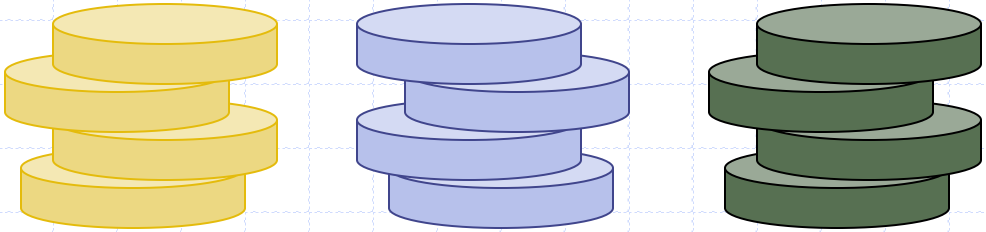
Stacks

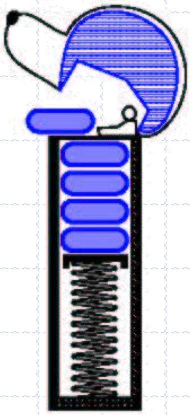


Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - **Data stored**
  - **Operations on the data**
  - **Error conditions associated with operations**

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Stack Interface in Java

- Java **interface** corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Different from the built-in Java class java.util.Stack

```
public interface Stack<E> {

  public int size();

  public boolean isEmpty();

  public E top()
      throws EmptyStackException;

  public void push(E element);

  public E pop()
      throws EmptyStackException;
}
```

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be "thrown" by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

Applications of Stacks

- Direct applications
  - Page-visited **history** in a Web browser
  - **Undo** sequence in a text editor
  - Chain of **method calls** in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

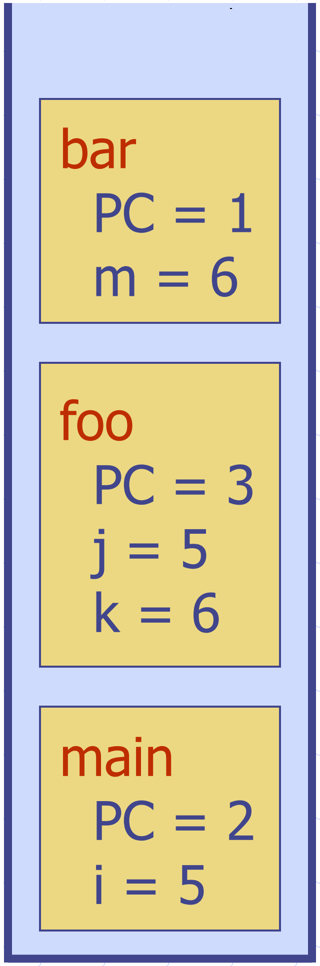
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

```
main() {
   int i = 5;
   foo(i);
}

foo(int j) {
   int k;
   k = j+1;
   bar(k);
}

bar(int m) {
   …
}
```

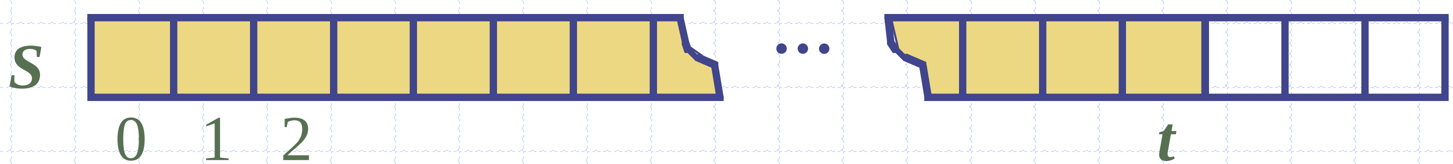


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

Algorithm *pop*()
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$

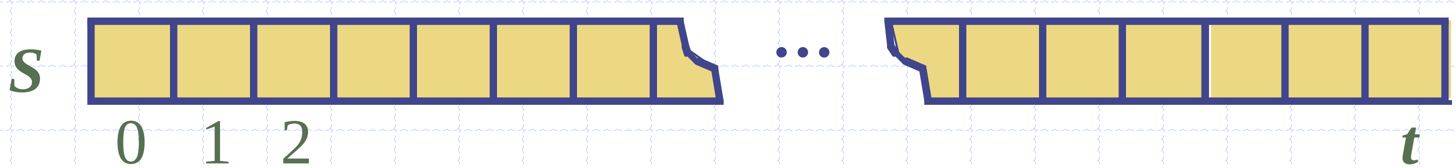


Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm ***push***(o)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$

http://www.cs.usfca.edu/~galles/visualization/StackArray.html



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - incorrect: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

Algorithm ParenMatch(*X*,*n*):

Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number

Output: **true** if and only if all the grouping symbols in *X* match

```
Let S be an empty stack
for i=0 to n-1 do
    if X[i] is an opening grouping symbol then
        S.push(X[i])
    else if X[i] is a closing grouping symbol then
        if S.isEmpty() then
            return false {nothing to match with}
        if S.pop() does not match the type of X[i] then
            return false {wrong type}
if S.isEmpty() then
    return true {every symbol matched}
else return false {some symbols were never matched}
```

HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Tag Matching Algorithm (in Java)

```
import java.io.*;
import java.util.Scanner;
import net.datastructures.*;
/** Simplified test of matching tags in an HTML document. */
public class HTML {
  /** Strip the first and last characters off a <tag> string. */
  public static String stripEnds(String t) {
    if (t.length() <= 2) return null;              // this is a degenerate tag
    return t.substring(1,t.length()-1);
  }
  /** Test if a stripped tag string is empty or a true opening tag. */
  public static boolean isOpeningTag(String tag) {
    return (tag.length() == 0) || (tag.charAt(0) != '/');
  }
```

Tag Matching Algorithm (cont.)

```
/** Test if stripped tag1 matches closing tag2 (first character is '/'). */
 public static boolean areMatchingTags(String tag1, String tag2) {
   return tag1.equals(tag2.substring(1)); // test against name after '/'
 }
 /** Test if every opening tag has a matching closing tag. */
 public static boolean isHTMLMatched(String[] tag) {
   Stack<String> S = new NodeStack<String>();     // Stack for matching tags
   for (int i = 0; (i < tag.length) && (tag[i] != null); i++) {
     if (isOpeningTag(tag[i]))
     S.push(tag[i]); // opening tag; push it on the stack
     else {
     if (S.isEmpty())
       return false;                    // nothing to match
     if (!areMatchingTags(S.pop(), tag[i]))
       return false;      // wrong match
     }
     }
   if (S.isEmpty()) return true; // we matched everything
   return false; // we have some tags that never were matched
 }
```

Tag Matching Algorithm (cont.)

```
public final static int CAPACITY = 1000; // Tag array size
/* Parse an HTML document into an array of html tags */
public static String[] parseHTML(Scanner s) {
  String[] tag = new String[CAPACITY]; // our tag array (initially all null)
  int count = 0;                              // tag counter
  String token;                               // token returned by the scanner s
  while (s.hasNextLine()) {
    while ((token = s.findInLine("<[^>]*>")) != null)  // find the next tag
      tag[count++] = stripEnds(token); // strip the ends off this tag
    s.nextLine(); // go to the next line
  }
  return tag; // our array of (stripped) tags
}
public static void main(String[] args) throws IOException { // tester
  if (isHTMLMatched(parseHTML(new Scanner(System.in))))
    System.out.println("The input file is a matched HTML document.");
  else
    System.out.println("The input file is not a matched HTML document.");
}
}
```

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stakcs:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top())
    doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

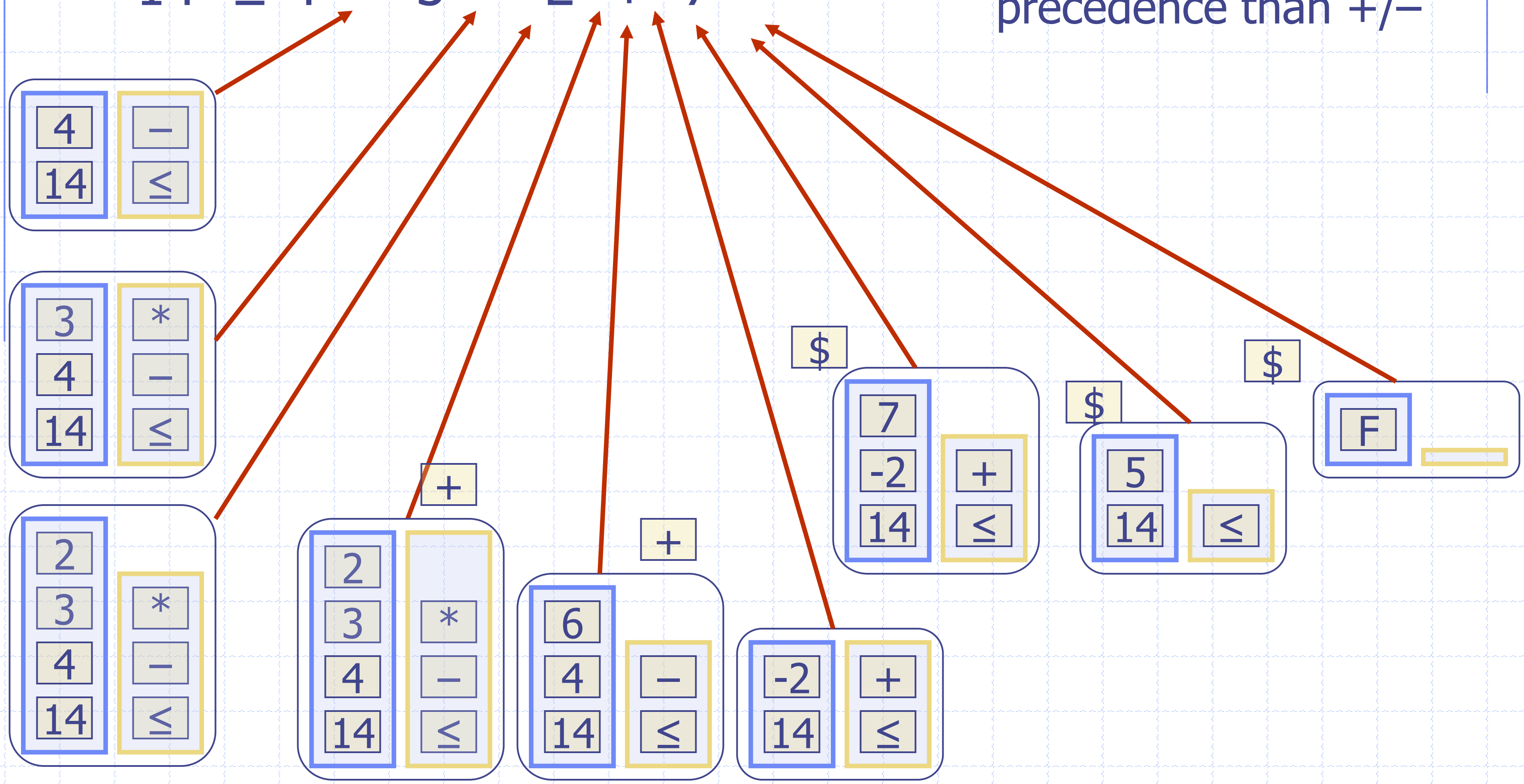
```
while there's another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

Slide by Matt Stallmann included with permission.

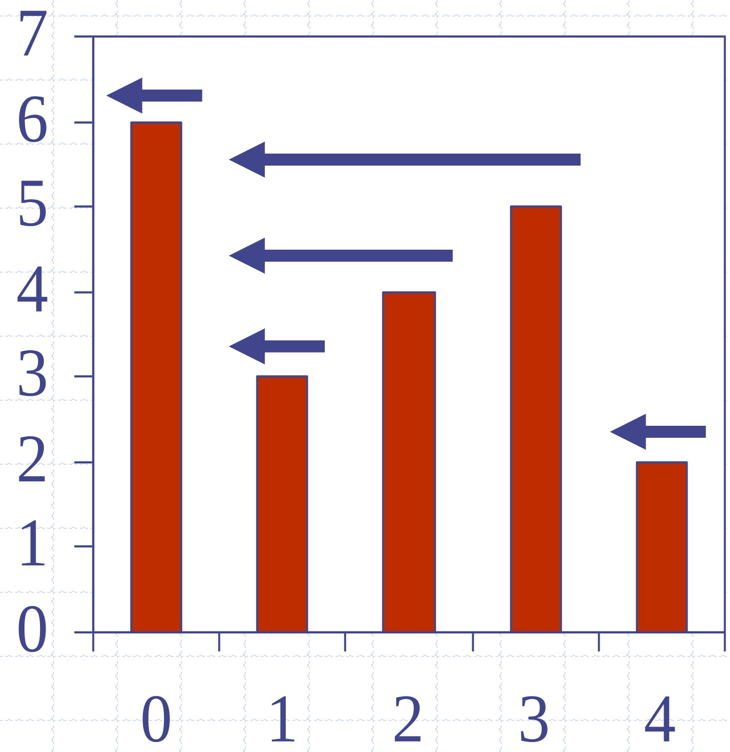
14 ≤ 4 – 3 * 2 + 7

Operator ≤ has lower precedence than +/–



Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

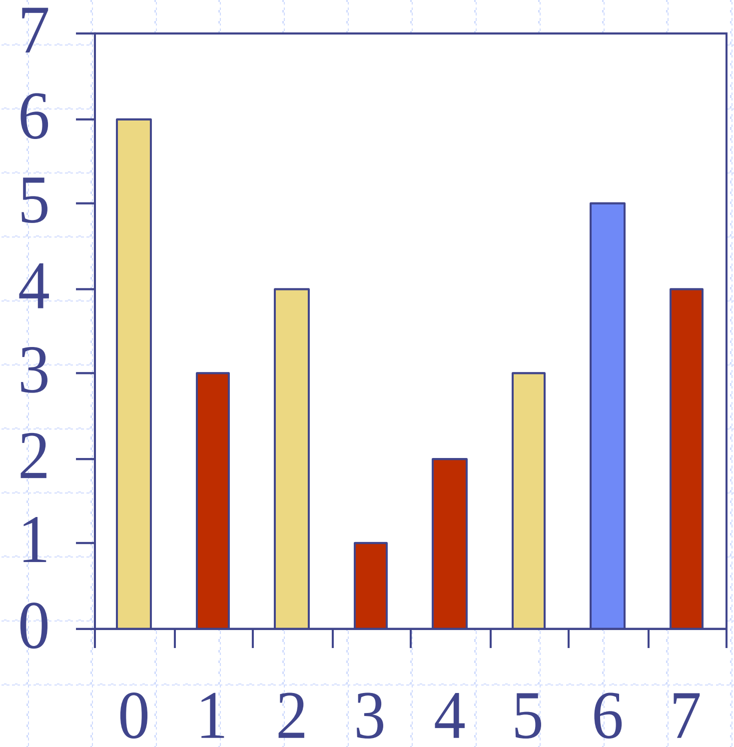
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	**#**
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \leq i \land X[i - s] \leq X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm ***spans1*** runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push i onto the stack



Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly once
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm ***spans2*** runs in $O(n)$ time

Algorithm *spans2*(X, n)	#
$S \leftarrow$ new array of n integers	n
$A \leftarrow$ new empty stack	1
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
while ($\neg A.isEmpty() \wedge$	
$X[A.top()] \leq X[i]$) **do**	n
$A.pop()$	n
if $A.isEmpty()$ **then**	n
$S[i] \leftarrow i + 1$	n
else	
$S[i] \leftarrow i - A.top()$	n
$A.push(i)$	n
return S	1