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Propagation of a Gaussian Wave

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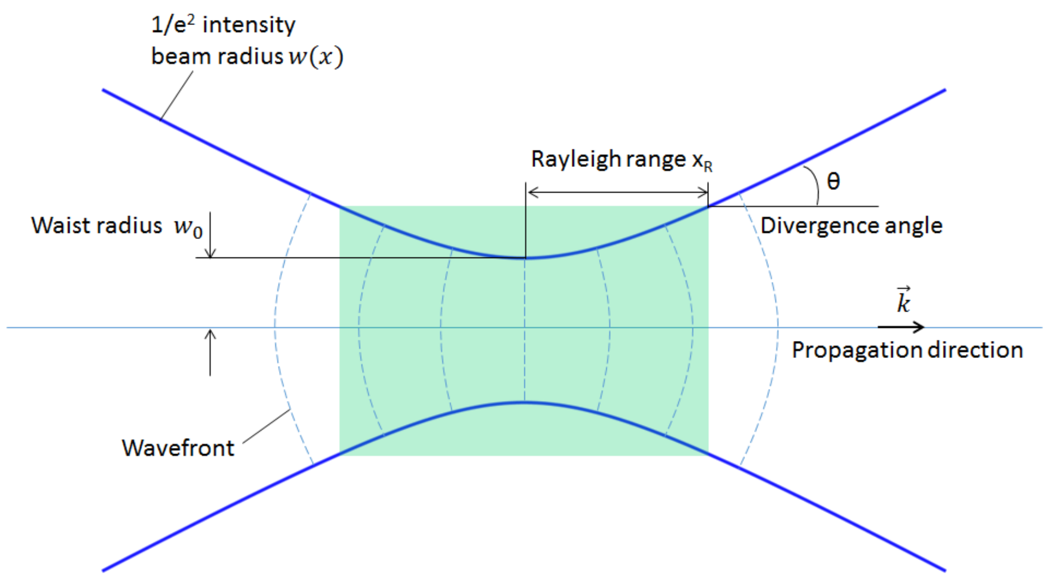
1.Introduction

The propagation of electromagnetic waves in free space is a fundamental topic in photonics and optical engineering. Among the wide variety of waveforms encountered in practical systems, the Gaussian beam plays a particularly important role due to its unique mathematical properties and its close correspondence to the output of real optical sources such as lasers. Unlike an ideal plane wave, which assumes infinite spatial extent and uniform phase distribution, a Gaussian beam exhibits a spatially varying amplitude that decays smoothly with radial distance from the beam center. This transverse distribution allows Gaussian beams to remain well-confined and to preserve their shape upon propagation, making them a suitable model for optical communication links, laser resonators, and imaging systems.

This project analyzed the spatial and directional evolution of the Gaussian beam, a fundamental mode of optical propagation. The simulation primarily investigates how the beam's intensity distribution varies both along the propagation axis and within the transverse plane. A key experimental objective involves modeling the interaction of the beam with a parabolic reflector placed at variable distances from the beam waist. By analyzing the characteristics of the reflected wave at different subsequent distances from the reflector, the simulation validates the theoretical principles governing beam manipulation and focusing. The study is supported by a comprehensive MATLAB implementation that computes the beam's fundamental properties, including the power, the continuous beam width evolution, and the instantaneous intensity profile across the propagation direction and slices of the transverse plane.

2.Gaussian Wave

Gaussian beams constitute one of the most important fundamental solutions to Maxwell's equations in optical and electromagnetic systems. They serve as accurate models for laser radiation, antenna aperture fields, optical communication beams, and general wave-propagation phenomena. In contrast to an ideal plane wave—whose electric and magnetic fields maintain a uniform amplitude across the transverse plane—a Gaussian beam exhibits a spatially varying amplitude whose intensity follows a Gaussian profile. Most of the beam power is confined within a narrow region around the propagation axis, and the transverse intensity distribution at any plane is circularly symmetric and centered about this axis.



At the reference plane $z = 0$, the complex field distribution of a Gaussian beam is expressed as:

$$U(r) = A_0 e^{-\rho^2/w_0^2},$$

where $\rho^2 = x^2 + y^2$ and w_0 denotes the beam waist, corresponding to the minimum spot size. A notable property of the Gaussian beam is that it can be represented as a continuous superposition of plane waves with different spatial-frequency components, k_x and k_y. This decomposition is obtained through the spatial Fourier transformation of the initial field distribution, enabling the beam to be analyzed and propagated efficiently in the spatial-frequency domain.

As the beam propagates in free space, it undergoes gradual divergence, and its spot size increases according to the characteristic Rayleigh distance:

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

which defines the distance over which the beam remains near its minimum width. Propagation over a distance z is modeled in the frequency domain by multiplying the spatial spectrum $U(k_x, k_y)$ by the propagation factor:

$$U_z(k_x, k_y) = U(k_x, k_y)\, e^{-jk_z z}$$

where $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$ or using the paraxial approximation $k_z = k - \frac{k_x^2 + k_y^2}{2k}$, k_z is the longitudinal wave number component. Applying the inverse Fourier transform yields the field distribution at the new propagation plane, completing the Fourier-optics method for Gaussian beam propagation.

3.Parameters for the Gaussian wave

We model the Gaussian beam with a wavelength $\lambda = 3.8$ mm and a beam waist $w_0 = 40$mm. Using these values, the relevant propagation parameters are computed according to the previously defined equations. A suitable number of sampling points N and an appropriate physical window size L , also selected to ensure that the spatial sampling interval dx exceeds the minimum required value for valid application of the paraxial approximation:

Parameter	Symbol	Value
Wavelength	λ	3.8 mm
Initial Beam Waist	w_0	40 mm
Rayleigh Distance	z_0	1.325 m
Number of Grid Points	N	1024
Wave Number	k	1653.46 m^{-1}
Physical Grid Size	L	3.0 m
Parabolic Reflector Focal Length	f	$-4 \times z_0$

3.1.Parameters & distances definition

Defining the parameters on MATLAB as follows:

```
lambda = 3.8e-3;      % wavelength (m)
w0     = 40e-3;       % beam waist (m)
k      = 2*pi/lambda; % wave number

z0 = pi * w0^2 / lambda;   % Rayleigh distance

% Required propagation distances:
z_list_before = [0, 0.5, 1];        % before reflector
z_list_after  = [1,4,6];            % after reflector (from mirror)
z_list_reflector=[3,4,5];           % mirror locations
f = -4*z0;                           % parabolic reflector focus
c=['r','b','g','k','m','c','g'];    % colors for plotting
axis_limit=300;
```

3.2.Grid settings

Setting the grid settings on MATLAB as follows:

```
N = 1024;              % points in x-y
L = 3.0;               % physical grid size (meters)
dx = L/N;              % sampling step
center = N/2 + 1;

z_max_est = N * dx^2 / lambda;

min_dx = (sqrt(2)*pi)/k;
if dx > min_dx
    fprintf('Sampling condition satisfied: dx = %e  >  %e\n', dx, min_dx);
else
    fprintf('Sampling condition may be violated: dx = %e  <=  %e\n', dx, min_dx);
end

x = (-N/2:N/2-1)*dx;
y = x;
[X,Y] = meshgrid(x,y);

% Spatial frequencies
fx = (-N/2:N/2-1)/(N*dx);
fy = fx;
[Fx, Fy] = meshgrid(fx, fy);

kx = 2*pi*Fx;
ky = 2*pi*Fy;
```

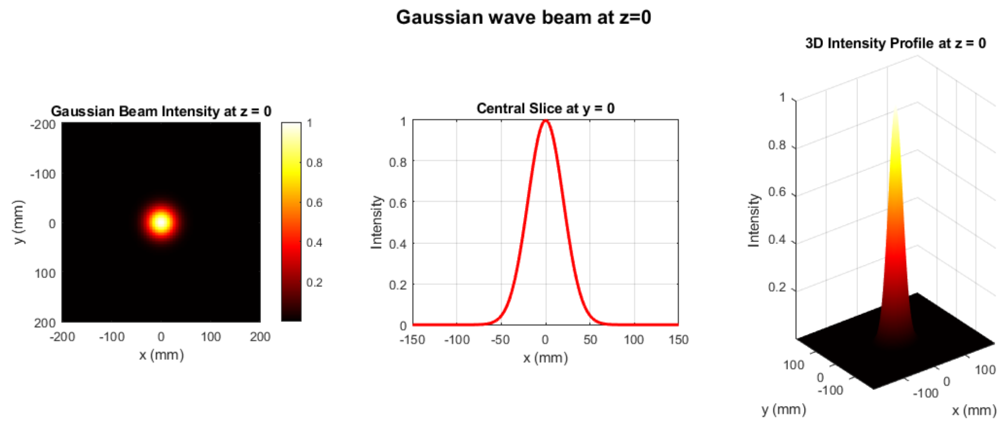
Note: The large number of sampling points and the extended physical window are chosen to ensure that the spatial sampling interval dx exceeds the minimum required for the validity of the paraxial approximation. In addition, increasing the window size improves the resolution in the spatial-frequency domain dk, since dx and dk are inversely related, consistent with the Fourier uncertainty principle.

4.Gaussian wave profile at the beam waist

We are expecting the highest intensity with lower beam width at the beam waist, MATLAB code modeling the gaussian wave at $z = z_o$:

```
U0 = exp(-(X.^2 + Y.^2)/(w0^2));
x_mm = x*1e3;
idx = find(x_mm >= -200 & x_mm <= 200);

figure(1);
set(gcf,'Position',[100 100 1400 450]);
sgtitle('Gaussian wave beam at z=0','FontWeight','bold');
    subplot(1,3,1);
        imagesc(x_mm(idx), x_mm(idx), abs(U0(idx,idx)).^2);
        title('Gaussian Beam Intensity at z = 0');
        xlabel('x (mm)');
        ylabel('y (mm)');
        axis image;
        colorbar;
        colormap('hot');

    subplot(1,3,2);
        plot(x*1e3, abs(U0(center,:)).^2,'r','LineWidth',2);
        title('Central Slice at y = 0');
        xlabel('x (mm)');
        ylabel('Intensity');
        xlim([-150 150]);
        grid on;
        pbaspect([1.3 1 1]); % adjusts width-to-height ratio

    subplot(1,3,3);
        surf(x_mm(idx), x_mm(idx), abs(U0(idx,idx)).^2);
        xlabel('x (mm)');
        ylabel('y (mm)');
        zlabel('Intensity');
        title('3D Intensity Profile at z = 0');
        shading interp;         % prevent shading for intensity visualization
        axis image;
        colormap hot;
        daspect([1 1.2 0.0014]); % scaling in X,Y,Z axes
```

Simulation Results:



As we see we have a gaussian distribution profile for the intensity with the radial distance.

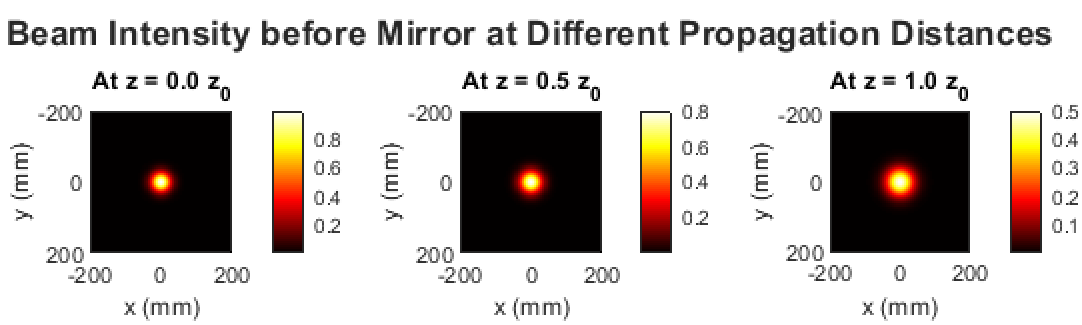
5.Gaussian wave propagation

In this part, we use MATLAB to simulate the propagation of a Gaussian beam over different distances z, illustrating how the beam's intensity evolves across the corresponding transverse planes, we are expecting the wave width to increase in parallel with decreasing in the amplitude of the intensity along with the increasing of distance (z).The following MATLAB code could model this propagation:

```
U0_f = fftshift(fft2(U0));

prop = @(z) exp(-1j*z*(k-((kx.^2 + ky.^2)/(2*k))));
figBefore2D = figure;
tL = tiledlayout(1, length(z_list_before), 'TileSpacing', 'compact', 'Padding', 'compact');
title(tL, 'Beam Intensity before Mirror at Different Propagation Distances',
'FontWeight','bold');

Power_before = zeros(1,length(z_list_before));  % initialize power array

for i = 1:length(z_list_before)
    z = z_list_before(i)*z0;
    Uz_f = U0_f .* prop(z);
    Uz   = ifft2(ifftshift(Uz_f));

    % Calculate total power before mirror
    Power_before(i) = sum(sum(abs(Uz).^2)) * dx^2;

    figure(figBefore2D);
```

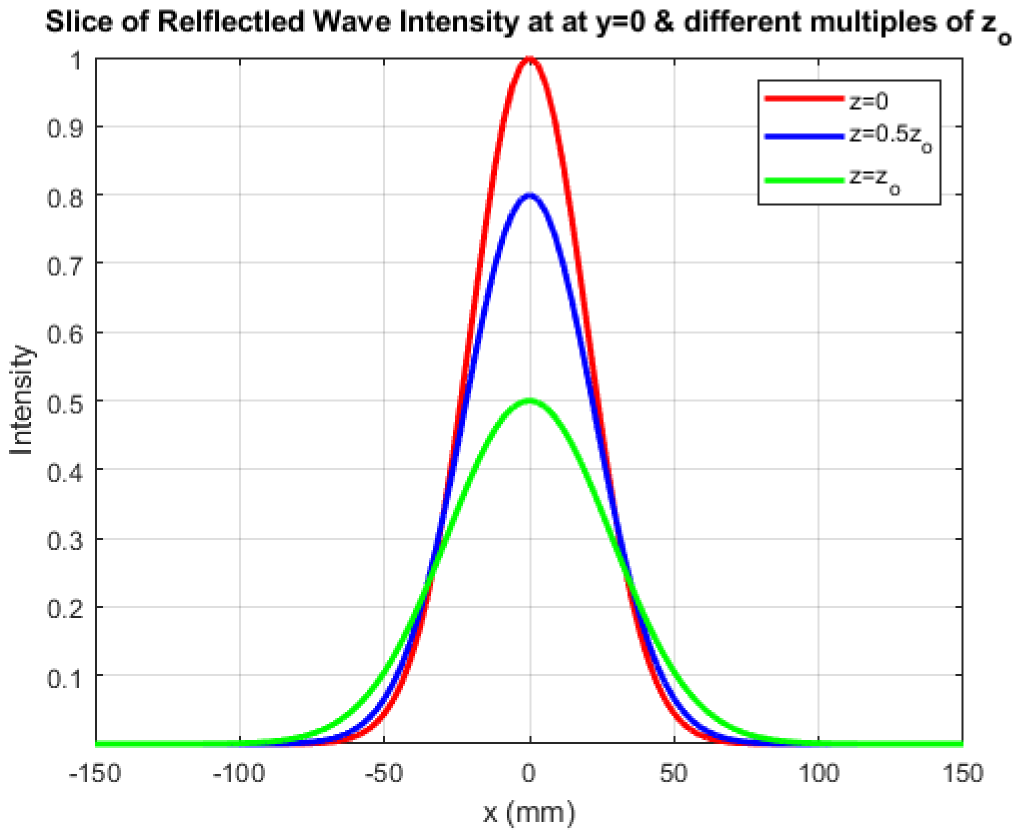
```
    ax = nexttile(i);

    % Plot intensity
    imagesc(ax, x_mm(idx), x_mm(idx), abs(Uz(idx,idx)).^2);
    axis(ax, 'image');
    xlabel(ax, 'x (mm)');
    ylabel(ax, 'y (mm)');
    title(ax, ['At z = ', num2str(z_list_before(i), '%.1f'), ' z_0']);
    colormap(ax, 'hot');
    colorbar(ax);



    if(i==1)
        Q = length(findobj('Type','figure')); % count existing figures
        figure(Q + 1);                        % open the next figure
    else
        figure(Q+1);
    end
        axis tight;
        plot(x*1e3, abs(Uz(center,:)).^2,c(i),'LineWidth',2);hold on;
        title('Slice of Relflectled Wave Intensity at at y=0 & different multiples of z_o');
        xlabel('x (mm)'); ylabel('Intensity');
        xlim([-150 150]);
        grid on;
end
legend('z=0','z=0.5z_o','z=z_o');
```

5.1. 2D Intensity profile at $z = 0, z = 0.5\ z_o$ & $z = z_o$



As expected, we can see a divergence in the intensity with increasing the distance.

5.2. Intensity Slices at $y = 0$, $z = 0, z = 0.5\ z_o$ & $z = z_o$



The figures illustrate the fundamental process of Gaussian beam divergence governed by the laws of diffraction. As the beam propagates along the z-axis, its width (w) increases, and its peak intensity (I) decreases to conserve power.

5.2.1.At z = 0 (Beam Waist)

- This point represents the beginning of the simulation and the location of the Initial Beam Waist (w_0).
- **Beam width ($\boldsymbol{w}$):** $w(0) = w_o = 40\ mm$ → (Minimum width).
- **Peak Intensity (I):** $I = I_{maximum} = 1\ watt/m^2$
- **Observation:** The beam is at its tightest focus. All the beam's energy is concentrated in the smallest possible area, resulting in the highest intensity. This is the starting point for divergence.

5.2.2. At $z = 0.5\, z_o$

- This point is halfway to the Rayleigh distance, where divergence has just begun.
- **Beam width (w):** $w(\frac{1}{2}z_o) \; = \; 1.118\, w_o$.
- **Peak Intensity (I):** Decreased slightly from z=0 → $I \approx 0.8$.
- **Observation:** The beam has experienced minimal divergence as the distance is less than z_o. The effect of spreading is noticeable, but the beam is still considered tightly focused within the near-field region.

5.2.3. At $z \; = \; z_o$ (Rayleigh Distance)

- This is the Rayleigh Distance, a crucial point in the Gaussian beam definition, separating the near-field from the far-field.
- **Beam Radius (w):** $w(z_o) = \sqrt{2w_o} \approx 1.414w_o$
- **Peak Intensity (I):** The peak intensity has halved compared to z=0.
- **Observation:** At this distance, the beam's cross-sectional area has doubled. The beam begins to enter the far-field region, where its wavefronts are primarily spherical, and its diameter begins to increase almost linearly with z. The spreading effect is significant.

6. Reflection of the gaussian wave

After propagating through free space, the Gaussian beam encounters a parabolic reflector, which plays a crucial role in reshaping the beam's phase profile and redirecting its propagation. A parabolic reflector introduces a spatially varying quadratic phase shift to the incident wavefront, effectively transforming the curvature of the beam. This phase modification is described by the reflector's transfer function

$$M(r) = \mathrm{e}^{(-j\,k\,\frac{x^2+y^2}{2f})},$$

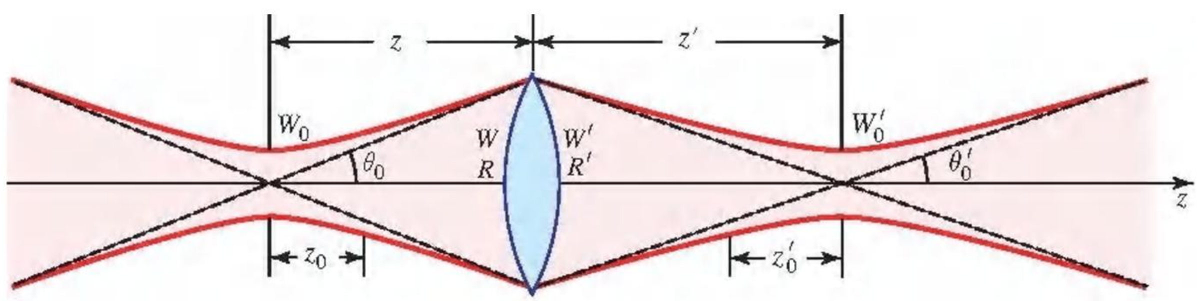
where f is the focal length of the parabolic surface and $f < 0$ for a reflective geometry. In this experiment, the focal length is chosen as $f = 4z_0$, ensuring that the reflector appropriately modifies the beam after it has propagated a significant distance relative to its Rayleigh range.

The reflector is applied at three different incident planes corresponding to propagation distances $z = 3z_0$, $4z_0$, and $5z_0$. At each location, we expect a different reflection shape as will be discussed later, the complex field distribution $U_{\text{in}}(r)$is multiplied by the mirror's transfer function to obtain the reflected field.

$$U_{\text{out}}(r) = M(r)\, U_{\text{in}}(r).$$

Following reflection, the modified beam is further propagated through free space to distances z_0, $4z_0$, and $6z_0$. This allows us to observe how the reflector influences the beam's focusing behavior, wavefront evolution, and intensity distribution over extended propagation ranges.

Since the position of the reflector influences the characteristics of the reflected beam, it is necessary to compute the reflected wave parameters for each reflector location using the appropriate equations. This allows us to accurately predict the behavior of the reflected wave.



NOTE: This figure is for a lens which is not our case, but same rules are applied, and we can think of this reflected beam as the transmitted beam in the figure but in opposite direction.

6.1. Boundary equations for the reflector

The boundary equations which defines the reflected beam shape:

$$R = z\sqrt{1+\left(\frac{z}{z_o}\right)^2}$$

$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$$

$$r = \frac{z_o}{z-f}$$

$$M_r = \left|\frac{f}{z-f}\right|$$

$$Magnification\ M = \frac{M_r}{\sqrt{1+r^2}}$$

$$Waist\ location \quad (z'-f) = M^2(z-f)$$

Where the R represents the radius of wavefront away from the beam waist, while R' represent the radius after reflection where positive R' represents diverging reflected wave and vice versa, z' represents the propagation distance after reflection.

Modeling such reflector on MATLAB can be as follows:

```
M = exp(-1j * k * (X.^2 + Y.^2) / (2*f));
% preallocations for the figure and power arrays
Power_after = zeros(length(z_list_reflector), length(z_list_after));
figAfter2D = gobjects(1, length(z_list_reflector));
%Uout_f_storage=zeros(length(z_list_reflector));
for j=1:length(z_list_reflector)
    z = z_list_reflector(j) * z0;
    Uin_f = U0_f .* prop(z);
    Uin   = ifft2(ifftshift(Uin_f));
    Uout = M .* Uin;
    Uout_f = fftshift(fft2(Uout));
    %Uout_f_storage(j) = Uout_f; % Store the reflected fiel

    figAfter2D(j) = figure;
    tL_after = tiledlayout(1, length(z_list_after), 'TileSpacing', 'compact', 'Padding',
'compact');
    title(tL_after, ['Reflected Beam Intensities after Mirror at z_{mirror} = ',
num2str(z_list_reflector(j)), ' z_0'], ...
        'FontWeight', 'bold');
    for i = 1:length(z_list_after)
        z = z_list_after(i)*z0;
        Uz_f = Uout_f .* prop(z);
        Uz   = ifft2(ifftshift(Uz_f));

        % Calculate total power
        Power_after(j,i) = sum(sum(abs(Uz).^2)) * dx^2;

        % subplot(1,length(z_list_after),i);
        figure(figAfter2D(j));
        ax = nexttile(i);
        imagesc(ax, x_mm(idx), x_mm(idx), abs(Uz(idx,idx)).^2);
        axis(ax, 'image');
        xlabel(ax, 'x (mm)');
        ylabel(ax, 'y (mm)');
        title(ax, ['At z = ', num2str(z_list_after(i), '%.1f'), ' z_0']);
        colormap(ax, 'hot');
        colorbar(ax);

        if(i==1)
            Q = length(findobj('Type','figure'));   % count existing figures
            fig(j)=figure(Q + 1);                    % open the next figure
            sgtitle(['Slice of Reflected Wave intensity from z_{mirror}=', ...
            num2str(z_list_reflector(j)),' z_o at different distances'], ...
           'FontSize', 12, 'FontWeight', 'bold');
        else
            figure(Q+1);
        end
```

```
        plot(x*1e3, abs(Uz(center,:)).^2,c(i),'LineWidth',2); hold on;
        xlabel('x (mm)'); ylabel('Intensity');
        xlim([-axis_limit axis_limit]);
        grid on;
    end
legend('At z=z_o','z=4z_o','z=6z_o','FontWeight','bold');
end
```

6.2.Reflector at $z = 3z_o$

For the Reflector $z = 3z_o$, we could predict the radius of the wavefront from the beam waist and the beam waist location using the above formulas:

$$for\ z = 3z_o\ \&\ f = 4z_o$$

$$R = z\left(1 + \left(\frac{z_o}{z}\right)^2\right) = 3z_o\left(1 + \left(\frac{z_o}{3z_o}\right)^2\right) = \frac{10}{3}z_o$$

$$\because \frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$$

$$R' = \frac{1}{20}z_o \rightarrow\ +ve\ sign \therefore we\ have\ diverging\ wave$$

$$, also\ for\ the\ beam\ location \rightarrow\ r = \frac{z_o}{z - f} = \frac{z_o}{3z_o - 4z_o} = -1$$

$$M_r = \left|\frac{f}{z - f}\right| = \left|\frac{4z_o}{3z_o - 4z_o}\right| = 4$$

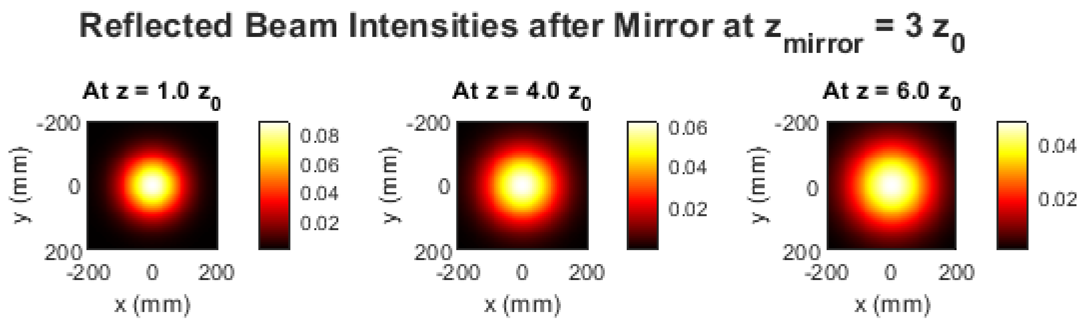
$$M = \frac{M_r}{\sqrt{1 + r^2}} = \frac{4}{\sqrt{1 + (-1)^2}} = \frac{4}{\sqrt{2}}$$

$$so\ the\ waist\ location\ (z' - f) = M^2(z - f)$$

$$(z' - f) = \left(\frac{4}{\sqrt{2}}\right)^2 (3z_o - 4z_o) = -8z_o$$

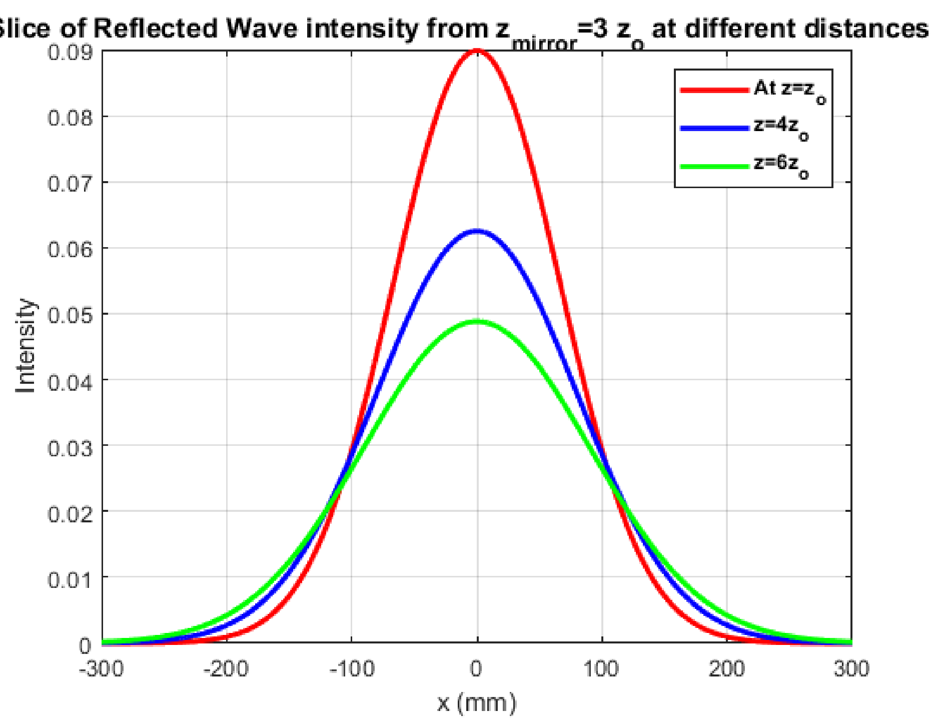
$$\therefore z' = -4z_o \rightarrow\ which\ has\ convientient\ sign\ (after\ the\ beam\ waist)$$

6.2.1. 2D Intensity profile for reflector at $z = 3z_o$ across different propagation distances for the reflected beam



As expected, we got a diverging wave with increasing the propagation distance.

6.2.2. Intensity Slice for reflector at $z = 3z_o$ across $y = 0$, $z = z_o$, $z = 4z_o$ & $z = 6z_o$.



As expected, we got a diverging wave beam with increasing the propagation distance z, increasing the width and decreasing amplitude of the intensity.

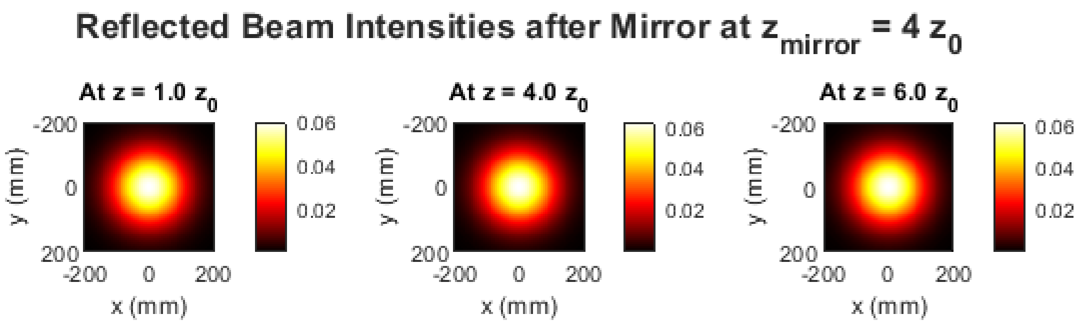
6.3. Reflector at $z = 4z_o$

Same calculations were done like the previous part to predict the radius of the reflected wave and the beam waist location giving the following results:

$$Reflected\ beam\ radius\ R' = -\frac{1}{68}z_o \rightarrow\ -ve\ sign$$
$$\therefore a\ converging\ reflected\ wave$$

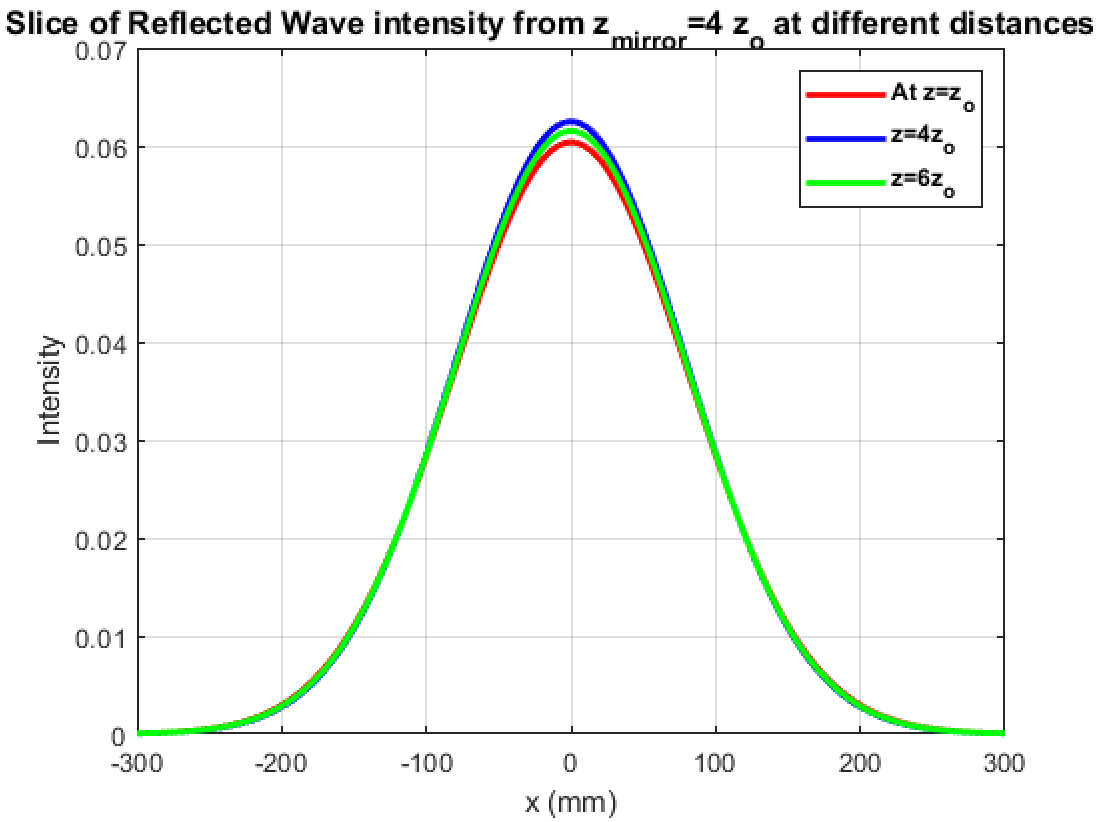
$$Beam\ location\ z' = 4z_o \rightarrow convienient\ sign\ (before\ the\ beam\ waist)$$

6.3.1. 2D Intensity profile for reflector at $z = 4z_o$ across different propagation distances for the reflected beam



As we expected, a very slightly converging wave till the beam waist at $z' = 4z_o$ then diverging again completing the gaussian wave profile.

6.3.2. Intensity Slice for reflector at $z = 4z_o$ across $y = 0$, $z = z_o$, $z = 4z_o$ & $z = 6z_o$.



As expected, the reflected beam starts to converge from $z = z_o$ until it reaches the new waist location at $z' = 4z_0$. At this point, the intensity attains its maximum value. Beyond this plane, the beam begins to diverge again, and its peak intensity decreases noticeably by the time it reaches $z' = 6z_0$.

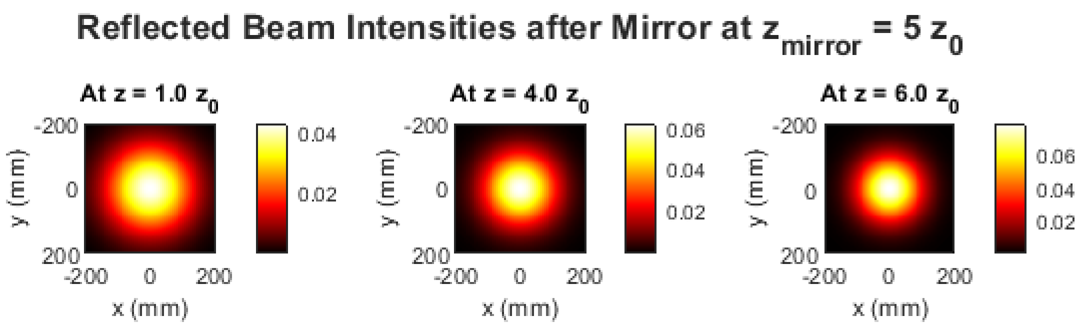
6.4.Reflector at $z = 5z_o$

Same calculations were done giving the following results:

$$Reflected\ beam\ radius\ R' = -\frac{3}{52}z_o \rightarrow\ -ve\ sign$$
$$\therefore a\ converging\ reflected\ wave$$

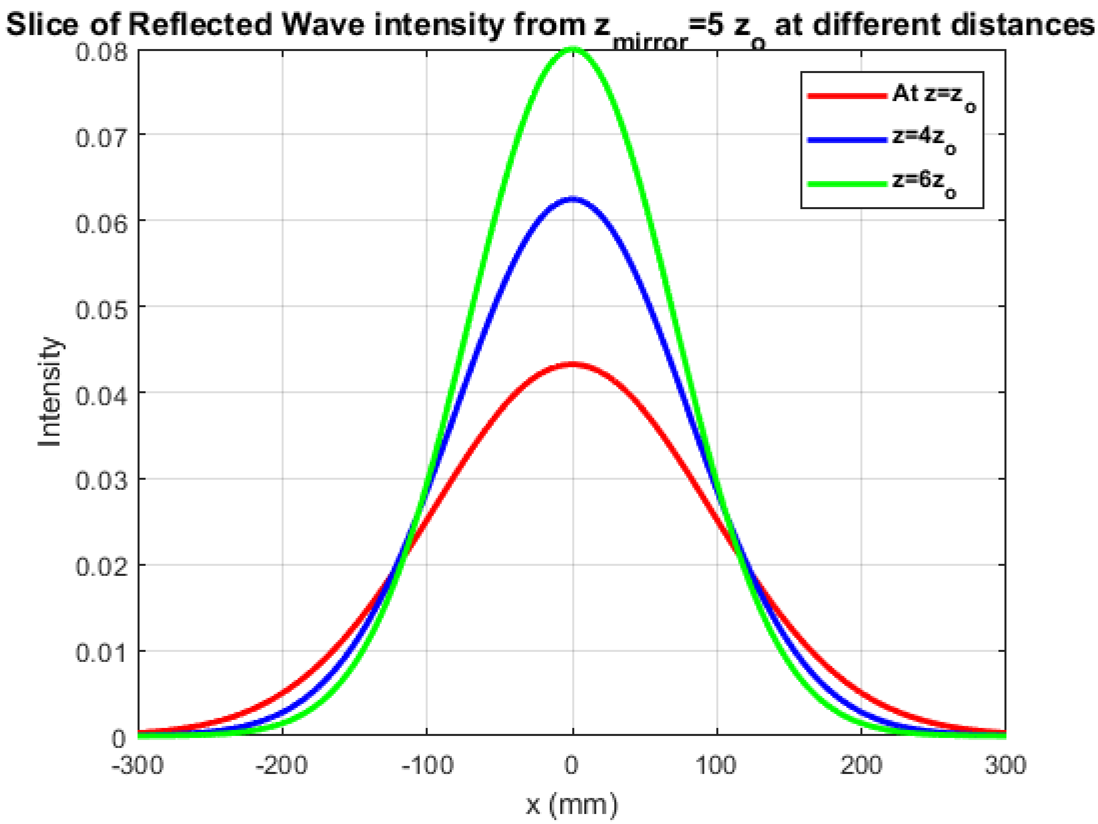
$$Beam\ location\ z' = 12z_o \rightarrow convienient\ sign\ (before\ the\ beam\ waist)$$

6.4.1. 2D Intensity profile for reflector at $z = 5z_o$ across different propagation distances for the reflected beam



As expected, we have a converging wave starting from the reflector all the way till $z' = 12z_o$ then starts to diverge again completing the gaussian wave profile.

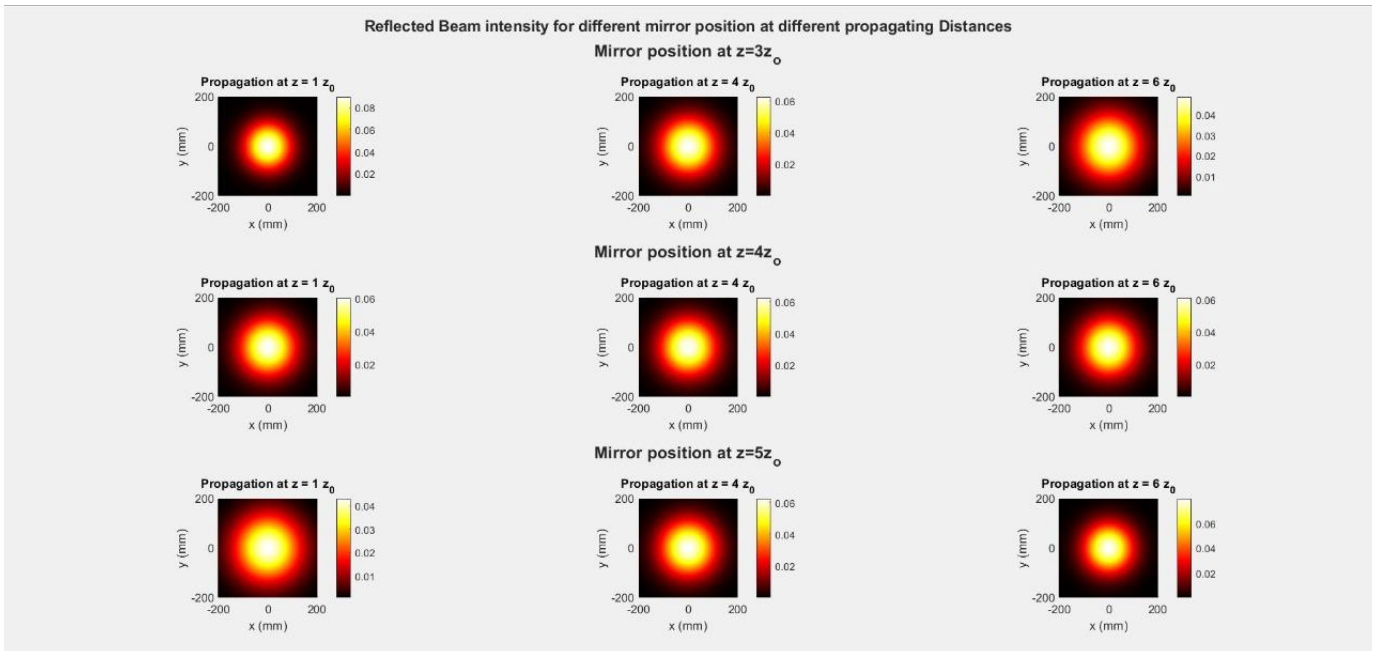
6.4.2. Intensity Slice for reflector at $z = 5z_o$ across $y = 0$, $z = z_o$, $z = 4z_o$ & $z = 6z_o$.



As expected, the beam undergoes convergence, characterized by an increase in amplitude accompanied by a reduction in beam width.

6.5.Reflector Conclusion

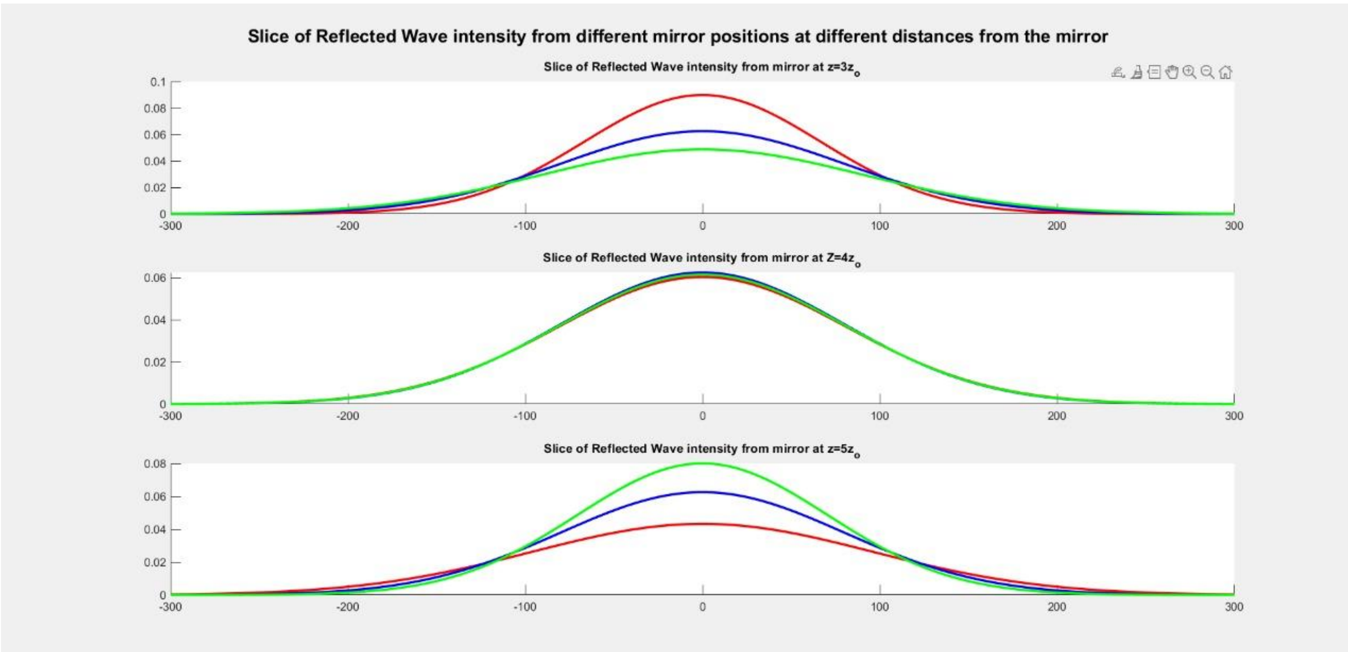
The numerical results and intensity maps presented in the previous Figures confirmed the theoretical predictions regarding how a parabolic mirror reshapes a Gaussian beam depending on its position relative to the Rayleigh distance. The reflected-beam intensity plots show clear differences in divergence, convergence, and peak intensity for mirror locations at $z = 3z_o, 4z_o, and\ 5z_o$.



For the reflector position $z = 3z_o$, the 2D intensity maps demonstrate that the beam expands immediately after reflection, consistent with the negative value of z'. This produces a rapid decrease in peak intensity, as seen from the red, blue, and green curves in the corresponding slice plot, which progressively flatten with propagation distance.

For the reflector position $z = 4z_o$, the figures show a nearly stationary beam waist at the mirror surface. The intensity slices for different propagation distances almost overlap, confirming the prediction that $M^2 = 0$ leads to a beam that neither strongly converges nor diverges immediately, but instead maintains a stable waist before eventually spreading.

For the reflector position $z = 5z_o$, the reflected beam shows a clear region of convergence before divergence. This is evident in the increased peak intensity at moderate propagation distances and the sharper central peak in the slice plot. The large positive value of z' predicted for this case aligns with the observed behavior of the beam narrowing before it expands again.



Overall, the visual results support the theoretical predictions showing that adjusting the mirror position allows precise control over the reflected Gaussian beam's waist location, intensity distribution, and propagation behavior. This demonstrates how a parabolic reflector can be used to tailor beam focusing properties in optical systems.

7.Power Calculations

∵ We are dealing with lossless media, so no power loss is expected.

Power could be calculated through this relation:

$$I = \frac{1}{2} I_o (\pi w_o^2) = \frac{1}{2} \times 1 \times \pi \times (40 \times 10^{-3})^2 = 2.5\, mwatt$$

Theoretical results are confirmed using matlab with the same power value all over the experiment:

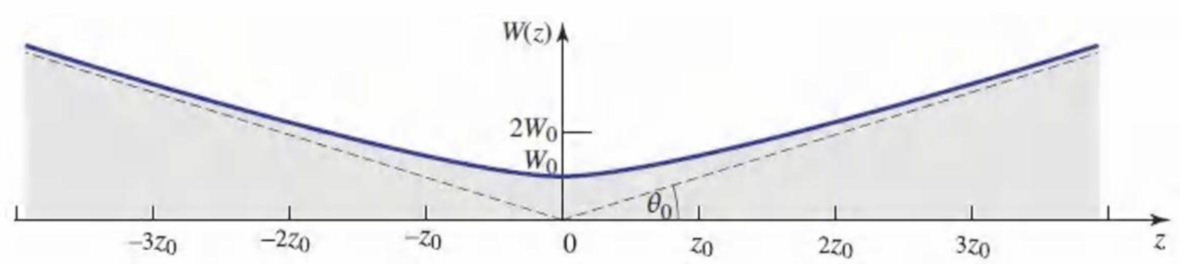
```
Power before mirror:
    0.0025    0.0025    0.0025

Power after mirror:
    0.0025    0.0025    0.0025
    0.0025    0.0025    0.0025
    0.0025    0.0025    0.0025
```

8.Wave width

he width of a Gaussian beam is commonly defined as the radial distance at which the intensity drops to $1/e^2 \approx 0.135$of its peak value. As previously discussed, the beam reaches its minimum width at the waist and then gradually expands as it propagates along the z-axis. This evolution is governed by the well-known relation

$$w(z) = w_0 \sqrt{1 + (\frac{z}{z_0})^2},$$

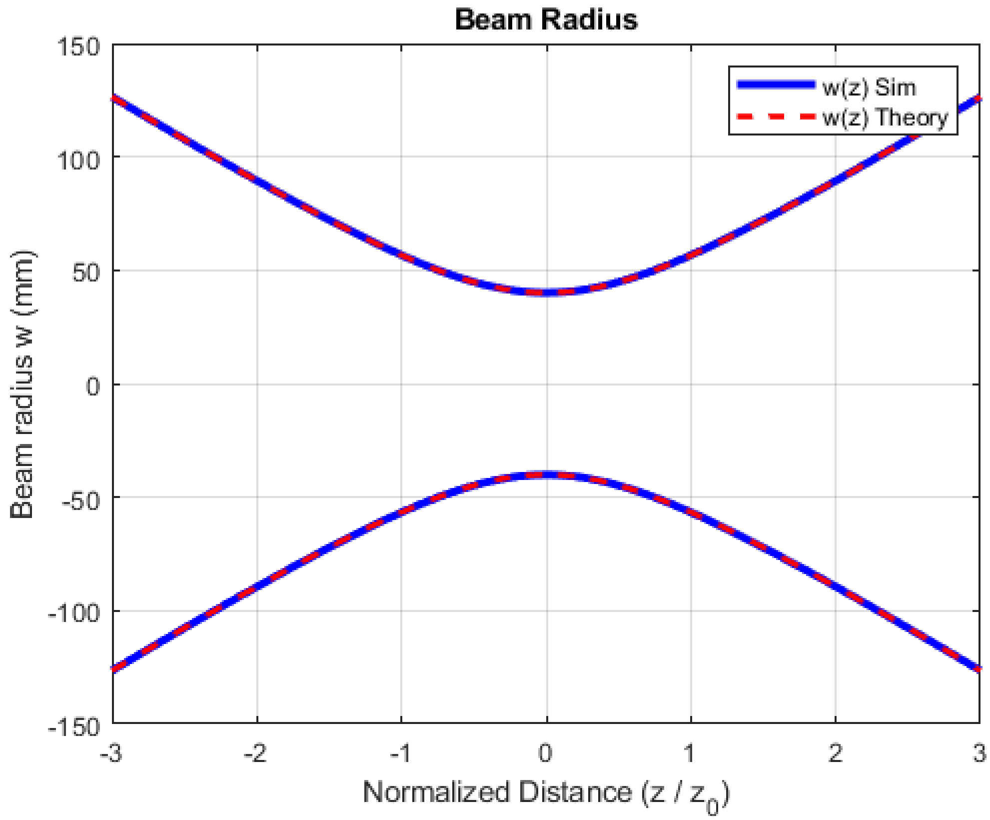
which predicts a monotonic increase in beam radius with distance from the waist.



To verify these theoretical predictions, MATLAB simulations can be performed to visualize the beam width and intensity distribution at different propagation distances, as demonstrated below.

```
z_norm_axis = linspace(-3,3,1001);
w = NaN(size(z_norm_axis));
Uprop_f_base = U0_f;

% Correct indices for center (N/2 + 1)
center_col = N/2 + 1;
center_row = N/2 + 1; % Adjusted to be consistent

for s = 1:length(z_norm_axis)
    z_current = z_norm_axis(s)*z0; % Physical propagation distance
    Uz_f = Uprop_f_base .* prop(z_current);
    Uz   = ifft2(ifftshift(Uz_f));

    int_slice = abs(Uz(center_row, :)).^2;

    I_max = max(int_slice);
    thr   = I_max * exp(-2); % 1/e^2 threshold

    % Find threshold crossing to the right of the peak
    % Note: center_col is now correct (index of x=0)
    right_part = int_slice(center_col:end);
    j = find(right_part <= thr, 1, 'first');

    if ~isempty(j)
        % Indices for interpolation (must be 1-based, global indices)
        i2 = center_col + j - 1;
        i1 = i2 - 1;

        % Linear interpolation to find the exact crossing point
        v1 = int_slice(i1);
        v2 = int_slice(i2);
        x1 = x(i1);
        x2 = x(i2);
        t = (thr - v1)/(v2 - v1);
        x_cross = x1 + t*(x2 - x1);

        % Beam width w(s) is the radius (distance from center x=0)
        w(s) = abs(x_cross);
    end
end

% Theoretical Calculation (w_t)
% Since z_norm_axis is z/z0, the correct formula is used directly.
wt = w0 * sqrt(1 + z_norm_axis.^2);
```

Plotted result:



As shown in the figure, the wave has a minimum width at $z = 0$ of $40\ mm$ then increases along the z axis.

9.Conclusion

In conclusion, this project successfully modeled the propagation and reflection of a Gaussian beam using Fourier-optics techniques. The MATLAB simulations accurately reproduced the theoretical behavior of Gaussian beams, including beam divergence, intensity evolution, and the dependence of beam width on propagation distance. The reflector analysis further demonstrated how a parabolic mirror reshapes the wavefront, creating a new beam waist whose position matched theoretical predictions. The intensity plots at various propagation distances confirmed the expected focusing and divergence patterns. Overall, the results validate both the theoretical framework and the numerical implementation used throughout the study.

10.Project full MATLAB code

Click here for full MATLAB code

11.Reference

Bahaa E. A. Saleh, and Malvin Carl Teich, “Fundamentals of Photonics”, 2nd Edition, John Wiley and sons, Inc., ISBN 978-0-471-35832-9.