COMP410 - Quiz 2

Question 1

Consider orthographic projection and suppose that the view volume is given by the intersection of the planes $z = 2$, $z = 4$, $y = 1$, $y = 2$, $x = 2$ and $x = 5$. Which of the following is the view normalization transformation that converts this projection (viewing) into orthographic projection with the default view volume, i.e., the cube centered at the origin with sides of length 2?

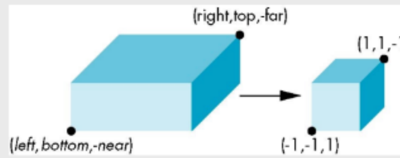
Answer 1

Normalization Transformation

- Two steps
  - Move center to origin

    $\boldsymbol{T}$ (-(left+right)/2, -(bottom+top)/2, (near+far)/2))
  - Scale to have sides of length 2

    $\boldsymbol{S}$ (2/(right-left), 2/(top-bottom), 2/(near-far))



$$\boldsymbol{N} = \boldsymbol{ST} = \begin{bmatrix} \frac{-2}{left - right} & 0 & 0 & -\frac{left + right}{left - right} \\ 0 & \frac{2}{top - bottom} & 0 & -\frac{top + bottom}{top - bottom} \\ 0 & 0 & \frac{2}{near - far} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that this transforms z=-near to z=-1 and z =-far to z=1.

Question 2

Suppose that the center of projection is at point $p = (1, 0, 0, 1)$. Which of the following is the matrix that gives the perspective projection onto the $x = 0$ plane?

Answer 2

We first translate the center of projection to the origin with $T(-1,0,0)$; then perform projection onto $x=-1$ plane and finally undo the translation with $T(1,0,0)$:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T(1,0,0)} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}}_{\text{projection to } x=-1} \underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T(-1,0,0)}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

Question 3

Suppose that the center of projection is at point $p = (1, 0, 0, 1)$. Which of the following is the matrix that gives the perspective projection onto the $x = y$ plane?

Answer 3

We first rotate the $x=y$ plane around z-axis 45° counter clockwise so that the $x=y$ plane is aligned with $x=0$ plane. With this rotation, the point $(1,0,0,1)$ moves to $(1/\sqrt{2}, 1/\sqrt{2}, 0, 1)$. Hence next to rotation, we move the point of projection to the origin with $T(-1/\sqrt{2}, -1/\sqrt{2}, 0)$. After applying the projection onto $x = -1/\sqrt{2}$ plane, we consequently undo the previous translation and rotation:

$$R_z(-45^\circ)\,T(1/\sqrt{2}, 1/\sqrt{2}, 0)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sqrt{2} & 0 & 0 & 0 \end{bmatrix} T(-1/\sqrt{2}, -1/\sqrt{2}, 0)\,R_z(45^\circ)$$

Question 4

Consider the perspective projection with COP at the origin and the projection plane at $z = -1$. Suppose also that the view volume is given by the frustum bounded by near plane $z = -2$ and far plane $z = -3$ with 120 degree field of view (as determined by the planes $x = \pm\sqrt{3}z$, $y = \pm\sqrt{3}z$).

Which of the following is the view normalization transformation that converts this projection (viewing) into orthographic projection with the default view volume, i.e., the cube centered at the origin with sides of length 2?

Answer 4

We first apply scaling along z-axes by $\sqrt{3}$ so that the view frustum is normalized into the one with 90 degrees of field. (so that $x = \pm\sqrt{3}z$ and $y = \pm\sqrt{3}$ become $x = \pm z$ and $y = \pm z$)
We then need to apply the normalization transformation N as given in the slides:

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \text{where } \alpha = -\frac{near + far}{far - near}$$

$$\beta = \frac{2 near \times far}{near - far}$$

this negative sign was missing in the slides; I corrected it

Here $\alpha = -\dfrac{(2+3)\cdot\sqrt{3}}{(3-2)\sqrt{3}} = -5$

$$\beta = \frac{2 \cdot 2(\sqrt{3}) \cdot 3(\sqrt{3})}{(2-3)\cdot\sqrt{3}} = -12\sqrt{3}$$

Note that all z values are scaled by $\sqrt{3}$ above.

So the result is:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & -12\sqrt{3} \\ 0 & 0 & -1 & 0 \end{bmatrix}}_{N} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{scale} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/\sqrt{3} & -12/\sqrt{3} \\ 0 & 0 & -\sqrt{3} & 0 \end{bmatrix}$$