

(Sheet (5))

Question (1) :-

$$F = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad v = \begin{bmatrix} 0.194 \\ 0.194 \\ 0.387 \\ 0.387 \end{bmatrix}$$

$$R = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0375 & 0 & 0.075 & 0 \\ 0 & 0.0375 & 0 & 0.075 \\ 0.075 & 0 & 0.15 & 0 \\ 0 & 0.075 & 0 & 0.15 \end{bmatrix}$$

⇒ For each Iteration :-

$$x^i = Fx + v$$

$$K = P^i H^T S^{-1}$$

$$P^i = F P F^T + Q$$

$$y = Z - Hx^i$$

Z - measurements

$$x = x^i + Ky$$

$$S = H P^i H^T + R$$

$$P = (I - KH) P^i$$

$$1) x_1 = \begin{bmatrix} -386.7642 \\ 295.1439 \\ -382.7690 \\ 292.4387 \end{bmatrix} \quad x_2 = \begin{bmatrix} -448.1163 \\ 354.2572 \\ -202.0062 \\ 161.6980 \end{bmatrix} \quad x_3 = \begin{bmatrix} -446.6460 \\ 365.8074 \\ -119.8366 \\ 101.6009 \end{bmatrix}$$

$$2) P_1 = \begin{bmatrix} 8.8424 & 0 & 8.7555 & 0 \\ 0 & 8.8424 & 0 & 8.7555 \\ 8.7555 & 0 & 13.6582 & 0 \\ 0 & 8.7555 & 0 & 13.6582 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 7.3486 & 0 & 4.1264 & 0 \\ 0 & 7.3486 & 0 & 4.1264 \\ 4.1264 & 0 & 3.4973 & 0 \\ 0 & 4.1264 & 0 & 3.4973 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 6.1212 & 0 & 2.4626 & 0 \\ 0 & 6.1212 & 0 & 2.4626 \\ 2.4626 & 0 & 1.5407 & 0 \\ 0 & 2.4626 & 0 & 1.5407 \end{bmatrix}$$

Question (2) :

$$1) n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$\theta = 0 \quad 1.25 \quad 2.5 \quad 3.75 \quad 5 \quad 6.25$$

$$\text{Noise} = 0.05 \quad -0.05 \quad 0.05 \quad -0.05 \quad 0.05 \quad -0.05$$

$$x_m = 1.05 \quad 0.265 \quad -0.751 \quad -0.87 \quad 0.334 \quad 0.95$$

$$y_m = 0.05 \quad 0.899 \quad 0.648 \quad -0.622 \quad -0.9 \quad -0.083$$

$$2) F = \begin{bmatrix} 1 & 0 & 1.25 & 0 \\ 0 & 1 & 0 & 1.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} 0.391 \\ 0.391 \\ 0.625 \\ 0.625 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.153 & 0 & 0.244 & 0 \\ 0 & 0.153 & 0 & 0.244 \\ 0.244 & 0 & 0.391 & 0 \\ 0 & 0.244 & 0 & 0.391 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1.05 \\ 0.05 \\ 1.15 \\ 0.35 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0.27 \\ 0.899 \\ -0.5451 \\ 0.9929 \end{bmatrix} \quad x_3 = \begin{bmatrix} -0.745 \\ 0.6634 \\ -0.7795 \\ -0.5972 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} -0.8745 \\ -0.6129 \\ 0.4172 \\ -1.1319 \end{bmatrix} \quad x_5 = \begin{bmatrix} 0.331 \\ -0.9075 \\ 1.4175 \\ 0.4275 \end{bmatrix} \quad x_6 = \begin{bmatrix} 0.966 \\ -0.082 \\ 0.0744 \\ 0.9239 \end{bmatrix}$$

n	0	1	2	3	4	5
x_t	1	0.315	-0.8	-0.82	0.284	0.999
y_t	0	0.949	0.598	-0.572	-0.959	-0.033
x_p	1.05	0.27	-0.745	-0.8745	0.331	0.966
y_p	0.05	0.899	0.663	-0.6129	-0.908	-0.082

$$\text{RMSE}_x = \sqrt{\frac{1}{6} \sum_{n=0}^5 (x_{\text{est}} - x_{\text{true}})^2} = 0.0495$$

$$\text{RMSE}_y = 0.0535$$

Question(3) :

$$1) n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$$P = 1.1 \quad 0.9 \quad 1.1 \quad 0.9 \quad 1.1 \quad 0.9 \quad 1.1$$

$$Q = 0.5 \quad 0.5 \quad 2.5 \quad 2.5 \quad 4.5 \quad 4.5 \quad 6.5$$

$$P^* = 2 \quad -2 \quad 2 \quad -2 \quad 2 \quad -2 \quad 2$$

$$2) F = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad z = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{bmatrix} \quad v = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.5 \\ 0.5 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad Q = \begin{bmatrix} \frac{1}{16} & 0 & \frac{1}{8} & 0 \\ 0 & \frac{1}{16} & 0 & \frac{1}{8} \\ \frac{1}{8} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{8} & 0 & \frac{1}{4} \end{bmatrix}$$

→ For each Iterations:

$$x' = Fz + v$$

$$x_1 = \begin{bmatrix} 0.7814 \\ 0.7757 \\ 1.1040 \\ 1.0983 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0.8214 \\ 0.4743 \\ 0.2031 \\ -0.1568 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0.2159 \\ 1.8140 \\ 0.1305 \\ 1.1805 \end{bmatrix}$$

$$P' = FPF^T$$

$$y = z - h(x')$$

$$S = HP'H^T, R$$

$$K = P'H^TS^{-1}$$

$$x = x' + Ky$$

$$x_4 = \begin{bmatrix} 0.6924 \\ 0.8720 \\ 0.1214 \\ -0.1846 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} -1.2130 \\ 2.3060 \\ -0.9139 \\ 1.8462 \end{bmatrix}$$

$$x_6 = \begin{bmatrix} -0.6949 \\ 0.7148 \\ -0.5462 \\ -0.9169 \end{bmatrix}$$

$$x_7 = \begin{bmatrix} -4.6147 \\ -3.2805 \\ -2.0335 \\ -3.2245 \end{bmatrix}$$

$$P = (I - KH)P'$$

$$3) \frac{RMSE}{P_x} = 2.26$$

$$\frac{RMSE}{P_y} = 1.82$$

4] $RMSE_x = 0.74$

$RMSE_y = 0.56$

6] The best technique in this case is to use the Lidar only, as it has the least RMSE.

Question (4) :- Please refer to class notes.