UNDERSTANDING PROGRAM EFFICIENCY: 1

(download slides and follow along on repl.it!)

COMP100 LECTURE 10

Today

- Measuring orders of growth of algorithms

- Big "Oh" notation

- Complexity classes

WANT TO UNDERSTAND EFFICIENCY OF PROGRAMS

- computers are fast and getting faster – so maybe efficient programs don't matter?
  - but data sets can be very large (e.g., in 2014, Google served 30,000,000,000,000 pages, covering 100,000,000 GB – how long to search brute force?)
  - thus, simple solutions may simply not scale with size in acceptable manner

- how can we decide which option for program is most efficient?

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- separate **time and space efficiency** of a program

- tradeoff between them:
  - can sometimes pre-compute results are stored; then use "lookup" to retrieve (e.g., memoization for Fibonacci)
  - will focus on time efficiency

WANT TO UNDERSTAND EFFICIENCY OF PROGRAMS

Challenges in understanding efficiency of solution to a computational problem:

▪ a program can be **implemented in many different ways**

▪ you can solve a problem using only a handful of different **algorithms**

▪ would like to separate choices of implementation from choices of more abstract algorithm

HOW TO EVALUATE EFFICIENCY OF PROGRAMS

- measure with a **timer**

- **count** the operations

- abstract notion of **order of growth**

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will argue that this is the most appropriate way of assessing the impact of choices of algorithm in solving a problem; and in measuring the inherent difficulty in solving a problem

TIMING A PROGRAM

- use time module

- recall that importing means to bring in that class into your own file

- **start** clock

- **call** function

- **stop** clock

```
import time

def c_to_f(c):
    return c*9/5 + 32

t0 = time.time()
c_to_f(100000)
t = time.time() - t0
print("t =", t, "s")
```

TIMING PROGRAMS IS INCONSISTENT

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- running time **varies between algorithms** ✓

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- time varies for different inputs but cannot really express a relationship between inputs and time ✘

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▪ assume these steps take
constant time:
- mathematical operations
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- assignments
- accessing objects in memory

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mysum → 1+3x ops

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STILL NEED A BETTER WAY

- timing and counting **evaluate implementations**

- timing **evaluates machines**

- want to **evaluate algorithm**

- want to **evaluate scalability**

- want to **evaluate in terms of input size**

STILL NEED A BETTER WAY

▪ Going to focus on idea of counting operations in an algorithm, but not worry about small variations in implementation (e.g., whether we take 3 or 4 primitive operations to execute the steps of a loop)

▪ Going to focus on how algorithm performs when size of problem gets arbitrarily large

▪ Want to relate time needed to complete a computation, measured this way, against the size of the input to the problem

▪ Need to decide what to measure, given that actual number of steps may depend on specifics of trial

NEED TO CHOOSE WHICH INPUT TO USE TO EVALUATE A FUNCTION

- want to express **efficiency in terms of size of input**, so need to decide what your input is

- could be an **integer**
  -- `mysum(x)`

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  `-- list_sum(L)`

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- could be **length of list**
  `-- list_sum(L)`

- **you decide** when multiple parameters to a function
  `-- search_for_elmt(L, e)`

DIFFERENT INPUTS CHANGE HOW THE PROGRAM RUNS

- a function that searches for an element in a list

```
def search_for_elmt(L, e):
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        if i == e:
            return True
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- when e is **not in list** → WORST CASE

- when **look through about half** of the elements in list → AVERAGE CASE

- want to measure this behavior in a general way

BEST, AVERAGE, WORST CASES

- suppose you are given a list `L` of some length `len(L)`

- **best case**: minimum running time over all possible inputs of a given size, `len(L)`
  - constant for `search_for_elmt`
  - first element in any list

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generally will focus on this case

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ORDERS OF GROWTH

Goals:

- want to evaluate program's efficiency when **input is very big**

- want to express the **growth of program's run time** as input size grows

- want to put an **upper bound** on growth – as tight as possible

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- do not need to be precise: **"order of" not "exact"** growth

- we will look at **largest factors** in run time (which section of the program will take the longest to run?)

- **thus, generally we want tight upper bound on growth, as function of size of input, in worst case**

MEASURING ORDER OF GROWTH: BIG OH NOTATION

- Big Oh notation measures an **upper bound on the asymptotic growth**, often called order of growth

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- Big Oh notation measures an **upper bound on the asymptotic growth**, often called order of growth

- **Big Oh or *O()*** is used to describe worst case
  - worst case occurs often and is the bottleneck when a program runs
  - express rate of growth of program relative to the input size
  - evaluate algorithm **NOT** machine or implementation

EXACT STEPS vs O()

```python
def fact_iter(n):
    """assumes n an int >= 0"""
    answer = 1
    while n > 1:
        answer *= n
        n -= 1
    return answer
```

- computes factorial

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■ computes factorial

■ number of steps: *1 + 5n + 1*

■ worst case asymptotic complexity: *O(n)*
  • ignore additive constants
  • ignore multiplicative constants

WHAT DOES *O(N)* MEASURE?

- Interested in describing how amount of time needed grows as size of (input to) problem grows

- Thus, given an expression for the number of operations needed to compute an algorithm, want to know asymptotic behavior as size of problem gets large

- Hence, will focus on term that grows most rapidly in a sum of terms

- And will ignore multiplicative constants, since want to know how rapidly time required increases as increase size of input

SIMPLIFICATION EXAMPLES

- drop constants and multiplicative factors
- focus on **dominant terms**

: $n^2 + 2n + 2$

: $n^2 + 100000n + 3^{1000}$

: $\log(n) + n + 4$

: $0.0001*n*\log(n) + 300n$

: $2n^{30} + 3^n$

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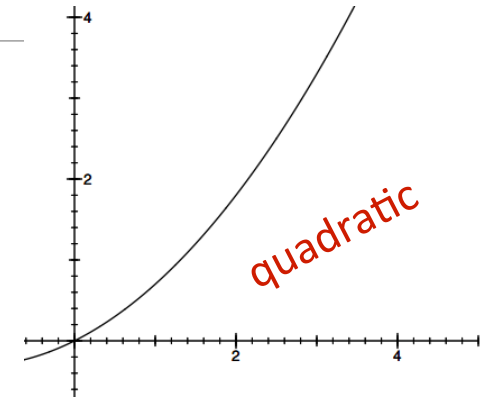- focus on **dominant terms**

O(n²) : $n^2 + 2n + 2$

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O(n) : $\log(n) + n + 4$

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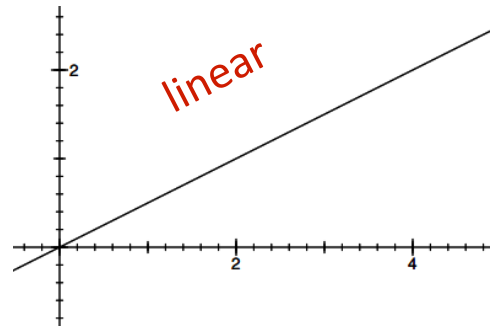
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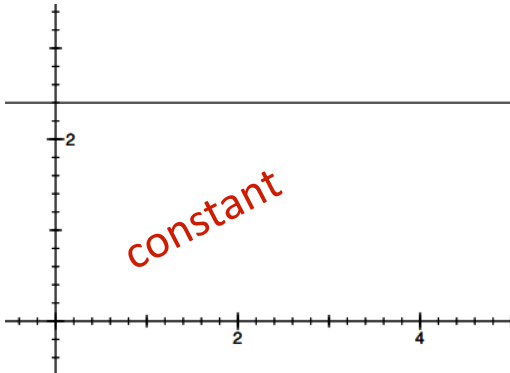
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$O(3^n)$: $2n^{30} + 3^n$

TYPES OF ORDERS OF GROWTH
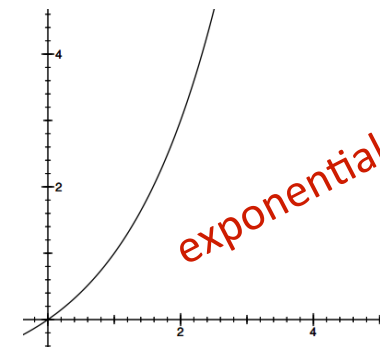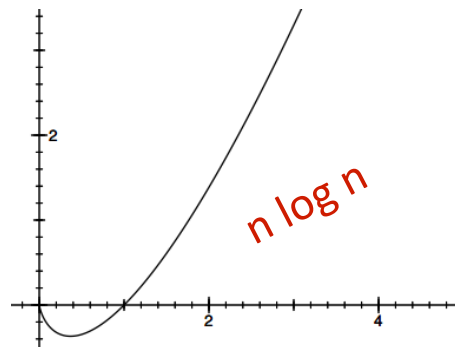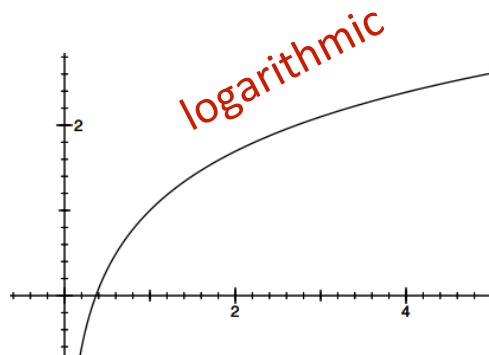
constant

linear

quadratic

logarithmic

n log n

exponential

ANALYZING PROGRAMS AND THEIR COMPLEXITY

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  - analyze statements inside functions
  - apply some rules, focus on dominant term

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Law of Addition for O():
  - used with **sequential** statements
  - O(f(n)) + O(g(n)) is O(f(n) + g(n))
  - for example,

```
for i in range(n):
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for j in range(n*n):
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    print('b')
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$O(n)$

$O(n*n)$

$O(n) + O(n*n)$

is O(n) + O(n*n) = O(n+n^2) = O(n^2) because of dominant term

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$O(n^2)$

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n loops, each O(n) ↗
O(n)*O(n)

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n loops, each O(n) →
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is O(n)*O(n) = O(n*n) = O(n^2) because the outer loop goes n times and the inner loop goes n times for every outer loop iter.
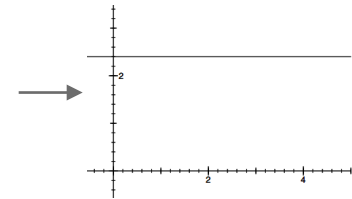
COMPLEXITY CLASSES

- *O(1)* denotes constant running time

- *O(log n)* denotes logarithmic running time

- *O(n)* denotes linear running time

- *O(n log n)* denotes log-linear running time

- *$O(n^c)$* denotes polynomial running time (c is a constant)

- *$O(c^n)$* denotes exponential running time (c is a constant being raised to a power based on size of input)

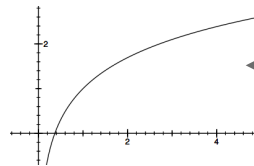# COMPLEXITY CLASSES ORDERED LOW TO HIGH
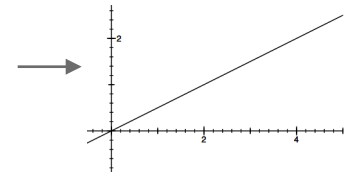
O(1)   :   constant

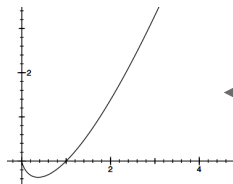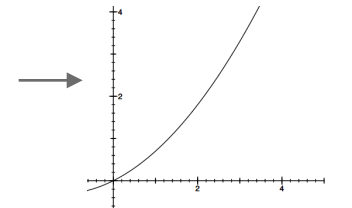O(log n)   :   logarithmic

O(n)   :   linear

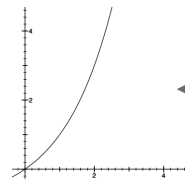O(n log n):   loglinear

O(n^c)   :   polynomial

c is a constant

O(c^n)   :   exponential

COMPLEXITY GROWTH

CLASS	n=10	= 100	= 1000	= 1000000
O(1)	1	1	1	1
O(log n)	1	2	3	6
O(n)	10	100	1000	1000000
O(n log n)	10	200	3000	6000000
O(n^2)	100	10000	1000000	1000000000000
O(2^n)	1024	12676506002282294014967032053 76	1071508607186267320948425049060001810561404811705533607443750388370351051124936122493198378815695858127594672917553146825187145285692314043598457757469857480393456777482423098542107460506237114187795418215304647498358194126739876755916554394607706291457119647768654216766042983165262438683720566806936937 6	Good luck!!

COMPLEXITY GROWTH

CLASS	n=10	= 100	= 1000	= 1000000
O(1)	1	1	1	1
O(log n)	1	2	3	6
O(n)	10	100	1000	1000000
O(n log n)	10	200	3000	6000000
O(n^2)	100	10000	1000000	1000000000000
O(2^n)	1024	1267650600228229401496703205376	107150860718626732094842504906001810561404811705533607443750383703510511249361224931983788156958581275946729175531468251871452856923140435984577574698574803934567774824230985421074605062371141877954182153046474983581941267398767559165543946077062914571196477686542167660429831652624386837205668069376	Good luck!!

Horrible Bad Fair Good Excellent

O(n!) O(2^n) O(n^2)

O(n log n)

O(n)

O(log n), O(1)
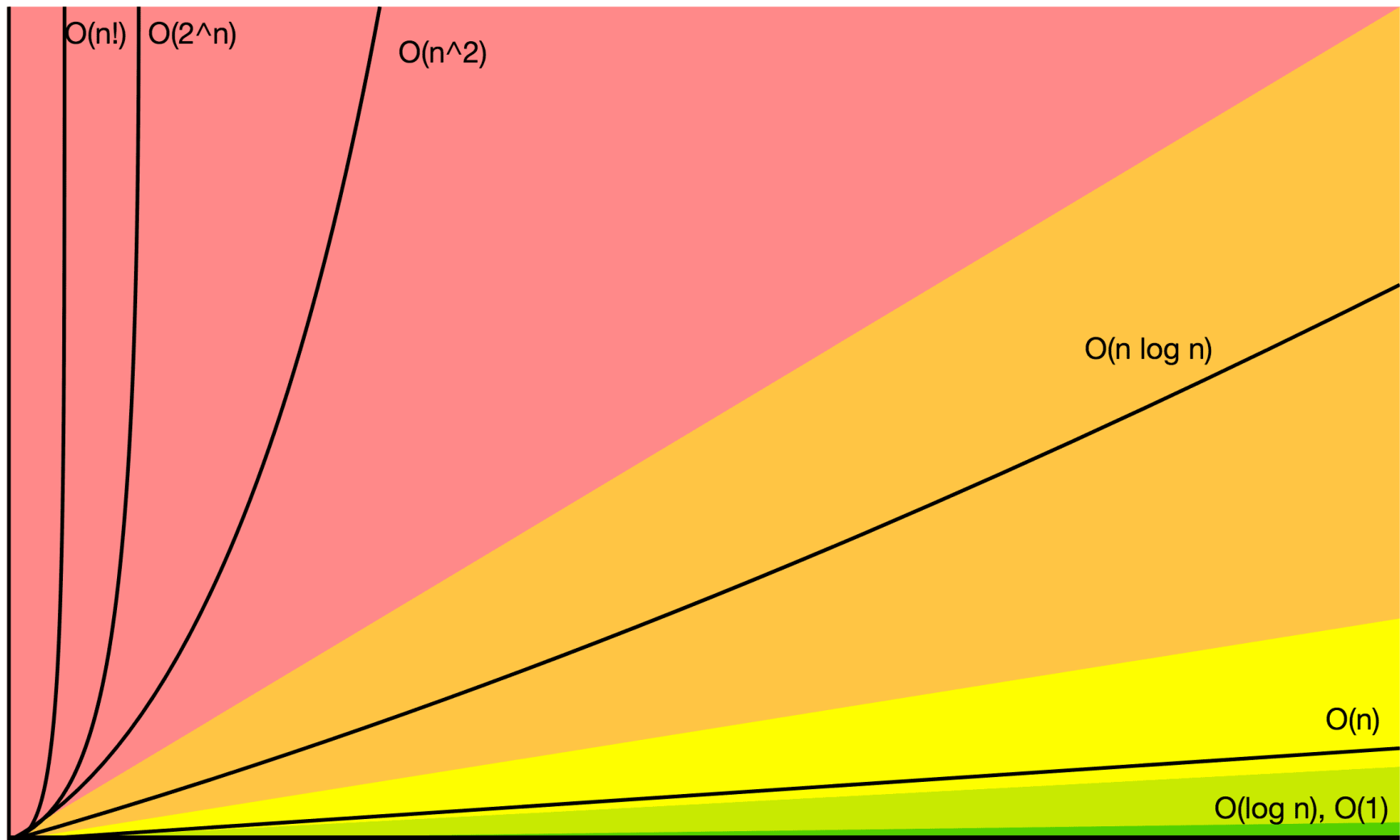
LINEAR COMPLEXITY

- Simple iterative loop algorithms are typically linear in complexity

LINEAR SEARCH
ON UNSORTED LIST

```python
def linear_search(L, e):
    found = False
    for i in range(len(L)):
        if e == L[i]:
            found = True
    return found
```

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- overall complexity is **O(n) – where n is len(L)**

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- overall complexity is **O(n) – where n is len(L)**

Assumes we can retrieve element of list in constant time

CONSTANT TIME LIST ACCESS

- if list is all ints
  - i^{th} element at
    - base + 4*i

base

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allocate fixed length, say 4 bytes
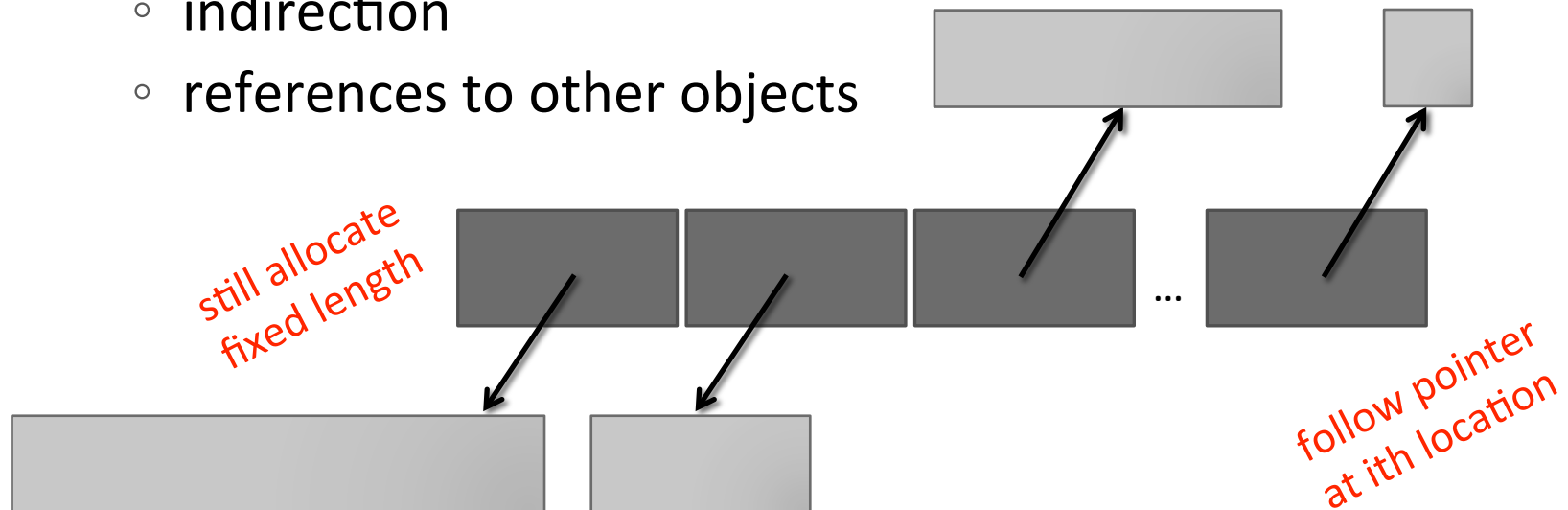
ith int

CONSTANT TIME LIST ACCESS

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- **if list is heterogeneous**
  - indirection
  - references to other objects

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allocate fixed length, say 4 bytes

ith int

- **if list is heterogeneous**
  - indirection
  - references to other objects

still allocate fixed length

follow pointer at ith location

LINEAR SEARCH ON SORTED LIST

```python
def search(L, e):
    for i in range(len(L)):
        if L[i] == e:
            return True
        if L[i] > e:
            return False
    return False
```

LINEAR SEARCH ON SORTED LIST

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def search(L, e):
    for i in range(len(L)):
        if L[i] == e:
            return True
        if L[i] > e:
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```

- must only look until reach a number greater than e

- O(len(L)) for the loop * O(1) to test if e == L[i]

- overall complexity is **O(n) – where n is len(L)**

- **NOTE:** order of growth is same, though run time may differ for two search methods

LINEAR SEARCH ON SORTED LIST

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    for i in range(len(L)):
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        if L[i] > e:
            return False
    return False
```

- must only look until reach a number greater than e

- O(len(L)) for the loop * O(1) to test if e == L[i]

worst case will need to look at whole list

- overall complexity is **O(n) – where n is len(L)**

- **NOTE:** order of growth is same, though run time may differ for two search methods

LINEAR COMPLEXITY

- searching a list in sequence to see if an element is present

- add characters of a string, assumed to be composed of decimal digits

```python
def addDigits(s):
    val = 0
    for c in s:
        val += int(c)
    return val
```

LINEAR COMPLEXITY

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- add characters of a string, assumed to be composed of decimal digits

```
def addDigits(s):
    val = 0
    for c in s:
        val += int(c)
    return val
```

- *O(len(s))*

LINEAR COMPLEXITY

- complexity often depends on number of iterations

```python
def fact_iter(n):
    prod = 1
    for i in range(1, n+1):
        prod *= i
    return prod
```

LINEAR COMPLEXITY

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def fact_iter(n):
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- number of times around loop is n

- number of operations inside loop is a constant (in this case, 3 – set i, multiply, set prod)
  - O(1 + 3n + 1) = O(3n + 2) = O(n)

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- number of operations inside loop is a constant (in this case, 3 – set i, multiply, set prod)
  - O(1 + 3n + 1) = O(3n + 2) = O(n)
- overall just *O(n)*

NESTED LOOPS

- simple loops are linear in complexity

- what about loops that have loops within them?

QUADRATIC COMPLEXITY

determine if one list is subset of second, i.e., every element of first, appears in second (assume no duplicates)

```python
def isSubset(L1, L2):
    for e1 in L1:
        matched = False
        for e2 in L2:
            if e1 == e2:
                matched = True
                break
        if not matched:
            return False
    return True
```

QUADRATIC COMPLEXITY

```python
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    for e1 in L1:
        matched = False
        for e2 in L2:
            if e1 == e2:
                matched = True
                break
        if not matched:
            return False
    return True
```

outer loop executed len(L1) times

each iteration will execute inner loop up to len(L2) times, with constant number of operations

O(len(L1)*len(L2))

worst case when L1 and L2 same length, none of elements of L1 in L2

O(len(L1)^2)

QUADRATIC COMPLEXITY

find intersection of two lists, return a list with each element
appearing only once

```python
def intersect(L1, L2):
    tmp = []
    for e1 in L1:
        for e2 in L2:
            if e1 == e2:
                tmp.append(e1)
    res = []
    for e in tmp:
        if not(e in res):
            res.append(e)
    return res
```

QUADRATIC COMPLEXITY

```python
def intersect(L1, L2):
    tmp = []
    for e1 in L1:
        for e2 in L2:
            if e1 == e2:
                tmp.append(e1)
    res = []
    for e in tmp:
        if not(e in res):
            res.append(e)
    return res
```

first nested loop takes *len(L1)*len(L2)* steps

second loop takes at most *len(L1)* steps

determining if element in list might take *len(L1)* steps

if we assume lists are of roughly same length, then

O(len(L1)^2)

O() FOR NESTED LOOPS

```python
def g(n):
    """ assume n >= 0 """
    x = 0
    for i in range(n):
        for j in range(n):
            x += 1
    return x
```

- computes n^2 very inefficiently
- when dealing with nested loops, **look at the ranges**
- nested loops, **each iterating n times**
- ***O(n^2)***

THIS TIME AND NEXT TIME

▪ have seen examples of loops, and nested loops

▪ give rise to linear and quadratic complexity algorithms

▪ next time, will more carefully examine examples from each of the different complexity classes