COMP 202 – Sample Final Name: _____

- By submitting this exam, you agree to fully comply with Koç University Student Code of Conduct, and accept any punishment as directed on the course syllabus in case of failure to comply. If you do not submit this assignment on time, you will receive no credit.
- Closed note, closed book. If you are stuck with a question, skip it and try again later.
- Show your work clearly, in order, and with proper explanation to get full points.
- You have 3 hours to take this 100 pts final. Good luck!

1. PROBLEM ONE [3 pts each=12pts]
   You have to sort a list that contains most of its elements in a sorted manner, and a few (constant number of) "random" elements. Discuss the runtime of each of the following sorting methods and pick the best suitable one. Consider all benefits and disadvantages, list all your assumptions.

   a) Quick sort

   b) Insertion sort

   c) Merge sort

   d) Heap sort

2. PROBLEM TWO [4 pts each = 20 pts]

a) In what condition on weights of edges in a connected weighted graph, there is exactly one minimum spanning tree?

b) Show that $3\,n \log n + 6\,n$ is $O(n \log n)$

c) Explain *informally* one method of making a tree balanced (i.e., name one of the balanced tree types and briefly describe how it achieves balance).

d) Use an example to explain why the single source all destinations shortest path algorithm (Dijkstra's algorithm) does not work for a graph that contains negative-weight edges.

e) What happens if there are negative-weight cycles?

3. PROBLEM THREE [32 pts]

a) [10 pts] You are given a sequence S of n integers each within range [0..n]. Give the pseudocode for an O(n) algorithm that checks if S contains equal elements, and returns a sequence of those elements that appear in S at least twice.

b) [3 pts] Explain why your algorithm runs in O(n) time.

c) [3 pts] Also explain the space requirement of your algorithm (other than the input sequence S), and remember that the time required *cannot* be less than the space required.

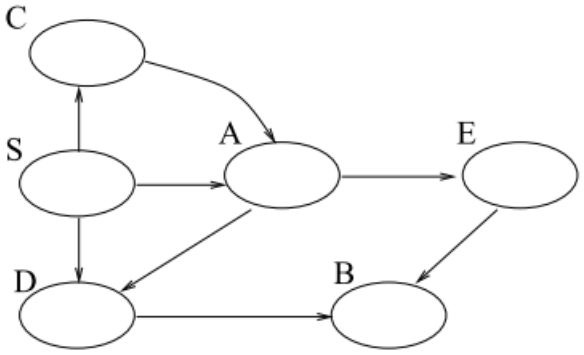
d) [16 pts] Now answer the same questions as above, but there is no restriction that all elements are within range [0..n]. You are allowed to run in *expected* O(n) time. Write down any assumptions you make.

4. PROBLEM FOUR [36 pts]

a) [12 pts] Consider the graph below in the following question. If you are iterating over a list of vertices (e.g., neighbors of a vertex), **iterate in the alphabetical order**.

Perform topological sort on this graph, assuming the vertices denote tasks, and the edges represent a “pre-requisite” information. Describe/Show each step clearly.

You may either use the forward method (start at a node with no incoming edges), or the backward method (start at a node with no outgoing edges). In any case, make sure to explain it and be consistent.



b) [24 pts] You are a project manager at the ministry of transport in Philippines. You are given the following data in the tabular form. You are asked to come up with a project that connects all the islands with a bridge (i.e, creates a connected component), with the least possible cost.

To do this, you need to first create and draw a graph out of the tabular data such that each vertex represents an island in Philippines, and labels and edges represent the cost (in millions of dollars) to build a bridge to connect two islands.

Then, you need to run a correct algorithm for this purpose. You do not need to know the name of the algorithm, but just use one of the ones we have seen in the class. Make sure you consistently use that algorithm.

Make sure to output the total cost of building bridges.

The table below uses the edge-list representation:

Island	Island	Cost of bridge ($ million)
Luzon	Romblon	7
Luzon	Jolo	23
Luzon	Coron	13
Luzon	Boracay	17
Panay	Romblon	12
Panay	Coron	2
Romblon	Jolo	21
Romblon	Coron	9
Jolo	Boracay	30
Jolo	Mindanao	12
Coron	Boracay	17
Boracay	Mindanao	28