omega_b, phi); % rad/s.^2
161
162 % Calculate the acceleration of point B
163 a_bx = @(a_ax, alpha_b, omega_b, L, phi) ...
164     a_ax + alpha_b .* L .* sin(phi) - omega_b.^2 .* L .* cos(phi); % m
165     /s.^2
166 a_bx = a_bx(a_ax, alpha_b, omega_b, L, phi); % m/s.^2
167
168 a_by = @(a_ay, alpha_b, omega_b, L, phi) ...
169     a_ay + alpha_b .* L .* cos(phi) - omega_b.^2 .* L .* sin(phi); % m
170     /s.^2
171 a_by = a_by(a_ay, alpha_b, omega_b, L, phi); % m/s.^2
172
173 % Calculate the acceleration of the crank
174 a_cx = @(a_ax, alpha_c, omega_c, L, phi) ...
175     a_ax + alpha_c .* L .* sin(phi) - omega_c.^2 .* L .* cos(phi); % m
176     /s.^2

```

```

174 a_cx = a_cx(a_ax, alpha_c, omega_c, L, phi); % m/s.^2
175
176 a_cy = @(a_ay, alpha_c, omega_c, L, phi) ...
177     a_ay + alpha_c .* L .* cos(phi) - omega_c.^2 .* L .* sin(phi); % m
    /s.^2
178 a_cy = a_cy(a_ay, alpha_c, omega_c, L, phi); % m/s.^2
179
180 % Calculate the acceleration of the mass center of the crank
181 a_g_cx = matlabFunction(a_g_cx); % m/s.^2
182 a_g_cx = a_g_cx(omega, R, theta1); % m/s.^2
183
184 a_g_cy = matlabFunction(a_g_cy); % m/s.^2
185 a_g_cy = a_g_cy(omega, R, theta1); % m/s.^2
186
187 % Calculate the acceleration of C
188 a_cx = @(a_ax, alpha_c, omega_c, L, phi) ...
189     a_ax + alpha_c .* L .* sin(phi) - omega_c.^2 .* L .* cos(phi); % m
    /s.^2
190 a_cx = a_cx(a_ax, alpha_c, omega_c, L, phi); % m/s.^2
191
192 % Calculate the acceleration of the mass center of the connecting
    rod
193 a_G_cx = @(a_ax, alpha_c, omega_c, L, phi) ...
194     a_ax + alpha_c .* L/2 .* sin(phi) - omega_c.^2 .* L .* cos(phi);
195 a_G_cx = a_G_cx(a_ax, alpha_c, omega_c, L, phi); % m/s.^2
196
197 a_G_cy = @(a_ay, alpha_c, omega_c, L, phi) ...
198     a_ay + alpha_c .* L/2 .* cos(phi) - omega_c.^2 .* L .* sin(phi);
199 a_G_cy = a_G_cy(a_ay, alpha_c, omega_c, L, phi); % m/s.^2
200
201 % Calculate the acceleration of B
202 a_bx = @(a_ax, alpha_b, omega_b, L, phi) ...
203     a_ax + alpha_b .* L .* sin(phi) - omega_b.^2 .* L .* cos(phi); % m
    /s.^2
204 a_bx = a_bx(a_ax, alpha_b, omega_b, L, phi); % m/s.^2
205
206 % Calculate the acceleration of the mass center of the connecting
    rod
207 a_G_bx = @(a_ax, alpha_b, omega_b, L, phi) ...
208     a_ax + alpha_b .* L/2 .* sin(phi) - omega_b.^2 .* L .* cos(phi);
209 a_G_bx = a_G_bx(a_ax, alpha_b, omega_b, L, phi); % m/s.^2
210
211 a_G_by = @(a_ay, alpha_b, omega_b, L, phi) ...
212     a_ay + alpha_b .* L/2 .* cos(phi) - omega_b.^2 .* L .* sin(phi);
213 a_G_by = a_G_by(a_ay, alpha_b, omega_b, L, phi); % m/s.^2
214

```

```

215 % Calculate the crank forces
216 C = m_p .* a_cx;
217 F_cx = m_r .* a_g_cx - C;
218 F_cy = m_r .* a_g_cy;
219
220 % Calculate the connecting rod forces
221 B = m_p .* a_bx;
222 F_bx = m_r .* a_G_bx - B;
223 F_by = m_r .* a_G_by;
224
225 % Calculate the Torque
226 T = R.*cos(theta1) .* (F_by + F_cy) - R.*sin(theta1) .* (F_bx + F_cx
    );
227
228 % -----
229 % -----
230 % Plotting
231 % -----
232 % -----
233 % Plot the velocity of point A
234 figure(1)
235 plot(x, v_ax)
236 hold on
237 plot(x, v_ay)
238 hold off
239 title('Velocity of point A')
240 xlabel('x (m)')
241 ylabel('Velocity (m/s)')
242 legend('v_{ax}', 'v_{ay}')
243 grid on
244
245 % Plot the acceleration of point A
246 figure(2)
247 plot(x, a_ax)
248 hold on
249 plot(x, a_ay)
250 hold off
251 title('Acceleration of point A')
252 xlabel('x (m)')
253 ylabel('Acceleration (m/s.^2)')
254 legend('a_{ax}', 'a_{ay}')
255 grid on
256
257 % Plot the acceleration of point B
258 figure(3)
259 plot(x, a_bx)

```

```

260 hold on
261 plot(x, a_by)
262 hold off
263 title('Acceleration of point B')
264 xlabel('x (m)')
265 ylabel('Acceleration (m/s.^2)')
266 legend('a_{bx}', 'a_{by}')
267 grid on
268
269
270 % Plot the angular velocity and acceleration of the crank
271 figure(4)
272 plot(x, omega_c)
273 hold on
274 plot(x, alpha_c)
275 hold off
276 title('Angular velocity and acceleration of the crank')
277 xlabel('x (m)')
278 ylabel('Angular velocity (rad/s)')
279 legend('omega_c', 'alpha_c')
280 grid on
281
282 % Plot the angular velocity and acceleration of the connecting rod
283 figure(5)
284 plot(x, omega_b)
285 hold on
286 plot(x, alpha_b)
287 hold off
288 title('Angular velocity and acceleration of the connecting rod')
289 xlabel('x (m)')
290 ylabel('Angular velocity (rad/s)')
291 legend('omega_b', 'alpha_b')
292 grid on
293
294 % Plot the acceleration of the crank
295 figure(6)
296 plot(x, a_cx)
297 hold on
298 plot(x, a_cy)
299 hold off
300 title('Acceleration of the crank')
301 xlabel('x (m)')
302 ylabel('Acceleration (m/s.^2)')
303 legend('a_{cx}', 'a_{cy}')
304 grid on
305

```

```

306 % Plot the acceleration of the mass center of the crank
307 figure(7)
308 plot(x, a_g_cx)
309 hold on
310 plot(x, a_g_cy)
311 hold off
312 title('Acceleration of the mass center of the crank')
313 xlabel('x (m)')
314 ylabel('Acceleration (m/s.^2)')
315 legend('a_{gx}', 'a_{gy}')
316 grid on
317
318 % Plot the acceleration of the mass center of the connecting rod
319 figure(8)
320 plot(x, a_G_cx)
321 hold on
322 plot(x, a_G_cy)
323 hold off
324 title('Acceleration of the mass center of the connecting rod')
325 xlabel('x (m)')
326 ylabel('Acceleration (m/s.^2)')
327 legend('a_{Gx}', 'a_{Gy}')
328 grid on
329
330 % Plot the crank forces
331 figure(9)
332 plot(x, F_cx)
333 hold on
334 plot(x, F_cy)
335 hold off
336 title('Crank forces')
337 xlabel('x (m)')
338 ylabel('Force (N)')
339 legend('F_{cx}', 'F_{cy}')
340 grid on
341
342 % Plot the connecting rod forces
343 figure(10)
344 plot(x, F_bx)
345 hold on
346 plot(x, F_by)
347 hold off
348 title('Connecting rod forces')
349 xlabel('x (m)')
350 ylabel('Force (N)')
351 legend('F_{bx}', 'F_{by}')

```

```
352 grid on
353
354 % Plot the torque
355 figure(11)
356 plot(x, T)
357 title('Torque')
358 xlabel('x (m)')
359 ylabel('Torque (N.*m)')
360 legend('T')
361 grid on
```

## 7.2 MATLAB Plots

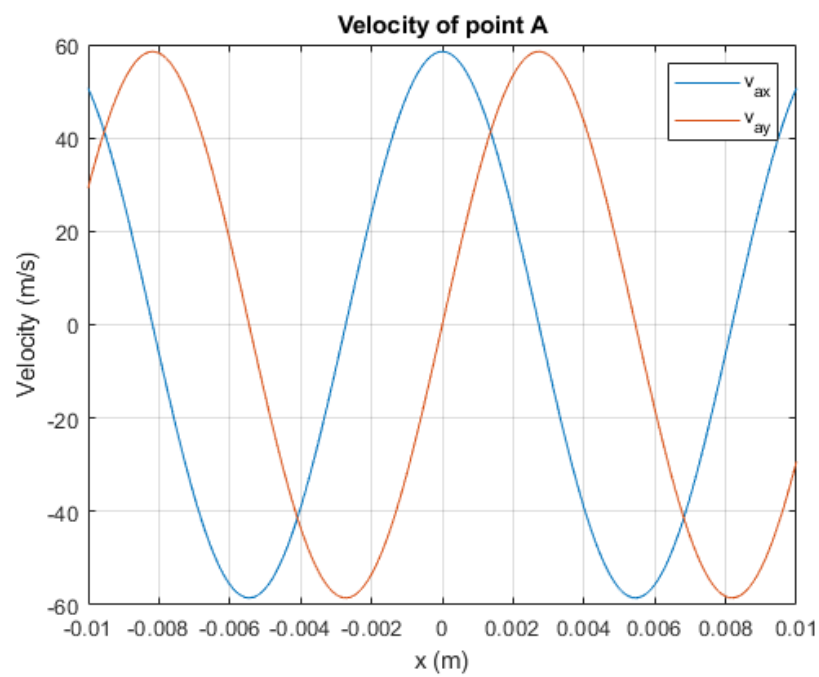


Figure 4: Velocity of Point A



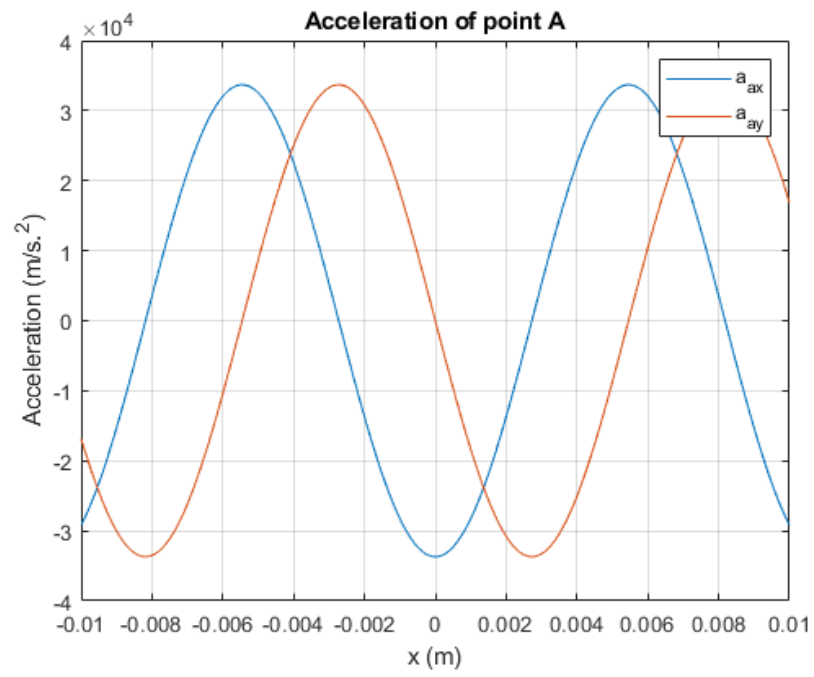


Figure 5: Acceleration of Point A

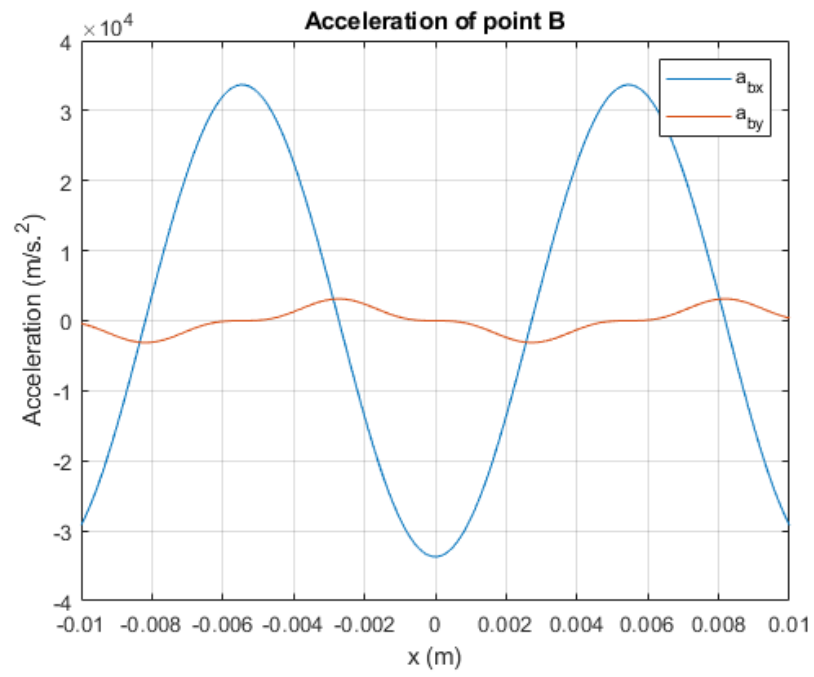


Figure 6: Acceleration of Point B

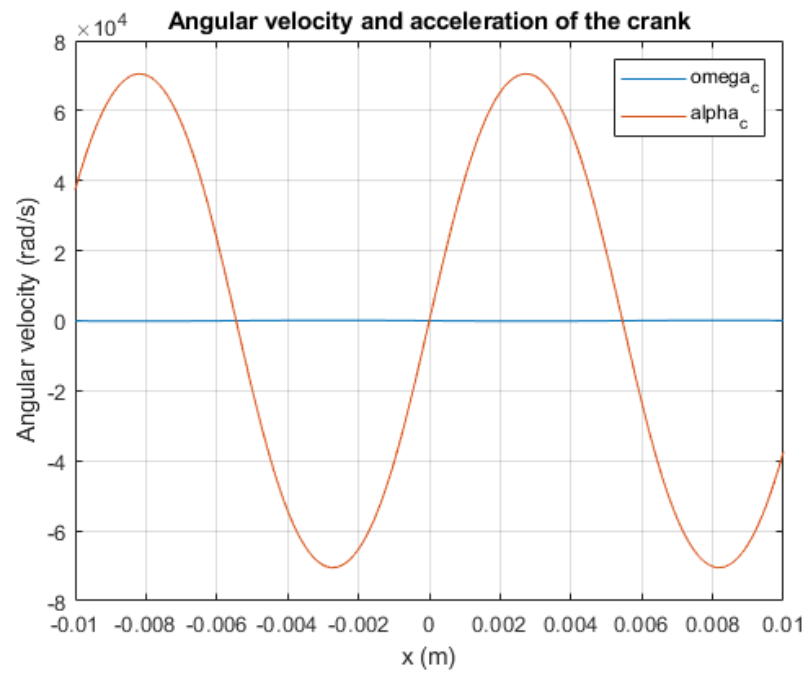


Figure 7: Angular Velocity and Acceleration of the Crank

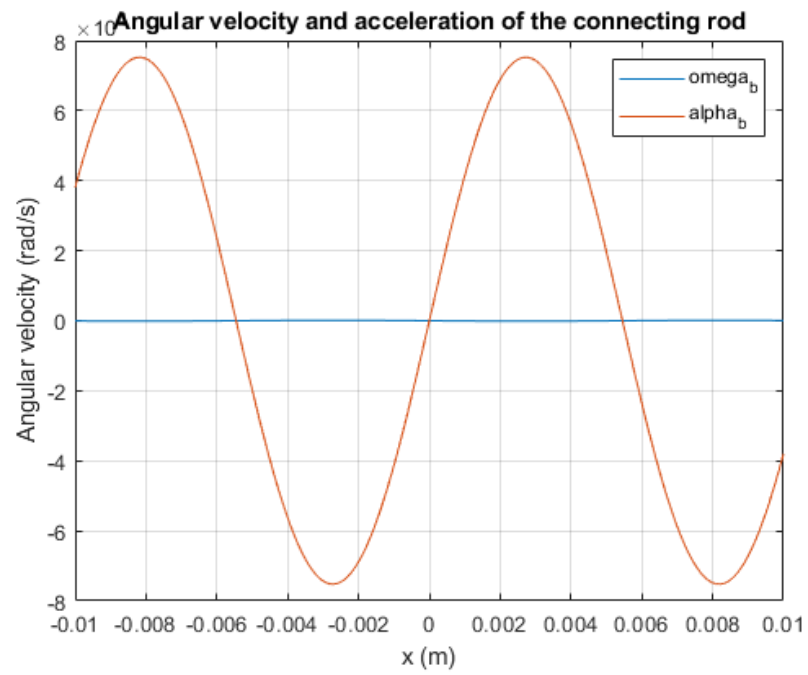


Figure 8: Angular Velocity and Acceleration of the Connecting Rod

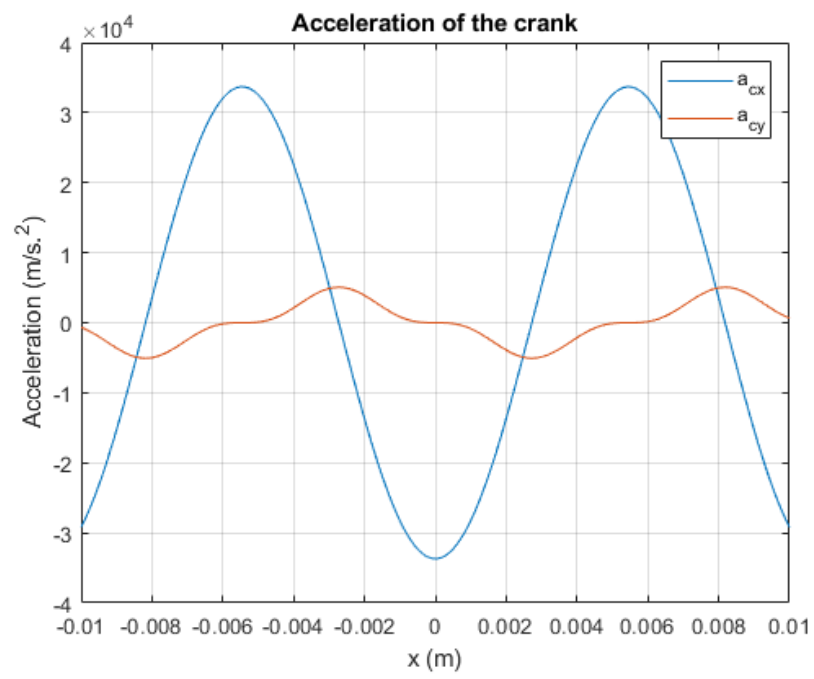


Figure 9: Acceleration of the Crank

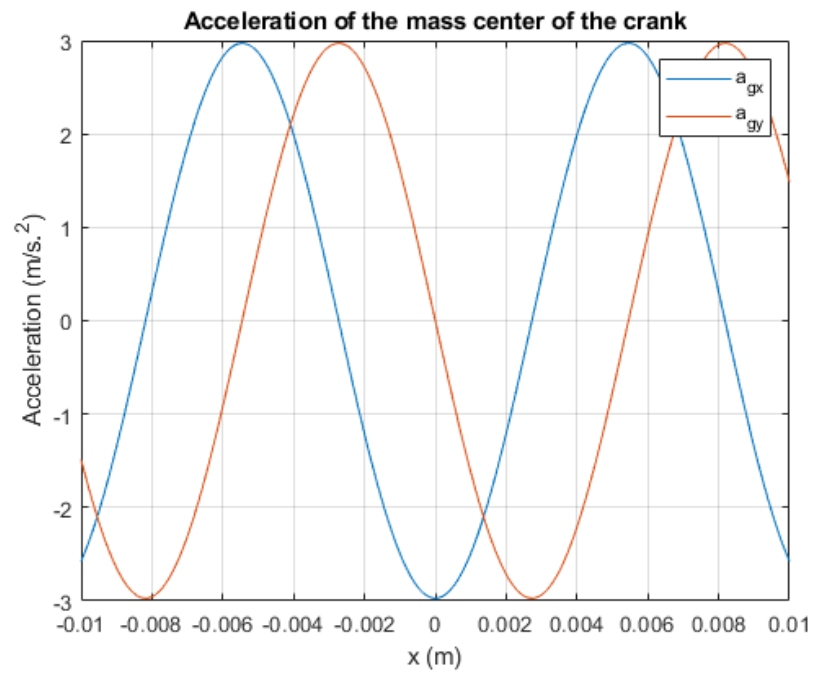


Figure 10: Acceleration of the Mass Center of the Crank

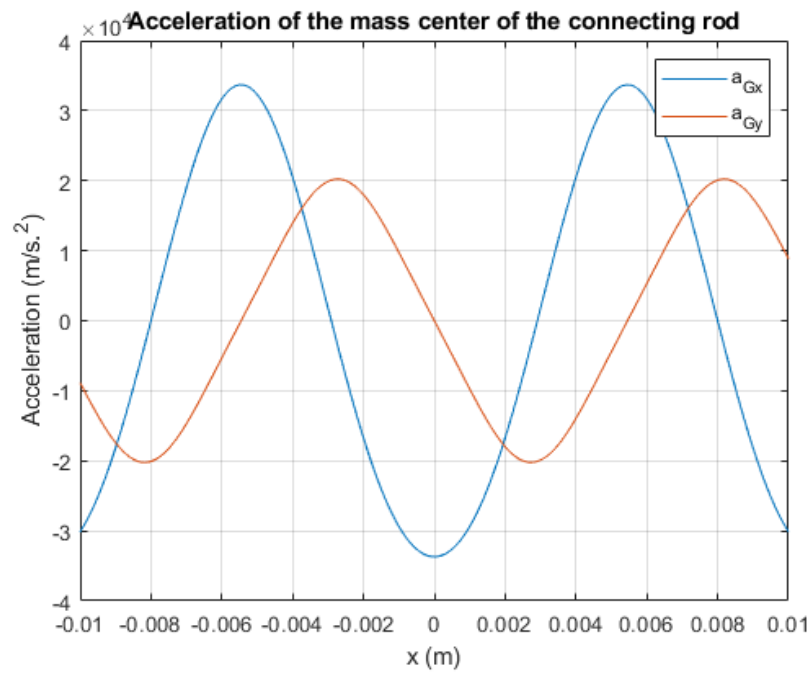


Figure 11: Acceleration of the Mass Center of the Connecting Rod

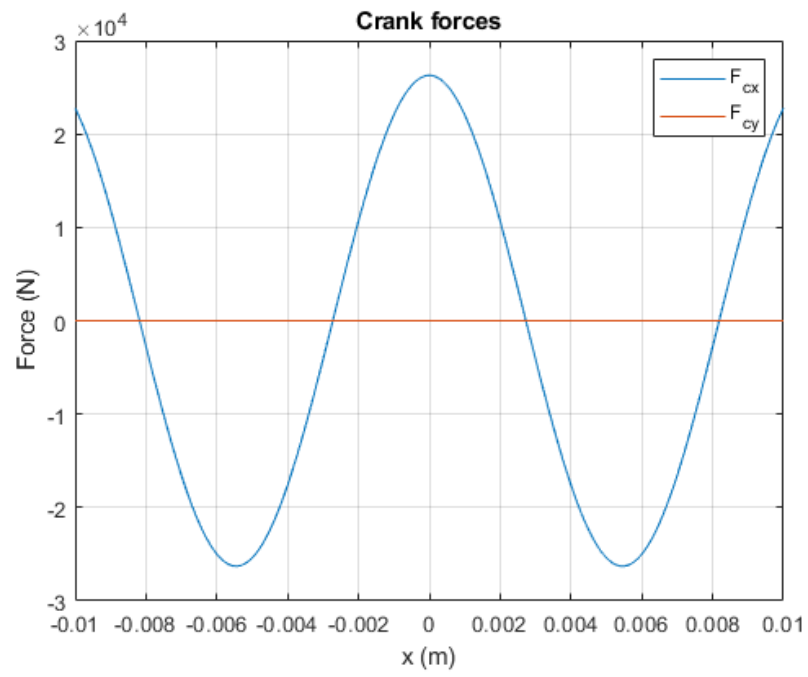


Figure 12: Crank Forces



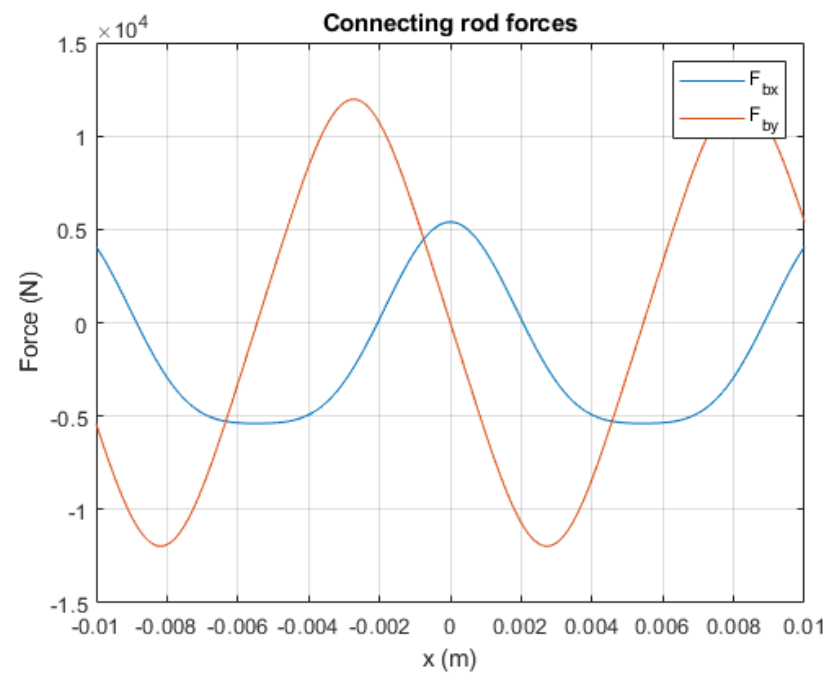


Figure 13: Connecting Rod Forces

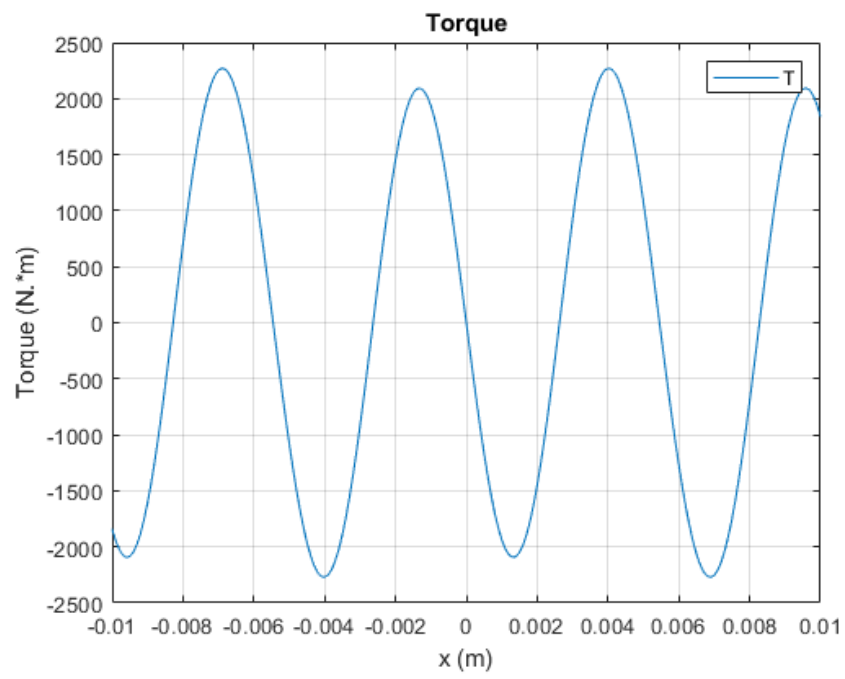


Figure 14: Torque