Experimental Evaluation of a Full Quaternion Based Attitude Quadrotor Controller

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***Abstract*—The aim of this article is to present a novel quaternion based control scheme for the attitude control problem of a quadrotor and experimentally evaluate its performance. A quaternion is a hyper complex number of rank 4 that can be utilized to avoid the inherent geometrical singularity when representing rigid body dynamics with Euler angles or the complexity of having coupled differential equations with the Direction Cosine Matrix (DCM). In the presented approach the novel contributions consist of: a) the quadrotor's attitude model and b) the proposed non–linear Proportional squared (P^2) control algorithm, which have been proposed and experimentally evaluated fully in the quaternion space, without any transformations nor calculations in the Euler angle nor the DCM spaces. The established control scheme is combined with a quaternion based Madwick Complementary filter for estimating the attitude quadrotor's responses. Multiple experimental results, including the case where external disturbances are acting on the quadrotor, are being presented for proving the efficiency and the robustness of the proposed novel quaternion based controller.**

I. Introduction

The area of Unmanned Aerial Vehicles (UAV) and especially the one of having the capability of Vertical Take–Off and Landing (VTOL) like the quadrotors, has been in the focus of numerous research and development efforts, mainly due to their efficiency in accomplishing complex missions in a large range of applications [1].

For achieving the desired performance most frequently the trajectory generation problem is being divided into two subproblems: a) the attitude problem and b) the translation problem, while as it has been proven [2, 3], these systems can be cascade interconnected. For the examined case of a quadrotor UAV, the position controller (translation) is generating reference attitude set–points for the attitude controller and thus the control problem of quadrotors has been confronted using several different approaches from research–leading teams worldwide, with famous works to include linear [4], and nonlinear controllers [5–7]. Although it has been proved that the aforementioned control strategies manage to stably navigate a quadrotor, the problem of designing optimized controllers that will be able to: a) provide fine–smoothed control actions for attitude stabilization and trajectory tracking, b) make use of model–knowledge for more accurate navigation, and c) preserve robustness against sudden and unpredicted external disturbances, is still an open challenge. One of the constrains that the control engineers are facing, when dealing with the attitude problem is still a fundamental problem in dynamics due to the fact that finite rotation of a rigid body does not obey the laws of vector addition, e.g. commutativity, while the attitude characteristics of the rigid body cannot be extracted by integrating the body's angular velocity.

However, when working with rotations, whether it's estimators or controllers, there has been one approach utilized more than any other, when creating models: the Newton–Euler equations [8], which is able to describe the combined translational and rotational dynamics of the rigid body. Although this modeling approach is considered a fundamental one, still it has three drawbacks. Firstly, it is solely based on Euler angles, which have the merit of being intuitive, but per definition these angles cannot define certain orientations as they suffer from singularities that result in a problem know as "gimbal lock" [9], which is the problem of losing one degree of freedom in a three-dimensional space in the case where two of the rotational axes align and locking together. Secondly, it is very computationally expensive. Calculating sines and cosines takes a lot of performance and can very fast become unmanageable especially if it's being implemented on low cost hardware. Thirdly, when creating estimators or controllers that need to use the Jacobian of the system states, the computational cost is even greater, as during these calculations some times all the matrix elements will have one or more sines or cosines to compute, which quickly can overwhelm the system. For overcoming these problems, three solutions can be followed: a) guarantee that the system will keep inside the bounds of Euler angels, b) the utilization of a Direction Cosine Matrix (DCM) approach, and c) the quaternion approach.

In the quaternion approach the previous mentioned limitations do not exist as one can directly translate a quaternion into a DCM and vice versa, however the quaternion and its corresponding derivative have four values and the only constraint is that it must be of unit length. This translates into a system of only four coupled differential equations (states), greatly decreasing the computational cost and keeping the overall complexity low [10]. Due to the fact that the quaternion is a complex number it's sometimes hard to get an intuitive feeling for what it represents, but the direct coupling to a DCM plus the direct relation to a rotation vector makes the translation easy and intuitive.

The novelty of this article stems from: a) experimentally evaluating of a full quaternion based non–linear P^2 controller for solving the attitude problem of a quadrotor from [11], b) designing and implementing a quaternion based controller on the quaternion based developed KFly autopilot, and c) experimentally evaluating the performance of the suggested control scheme under external disturbances.

The rest of the article is structured as it follows. In Section II the novel full quaternion based control scheme is

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established and in Section III experimental results that prove the efficiency of the proposed scheme are depicted. Finally, in Section IV conclusions are drawn.

II. Controller Synthesis

In this section a feedback control scheme for the attitude stabilization of the quadrotor aircraft will be implemented based on the quaternion mathematics and theoretcal results from [11].

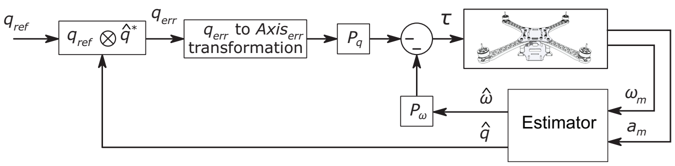

Fig. 1. Block diagram of the full nonlinear P^2 quaternion based control scheme combined with the Madgwick complementary filter

In the proposed approach from [11], an inner loop consisting of an proportional controller P_ω for the angular velocity and an outer loop proportional controller P_q for the angular velocity reference tracking, have been effectively combined for creating a non-linear P^2-controller for the attitude regulation problem. The overall mathematical formulation of the proposed P^2 control scheme is being denoted as it follows:

$$\tau = -P_q \cdot \begin{bmatrix} q_1^{err} \\ q_2^{err} \\ q_3^{err} \end{bmatrix} - P_\omega \cdot \begin{bmatrix} \hat{\omega}_x \\ \hat{\omega}_y \\ \hat{\omega}_z \end{bmatrix} \tag{1}$$

where $\hat{\omega} = [\hat{\omega}_x,\ \hat{\omega}_y,\ \hat{\omega}_z]^T$ are the estimated angular velocity. It should be noted that equation (1) is derivation free (using $\hat{\omega}$ as dampening) and can be straight forward to be implemented with very low computational cost. Moreover, the noise immunity of this design is only as good as the estimated quaternion and angular velocity, since the suggested scheme will directly amplify noise just as much as the corresponding errors.

III. System Configuration and Experimental Results

A. System Configuration

The performance of the proposed scheme has been evaluated by utilizing the quadrotor platform presented in Figure 2. The main control unit utilized in this system is named the KFly platform, which has been designed and developed at Luleå University of Technology in Sweden for satisfying the need of a system being capable of: a) performing estimation on a large number of states with advanced estimation algorithms, b) executing high performance control algorithms, and c) having a low cost and small size. The Micro Controller Unit (MCU) driving the system is ST's STM32F405RG, an 32-bit ARM Cortex-M4F CPU featuring a Digital Signal Processing (DSP) core and a Floating Point Unit (FPU) – making it very efficient performing both fixed point and floating point calculations.

KFly utilizes three main sensor chips on board which are mainly used for estimation and control. The first one is the Invensense's MPU-6050 Accelerometer and Gyroscope System On a Chip (SOC) for measuring acceleration and angular velocity, with a full scale range of ± 16 g and ± 2000 degrees/s. It has a sampling rate at 200 Hz and running each sample trough an internal low-pass filter with an 44 Hz cut-off frequency. Both the accelerometer and the gyroscope has resonance frequencies well outside (above 30 kHz) the vibrations created by the air frame. The second one is the Honeywell's HMC5883L Magnetometer for detecting North/South/East/West with 5 milli-Gauss resolution. Sampling is done at 75 Hz. The third sensor is a MEAS Switzerland's MS5611-01BA03 Barometric Pressure Sensor used as an altimeter with an altitude resolution of 10 cm at a sampling of 10 Hz. Finally, the system features the FreeRTOS[1] real time kernel for ease of use and fast application development. FreeRTOS handles the communication with the hardware, the mutual exclusions (mutex) for the communication buses, Interrupt Service Routine (ISR) handler synchronization and data processing synchronization guaranteeing the real time requirements.

Fig. 2. The utilized quadrotor experimental platform with the KFly platform on the top

B. Experimental Results

The proposed quaternion based controller have been experimentally evaluated under two fundamental tracking test cases, which were the quaternion based q_1 (pitch) tracking of variable time and amplitude step set–points, with and without the effect of external disturbances. The symmetrical design of the quadrotor, allows the direct generalization of the presented results to roll and yaw. Due to the fundamental properties of the quadrotors, it is expected that yaw will exhibit a slower response. Having this under consideration a single free rotation axis setup has been utilized to experimentally evaluate the performance of the proposed scheme.

The gains of the nonlinear P^2 controller have been set as: $P_q = 5$ and $P_\omega = 0.1$ based on ad-hoc tuning, while standard gains for the Madgwick Complementary filter have been utilized from [12].

Initially, the case of no external disturbances have been considered, with the optimum goal of tracking a quaternion based variable step reference. The experimental results obtained for the quadrotor's pitch response, combined with the applied P^2 control effort (differential throttle) are depicted in Figure 3. In Figures 4 and 5 the noisy raw measurements generated from the accelerometers and the filtered angular

[1] http://www.freertos.org/

velocities are presented respectively. In each iteration of the algorithm, these measurements are been utilized to produce the quaternion estimation, presented in Figure 3. This estimation is being further utilized for calculating the $\boldsymbol{q}_{err}$ and calculating the control action.

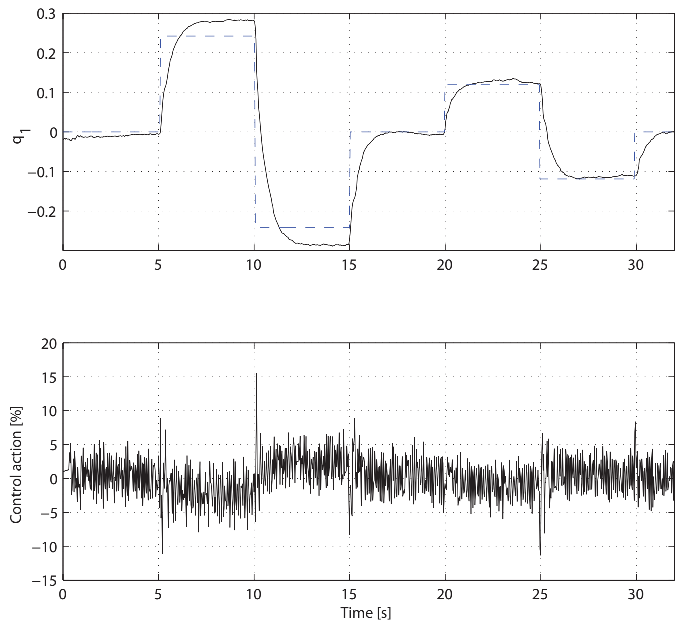

Fig. 3. Experimental variable step reference tracking response for q_1(pitch) and corresponding differential control effort

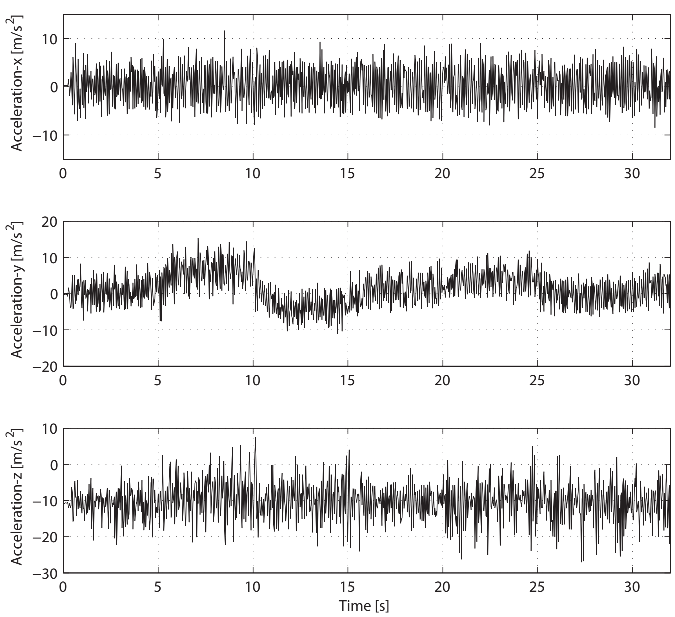

Fig. 4. Measured noisy raw accelerations

The second set of experimental results, considers the case where sudden external disturbances are acting on the vehicle. These disturbances simulate the case of sudden and strong wind gusts acting on the quadrotor and have been generated by exciting the frame with a small vibration hammer. As before, in this case the goal has been to track the same quaternion based variable step reference. The experimental results obtained for the quadrotor's pitch response, combined with the applied P^2 control effort, for the case of external disturbances, are depicted in Figure 6. In Figures 7 and 8 the noisy raw measurements generated from the accelerometers and the filtered angular velocities are presented respectively. As before, these measurements have been utilized to produce the quaternion estimation, presented in Figure 6.

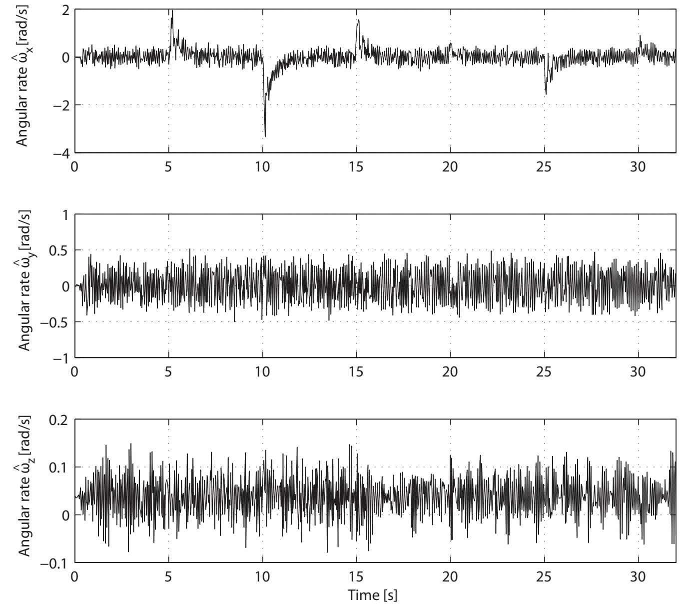

Fig. 5. Filtered angular velocities

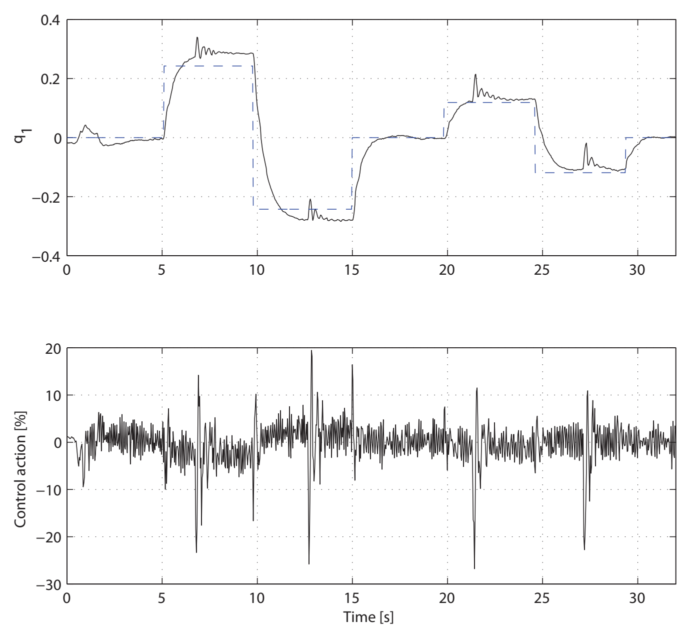

Fig. 6. Experimental variable step reference tracking response for q_1 (pitch) under externally acting disturbances and corresponding differential control effort

From all the experimental cases examined, it has been shown that the proposed scheme is able to provide a fast and accurate tracking of a variable step reference. Even in the case

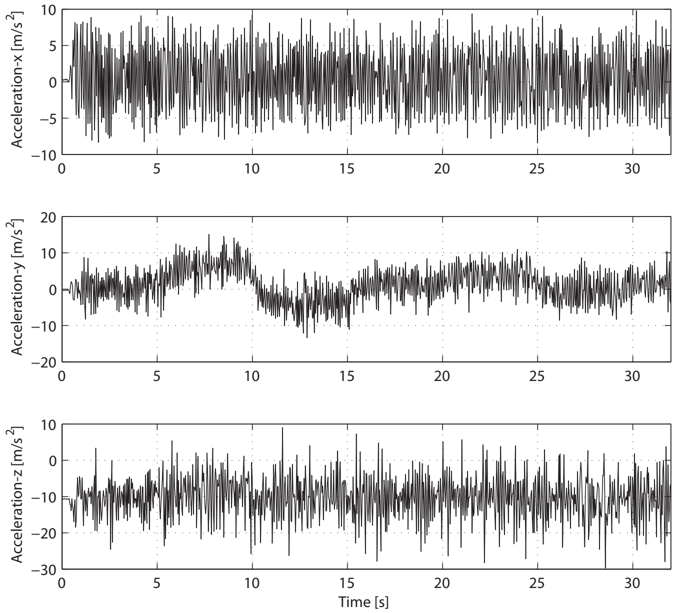

Fig. 7. Measured noisy raw accelerations under the effect of external disturbances

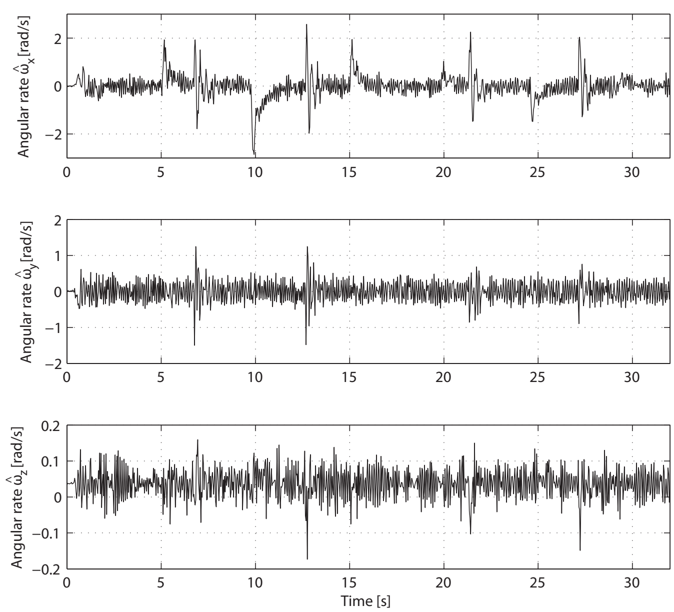

Fig. 8. Filtered angular velocities under the effect of external disturbances

where strong external disturbances are acting on the quadrotor, still the proposed controller manages to attenuate them in a short time window. In the presented experimental results there is a small constant offset error, which is evident in the case of large reference angles. This effect is due to the fact that the utilized frame has not been fully balanced and the center of gravity does not coincide with the center of rotation. This issue can be further addressed by the insertion of an additional integral term in the rate controller, which will drive the error to zero by using the inherit dynamics of the quadrotor. Moreover, it should be highlighted that due to the existing vibrations in the quadrotor's frame, highly corrupted measurements have been generated and some of it is transfered to the control signal via the angular rate gain. However, in all the cases the Madgwick complementary filter managed to produce a smooth estimation of the quaternion and only $\pm$ 5 % control signal noise.

IV. Conclusions

In this article a novel quaternion based control scheme for the attitude control problem of a quadrotor has been presented and experimentally evaluated. In the presented approach the complete design of the quadrotor's model, the Madgwick complementary filter estimator, and the P^2 controller have been performed in the quaternion space, without any transformations nor calculations in the Euler angle nor DCM spaces. Multiple experimental results, including the case where strong external disturbances were acting on the system, have been presented that proved the overall efficiency and the robustness of the proposed quaternion based controller.

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