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sheet (1)

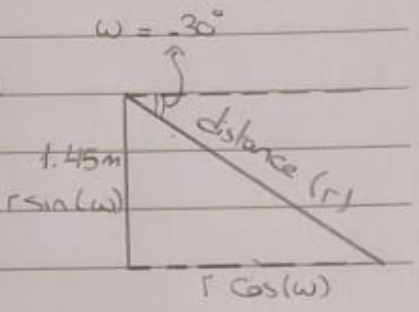
Question (1) :-

1) For a single layer :- no. of generated points = $\frac{360}{0.36}$ = 1000 Point/layer

Total no. of points = 12 Layers x 1000 Point/Layer = 12 000 Point

2) Degree interVal = $\frac{(-30)-(-6)}{12}$ = $-2°$

3) laser layer that will hit ground first is 12



The distance (r) = $\left|\frac{1.45}{\sin(-30°)}\right|$ = 2.9 m

4) laser layer that will hit ground last is 1

The distance (r) = $\left|\frac{1.45}{\sin(-6°)}\right|$ = 13.87 m

5) @ layer 1 :- r = 13.87 m (rejected)
@ layer 2 :- r = 10.42 m (rejected)
@ layer 3 :- r = 8.35 m (Accepted)

∴ 10 layers only will produce points

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Question (2) :-

1] Model using P1, P5 :-

↳ Equation :- $-3.5x + 2.5y + 4.6 = 0$

$35x - 25y - 46 = 0$

↳ Distances :- P1 → d = 0 P5 → d = 0

P2 → d = 0 P6 → d = 0.23

P3 → d = 0.3 P7 → d = 0.7

P4 → d = 0.35 P8 → d = 0.8

↳ no. of outliers = 2

2] Model using P2, P7 :-

↳ Equation :- $-3.1x + 3.1y + 0.62 = 0$

$5x - 5y - 1 = 0$

↳ Distances :- P1 → d = 0.28 P5 → d = 0.42

P2 → d = 0 P6 → d = 0.28

P3 → d = 0.14 P7 → d = 0

P4 → d = 0.71 P8 → d = 0.28

↳ no. of outliers = 2

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3] For the best model, we should consider all 28 (8C2) cases.

After checking all cases, we got that :-

→ The least no of outliers are 1

→ The models that will have the least no of outliers are :-
Model using P_2 and P_6
Model using P_2 and P_8

→ Number of iterations needed = 28 - 2 + 1 = 27

4] least squares model :-

$$\begin{bmatrix} b \\ m \end{bmatrix} = (X^T X)^{-1} X^T y$$

where :- b = y intercept
m = slope

→ $X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3.1 & 4.1 & 4.9 & 5.1 & 5.6 & 6.1 & 7.2 & 8.5 \end{bmatrix} \begin{bmatrix} 1 & 3.1 \\ 1 & 4.1 \\ 1 & 4.9 \\ 1 & 5.1 \\ 1 & 5.6 \\ 1 & 6.1 \\ 1 & 7.2 \\ 1 & 8.5 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 44.6 \\ 44.6 & 269.1 \end{bmatrix}$$

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$$\rightarrow (X^T X)^{-1} = \begin{bmatrix} 1.644 & \frac{-1115}{4091} \\ \frac{-1115}{4091} & \frac{200}{4091} \end{bmatrix}$$

$$\rightarrow X^T y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3.1 & 4.1 & 4.9 & 5.1 & 5.6 & 6.1 & 7.2 & 8.5 \end{bmatrix} \begin{bmatrix} 2.5 \\ 3.9 \\ 4.5 \\ 5.9 \\ 6 \\ 6.3 \\ 7 \\ 8.7 \end{bmatrix}$$

$$= \begin{bmatrix} 44.8 \\ 272.26 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} b \\ m \end{bmatrix} = (X^T X)^{-1} X^T y = \begin{bmatrix} 1.644 & \frac{-1115}{4091} \\ \frac{-1115}{4091} & \frac{200}{4091} \end{bmatrix} \begin{bmatrix} 44.8 \\ 272.26 \end{bmatrix}$$

$$= \begin{bmatrix} -0.553 \\ 1.0999 \end{bmatrix}$$

→ Line equation :- $1.0999x - y - 0.553 = 0$

→ Distances :-

P1 → d = 0.24
P2 → d = 0.04
P3 → d = 0.23
P4 → d = 0.57
P5 → d = 0.26
P6 → d = 0.09
P7 → d = 0.25
P8 → d = 0.06

∴ we have 1 outlier

Q2.3

Combination of points				Line Equation		Distance								Number of outliers
Point 1		Point 2		Slope	Y-intercept	P1	P2	P3	P4	P5	P6	P7	P8	
3.1	2.5	4.1	3.9	1.400	-1.840	0.00	0.00	0.30	0.35	0.00	0.23	0.72	0.79	2
3.1	2.5	4.9	4.5	1.111	-0.944	0.00	0.19	0.00	0.79	0.48	0.31	0.04	0.13	2
3.1	2.5	5.1	5.9	1.700	-2.770	0.00	0.15	0.54	0.00	0.38	0.66	1.25	1.51	4
3.1	2.5	5.6	6	1.400	-1.840	0.00	0.00	0.30	0.35	0.00	0.23	0.72	0.79	2
3.1	2.5	6.1	6.3	1.267	-1.427	0.00	0.08	0.17	0.54	0.21	0.00	0.43	0.40	2
3.1	2.5	7.2	7	1.098	-0.902	0.00	0.20	0.02	0.81	0.51	0.34	0.00	0.18	2
3.1	2.5	8.5	8.7	1.148	-1.059	0.00	0.17	0.04	0.72	0.41	0.23	0.14	0.00	2
4.1	3.9	4.9	4.5	0.750	0.825	0.52	0.00	0.00	1.00	0.78	0.72	0.62	1.20	6
4.1	3.9	5.1	5.9	2.000	-4.300	0.27	0.00	0.45	0.00	0.40	0.72	1.39	1.79	5
4.1	3.9	5.6	6	1.400	-1.840	0.00	0.00	0.30	0.35	0.00	0.23	0.72	0.79	2
4.1	3.9	6.1	6.3	1.200	-1.020	0.13	0.00	0.23	0.51	0.19	0.00	0.40	0.31	1
4.1	3.9	7.2	7	1.000	-0.200	0.28	0.00	0.14	0.71	0.42	0.28	0.00	0.28	2
4.1	3.9	8.5	8.7	1.091	-0.573	0.21	0.00	0.18	0.61	0.31	0.15	0.19	0.00	1
4.9	4.5	5.1	5.9	7.000	-29.800	1.50	0.71	0.00	0.00	0.48	0.93	1.92	2.97	6
4.9	4.5	5.6	6	2.143	-6.000	0.79	0.47	0.00	0.41	0.00	0.33	1.03	1.49	5
4.9	4.5	6.1	6.3	1.500	-2.850	0.39	0.33	0.00	0.61	0.25	0.00	0.53	0.67	3
4.9	4.5	7.2	7	1.087	-0.826	0.03	0.18	0.00	0.80	0.50	0.34	0.00	0.19	2
4.9	4.5	8.5	8.7	1.167	-1.217	0.07	0.22	0.00	0.76	0.44	0.26	0.12	0.00	2
5.1	5.9	5.6	6	0.200	4.880	2.94	1.77	1.33	0.00	0.00	0.20	0.67	2.08	5
5.1	5.9	6.1	6.3	0.400	3.860	2.41	1.49	1.23	0.00	0.09	0.00	0.24	1.34	4
5.1	5.9	7.2	7	0.524	3.229	2.08	1.31	1.15	0.00	0.14	0.11	0.00	0.90	4
5.1	5.9	8.5	8.7	0.824	1.700	1.35	0.91	0.95	0.00	0.24	0.33	0.49	0.00	4
5.6	6	6.1	6.3	0.600	2.640	1.71	1.03	0.93	0.17	0.00	0.00	0.03	0.82	4
5.6	6	7.2	7	0.625	2.500	1.64	0.99	0.90	0.18	0.00	0.01	0.00	0.75	4
5.6	6	8.5	8.7	0.931	0.786	0.86	0.51	0.62	0.27	0.00	0.12	0.36	0.00	3
6.1	6.3	7.2	7	0.636	2.418	1.60	0.95	0.87	0.20	0.02	0.00	0.00	0.74	4
6.1	6.3	8.5	8.7	1.000	0.200	0.57	0.28	0.42	0.42	0.14	0.00	0.28	0.00	3
7.2	7	8.5	8.7	1.308	-2.415	0.52	0.58	0.31	1.00	0.66	0.45	0.00	0.00	5

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Question (3) :-

[1] a) $\vec{P} \times \vec{q} = \begin{vmatrix} i & j & K \\ -1 & 2 & 0 \\ 3 & 1 & 4 \end{vmatrix} = 8i + 4j - 7K$

b) $\vec{P} \times \vec{q} = \begin{vmatrix} i & j & K \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{vmatrix} = -K$

[2] a) Equation of the plane :- $5x + 4y - 6z - 3 = 0$

d) Distance $= \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|15 + 4 - 24 - 3|}{\sqrt{25 + 16 + 36}}$

$= \frac{8}{\sqrt{77}} \approx 0.912$