

Sheet (3)

Question (1) :-

1) The area covered by the camera = area of circular sector = $\frac{1}{2} r^2 \theta$ (radian)

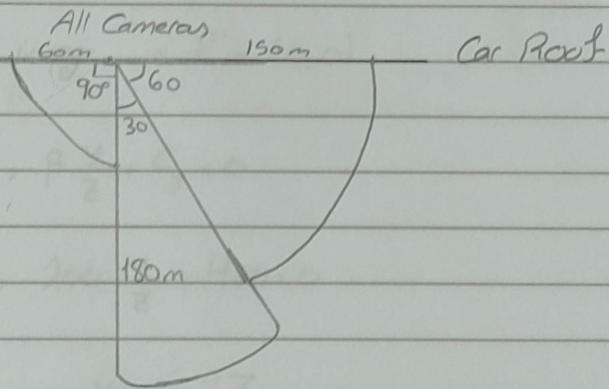
$$\rightarrow \text{Camera (1)} \therefore \text{Area} = \frac{1}{2} (180)^2 \left(\frac{\pi}{180} \times 30 \right) = 8482.3 \text{ m}^2$$

$$\rightarrow \text{Camera (2)} \therefore \text{Area} = \frac{1}{2} (150)^2 \left(\frac{\pi}{180} \times 60 \right) = 11781 \text{ m}^2$$

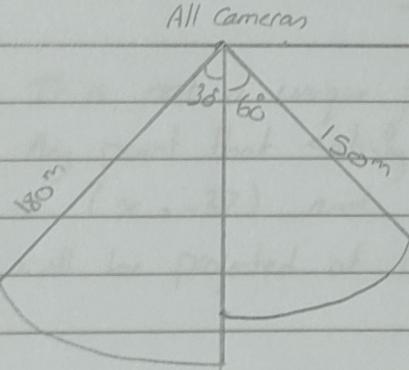
$$\rightarrow \text{Camera (3)} \therefore \text{Area} = \frac{1}{2} (60)^2 \left(\frac{\pi}{180} \times 90 \right) = 2827.5 \text{ m}^2$$

Camera (2) covers the largest area.

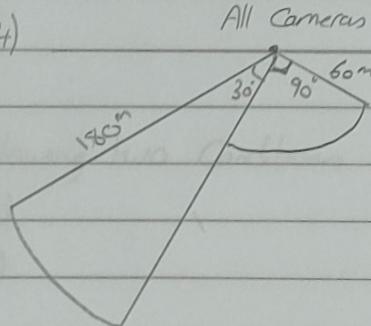
2)



3)



4)



Question (2) :-

1] a. $P = (30, 20, 50) \rightarrow P' = (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$
 $= (360, 480)$

b. $P = (10, 10, 10) \rightarrow P' = (400, 600)$

c. $P = (-30, -20, 50) \rightarrow P' = (240, 320)$

2] Any point that will have P' outside the image size (600x800)

Example : $P = (300, 200, 50) \rightarrow P' = (900, 1200) > \text{image size}$

3] At top left corner of image $\rightarrow P' = (0,0)$

$$\alpha \frac{x}{z} + c_x = 0, \beta \frac{y}{z} + c_y = 0$$

$$100 \frac{x}{z} + 300 = 0, 100 \frac{y}{z} + 400 = 0$$

$$x = -3z$$

$$y = -2z$$

\Rightarrow It is not a unique point.

Any point that satisfy the following two conditions

$$(x = -3z) \text{ and } (y = -2z)$$

will be projected at $P' = (0,0)$

4] Yes.

Example 2: in the previous answer we said that, if a point satisfy ($x_1=32$) and ($y_1=22$) then they will be projected at $P' = (0,0)$

$$\text{Put } z_1=1 \rightarrow P_1 = (-3, -2, 1)$$

$$z_2=2 \rightarrow P_2 = (-6, -4, 2)$$

$$z_3=3 \rightarrow P_3 = (-9, -6, 3)$$

P_1, P_2, P_3 will be projected at $P' = (0,0)$

→ check if they are collinear by using triangle inequality :-

$$a = \sqrt{14} \quad b = 2\sqrt{14} \quad c = \sqrt{14}$$

$$\text{as } a+c=b$$

∴ P_1, P_2, P_3 are collinear

2. Calculating the triangle sides a, b and c by the equations:

$$a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$b = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$$

$$c = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$$

Now apply one of the following methods:

triangle inequality: $a + b > c \quad a + c > b \quad b + c > a$

if all the inequalities are true then the points are not collinear.

Second method is by calculating the area by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

If the area equal 0 then the points are collinear (s - half the perimeter).

Question (3) :-

a. using the Constant Velocity model :-

$$d(t+\Delta t) = d(t) - v_0 \Delta t$$

$$0 = 25 - (30 \times \frac{5}{18}) \Delta t$$

$\Delta t = 3$ second

b. using the Constant Velocity model :-

$$d(t+\Delta t) = d(t) - v(t) \Delta t - \frac{1}{2} a_0 \Delta t^2$$

$$0 = 25 - (30 \times \frac{5}{18}) \Delta t - \frac{5}{2} \Delta t^2$$

$\Delta t = 1.91$ sec

c. using a hybrid model :-

$$\text{At constant deceleration :- } v(t+\Delta t) = v(t) - a_0 \Delta t$$

$$v' = (30 \times \frac{5}{18}) - 5 \times 1$$

$$v' = \frac{10}{3} \text{ m/s}$$

$$d(t+\Delta t) = d(t) - v(t) \Delta t - \frac{1}{2} a_0 \Delta t^2$$

$$d' = 25 - (30 \times \frac{5}{18}) \times 1 - \frac{1}{2} \times 5 \times (1)^2 \\ = \frac{85}{6} \text{ m}$$

$$\text{At constant velocity :- } d(t+\Delta t) = d(t) - v_0 \Delta t$$

$$0 = \frac{85}{6} - \frac{10}{3} \Delta t$$

$$\Delta t = 4.25 \text{ sec}$$

$$\therefore \text{Total TTC} = 1 + 4.25 = 5.25 \text{ seconds}$$

d. At hybrid Model 2-

→ At Constant deceleration $\therefore \Delta t = 1 \text{ sec}$ $a = ??$

$$v' = (30 \times \frac{5}{18}) - ax1$$

$$v' = \frac{25}{3} - a$$

$$\begin{aligned} d' &= 25 - (30 \times \frac{5}{18}) \times 1 - \frac{1}{2} a (1)^2 \\ &= \frac{50}{3} - \frac{1}{2} a \end{aligned}$$

→ At Constant Velocity $\therefore d(t+\Delta t) = d(t) - v_0 \Delta t$

$$0 = d' - v' \Delta t$$

$$0 = \frac{50}{3} - \frac{a}{2} - \frac{25}{3} \Delta t + a \Delta t$$

↓
Equ. (1)

At Constant deceleration $\therefore d(t+\Delta t) = d(t) - v(t) \Delta t - \frac{a_0}{2} \Delta t^2$

$$0 = 25 - \frac{25}{3} \Delta t - \frac{1}{2} a \Delta t^2 \rightarrow \text{Equ. (2)}$$

By using Equ. (1) and Equ. (2) \therefore

$$\text{We got } 75 \Delta t^3 - 525 \Delta t + 225 = 0$$

$$\text{e. } \Delta t = 0.398 \text{ sec.} \rightarrow a = 1.75 \text{ m/s}^2$$

$$\Delta t = 0.44 \text{ sec.} \rightarrow a = 217 \text{ m/s}^2$$

Question (4) :-

1) We have 15 iterations

$$\text{Points} \quad d_o \quad d_i \quad h_{ij} = \frac{d_o}{d_i}$$

1,2	28.3	14.1	2
1,3	72.8	51	1.4
1,4	100	70	1.4
1,5	72.8	51	1.4
1,6	28.3	14.1	2
2,3	50	40	1.25
2,4	82.5	60.8	1.36
2,5	64	44.7	1.43
2,6	40	20	2
3,4	36	22.4	1.6
3,5	40	20	2
3,6	64	44.7	1.43
4,5	36	22.4	1.6
4,6	82.5	60.8	1.36
5,6	50	40	1.25

2) → by using the average (mean) $\bar{h}_{ij} = \frac{h_{ij}}{h_o}$

$$\text{mean } \left(\frac{h_{ij}}{h_o} \right) = 1.57 \quad \text{TTC} = \frac{-\Delta t}{1 - \frac{h_{ij}}{h_o}} = \frac{-\frac{1}{20}}{1 - 1.57} = 0.088 \text{ sec}$$

→ by using the median \tilde{h}_{ij}

$$\text{median } \left(\frac{h_{ij}}{h_o} \right) = 1.43 \quad \text{TTC} = \frac{-\Delta t}{1 - \frac{h_{ij}}{h_o}} = \frac{-\frac{1}{20}}{1 - 1.43} = 0.1163 \text{ sec}$$