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1 Objectives

The behavior of beams under different loading conditions is a fundamental aspect of structural engineering. Understanding how beams deflect when subjected to varying loads is crucial in designing safe and efficient structures. In this experiment, the focus is on two specific beam configurations: the cantilever and the simply supported beam. The deflection of these beams will be investigated both theoretically and experimentally.

The objectives for this experiment are outlined as follows:

1. **Measurement of Deflection:** Experimentally acquire accurate values of deflection for both a cantilever and a simply supported beam under varying loads to understand their structural behaviors.

2. **Comparison of Experimental and Theoretical Results:** Analyze and compare the obtained experimental deflection values with the theoretically predicted values for both the cantilever and the simply supported beam to assess the accuracy and reliability of theoretical models in predicting structural deflections.

These objectives aim to provide a comprehensive understanding of how different types of beam structures behave under applied loads and to evaluate the alignment between theoretical predictions and actual experimental observations in real-world scenarios.

2 Procedure

This experiment consists of two parts: the cantilever beam testing and the simply supported beam testing.

2.1 Cantilever Beam Testing

1. **Step 1:** Check the initial readings of the electrical potentiometers to ensure they are zero.

2. **Step 2:** Apply loads in increments of 2 lb (8.9 N) at the mid-span ($P1$) up to a total load of 10 lb. Record the vertical deflections at both the mid-span ($P1$) and at the end of the cantilever beam ($P2$) for each load.

3. **Step 3:** Remove the applied load.

4. **Step 4:** Apply the same set of loads to the end of the beam ($P2$) and measure the deflections at the mid-span ($P1$) and at the end of the cantilever beam ($P2$).

2.2 Simply Supported Beam Testing

1. **Step 1:** Ensure that the initial readings of the electrical potentiometers are zero.

2. **Step 2:** Apply loads in increments of 10 lb (44.482 N) at the mid-span of the beam ($P3$) up to a total load of 40 lb (177.928 N). Record the vertical deflections at both the mid-span ($P3$) and at the quarter-span of the beam ($P4$) for each load.

3. **Step 3:** Remove the applied load.

4. **Step 4:** Apply the same set of loads to the quarter-span ($P4$) and measure the deflections at the mid-span ($P3$) and at the quarter-span of the beam ($P4$).

Throughout the experiment, ensure accurate recording of all measurements, maintain the safety protocols, and perform the tests as per the specified loading conditions for each beam configuration.

2.3 Free-Body Diagrams

This section contains the free-body diagrams for the each beam and each loading condition.

2.3.1 Cantilever Beam Load Applied at the Mid-Span

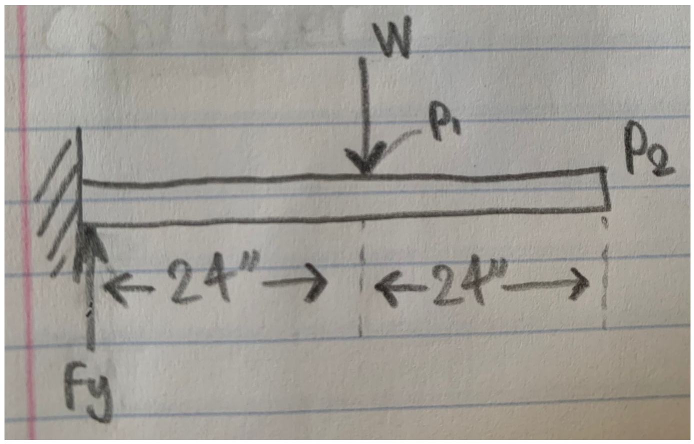


Figure 1: Free-body diagram of the cantilever beam with the load applied at the mid-span.

2.3.2 Cantilever Beam Load Applied at the End

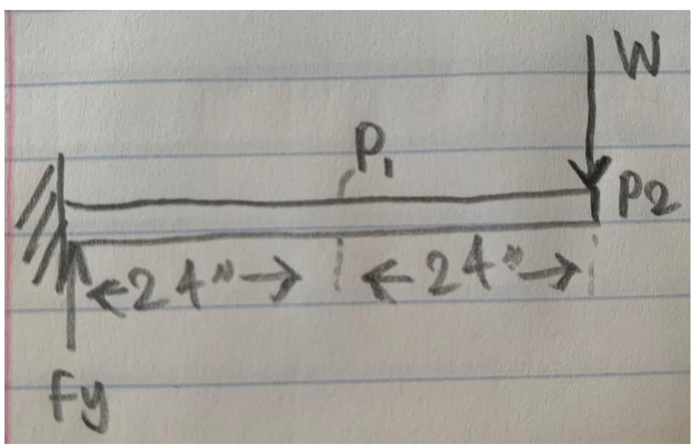


Figure 2: Free-body diagram of the cantilever beam with the load applied at the end.

2.3.3 Simply Supported Beam Load Applied at the Mid-Span

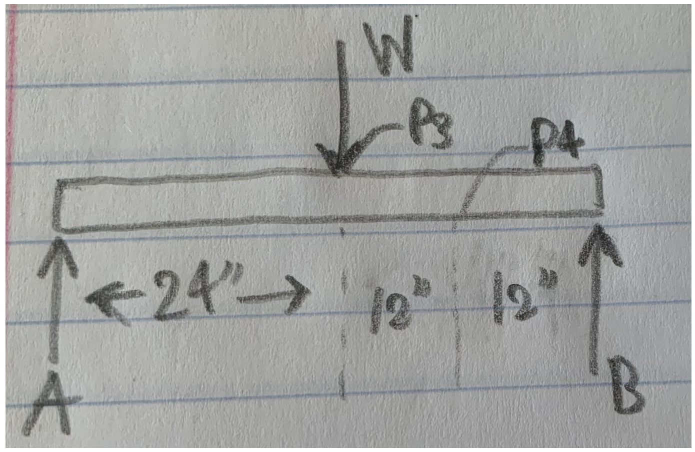


Figure 3: Free-body diagram of the simply supported beam with the load applied at the mid-span.

2.3.4 Simply Supported Beam Load Applied at the Quarter-Span

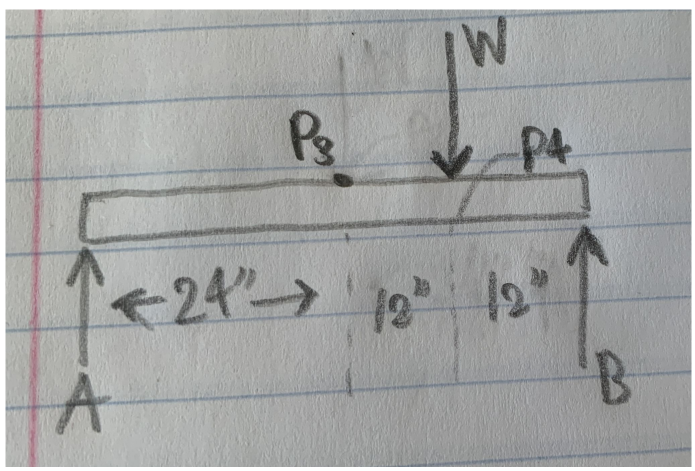


Figure 4: Free-body diagram of the simply supported beam with the load applied at the quarter-span.

3 Results

This section contains theoretical derivation of the deflection equations for each beam and each loading condition, the experimental results, and the plots and summary tables of the theoretical and experimental results in the comparison section.

3.1 Cantilever Beam Load Applied at the Mid-Span

3.1.1 Theoretical Results

The calculations for the theoretical deflection of the cantilever beam with the load applied at the mid-span are shown in figure 5.

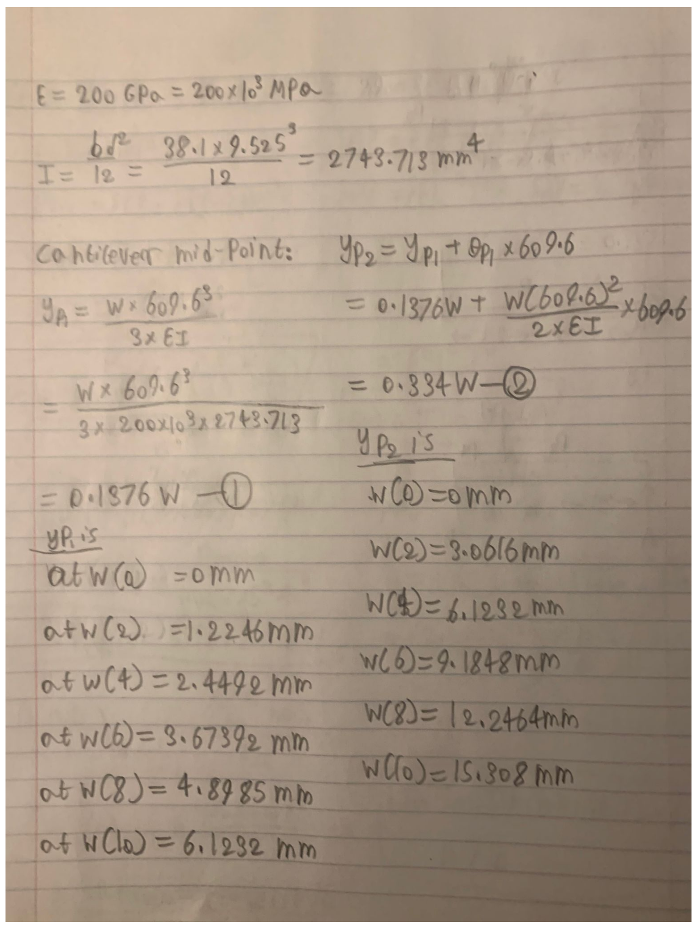


Figure 5: Theoretical deflection of the cantilever beam at the mid-span

3.1.2 Experimental Results

The experimental results for the deflection of the cantilever beam with the load applied at the mid-span are shown in table 1.

Table 1: Experimental deflection of the cantilever beam at the mid-span

Load (lb)	Deflection P1 (in)	Deflection P2 (in)
2	-0.096259843	-0.305385827
4	-0.19707874	-0.616444882
6	-0.29834252	-0.917594488
8	-0.413007874	-1.239968504
10	-0.513062992	-1.530185039

3.1.3 Comparison of Theoretical and Experimental Results

To compare the theoretical and experimental results, the theoretical deflection values and the experimental deflection values are shown in table 2.

Table 2: Comparison of theoretical and experimental deflection of the cantilever beam at the mid-span

Load (lb)	Theoretical Deflection (in)	Experimental Deflection (in)	Percentage Error (%)
0	0	0	0
2	-0.120525977	-0.096259843	20.13
4	-0.241051954	-0.19707874	18.24
6	-0.361577931	-0.29834252	17.49
8	-0.482103908	-0.413007874	14.33
10	-0.602629885	-0.513062992	14.86

The difference is also plotted in figure 6.

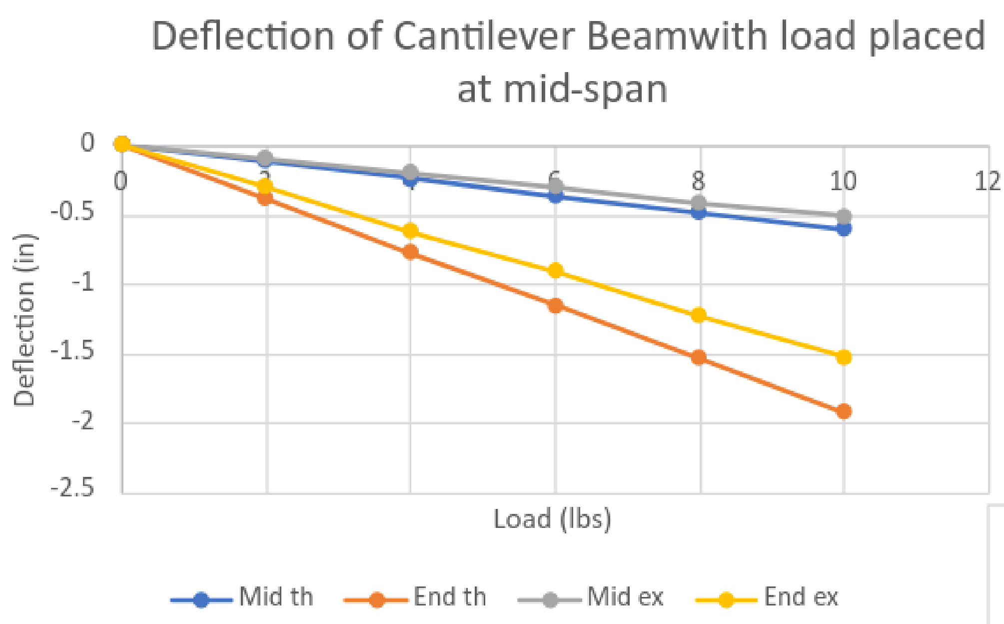
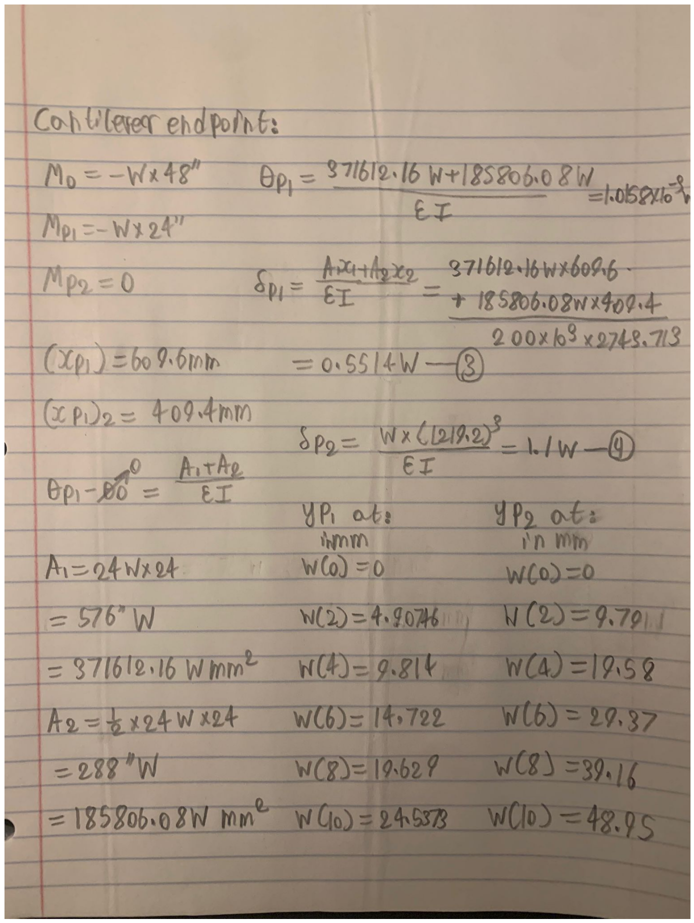


Figure 6: Comparison of theoretical and experimental deflection of the cantilever beam at the mid-span

3.2 Cantilever Beam Load Applied at the End

3.2.1 Theoretical Results

The calculations for the theoretical deflection of the cantilever beam with the load applied at the end are shown in figure 7.


Cantilever endpoint:

$M_0 = -W \times 48''$

$M_{P1} = -W \times 24''$

$M_{P2} = 0$

$(x_{P1}) = 609.6mm$

$(x_{P1})_2 = 409.4mm$

$\theta_{P1} - \cancel{\theta_0}^{0} = \frac{A_1 + A_2}{EI}$

$A_1 = 24W \times 24$

$= 576'' W$

$= 371612.16\, W mm^2$

$A_2 = \frac{1}{2} \times 24W \times 24$

$= 288'' W$

$= 185806.08 W\, mm^2$

$\theta_{P1} = \frac{371612.16\, W + 185806.08\, W}{EI} = 1.0158 \times 10^{-9}$

$\delta_{P1} = \frac{A_1 x_1 + A_2 x_2}{EI} = \frac{371612.16 W \times 609.6 + 185806.08 W \times 409.4}{200 \times 10^3 \times 2743.713}$

$= 0.5514 W$ — (3)

$\delta_{P2} = \frac{W \times (1219.2)^3}{EI} = 1.1 W$ — (4)

y_{P1} at: in mm	y_{P2} at: in mm
$W(0) = 0$	$W(0) = 0$
$W(2) = 4.90746$	$W(2) = 9.79$
$W(4) = 9.814$	$W(4) = 19.58$
$W(6) = 14.722$	$W(6) = 29.37$
$W(8) = 19.629$	$W(8) = 39.16$
$W(10) = 24.5373$	$W(10) = 48.95$

Figure 7: Theoretical deflection of the cantilever beam at the end

3.2.2 Experimental Results

The experimental results for the deflection of the cantilever beam with the load applied at the end are shown in table 3.

Table 3: Experimental deflection of the cantilever beam at the end

Load (lb)	Deflection P1 (in)	Deflection P2 (in)
2	-0.038511811	-0.085669291
4	-0.078137795	-0.183468504
6	-0.120314961	-0.297409449
8	-0.161791339	-0.398330709
10	-0.205374016	-0.506389764

3.2.3 Comparison of Theoretical and Experimental Results

To compare the theoretical and experimental results, the theoretical deflection values and the experimental deflection values are shown in table 4.

Table 4: Comparison of theoretical and experimental deflection of the cantilever beam at the end

Load (lb)	Theoretical Deflection (in)	Experimental Deflection (in)	Percentage Error (%)
0	0	0	0
2	-0.38568	-0.30539	20.8195
4	-0.77137	-0.61644	20.084
6	-1.15705	-0.91759	20.6953
8	-1.154273	-1.23997	19.625
10	-1.923842	-1.53019	20.6507

The difference is also plotted in figure 8.

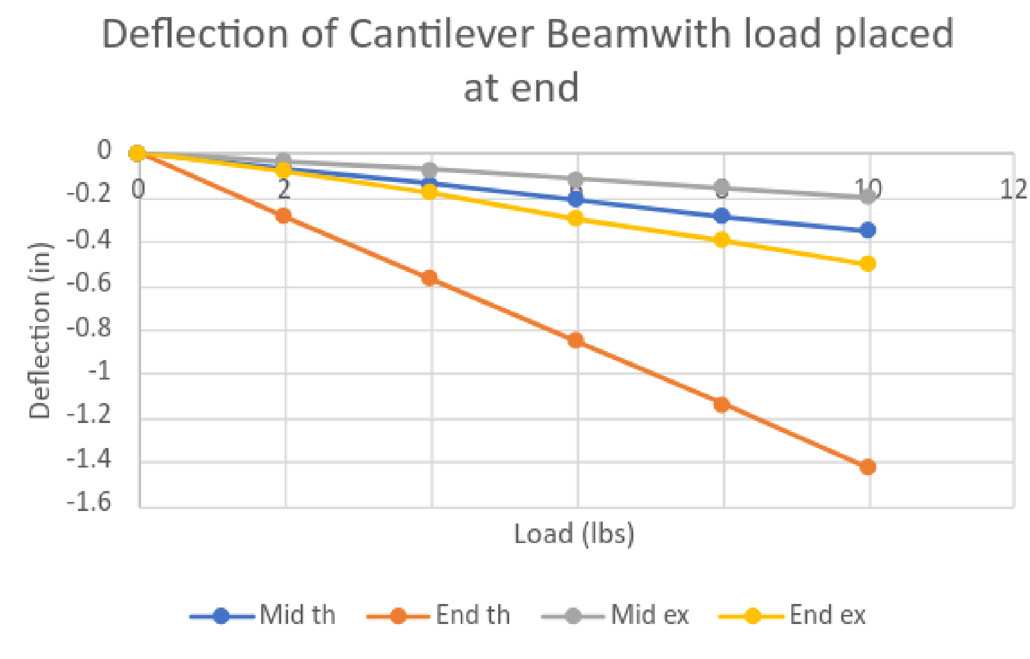


Figure 8: Comparison of theoretical and experimental deflection of the cantilever beam at the end

3.3 Simply Supported Beam Load Applied at the Mid-Span

3.3.1 Theoretical Results

The calculations for the theoretical deflection of the simply supported beam with the load applied at the mid-span are shown in figure 9.

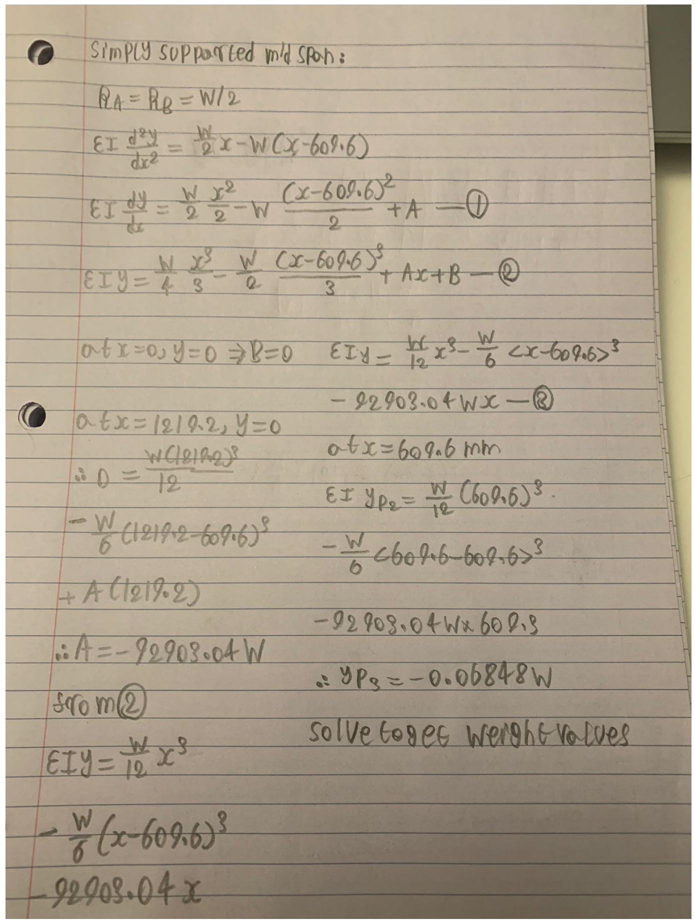


Figure 9: Theoretical deflection of the simply supported beam at the mid-span

3.3.2 Experimental Results

The experimental results for the deflection of the simply supported beam with the load applied at the mid-span are shown in table 5.

Table 5: Experimental deflection of the simply supported beam at the mid-span

Load (lb)	Deflection P1 (in)	Deflection P2 (in)
10	-0.073925197	-0.068783465
20	-0.152350394	-0.13619685
30	-0.23169685	-0.202275591
40	-0.313082677	-0.268098425

3.3.3 Comparison of Theoretical and Experimental Results

To compare the theoretical and experimental results, the theoretical deflection values and the experimental deflection values are shown in table 6.

Table 6: Comparison of theoretical and experimental deflection of the simply supported beam at the mid-span

Load (lb)	Theoretical Deflection (in)	Experimental Deflection (in)	Percentage Error (%)
0	0	0	0
10	-0.120525977	-0.073925197	38.66451143
20	-0.241051954	-0.152350394	36.79769396
30	-0.361577931	-0.23169685	35.92063273
40	-0.482103908	-0.313082677	35.05908748

The difference is also plotted in figure 10.

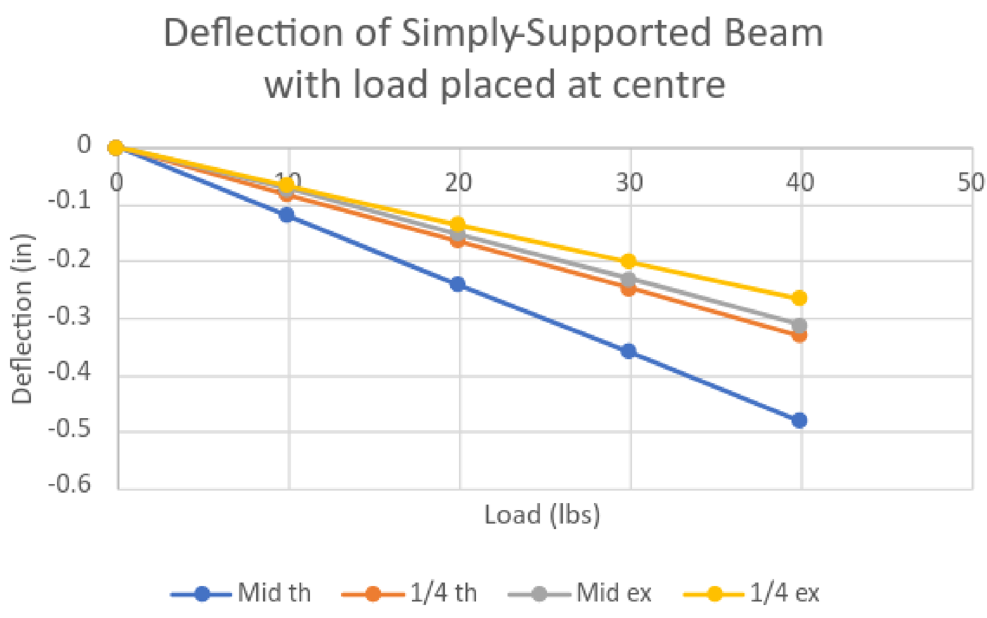
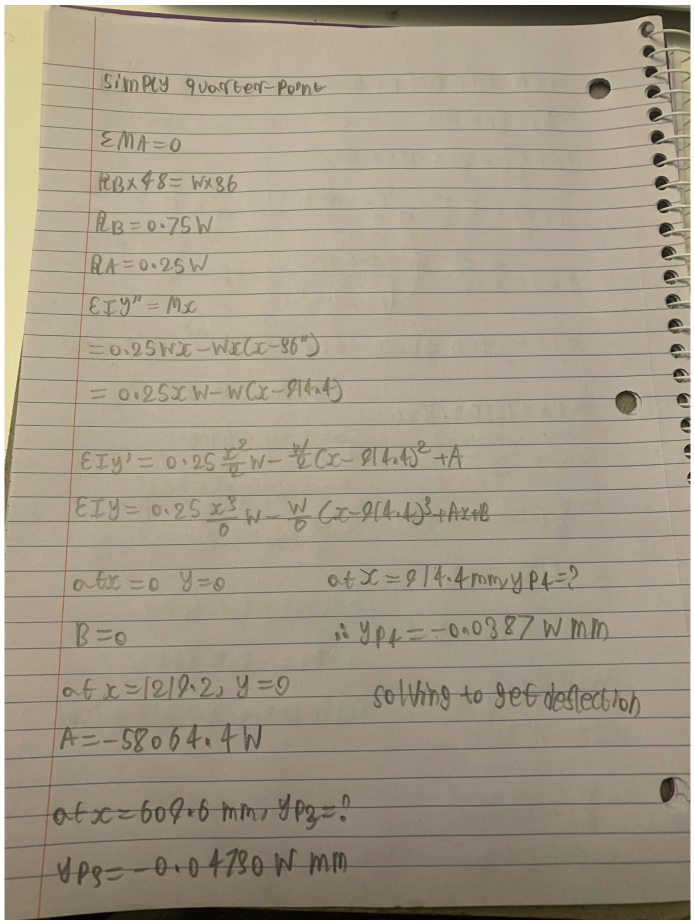


Figure 10: Comparison of theoretical and experimental deflection of the simply supported beam at the mid-span

3.4 Simply Supported Beam Load Applied at the Quarter-Span

3.4.1 Theoretical Results

The calculations for the theoretical deflection of the simply supported beam with the load applied at the quarter-span are shown in figure 11.



simply quarter-point

$\Sigma M_A = 0$

$R_B \times 48 = W \times 36$

$R_B = 0.75W$

$R_A = 0.25W$

$EIy'' = M_x$

$= 0.25Wx - Wx(x - 36'')$

$= 0.25xW - W(x - 914.4)$

$EIy' = 0.25\frac{x^2}{2}W - \frac{W}{2}(x - 914.4)^2 + A$

$EIy = 0.25\frac{x^3}{6}W - \frac{W}{6}(x - 914.4)^3 + Ax + B$

at $x = 0$ $y = 0$

$B = 0$

at $x = 1219.2$, $y = 0$

$A = -58064.4W$

at $x = 914.4$ mm, $y_{P4} = ?$

$\therefore y_{P4} = -0.0387$ W mm

solving to get deflection

at $x = 609.6$ mm, $y_{P3} = ?$

$y_{P3} = -0.04730$ W mm

Figure 11: Theoretical deflection of the simply supported beam at the quarter-span

3.4.2 Experimental Results

The experimental results for the deflection of the simply supported beam with the load applied at the quarter-span are shown in table 7.

Table 7: Experimental deflection of the simply supported beam at the quarter-span

Load (lb)	**Deflection P1 (in)**	**Deflection P2 (in)**
10	-0.018897638	-0.018897638
20	-0.039370079	-0.039370079
30	-0.05984252	-0.05984252
40	-0.080314961	-0.080314961

3.4.3 Comparison of Theoretical and Experimental Results

To compare the theoretical and experimental results, the theoretical deflection values and the experimental deflection values are shown in table 8.

Table 8: Comparison of theoretical and experimental deflection of the simply supported beam at the quarter-span

Load (lb)	**Theoretical Deflection (in)**	**Experimental Deflection (in)**	**Percentage Error (%)**
0	0	0	0
10	-0.120525977	-0.018897638	24.3305
20	-0.241051954	-0.039370079	23.628
30	-0.361577931	-0.05984252	23.452
40	-0.482103908	-0.080314961	23.332

The difference is also plotted in figure 12.

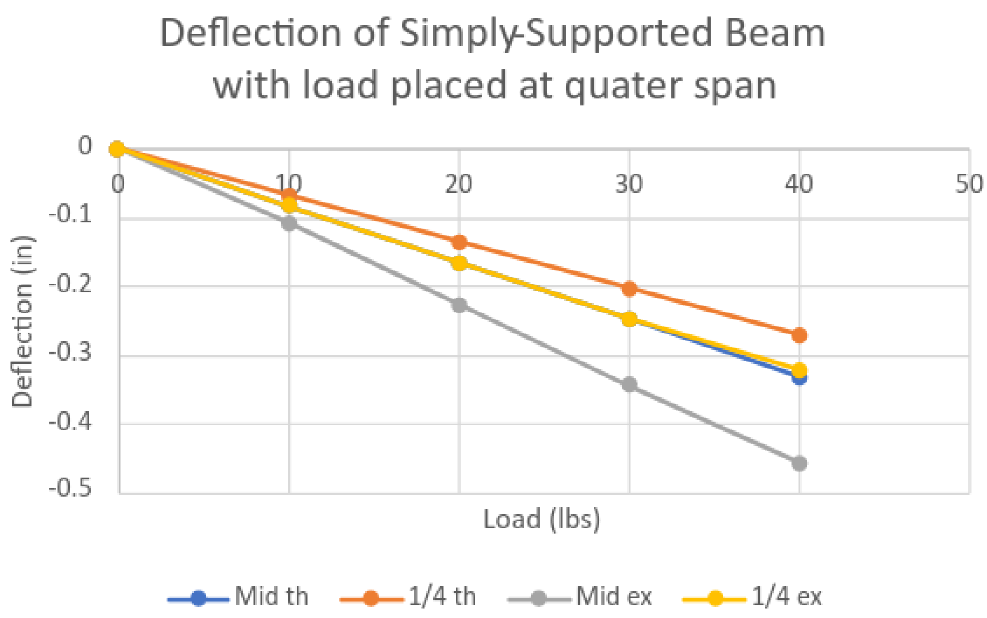


Figure 12: Comparison of theoretical and experimental deflection of the simply supported beam at the quarter-span

4 Discussion

4.1 Analysis of Results

The comparison of the theoretical and experimental results showed that the percentage error is around 20% for most the cases. This is a significant difference, the reason can be found in the sources of error section.

4.2 Sources of Error

The sources of error for this experiment are listed as follows:

1. **Material Properties:** The theoretical calculations assume that the beam is made of a homogeneous material with uniform properties. However, the actual beam is made of a composite material with varying properties. This difference in material properties can cause the theoretical calculations to be inaccurate.

2. **Beam Geometry:** The theoretical calculations assume that the beam is a perfect rectangle with uniform cross-section. However, the actual beam is not a perfect rectangle and has varying cross-section along its length. This difference in beam geometry can cause the theoretical calculations to be inaccurate.

3. **Experimental Errors:** The experimental measurements are subject to errors due to the limitations of the measuring instruments. This can cause the experimental results to be inaccurate.

5 Conclusions

5.1 Summary

The deflection of beams under different loading conditions is a fundamental aspect of structural engineering. In this experiment, the deflection of two different beam configurations, the cantilever and the simply supported beam, were investigated both theoretically and experimentally. The deflection of these beams were measured under varying loads and the theoretical and experimental results were compared to assess the accuracy and reliability of the theoretical models in predicting structural deflections.

The objectives for this experiment were outlined as follows:

1. **Measurement of Deflection:** Experimentally acquire accurate values of deflection for both a cantilever and a simply supported beam under varying loads to understand their structural behaviors.

2. **Comparison of Experimental and Theoretical Results:** Analyze and compare the obtained experimental deflection values with the theoretically predicted values for both the cantilever and the simply supported beam to assess the accuracy and reliability of theoretical models in predicting structural deflections.

The results of this experiment showed that the theoretical and experimental deflection values are not in agreement. The percentage error between the theoretical and experimental results was around 20% for most of the cases. This difference in the results can be attributed to the following sources of error: the difference in material properties, the difference in beam geometry, and the experimental errors.

5.2 Maxwell's Law of Reciprocal Deflections

Maxwell's law of reciprocal deflections states that the deflection at any point in a beam is equal to the deflection at the same point in a beam with the loading and supports interchanged. This law was verified in this experiment by comparing the deflection of the cantilever beam with the load applied at the mid-span and the deflection of the cantilever beam with the load applied at the end. The deflection values at the mid-span ($P1$) and at the end of the cantilever beam ($P2$) were measured for both loading conditions. The theoretical deflection values were also calculated for both loading conditions. The comparison of the theoretical and experimental results showed that the deflection values at the mid-span ($P1$) and at the end of the cantilever beam ($P2$) are equal for both loading conditions. This verifies Maxwell's law of reciprocal deflections.