

COMP 341 Intro to AI Decision Networks



Asst. Prof. Barış Akgün
Koç University

Recap

- Uncertainty
 - The real world is uncertain to an agent!
 - Use probabilistic models for representation – **Joint Distribution**
- Bayesian Networks
 - An intuitive way of representing uncertainty with local conditional distributions
- Inference in BNs: $P(X_q | x_{e_1}, \dots, x_{e_k})$

Stuff you care about $\xrightarrow{\quad}$ $P(X_q | x_{e_1}, \dots, x_{e_k})$ $\xleftarrow{\quad}$ *Stuff you already know*
- Exact Inference: Enumeration
- Approximate Inference: Sampling

Making Decisions

- What do we do with the outcome of an inference query?
- Would you take an umbrella when:
 - No info
 - It is cloudy
 - Forecast says rain
 - It is cloudy and forecast says rain
- Model it as a BN
- Ask the query
- Do inference
- Is the calculated probability enough?

Our Definition of AI

- “Science of making **rational agents**”: A **rational** agent selects actions that maximize its (expected) utility
- Utilities in the umbrella example:

Real Weather	Umbrella Decision	Utility
rain	take	
rain	leave	
sunny	take	
sunny	leave	

- Expected utility: probability of the outcome X utility of the outcome
- Question: How to put utilities into Bayesian Networks?

Decision Networks: Representation

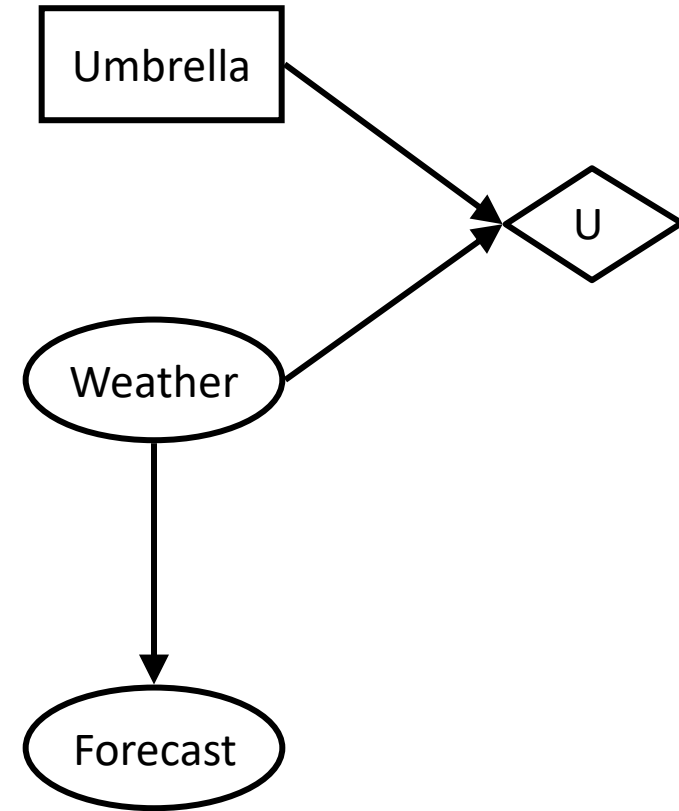
- New node types in the BNs:

- In addition to regular nodes
- Actions (rectangles, cannot have parents, can be parents, act as observed evidence)
- Utility node (diamond, depends on action and chance nodes)

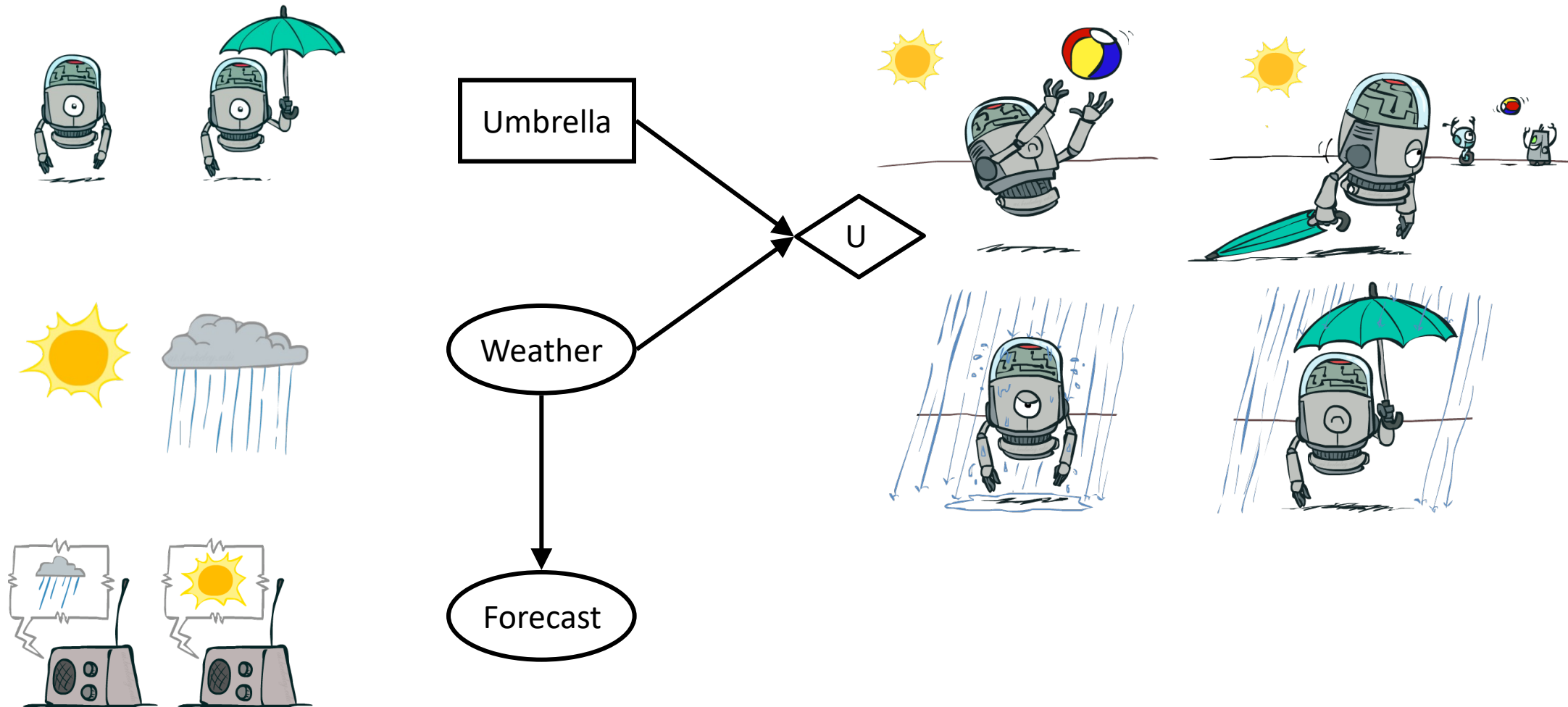


- Info in the nodes:

- CPTs
- Available Action List
- Utility Table



Decision Networks



Decision Networks: Expected Utility

Chose actions to “Maximize Expected Utility”

- Expected Utility (EU) of an action

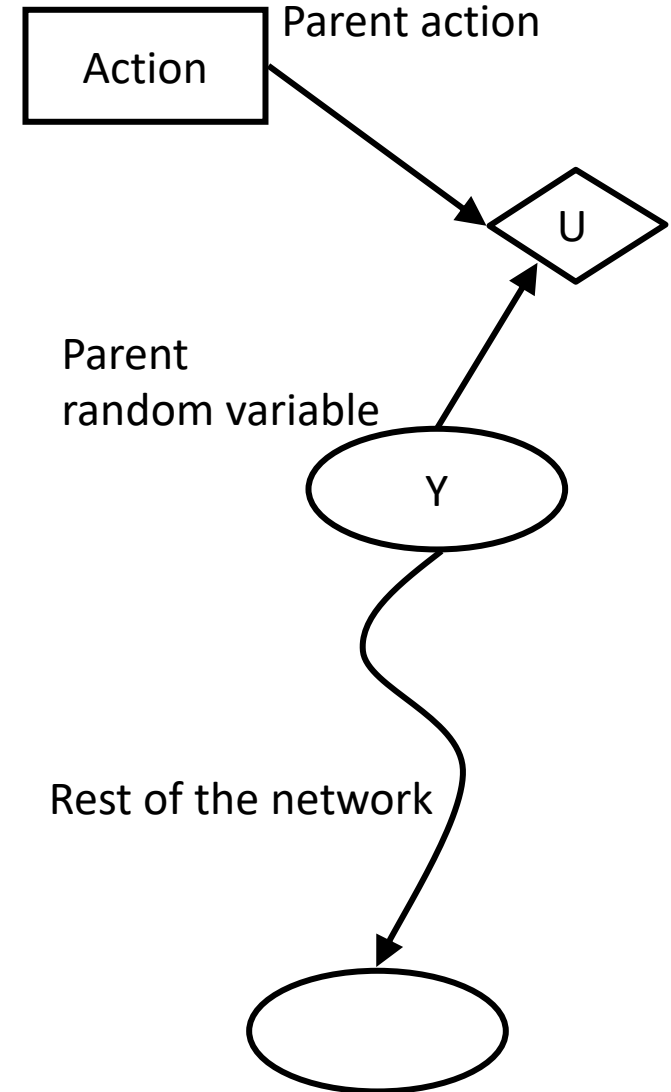
$$EU(action|evidence) = \sum_{y \in Y} P(y_i|evidence)U(y_i, action)$$

- How to calculate $P(y_i|evidence)$?
 - Any inference method we have seen so far!

- Maximum Expected Utility (MEU)

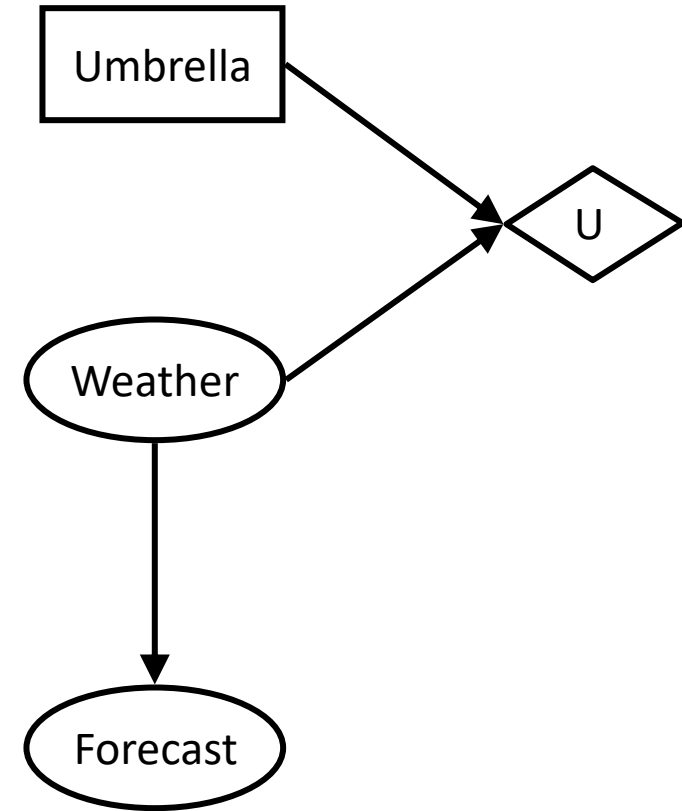
$$MEU(evidence) = \max_a (EU(a|evidence))$$

- Selected Action is $\operatorname{argmax}_a (EU(a|evidence))$



Decision Networks

- Action selection
 - Instantiate all evidence
 - Calculate posterior for all parents of utility node, given the evidence (inference part)
 - Set action node(s) each possible way
 - Calculate expected utility for each action
 - Choose the maximizing action



Decision Networks

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

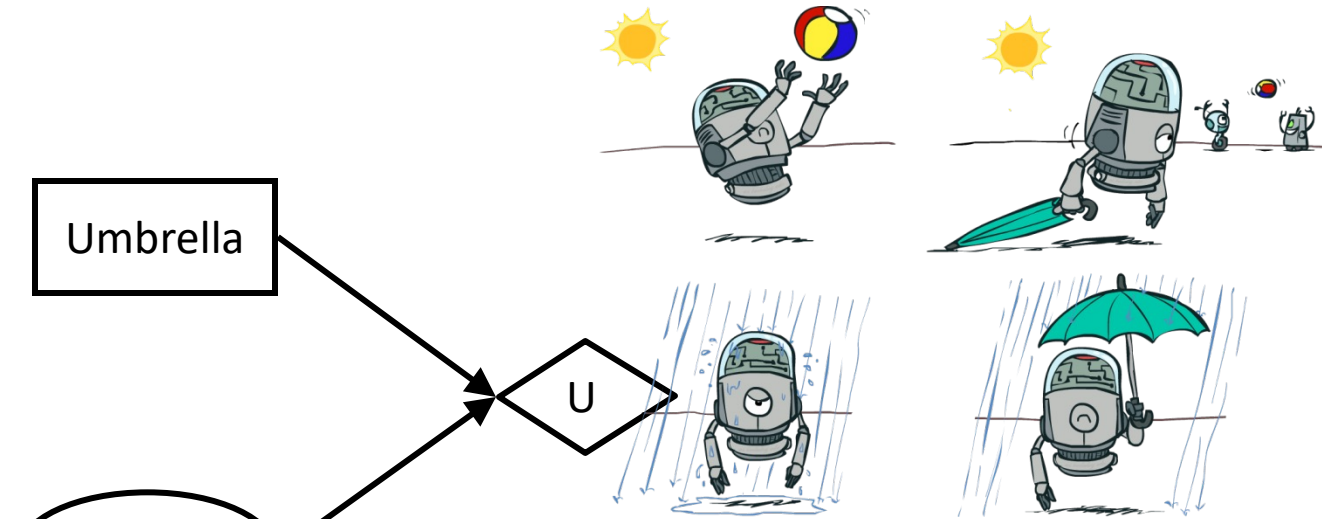
$$\begin{aligned} EU(\text{leave}) &= \sum_w P(w)U(\text{leave}, w) \\ &= 0.7 \cdot 100 + 0.3 \cdot 0 = 70 \end{aligned}$$

Umbrella = take

$$\begin{aligned} EU(\text{take}) &= \sum_w P(w)U(\text{take}, w) \\ &= 0.7 \cdot 20 + 0.3 \cdot 70 = 35 \end{aligned}$$

Optimal decision = leave

$$MEU(\emptyset) = \max_a EU(a) = 70$$

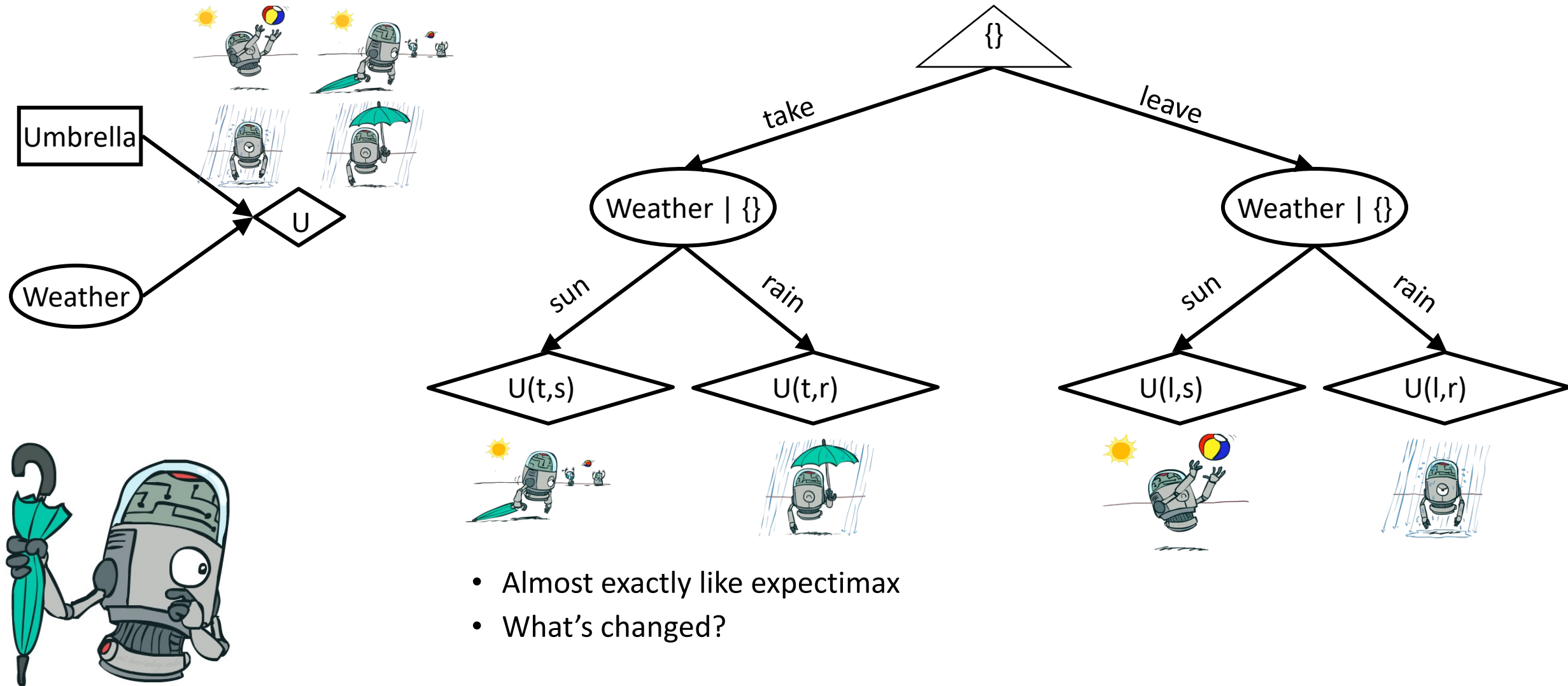


W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

What if W=rain?

Decisions as Outcome Trees



Example

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

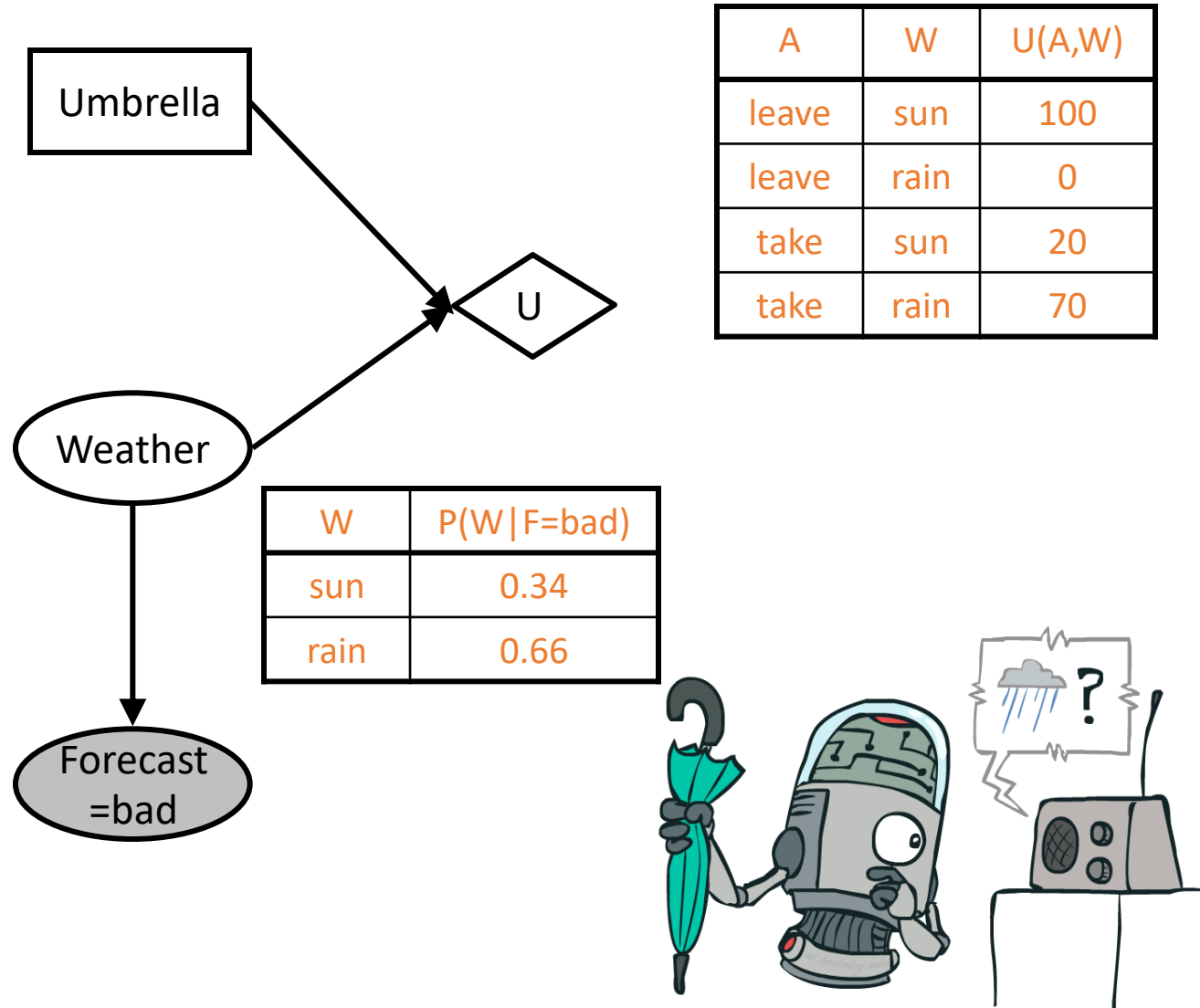
$$\begin{aligned} EU(\text{leave}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{leave}, w) \\ &= 0.34 \cdot 100 + 0.66 \cdot 0 = 34 \end{aligned}$$

Umbrella = take

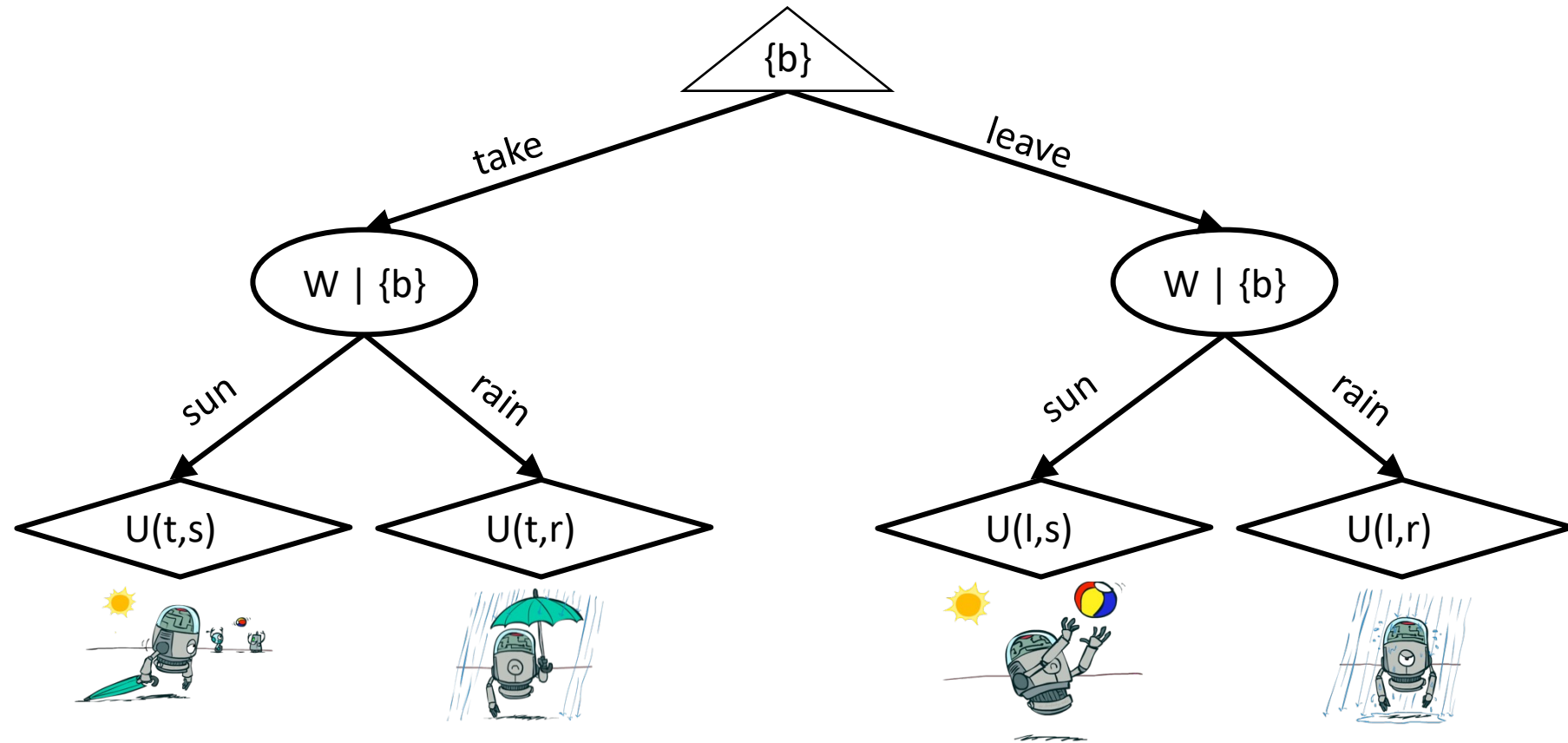
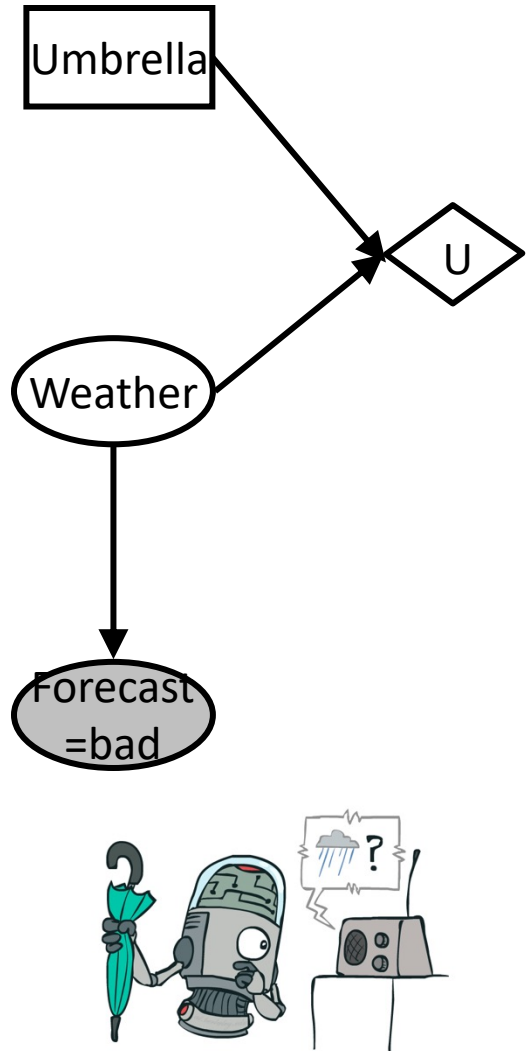
$$\begin{aligned} EU(\text{take}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{take}, w) \\ &= 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \end{aligned}$$

Optimal decision = take

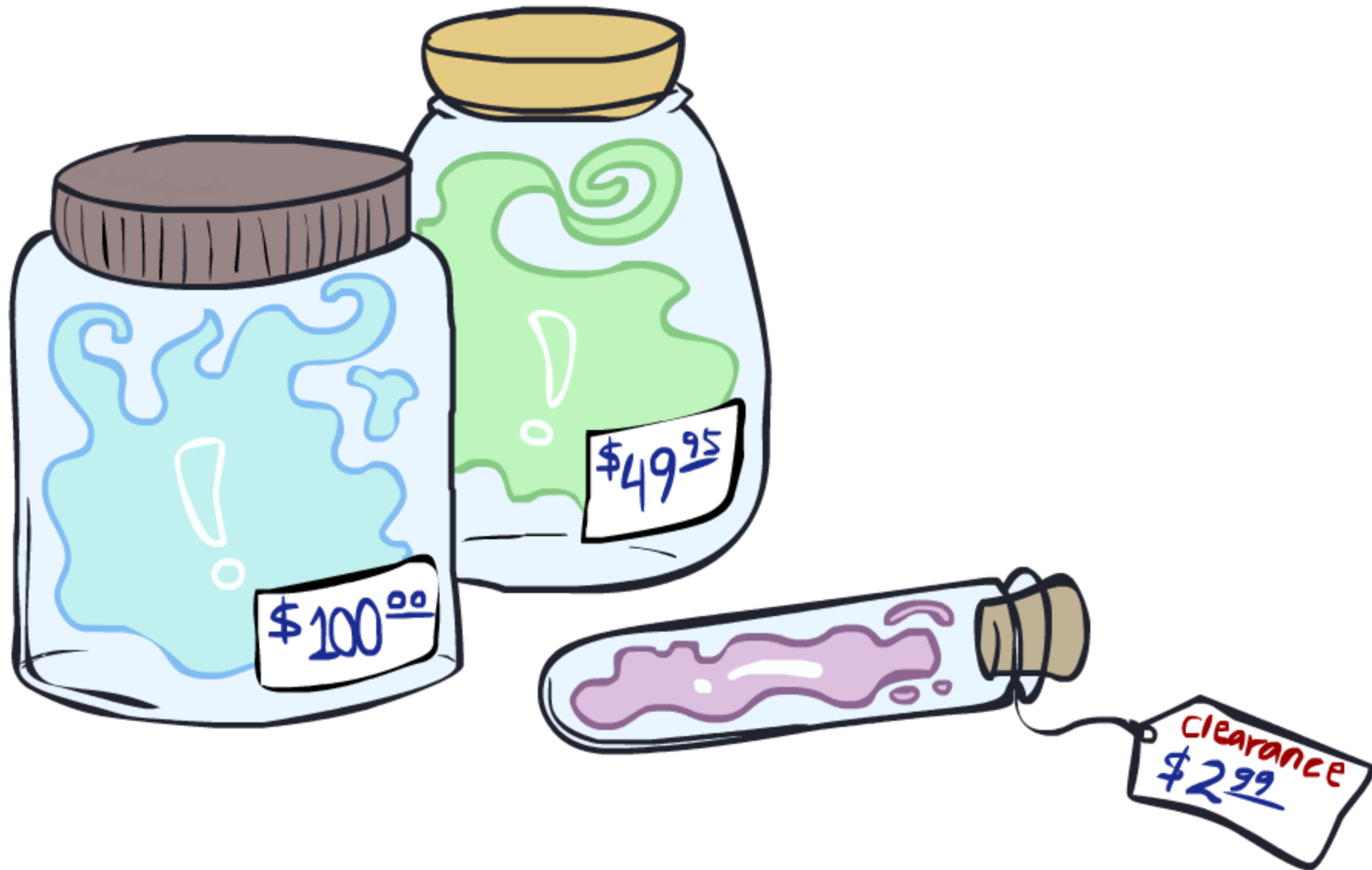
$$MEU(F = \text{bad}) = \max_a EU(a|\text{bad}) = 53$$



Decisions as Outcome Trees

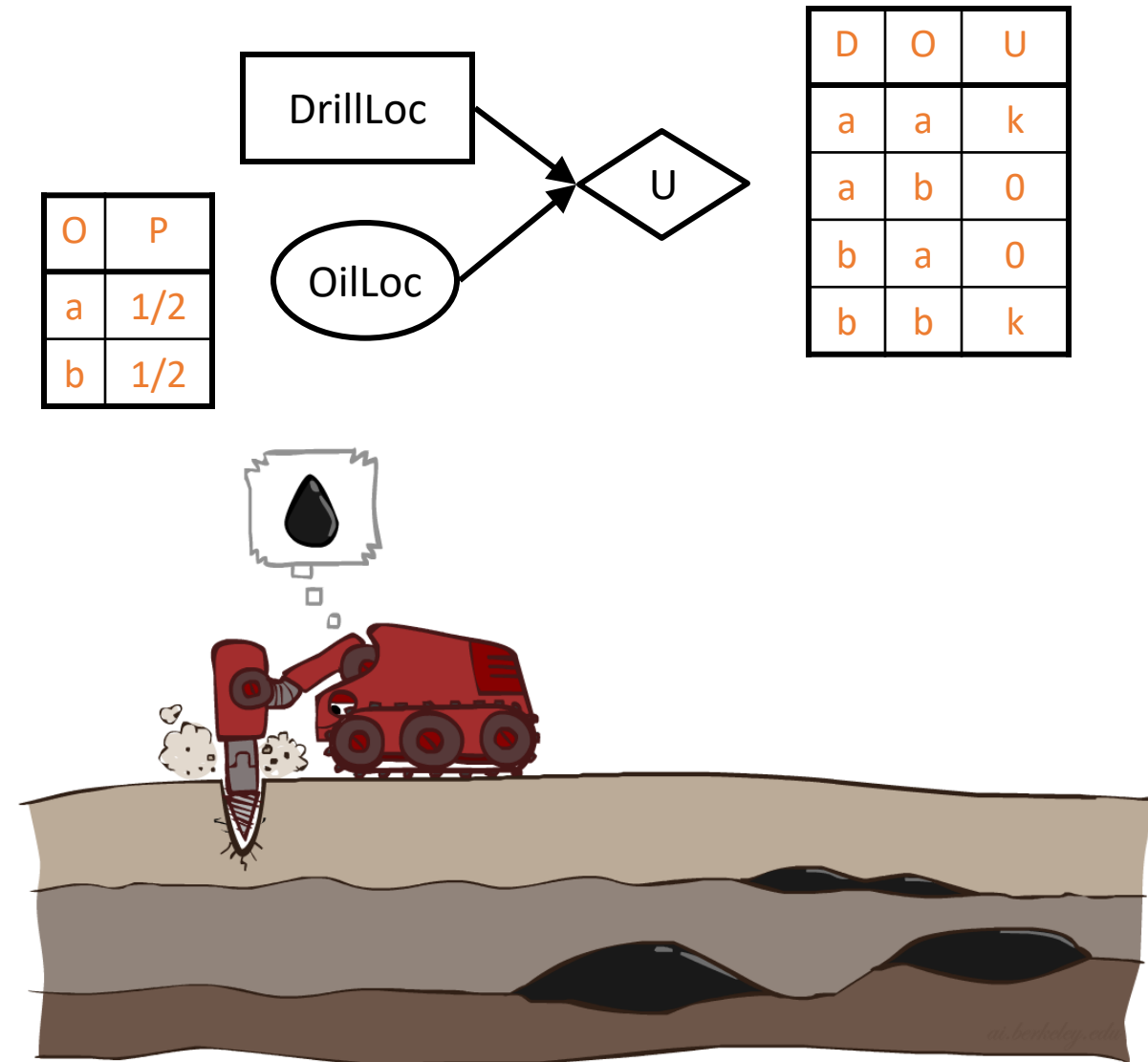


Value of Information



Value of Information

- Idea: compute value of acquiring evidence
 - Can be done directly from decision network
- Example: buying oil drilling rights
 - Two blocks A and B, exactly one has oil, worth k
 - You can drill in one location
 - Prior probabilities 0.5 each, & mutually exclusive
 - Drilling in either A or B has $EU = k/2$, $MEU = k/2$
- Question: what's the **value of information** of O?
 - Value of knowing which of A or B has oil
 - Value is expected gain in MEU from new info
 - Survey may say "oil in a" or "oil in b", prob 0.5 each
 - If we know OilLoc, MEU is k (either way)
 - Gain in MEU from knowing OilLoc?
 - $VPI(OilLoc) = k/2$
 - Fair price of information: $k/2$



VPI Example

MEU with no evidence

$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$

MEU if forecast is bad

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$

MEU if forecast is good

$$\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95$$

Forecast distribution

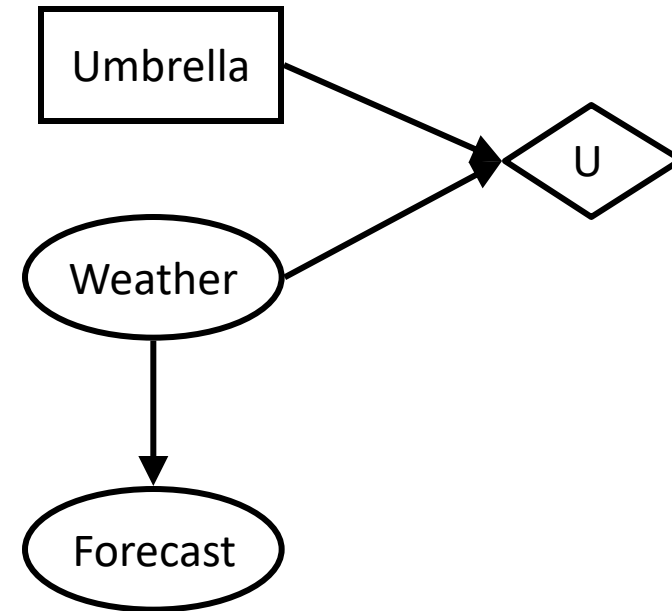
F	P(F)
good	0.59
bad	0.41



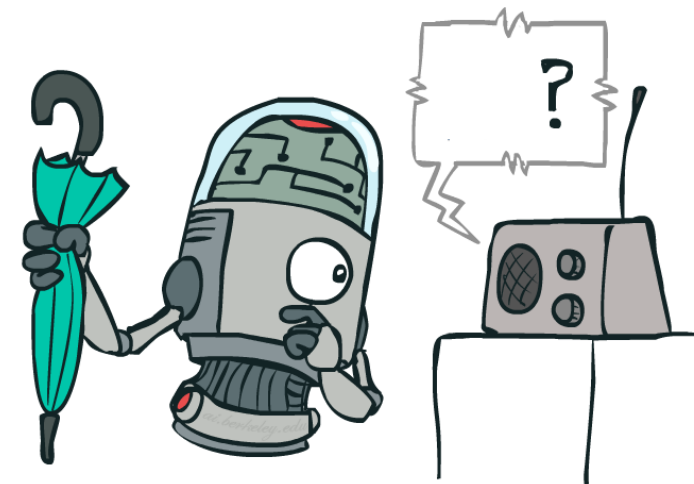
$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\text{VPI}(E'|e) = \left(\sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



Value of Information

- Assume we have evidence $E=e$. Value if we act now:

$$MEU(e) = \max_a \sum_s P(s|e) U(s, a)$$

- Assume we see that $E' = e'$. Value if we act then:

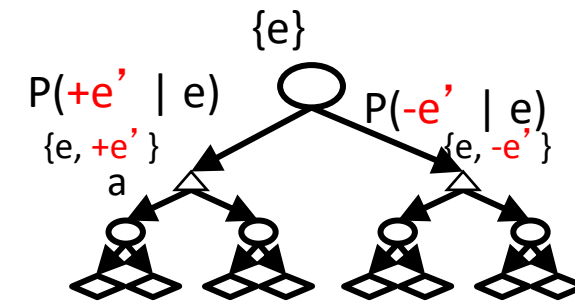
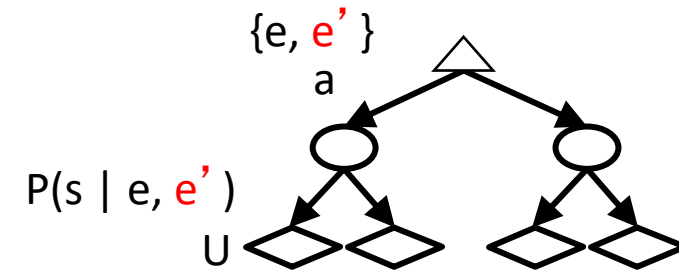
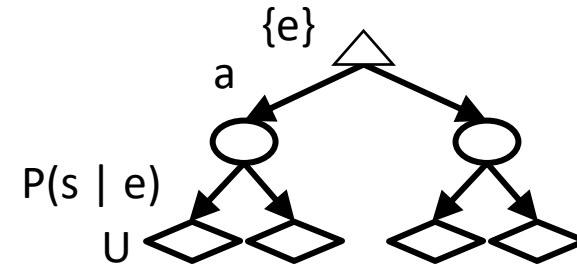
$$MEU(e, e') = \max_a \sum_s P(s|e, e') U(s, a)$$

- BUT E' is a random variable whose value is **unknown**, so we don't know what e' will be
- Expected value if E' is revealed and then we act:

$$MEU(e, E') = \sum_{e'} P(e'|e) MEU(e, e')$$

- Value of information: **how much MEU goes up by revealing E' first then acting, over acting now**

$$VPI(E'|e) = MEU(e, E') - MEU(e)$$



What do we need to “Infer”?

$$\text{MEU}(e) = \max_a \sum_s \boxed{P(s|e)} U(s, a)$$

$$\text{MEU}(e, e') = \max_a \sum_s \boxed{P(s|e, e')} U(s, a)$$

$$\text{MEU}(e, E') = \sum_{e'} \boxed{P(e'|e)} \text{MEU}(e, e')$$

$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e)$$

Careful, we need all of the highlighted distributions to calculate VPI!

VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$



- Nonadditive

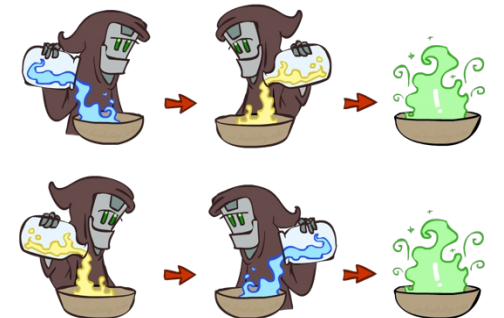
(think of observing E_j twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$



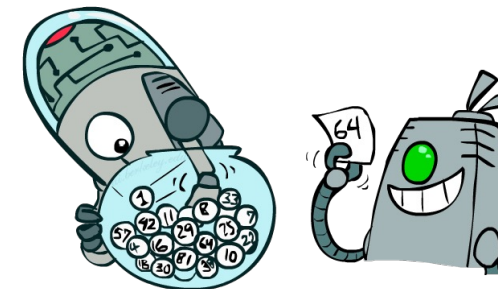
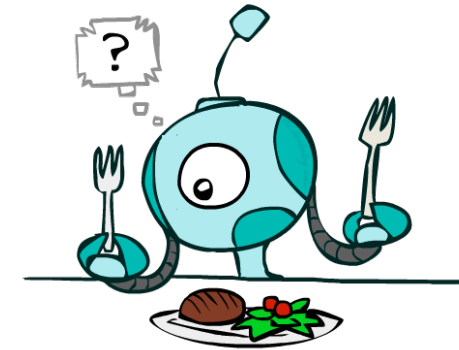
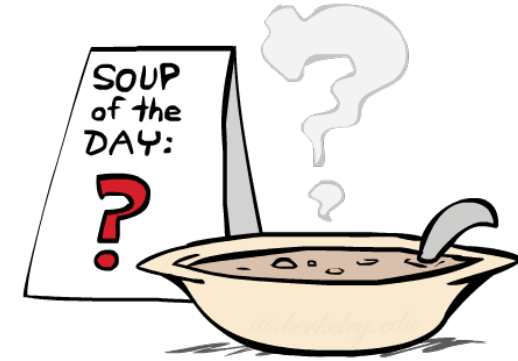
- Order-independent

$$\begin{aligned} \text{VPI}(E_j, E_k|e) &= \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \\ &= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \end{aligned}$$



Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?
- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?
- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



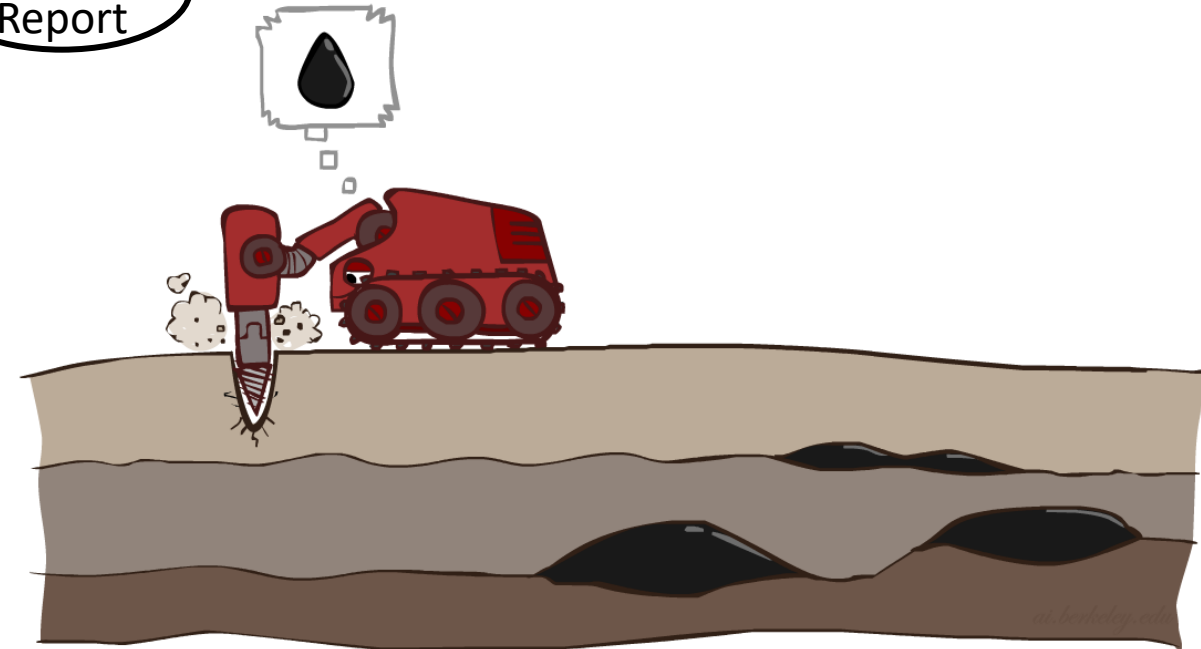
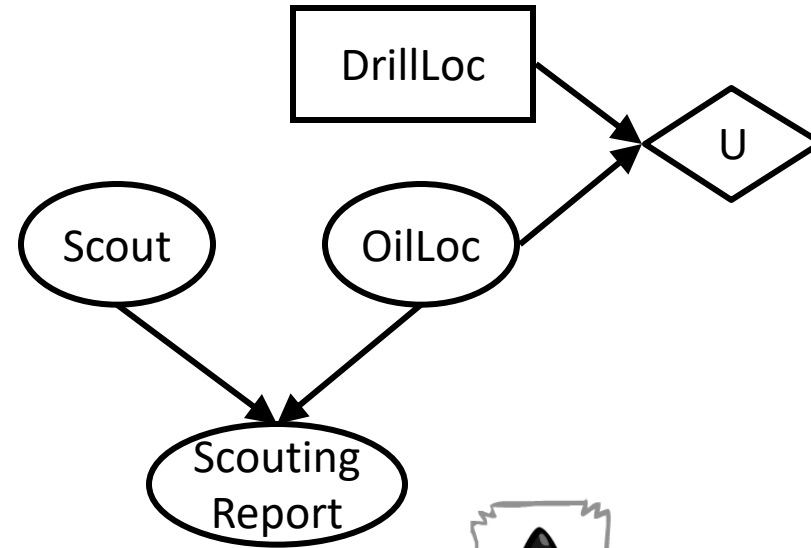
Value of Imperfect Information?



- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

VPI Question

- $VPI(\text{OilLoc})$?
- $VPI(\text{ScoutingReport})$?
- $VPI(\text{Scout})$?
- $VPI(\text{Scout} \mid \text{ScoutingReport})$?
- Generally:
If $\text{Parents}(U) \perp\!\!\!\perp Z \mid \text{CurrentEvidence}$
Then $VPI(Z \mid \text{CurrentEvidence}) = 0$



Additional Notes

- Action nodes as parents to variable nodes
 - Treat as evidence when going over actions
- Utility nodes having multiple random variable parents
 - Calculate posteriors over each parent to calculate the EU
- Utility nodes having multiple action parents
 - Instantiate all possible action combinations and max wrt these combinations
- Multiple utility nodes
 - Separate actions: Treat them individually
 - Overlapping actions: Max over the sum of EUs