HW2

Sunday, November 6, 2022 5:16 AM

Omar Abdelmotaleb

I pledge my honor that I have abided by the Stevens Honor System. Omar A.

CS559-B HW2

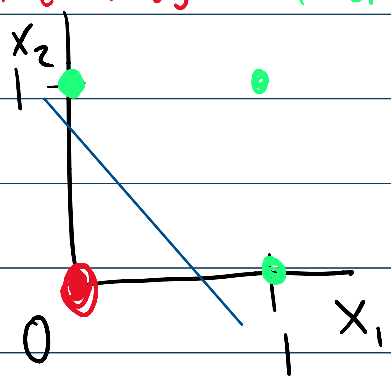
Due: Nov. 2nd, 2022

Problem 1 (30pt): [Perceptron Algorithm]

In this problem, we learn the linear discriminant function for boolean OR function. Suppose we have two dimensional $x = (x_1, x_2)$, x_1 and x_2 can be either 0 (false) or 1 (true). The boolean OR function is defined as: $f(x_1, x_2) = x_1$ **OR** x_2. Specifically, $f(0,0) =$ **false**, $f(1,0) =$ **true**, $f(0,1) =$ **true**, and $f(1,1) =$ **true** where **true** can be treated as *positive class* and **false** can be treated as *negative class*. You can think of this function as having 4 points on the 2D plane (x_1 being the horizontal axis and x_2 being the vertical axis): $P_1 = (0,1), P_2 = (1,1), P_3 = (1,0), P_4 = (0,0)$, P_1, P_2, P_3 in *positive class* and P_4 in *negative class*.

(1) [5pt] For boolean OR function, is the negative class and positive class linearly separable?

Negative class Positive class



The negative and positive class is linearly separable.

(2) [25pt] Is it possible to apply the **perceptron algorithm** to obtain the linear decision boundary that correctly classify both the positive and negative classes? If so, write down the updation steps and the obtained linear decision boundary. (You may assume the initial decision boundary is $x_2 = \frac{1}{2}$, and sweep the 4 points in clockwise order, i.e., $(P_1, P_2, P_3, P_4, P_1, P_2, \ldots)$, note that you **can not** write down the arbitrary linear boundary without updation steps.)

It is possible to obtain.

Initial weights are 0, $w_0 = \frac{1}{2}$

Predict P1, P2, P3 to be positive
P4 to be negative

Positive if >0, negative if <0

P1: $(\underline{0}, \underline{1})$

$$y = 0 \cdot \underline{0} + 0 \cdot \underline{1} - \frac{1}{2} = 0 + 0 - \frac{1}{2} = -\frac{1}{2} < 0$$

negative

Belongs to positive class, incorrect prediction

Update weights:

$$w_0 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$w = 0 + 1 = 1$$

Using new weights

$$y = 1 \cdot 0 + 1 \cdot 1 + \frac{1}{2} = 1.5 > 0$$

$y = 1 \cdot \underline{0} + 1 \cdot 1 + \frac{1}{2} = 1.5 > 0$
positive

Correct prediction, no update needed

P2: $(\underline{1}, \underline{1})$

$y = 1 \cdot \underline{1} + 1 \cdot \underline{1} + \frac{1}{2} = 2.5 > 0$
positive

Belongs to positive class, correct prediction
no update needed

P3: $(\underline{1}, \underline{0})$

$y = 1 \cdot 1 + 1 \cdot 0 + \frac{1}{2} = 1.5 > 0$
positive

Belongs to positive class, correct prediction

no update needed

P4: $(\underline{0}, \underline{0})$

$$y = 1 \cdot \underline{0} + 1 \cdot \underline{0} + \frac{1}{2} = 0.5 > 0$$

positive

Belongs to negative class, incorrect prediction

Update weights

$$w_0 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$w = 1 - 0 = 1$$

Using new weights

$$y = 1 \cdot \underline{0} + 1 \cdot \underline{0} - \frac{1}{2} = -\frac{1}{2} < 0$$

negative

correct prediction, no update needed

Final weights: $w = 1$, $w_0 = -\frac{1}{2}$

Testing with the inputs using final weights:

Testing with the inputs using final weights.

P1: $(0, 1)$ Positive

$$y = 1\cdot 0 + 1\cdot 1 - \frac{1}{2} = \frac{1}{2} > 0$$ ✓

P2: $(1, 1)$ Positive

$$y = 1\cdot 1 + 1\cdot 1 - \frac{1}{2} = 1.5 > 0$$ ✓

P3: $(1, 0)$ Positive

$$y = 1\cdot 1 + 1\cdot 0 - \frac{1}{2} = \frac{1}{2} > 0$$ ✓

P4: $(0, 0)$ Negative

$$y = 1\cdot 0 + 1\cdot 0 - \frac{1}{2} = -\frac{1}{2} < 0$$ ✓

Decision boundary:

$$y = w \cdot x_1 + w \cdot x_2 + w_0$$

$$y = x_1 + x_2 - \frac{1}{2}$$

Learning rule:

$$x_1 + x_2 = \frac{1}{2}$$