Names: Omar Abdelmotaleb and Benjamin Singleton
Pledge: I pledge my honor that I have abided by the Stevens Honor System. -Omar Abdelmotaleb
Pledge: I pledge my honor that I have abided by the Stevens Honor System. -Benjamin Singleton

Problem 1. (10 points) Use the pumping lemma to show that the following languages are not regular.

a) $\{0^i 1^j : i < j\}$
b) $\{0^i 1^j : i > j\}$

1. a) In the case of a string such as 00111, x could be set to equal epsilon, y equals 00, and z equals 111. The length of y is 2 which satisfies |y|>0 and similarly, |xy| would be 2 which when choosing an arbitrary pumping length of 2, would also satisfy |xy|<=p. However, pumping the string would produce a string not within the language. For instance, pumping xyz once would create the string 0000111, where i > j which contradicts i < j and similarly pumping twice would produce 000000111 where the inequality is only heightened. As such, since the string is not within the language, the language is not regular.

b) In the case of a string such as 00011, x could be set to equal 000, y equals 11, and z equals epsilon. The length of y equals 2, which satisfies |y|>0 and similarly, |xy| would be 5 which when choosing an arbitrary pumping length of 7, would also satisfy |xy| <=p. However, pumping the string would be a string not within the language. For instance, pumping xyz once would create the string 0001111, where i < j which contradicts i > j and similarly pumping twice would produce 000111111 where the inequality is only heightened. As such, since the string is not within the language, the language is not regular.

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Problem 2. (10 points) Prove that the language $B = \{0^i 1^j : i \neq j\}$ is not regular. Do not use the pumping lemma. Instead, express B as the result of regular operations between the non-regular language $\{0^i 1^i : i \geq 0\}$ and a regular language.

2.
$A = \{1^i 1^j : i \geq 0, i \neq j\}$
$B = \{0^i 1^j : i \neq j\}$
$C = \{0^i 1^i : i \geq 0\}$

Given C that is irregular and B that is irregular,
$A^R - C^I = B$
$\{1^i 1^j : i \geq 0, i \neq j\} - \{0^i 1^i : i \geq 0\} = \{0^i 1^j : i \neq j\}$
Since A is regular and C is irregular than B must be irregular

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Problem 3. (10 points) The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or greater. If p is a pumping length for regular language A, then so is any length $p' \geq p$. The ***minimum pumping length*** for A is the smallest p that is a pumping length of A.

For example, the pumping length of 01^* cannot be 1 because the string $s = 0$ of length 1 cannot be pumped to give another string in the language. But any string of length 2 or more can be pumped by choosing $x = 0, y = 1, and\ z$ to be the rest of the string.

What is the minimum pumping length for each of the following languages? Justify your answer in each case.

1. 0001^*
2. 0^*1^*
3. $0^*1^*0^*1^* \cup 10^*1$
4. $(01)^*$
5. $1^*\,01^*\,01^*$

3.

1. The minimum pumping length of the language is 4. x would equal 000, y would equal 1, and z would be the rest of the string which could be any amount of 1s. If the pumping length was any smaller the 3 necessary 0s or the 1 would not be able to be in |xy|.

2. The minimum pumping length of the language is 1. x can be set to ε, y can be set to either 0 or 1, and z can be set whatever the rest of the string is chosen to consist of.

3. The minimum pumping length is 2. Each symbol on the left side of the union is starred, meaning that the minimum pumping length of that side is 1 as x could equal ε, y could equal 0 or 1, and z would equal the rest of the string. On the right side, the minimum would be 2, with x equal to 1, y equal to 0, and z equal to the rest of the string. A string with the length of the left side's minimum pumping length would adhere to |xy| <= p when p is the right side's minimum pumping length, but not vice versa. As such, the language's overall minimum pumping length is 2 so that all strings are accepted.

4. The minimum pumping length is 2. x would equal x, y would equal 01, and z would be the rest of the string. Because 0 and 1 are both within the star, the language would consist of iterations of 01. If the pumping length was less than two, then the pair 01 wouldn't fit.

5. The minimum pumping length would be 3. x would be equal to 10, y would equal 1, and z would be 0 followed by any amount of 1s. The pumping length could not be any shorter because the string needs to contain exactly two 0s. The first is within x and the second is within y and to maintain the language a 1^* must be pumped. The established language adheres to these rules.

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Problem 4. (20 points)

a) (7 points) Show that the language
$$L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = 1 \Rightarrow j = k\}$$
satisfies the three conditions of the pumping lemma. Hint: set the pumping threshold to 2 and argue that every string in L can be divided into three parts to satisfy the conditions of the pumping lemma.

b) (8 points) Prove that L is not regular. Note that $L = b^*c^* \cup aaa^*b^*c^* \cup \{ab^ic^i : i \geq 0\}$, and use the fact that regular languages are closed under complement and difference.

c) (5 points) Explain why parts (c) and (d) do not contradict the pumping lemma.

4.a)

Assuming that p=2, in the case of a^i when i = 0, the first part of the pumping lemma can be satisfied by setting x equal to epsilon, y equal to either b or c, and z equal to the rest of the string. Pumping this string would always produce a string within the language as you'd be adding either additional b's or c's, which don't have to be equal in this case.
For the second part, |y| > 0, |y| would equal 1 and so it satisfies this condition.
And in the third part, |xy|<=p, |x| would be 0 as it's epsilon and as previously stated |y| = 1, so |xy| = 1 and p=2, so |xy|<=p.

In the case of a^i when i = 1, the first part of the pumping lemma can be satisfied by setting x equal to epsilon, y equal to a, and z equal to the rest of the string where the number of a's and the number of b's equal one another, i.e. j=k. Pumping this string would always produce as a string within the language as you'd be adding additional a's. This is the only option as if you attempted to pump a "b" or "c" the string would not have an equal number of b's and c's while there was a single a.
For the second part, |y| > 0, y is set to just a so |y| equals 1, which satisfies the condition.
And for the third part, |xy| <= p, |x| would be 0 as it's epsilon and as previously stated |y| = 1, so |xy| =1 and p=2, so |xy| <= p.

And lastly for the third possible case of a^i when i > 1, for the first part x could equal epsilon, y would equal either aa, b, or c, and z would equal the rest of the string. Because i does not equal 1, j and k do not have to be equal and a can be any value greater than 1 as well. As such, pumping any single letter still produces a string within the language.
For the second part, |y| > 0, y equals either aa, b, or c, the lengths of which are all greater than 0 so this part is satisfied.
And for the third part, |xy| <= p, |x| would be 0 as it's epsilon while y would either equal 2 or 1 depending on whether aa was chosen or b or c was chosen. In both cases, |xy| <= 2, so the condition is satisfied.

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b) The third part of L, $\{ab^i c^i : i \geq 0\}$, defines an infinite language that does not have an fsa. Because fsa's don't have any memory, there's no way to keep track of how many b's have occurred to see if there's a matching number of c's. Since no fsa can be created for the third part it is not regular, and because regular languages are closed under union, the whole language is not regular.

c) The pumping lemma can be used to test if a language is not regular; however, if a language does meet all of its conditions the language is not necessarily irregular. If a language meets all conditions of the lemma, then further proofs are required to deem it irregular. As such, the language L can satisfy the pumping lemma as displayed in part a but not be regular as explained in part b.