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CSI2110 (Fall 2021)
Assignment 8 (3%) – 12 points

Due: Thursday Nov 18, 11:59PM

Late assignment policy: 1min-24hs late are accepted with 30% off; no assignments accepted after that.

Question 1. ***[7 points]***

Consider the following Hash table where insertions are done using the hash function

$$h(k) = 3*k + 1 \bmod 11.$$

Insert the following keys 10, 21, 3, 14, 18, 7 in an empty Hash table with 11 spots, **mark the positions (underline/square/circle a key on the position)** that are probed for each insertion (in the given table show the Hash table after each insertion). After all insertions, what is the **average number of probes** (comparisons) needed to search for an existing key in this table? In each part, collisions are solved by each of the following 3 methods specified in the part.

Hint: Here is an example when inserting different keys 1, 3 and 8 with linear probing. Each row shows the table after the insertion of each key. For each insertion we underline the positions that were probed; since every element was inserted in their home address the number of probes for each element is 1 and the average number of probes is (1+1+1)/3 = 1.

0	1	2	3	4	5	6	7	8	9	10
				1						
				1						**3**
			8	1						3

1. **[2 point]** Linear probing

0	1	2	3	4	5	6	7	8	9	10	
									10		1
									10	21	2
3									10	21	2

3	14								10	21	3
3	14	18							10	21	3
3	14	18	7						10	21	4

Average number of probes = (1+2+2+3+3+4)/6 = 2.5 probes

2. **[2 points]** Quadrating probing

0	1	2	3	4	5	6	7	8	9	10	
									10		1
									10	21	2
3									10	21	3
3		14							10	21	3
3	18	14							10	21	2
3	18	14	7						10	21	3

Average number of probes = (1+2+2+3+2+3)/6 = 2.17 probes

3. **[3 points]** Double hashing with secondary hash function: d(k) = 4 – (k mod 3).
Double hashing is done by $h_j(k) = [h(k) + j^2 \cdot d(k)] \bmod 11$.

10 = (9+

0	1	2	3	4	5	6	7	8	9	10
									10	
		21							10	
		21							10	3
	14	21							10	3
18	14	21							10	3
18	14	21	7						10	3

K	h(K)	d(K)	Probes	
10	9	3	9	1
21	2	4	9 2	2
3	10	4	10	1
14	1	2	10 1	2
18	0	4	0	1
7	3	3	0 3	2

Average number of probes = (1+2+1+2+1+2)/6 = 1.5 probes

Question 2. [5 points] Consider a hash table that is constructed with double hashing:

$h(k) = [h(k) + j*d(k)] \bmod 11$

j x d(K)?

The primary hash function is h(k) = (2k + 1) mod 11 and the secondary hash function is d(k) = (k div 2)+1 where div is integer division (e.g. 10 mod 3 = 1 and 10 div 3 =3).

1. **[2 point]** Insert the keys 13 and then 11 in the following hash table and describe the process indicating for each key the positions probed.

① cal the h(k)
② If occupied cal d(k)
③ Then cal the double hashing, if occupied repeat from step ②

0	1	2	3	4	5	6	7	8	9	10
					2	19	3			
					2	19	3			13
	11				2	19	3			13

K	$h_i(k)$	d(k)	Probes	
13	10	8	5,10	2
11	1	6	1	1

2. **[1 point]** Delete the key 2 in the hash table below and describe the process.

0	1	2	3	4	5	6	7	8	9	10
	11				13		2			
	11				13		avail			

→ probe until found (5...)
① Search key 2 → 5
② Delete & replace with avail so we can tell that it was an occupied cell

3. **[1 point]** Search the key 11 in the hash table and describe the process.

0	1	2	3	4	5	6	7	8	9	10
16	AVAILABLE	AVAILABLE			2		3	11		

h(11) = (22+1) mod 11 = 1
= available (keep prob
d(11) = (11 div 2) + 1
= 6
$h_1(11)$ = (1 + 1·6) mod 11
= 7 → 3 (not 11 keep probing)
$h_2(11)$ = (1 + 2·6) mod 11 = 2 → avail
keep probing
$h_3(11)$ = (1 + 3·6) mod 11
= 8 → 11 (found)
4 probes

4. **[1 point]** Insert the key 25 in the hash table, describe the process.

0	1	2	3	4	5	6	7	8	9	10
16	AVAILABLE	AVAILABLE		25	2		3	11	4	

h(25) = (51) mod 11 = 7, occupied, find d(k) & keep probing
d(25) = 13
$h_1(25)$ = (7 + 13) mod 11
= 9, occupied, keep probing
$h_2(25)$ = (7 + 26) mod 11
= 0, occupied, keep probing
$h_3(25)$ = (7 + 39) mod 11
= 2, occupied, keep probing
$h_4(25)$ = (7 + 52) mod 11
= 4, not occupied
then insert at index 4
5 probes