Names: Omar Abdelmotaleb and Benjamin Singleton
Pledge: I pledge my honor that I have abided by the Stevens Honor System. -Omar Abdelmotaleb
Pledge: I pledge my honor that I have abided by the Stevens Honor System. -Benjamin Singleton

Problem 1. (5 points) If G is a CFG in Chomsky Normal Form, show that a string of terminal symbols of length $n \geq 1$ is generated by the application of exactly $2n - 1$ rules of G.

DEFINITION **2.8**

A context-free grammar is in ***Chomsky normal form*** if every rule is of the form

$$A \rightarrow BC$$
$$A \rightarrow a$$

where a is any terminal and A, B, and C are any variables—except that B and C may not be the start variable. In addition, we permit the rule $S \rightarrow \varepsilon$, where S is the start variable.

From the start state, it will take n-1 applications of the first rule of CNF, A → BC, to create a string of variables and terminals. It will then take n applications of the second rule, A → a, to switch each remaining variable to a terminal symbol. In total that will be (n-1)+n or 2n-1 applications of CNF rules.

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Problem 2. (10 points) Let $G = (V, \Sigma, R, S)$ be a CFG where $V = \{S, T, U\}$, $\Sigma = \{0, \#\}$, and R is the set of rules:

$$
\begin{aligned}
S &\to TT \mid U \\
T &\to 0T \mid T0 \mid \# \\
U &\to 0U00 \mid \#
\end{aligned}
$$

Describe the language $L(G)$ in English and prove that it is not regular.

The language L(G) can be described by
$\{0^a\#0^b0^c\#0^d \;:\; a,b,c,d \geq 0\} \cup \{0^i\#0^{2i} : i \geq 0\}$

The second part of L(G), $\{0^i\#0^{2i} : i \geq 0\}$, can be represented by the string s=$0^p\#0^{2p}$ where p is the pumping length. By the pumping lemma, |xy| <= p and |y| >0 so therefore, y=0^k. By pumping y=0^k we get $0^{p+1}\#0^{2p}$ which isn't in the language because p is not two times larger than p+1. Therefore, it contradicts the pumping lemma and as such $\{0^i\#0^{2i} : i \geq 0\}$ is not regular. Because $\{0^i\#0^{2i} : i \geq 0\}$ is non-regular, the union of $\{0^i\#0^{2i} : i \geq 0\}$ and $\{0^a\#0^b0^c\#0^d \;:\; a,b,c,d \geq 0\}$ will result in a non-regular language, making L(G) which is the union of these two languages non-regular.

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Problem 3. (10 points) Give a CFG for $\{a^i b^i c^k d^k : \ i, k \geq 0\} \cup \{a^i b^k c^k d^i : 1, k \geq 0\}$. Is your grammar ambiguous?

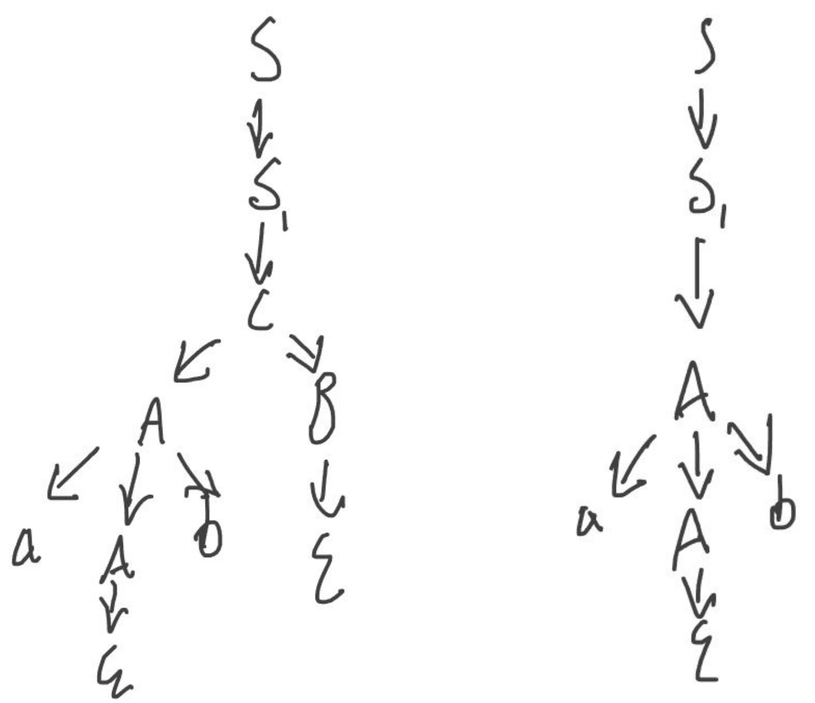
$L1 = \{a^i b^i c^k d^k : \ i, k \geq 0\}$
$L2 = \{a^i b^k c^k d^i : i, k \geq 0\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow A \mid B \mid C \mid \varepsilon$
$A \rightarrow aAb \mid \varepsilon$
$B \rightarrow cBd \mid \varepsilon$
$C \rightarrow AB \mid \varepsilon$

$S_2 \rightarrow aS_2d \mid D \mid \varepsilon$
$D \rightarrow bDc \mid \varepsilon$

The grammar is ambiguous. For instance, the string “ab” can be generated by both of the following parse trees.



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Problem 4. (15 points) Let $L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$ and $L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$. For each language, either give a CFG for it, or prove that it is not a CFL.

L_{add} CFG:

S → AB
A → aAb | ε
B → bBc | ε

L_{mult} is not a CFL. To prove so we use the pumping lemma for CFL. With the string $s = a^p b^{pp} c^p$, by the pumping lemma |vxy|<=p and |vy|>0 so therefore we have the following cases:

1) v=a^k, x=b^{k*k}, and y=c^k where 1<=k<=p. By pumping the string we get $a^{k+p}b^{k*k}c^{k+p}$. (k+p) x (k+p) does not equal k^2 and so this string is not in the language
2) v=a^k, x=**ε**, and y=b^{k*k} where 1<=k<=p. By pumping the string we get $a^{k+p}b^{k*k+p}c^k$. (k+p) x k does not equal (k^2+p) and so this string is not in the language.
3) v=b^{k*k}, x=**ε**, and y=c^k where 1<=k<=p. By pumping the string we get $a^k b^{k*k+p} c^{k+p}$. k x (k+p) does not equal (k^2+p) and so this string is not in the language.

Since, in all cases, the pumped string is not in the language, it follows that the L_{mult} is not a CFL.