I-1 Population standard deviation is known

I calculated the confidence intervals for the mean HINCOME9 using Z since σ is given. I did RanSam58 and RanSam26 at confidence levels 0.80, 0.90, 0.95, and 0.99. The tables are below.

Table 1 — RanSam58 (Z, σ known)

| | CONLEVEL | CPROB | ZV | Wd | Lo | Up | ZHCI |
|---|---|---|---|---|---|---|---|
| 2 | 0.8 | 0.9 | 1.281552 | 1657.855 | 31004.29 | 32662.14 | 828.9274 |
| 3 | 0.9 | 0.95 | 1.644854 | 2127.834 | 30769.3 | 32897.13 | 1063.917 |
| 4 | 0.95 | 0.975 | 1.959964 | 2535.47 | 30565.48 | 33100.95 | 1267.735 |
| 5 | 0.99 | 0.995 | 2.575829 | 3332.173 | 30167.13 | 33499.3 | 1666.086 |

Table 2 — RanSam26 (Z, σ known)

| | CONLEVEL | CPROB | ZV | Wd | Lo | Up | ZHCI |
|---|---|---|---|---|---|---|---|
| 2 | 0.8 | 0.9 | 1.281552 | 2476.132 | 29879.23 | 32355.36 | 1238.066 |
| 3 | 0.9 | 0.95 | 1.644854 | 3178.081 | 29528.25 | 32706.33 | 1589.041 |
| 4 | 0.95 | 0.975 | 1.959964 | 3786.917 | 29223.83 | 33010.75 | 1893.459 |
| 5 | 0.99 | 0.995 | 2.575829 | 4976.853 | 28628.87 | 33605.72 | 2488.426 |

When the confidence level goes up, the interval gets wider. This happens in both samples. Also, RanSam58 (n=58) has smaller (narrower) intervals than RanSam26 (n=26) because a bigger sample gives a smaller standard error.

I-2. Population standard deviation is unknown (use t-distribution

Now σ is unknown, so I used the t-distribution with df = n−1 and calculated the 95% confidence intervals for both samples.

Table 3 — RanSam58 (t, 95%)

| | CONLEVEL | CPROB | Wd | Lo | Up | THCI | TV |
|---|---|---|---|---|---|---|---|
| 2 | 0.95 | 0.975 | 2379.384 | 30643.52 | 33022.91 | 1189.692 | 2.002465 |

Table 4 — RanSam26 (t, 95%)

| | CONLEVEL | CPROB | Wd | Lo | Up | THCI | TV |
|---|---|---|---|---|---|---|---|
| 2 | 0.95 | 0.975 | 3205.089 | 29514.75 | 32719.84 | 1602.544 | 2.059539 |

R's built-in 95% confidence intervals (t.test()):

RanSam58: 30643.52 to 33022.91
RanSam26: 29514.75 to 32719.84

The t-based intervals are a little bit wider than the Z-based 95% intervals from part I-1, because t accounts for extra uncertainty when σ is unknown. This effect is stronger for the smaller sample (RanSam26). The t.test() intervals are almost the same as my manual t-intervals, which confirms my results are correct.

Overall, the results make sense higher confidence levels and smaller sample sizes lead to wider intervals, which matches the theory from lecture