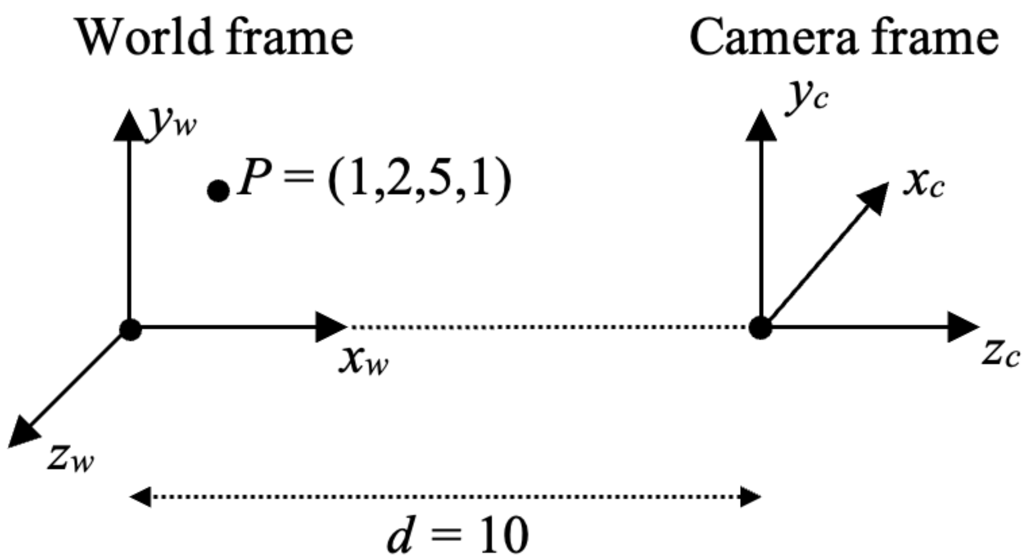
COMP410 - Quiz 1

Question 1

Consider the figure below, where the homogeneous coordinates of P are given as $(1, 2, 5, 1)$ in the world frame. Which of the following is the homogeneous coordinates of P in the camera frame? Note that the distance d between the origins of the two frames is 10, and the coordinate axes of the frames are pairwise parallel.



Options

a. $(-9, -5, 2, 1)$
b. $(1, 5, 2, 1)$
c. $(-5, 2, -9, 1)$ (correct)
d. $(-9, -5, 2, 1)$
e. $(9, 5, 2, 1)$
f. $(-10, 0, 0, 0)$
g. $(-9, 5, 2)$
h. $(10, 0, 0, 0)$

Answer 1

The modelview transformation (from world to camera) is $R_y(-90°)$ followed by $T(0,0,-10)$:

$$R_y(\underbrace{-90°}_{\text{clockwise}})$$

$$M = T(0,0,-10) \cdot R_y(-90°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(This is actually the transformation with which you can transform the world frame which is initially the same as camera frame to the new world frame.)

To get the coordinates in camera frame, we need to multiply it with this modelview transformation:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ -9 \\ 1 \end{bmatrix}$$

Question 2

Which of the following transformation pairs can not in general be concatenated in arbitrary order with matrix multiplication (so that the overall result is affected by the order)? In other words, mark the transformation pairs that can not commute in general. You are expected to mark all such pairs.

Options

a. Scale, Scale
b. Rotation, Rotation
c. Translation, Translation
d. Scale, Rotation
e. Rotation, Translation
f. Scale, Translation

Answer 2

Formally you can figure this out by writing the matrices of these standard transformations and then checking whether the multiplication operation commutes or not.

It is also possible to see this by geometric interpretation; for instance consider (scale, rotation) pair; hence S·R versus R·S. We can interpret these concatenations in the order of appearance. For example, in the case of R·S, when we apply first R around some axis, then since the local coordinate axes also rotate, the scale operation would perform on the rotated axes, which is not the case for R·S.

Question 3

Consider rotating an object clockwise by 45 degrees around the fixed point $(1, 1, 1, 1)$ and the vector $(1, 1, 1, 0)$. Which of the following is the corresponding 4×4 transformation matrix?

Options

a. $T(1, 1, 1)\, R_y(60°)\, R_z(-\ 45°)\, R_y(-\ 60°)\, T(-\ 1, -\ 1, -\ 1)$
b. $T(1, 1, 1)\, R_x(60°)\, R_z(-\ 45°)\, R_x(-\ 60°)\, T(-\ 1, -\ 1, -\ 1)$
c. $R_Z(-\ 45°)\, T(-\ 1, -\ 1, -\ 1)$
d. $T(1, 1, 1)\, R_Z(-\ 45°)\, R_x(45°)\, R_y(-\ 45°)\, R_x(45°)\, R_z(-\ 45°)\, T(-\ 1, -\ 1, -\ 1)$
e. $T(1, 1, 1)\, R_y(-\ 90°)\, R_z(45°)\, R_x(-\ 45°)\, R_z(-\ 45°)\, R_y(90°)\, T(-\ 1, -\ 1, -\ 1)$
f. $T(1, 1, 1)\, R_Z(-\ 45°)\, R_x(45°)\, R_y(-\ 45°)\, R_x(-\ 45°)\, R_z(45°)\, T(-\ 1, -\ 1, -\ 1)$
g. $T(1, 1, 1)\, R_Z(-\ 90°)\, R_y(45°)\, R_X(-\ 45°)\, R_y(-\ 45°)\, R_z(90°)\, T(-\ 1, -\ 1, -\ 1)$

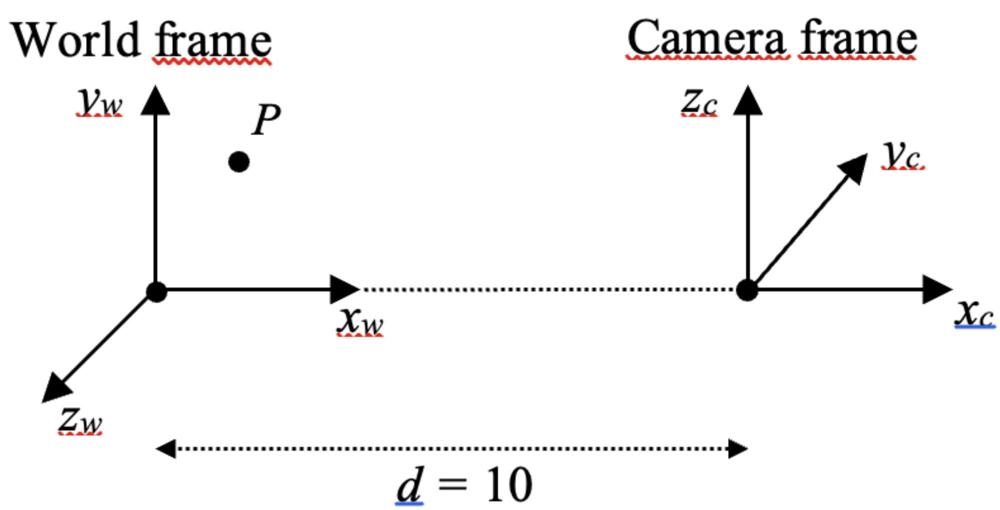
Answer 3

- First, we need to move the fixed point $(1,1,1,1)$ to origin with $T(-1,-1,-1)$.
- Next, we need to align the axis of rotation $(1,1,1,0)$ with one of the coordinate axis, say y-axis.
- This can be achieved by first rotating around z-axis with +45 degrees (counter-clockwise) and then rotating around x-axis with -45 degrees (clockwise).
- Once the rotation axis is aligned with y-axis, we need to apply -45 degrees (clockwise) of rotation around y-axis.
- Lastly, we need to undo all the transformations applied. Hence:

$$T(1,1,1)\,R_z(-45^\circ)\,R_x(45^\circ)\,R_y(-45^\circ)\,R_x(-45^\circ).R_z(45^\circ)\,T(-1,-1,-1)$$

Question 4

Consider the figure below. Which of the following is the affine transformation that relates the homogeneous coordinates of a point P given in camera frame to its coordinates in world frame? Note that the distance d between the origins of the two frames is 10, and the coordinate axes of the frames are pairwise parallel. Note also that the transformation is from camera to world, not from world to camera.



Answer 4

We need to figure out how world frame is transformed to camera frame (i.e., the inverse of the standard modelview transformation).

To achieve this:

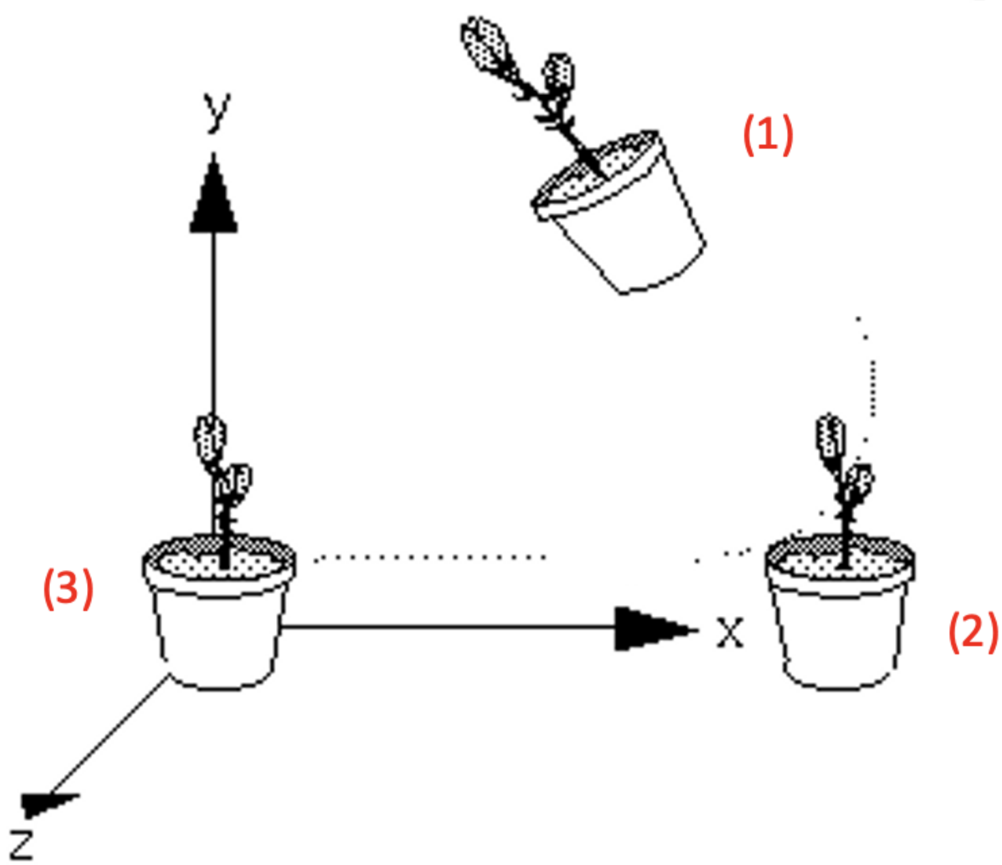
- First translate 10 units along x-axis
- Then rotate around x-axis (-90) degrees (clockwise)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 5

Consider the figure below, where the object is first transformed from position **(1)** to position **(2)** by 45 degrees clockwise rotation around one of the coordinate axis and then to position **(3)** by translation of 5 units along another coordinate axis. Which of the following is the 4×4 affine transformation matrix that moves the object accordingly?



Answer 5

The transformation is rotation around z-axis by -45 degrees, followed by translation of -5 units along x-axis:

$$\text{Transformation}: \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & -5 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$