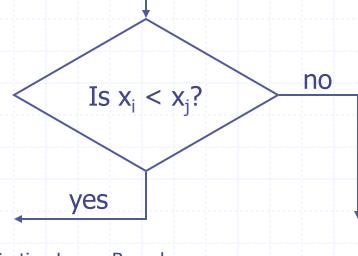
Sorting Lower Bound



Comparison-Based Sorting

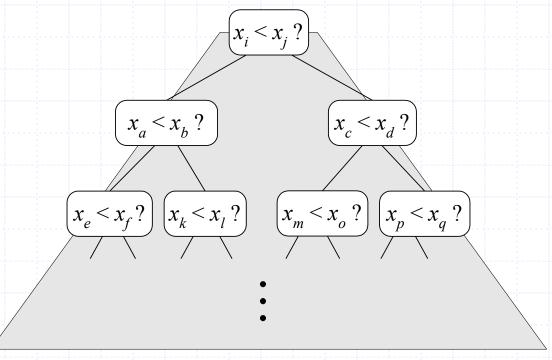


- Many sorting algorithms are comparison based.
 - They sort by making comparisons between pairs of objects
 - Examples: bubble-sort, selection-sort, insertion-sort, heap-sort, merge-sort, quick-sort, ...
- ◆ Let us therefore derive a lower bound on the running time of any algorithm that uses comparisons to sort n elements, x₁, x₂, ..., xₙ.



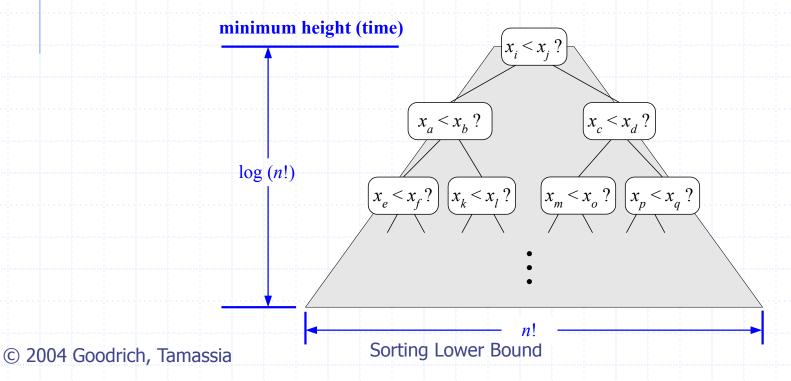
Counting Comparisons

- Let us just count comparisons then.
- Each possible run of the algorithm corresponds to a root-to-leaf path in a decision tree



Decision Tree Height

- The height of the decision tree is a lower bound on the running time
- Every input permutation must lead to a separate leaf output
- ◆ If not, some input ...4...5... would have same output ordering as ... 5...4..., which would be wrong
- ♦ Since there are $n!=1\cdot 2\cdot ...\cdot n$ leaves, the height is at least log (n!)



4

The Lower Bound



- Any comparison-based sorting algorithms takes at least log (n!) time
- Therefore, any such algorithm takes time at least

$$\log (n!) \ge \log \left(\frac{n}{2}\right)^{\frac{n}{2}} = (n/2)\log(n/2).$$

That is, any comparison-based sorting algorithm must run in $\Omega(n \log n)$ time.