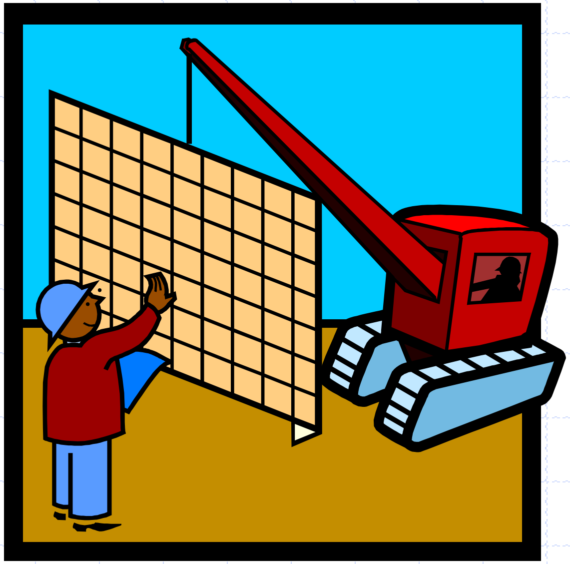
Array Lists



The Array List ADT

- The Array List ADT extends the notion of array by storing a sequence of arbitrary objects
- An element can be accessed, inserted or removed by specifying its index (number of elements preceding it)
- An exception is thrown if an incorrect index is given (e.g., a negative index)

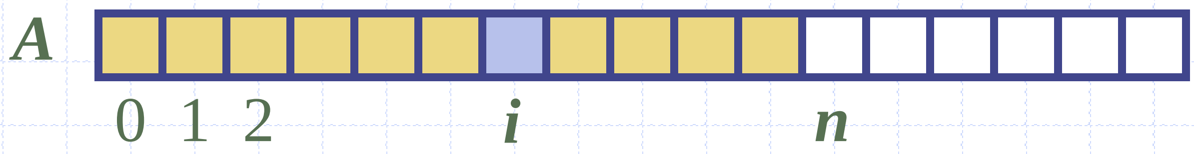
- Main methods:
  - get(integer i): returns the element at index i without removing it
  - set(integer i, object o): replace the element at index i with o and return the old element
  - add(integer i, object o): insert a new element o to have index i
  - remove(integer i): removes and returns the element at index i
- Additional methods:
  - size()
  - isEmpty()

Applications of Array Lists

- Direct applications
  - Sorted collection of objects (elementary database)
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

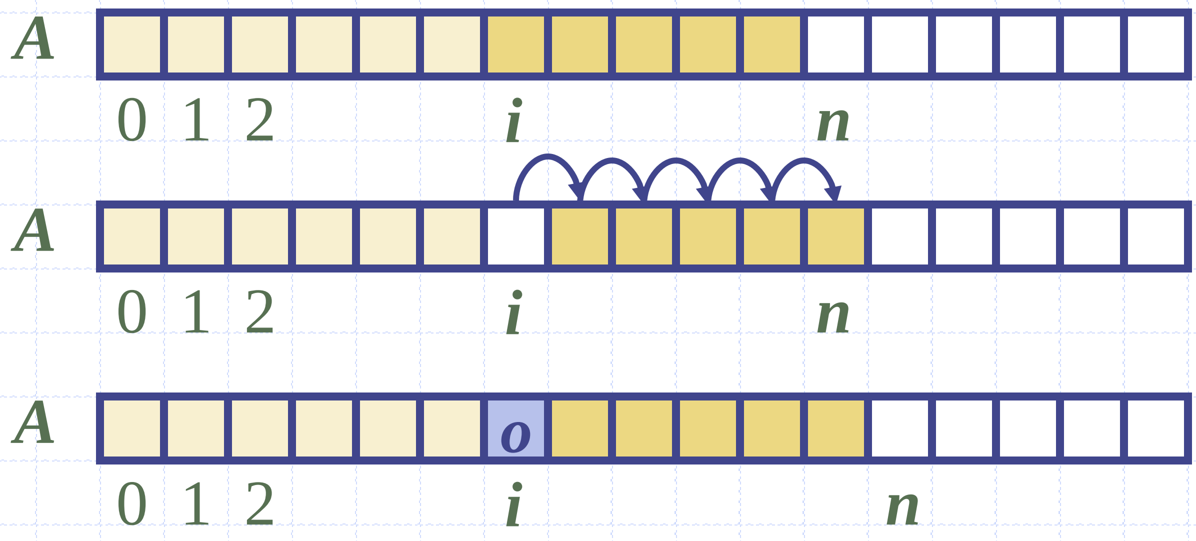
Array-based Implementation

- Use an array A of size N
- A variable n keeps track of the size of the array list (number of elements stored)
- Operation ***get***(i) is implemented in $O(1)$ time by returning $A[i]$
- Operation ***set***(i,o) is implemented in $O(1)$ time by performing $t = A[i]$, $A[i] = o$, and returning t.



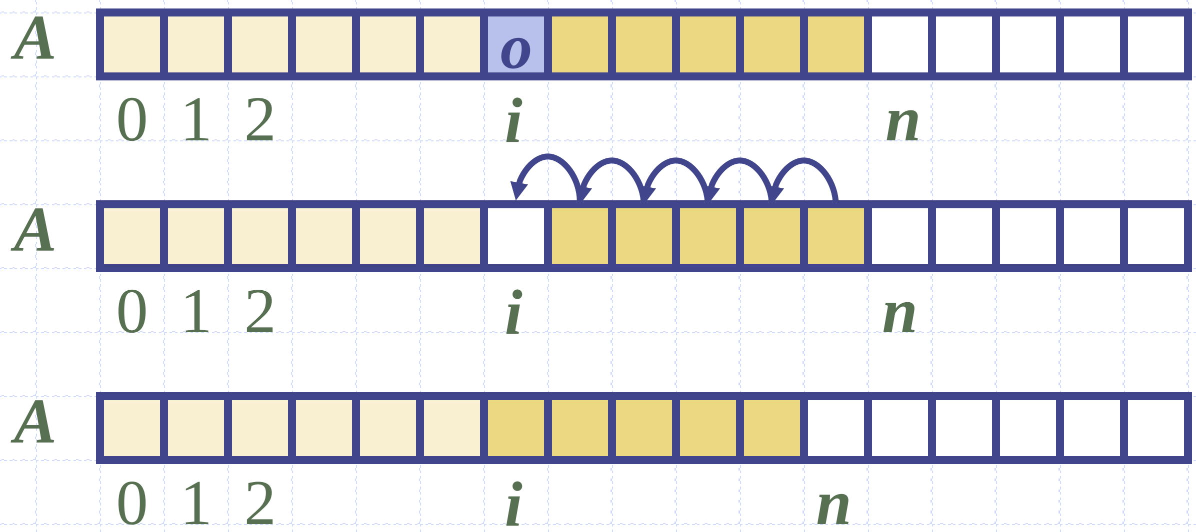
Insertion

- In operation ***add***(i, o), we need to make room for the new element by shifting forward the $n - i$ elements $A[i], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Element Removal

- In operation ***remove***(i), we need to fill the hole left by the removed element by shifting backward the $n - i - 1$ elements $A[i + 1], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Performance

- In the array based implementation of an array list:
  - The space used by the data structure is $O(n)$
  - ***size***, ***isEmpty***, ***get*** and ***set*** run in $O(1)$ time
  - ***add*** and ***remove*** run in $O(n)$ time in worst case
- If we use the array in a circular fashion, operations ***add***$(0, x)$ and ***remove***$(0, x)$ run in $O(1)$ time
- In an ***add*** operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Growable Array-based Array List

- In an add(o) operation (without an index), we always add at the end
- When the array is full, we replace the array with a larger one
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

Algorithm $add(o)$
  if $t = S.length - 1$ **then**
    $A \leftarrow$ **new array of size …**
    for $i \leftarrow 0$ **to** $n-1$ **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $n \leftarrow n + 1$
  $S[n-1] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n add(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an add operation the average time taken by an add over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$

$$n + c(1 + 2 + 3 + \ldots + k) =$$

$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of an add operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$
$$n + 2^{k+1} - 1 =$$
$$3n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of an add operation is $O(1)$

geometric series