

W1 - introduction

① What is CV

- ❖ Automatic understanding of images and video.

it's not:

- ① Image Processing
- ② Pattern Recognition
- ③ Computer Graphics

CV emulate human vision

- Process images and extract information using context
- Make decisions using this information

② Human Visual System

❖ But still is not perfect:

- ❖ Suffers from illusions
- ❖ Ignores many details
- ❖ Ambiguous description of the world



③ CV challenges

① Viewpoint variation

② Background clutter

③ Illumination

④ Occlusion



⑤ Deformation

⑥ Intra-class variation

⑦ Brief history (started in 1966)

⑤ CV tasks

① ♦ Reorganization

- ❖ Segment (cluster) pixels with similar properties

② ♦ Reconstruction *complete the image*

- ❖ Recover 3D information from data

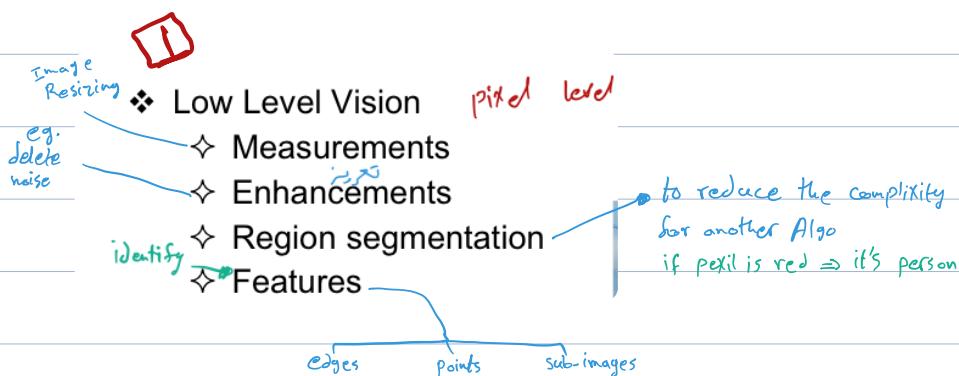
③ ♦ Recognition

- ❖ Detect and identify objects

④ ♦ Understanding

- ❖ What is happening in the scene?

Levels of CV tasks:-



② Mid Level Vision

- ❖ Reconstruction *recognize the objects*
- ❖ Depth
- ❖ Motion Estimation

③ High Level Vision

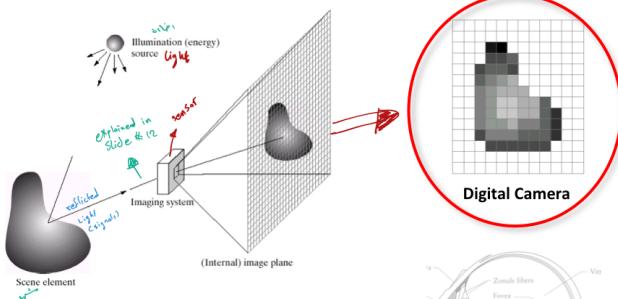
- ❖ Category detection
- ❖ Activity recognition
- ❖ Deep understandings
- ❖ Pose estimation

*car moving
person ...
horse ...*

⑥ CV application

W2 - Image Representation (IR)

Image Acquisition كتاب



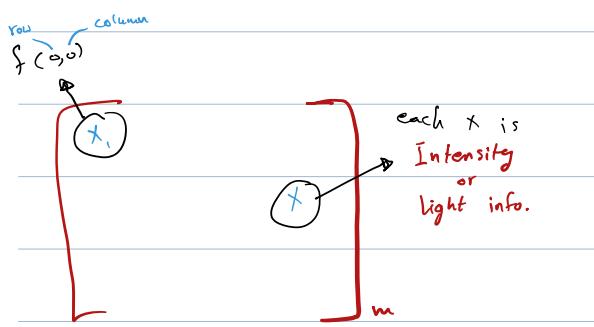
Images are typically generated by illuminating a scene and absorbing the energy reflected by the objects in that scene



- ❖ Image can be represented in two domains:
 - ① Spatial domain
 - ② Frequency domain

① IR in the Spatial Domain

- ❖ A **(digital) image** is a 2D array of intensity values, $f(x, y)$, which represents 2D intensity function discretized both in spatial coordinates (**spatial sampling**) and brightness (**quantization**) values.



Converting Signals into digital form needs two steps:

- ① **Sampling:** on X axis
How many pixels in the image and their locations
Measuring at uniformly spaced locations
- ② **Quantization:** on Y axis
How many unique colors in the image

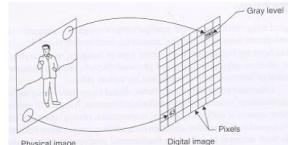
- ❖ The transition between continuous values of the image function and its digital equivalent is called quantization.



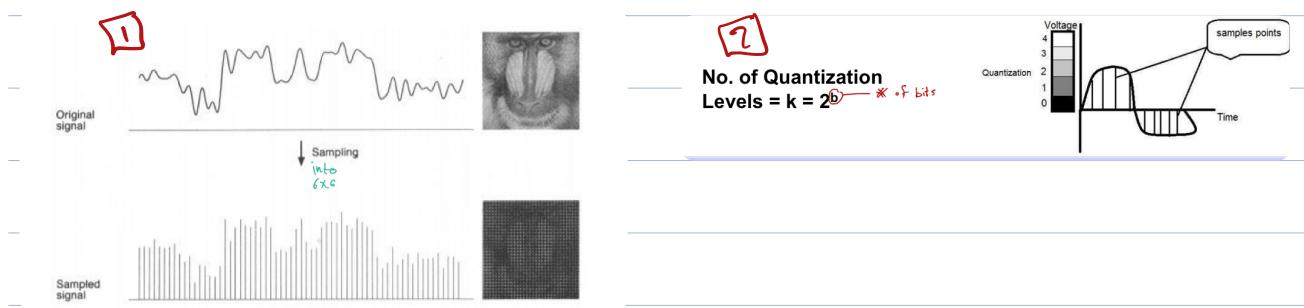
$$X = \# \text{ of samples} = \# \text{ of pixels}$$

$$Y = \text{the value of each sample (pixels)}$$

$$\# \text{ of } X \leq Y_s$$



Slide #12



* Spatial Resolution

Depends on number of pixels of the image and bit depth

more pixels \Rightarrow more details

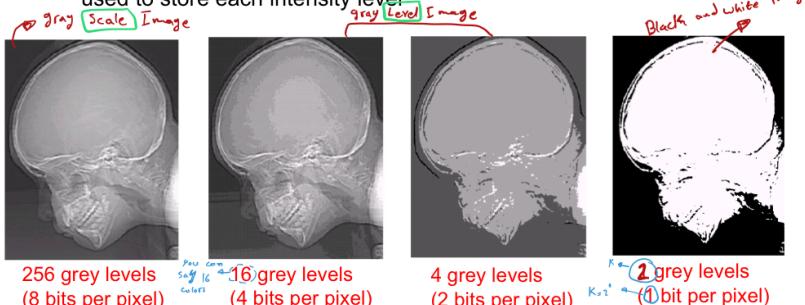
Total bits necessary to represent an image =
No. of rows x No. of cols x Bit depth

~~No. of pixels~~

space memory to
store pixel color
||
size of pixel to
store color value

* Intensity level Resolution

Intensity level resolution is usually given in terms of the number of bits used to store each intensity level



Based on the intensity levels and number of channels,

we can categorize images into:

1 bit/pixel ✓ Binary image (0, 1)

8 bits/pixel ✓ Gray-scale (or gray-tone) image (0, 255)

24 bits/pixel ✓ Vector-Valued Images Color Image

② Image values and Basic Statistics

1

- ❖ Mean (i.e., the "average gray level") of image I :

$$\mu_I = \frac{1}{N_{cols} \cdot N_{rows}} \sum_{x=1}^{N_{cols}} \sum_{y=1}^{N_{rows}} I(x, y)$$

❖ Mean is a measure of image Brightness

2

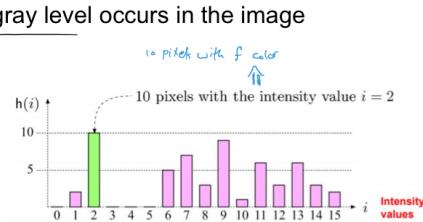
- ❖ Variance of image I :

$$\sigma_I^2 = \frac{1}{(N_{cols} \cdot N_{rows} - 1)} \sum_{x=1}^{N_{cols}} \sum_{y=1}^{N_{rows}} [I(x, y) - \mu_I]^2$$

3

- ❖ Histogram captures the distribution of gray levels in the image.

- ❖ How frequently each gray level occurs in the image



why we use histograms??

- ❖ Histograms help detect image acquisition issues
- ❖ Problems with image can be identified on histogram
 - ❖ Over and under exposure
 - ❖ Contrast
 - ❖ Dynamic Range
 - ❖ Image defects
 - ❖ Effect of image compression

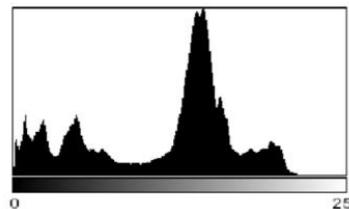
Detecting Bad Exposure using Histograms

→ in the color values

- ❖ Are intensity values spread (good) out or bunched up (bad)

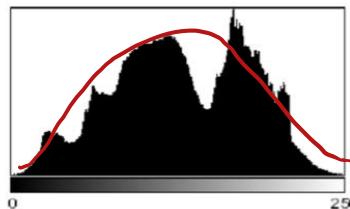


Image



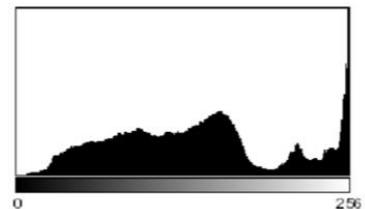
(a)

Underexposed



(b)

Properly Exposed
Balanced
nice bell curve



(c)

Overexposed

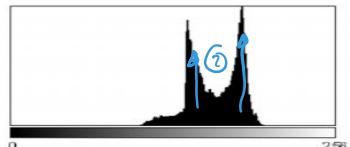
Histogram

Histograms and Contrast

- ❖ The **contrast** of a grayscale image indicates how easily objects in the image can be distinguished
 - ❖ Good Contrast: Widely spread intensity values + large difference between min and max intensity values

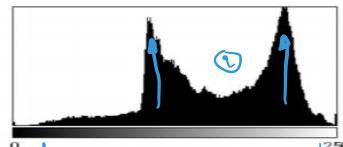


Image



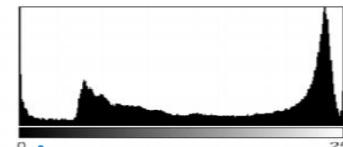
(a)

Low contrast



(b)

Normal contrast



(c)

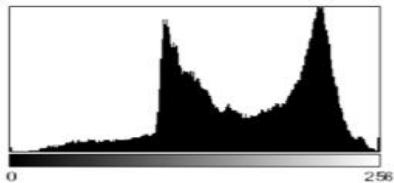
High contrast

Histogram

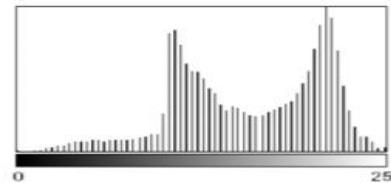
Histograms and Dynamic Range

How many unique color in image

- ❖ **Dynamic Range:** Number of distinct pixels in image

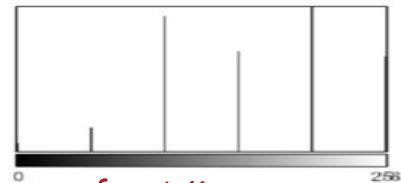


(a)



(b)

High Dynamic Range
**Low Dynamic Range
(64 intensities)**



6 colors
(c)

**Extremely low
Dynamic Range
(6 intensity values)**

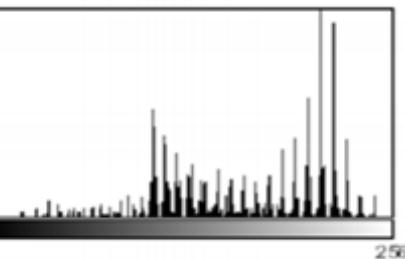
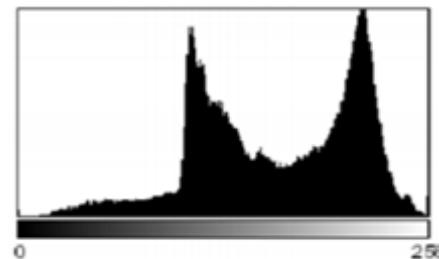
Image Defects: Effect of Image Compression

- ❖ Example: in GIF compression, dynamic range is reduced to only few intensities (quantization)

Original Image



Original Histogram



Histogram after GIF
after compression

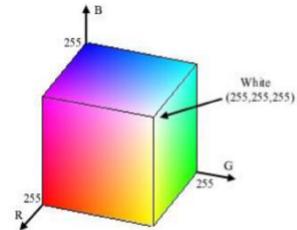
③ Color and Color Images

popular models:-

① RGB

◇ Can have $256(R) \times 256(G) \times 256(B)$ different colors.

→ Intensity of a pixel in a RGB space is given by the mean $M = (R+G+B)/3$

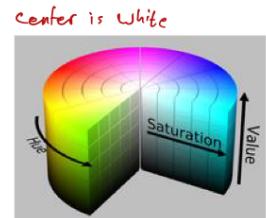


② HSV / HSI

Hue: Which pure color?

مُسْتَوْجَع
Saturation: How white the color is?

Value / Intensity: How Dark the color is? (Brightness)



Hue:

Red = $0^\circ \rightarrow 60^\circ$
 Yellow = $61^\circ \rightarrow 120^\circ$
 Green
 Cyan
 Blue
 Magenta

Saturation:

range [0, 1]

0 = white

1 = primary color

Brightness:

range [0, 100]

0 = Black

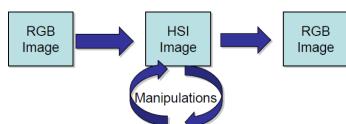
100 = reveals the most color

→ RGB is great for color generation, but HSV is great for color description

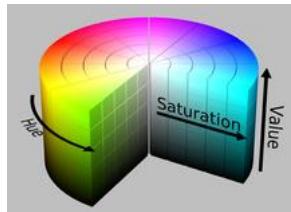
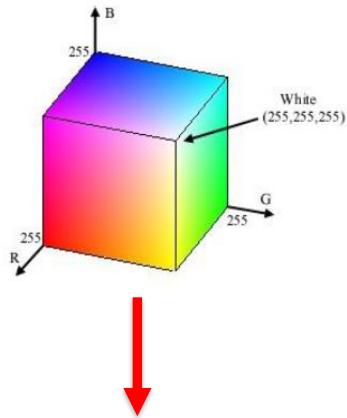
So we use HSV in manipulating

❖ In order to manipulate an image under the HSI model we:

- ◊ First convert it from RGB to HSI
- ◊ Perform our manipulations under HSI
- ◊ Finally convert the image back from HSI to RGB



Converting From RGB To HSV/HSI



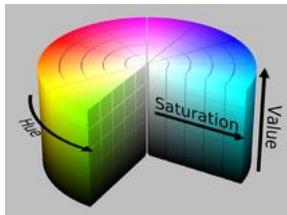
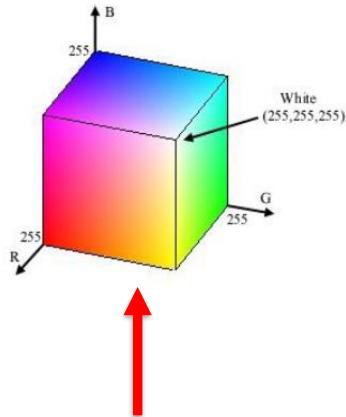
$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2} [(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{\frac{1}{2}}} \right\}$$

$$I = \frac{1}{3}(R + G + B)$$

Converting From HSI To RGB



– RG sector ($0 \leq H < 120^\circ$)

$$R = I \left[1 + \frac{S \cos H}{\cos(60 - H)} \right], G = 3I - (R + B), B = I(1 - S)$$

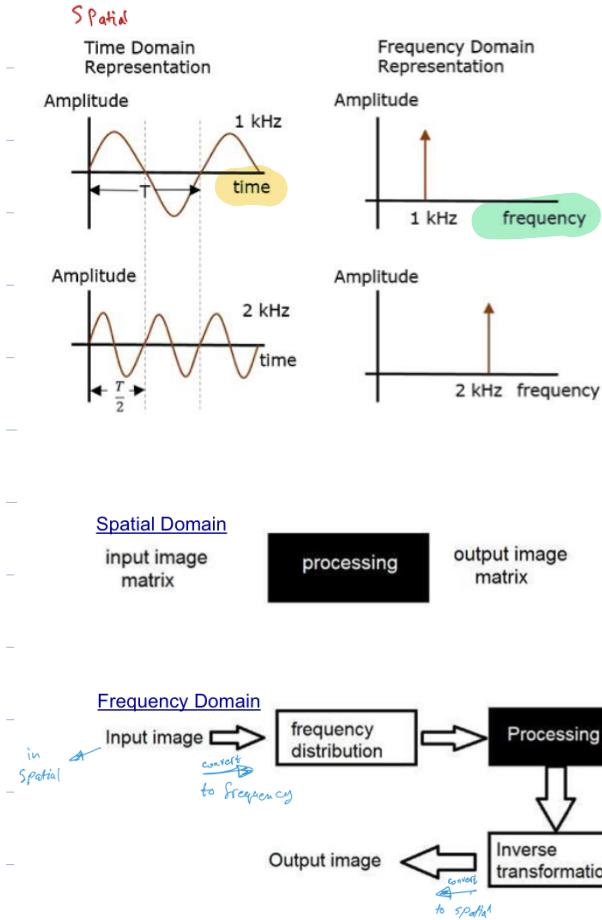
– GB sector ($120^\circ \leq H < 240^\circ$)

$$R = I(1 - S), G = I \left[1 + \frac{S \cos(H - 120)}{\cos(H - 60)} \right], B = 3I - (R + G)$$

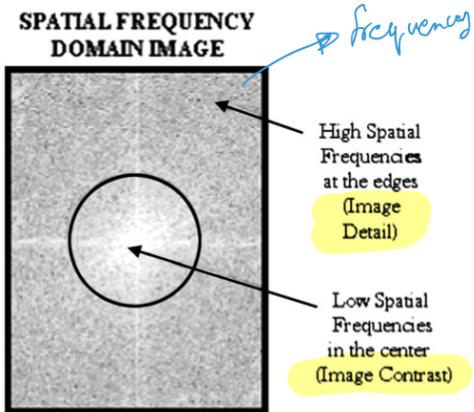
– BR sector ($240^\circ \leq H < 360^\circ$)

$$R = 3I - (G + B), G = I(1 - S), B = I \left[1 + \frac{S \cos(H - 240)}{\cos(H - 180)} \right]$$

④ IIR in the Frequency Domain



- Whereas in frequency domain, we deal with the rate at which the pixel values are changing in spatial domain.
- The term frequency in an image tells about the rate of change of pixel values. \rightarrow in spatial domain



Why we need a domain other than spatial domain ?

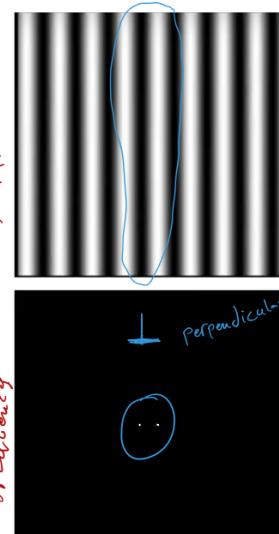
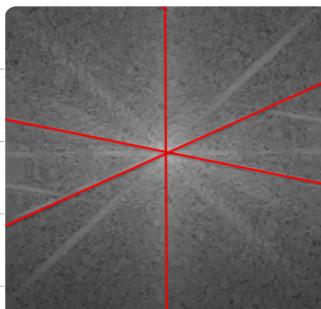
- Sometimes image processing/computer vision tasks are best performed in a domain other than the spatial domain.

*in time and quality
and simplest Algo.*

High frequencies correspond to
the smooth areas in the image?

False

\Rightarrow high freq = Sharp areas and edges
 \Rightarrow low freq = Smooth areas Why



to convert signal from time domain into frequency domain
we use transforms

Some kind of transforms:-

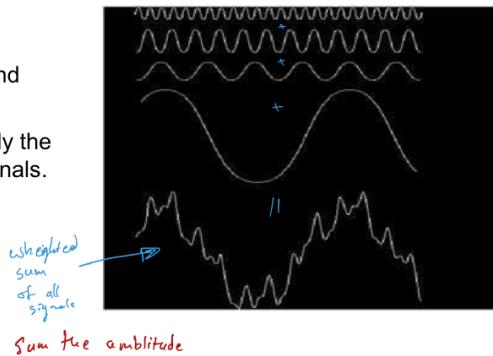
① Fourier Series

❖ Fourier series

- ◊ Periodic signals can be represented into sum of Sines and Cosines when multiplied with a certain weight.

❖ Fourier series

- ◊ The signals are sines and cosines.
- ◊ The last signal is actually the sum of all the above signals.
- ◊ This was the idea of the Fourier.



② Fourier transform

- ❖ **Fourier transform:** the non periodic signals whose area under the curve is finite can also be represented into integrals of the sines and cosines after being multiplied by a certain weight.

- ◊ Images are non – periodic, so Fourier transform is used to convert them into frequency domain.

Discrete Fourier
Transform

- ❖ The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image.

◆ 2D Discrete Fourier Transform

freq. \rightarrow

$$I(u, v) = \frac{1}{N_{\text{cols}} \cdot N_{\text{rows}}} \sum_{x=0}^{N_{\text{cols}}-1} \sum_{y=0}^{N_{\text{rows}}-1} I(x, y) \cdot \exp \left[-i2\pi \left(\frac{xu}{N_{\text{cols}}} + \frac{yv}{N_{\text{rows}}} \right) \right]$$

mean

spatial

Complex

Base function

◆ Inverse Discrete Fourier Transform maps Fourier transform I back into the spatial domain:

Spatial \rightarrow

$$I(x, y) = \sum_{u=0}^{N_{\text{cols}}-1} \sum_{v=0}^{N_{\text{rows}}-1} I(u, v) \exp \left[i2\pi \left(\frac{xu}{N_{\text{cols}}} + \frac{yv}{N_{\text{rows}}} \right) \right]$$

freq.

* notes about DFT:-

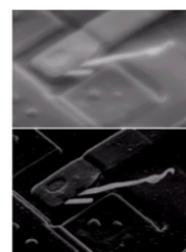
- $F(0,0)$ is called dc component

- the values of DFT are complex

- to visually analyze a DFT, we compute Fourier spectrum
(the magnitude of $F(u, v)$)

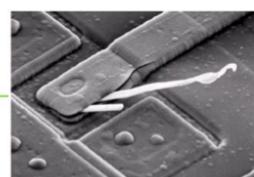
Keep Smooth area

Low Frequencies only:



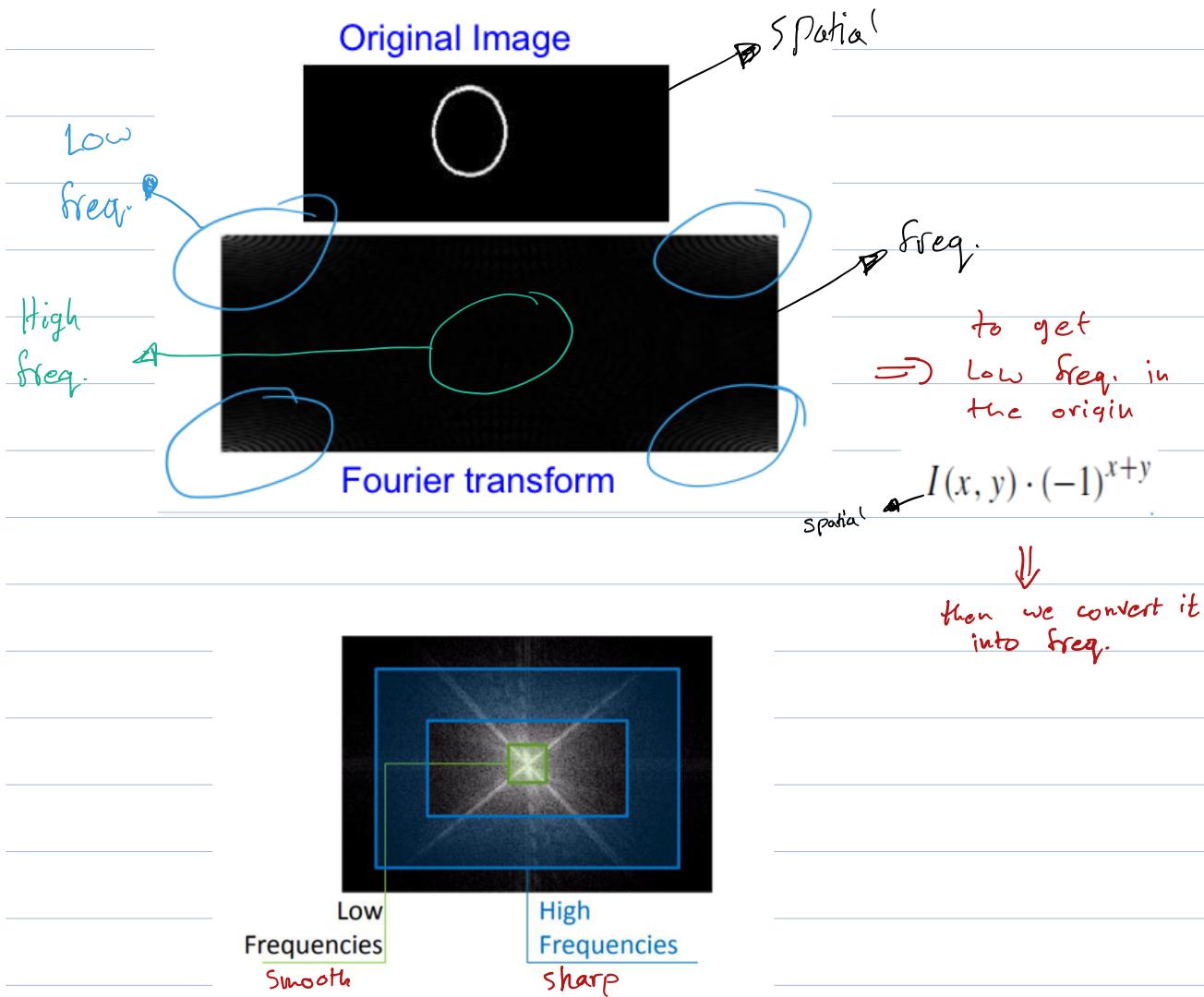
Keep edge and sharp area

High Frequencies only:



THE ARC.

- ❖ Origin in image I and Fourier transform I is in upper left corner



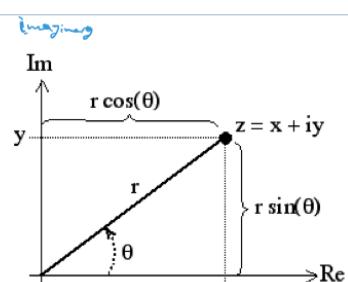
- ❖ Allows to plot ordered pairs of the form (a, b) , where a and b are real numbers

$$z = a + ib$$

- Also called "Rectangular Coordinates"

- ❖ **Polar Form:** Rather than using a and b , the polar coordinates use r and θ in their ordered pairs.

- ❖ The general form for polar numbers is as follows: $re^{i\theta}$



$$r = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$

Discrete Fourier transform (TF = fft2())

- ❖ **Magnitude** (spectrum) of the FT

`abs(TF)`

$$|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{1/2}$$

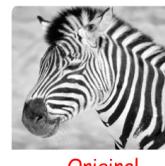
- ❖ **Power spectrum** (spectral density)

$$P(u, v) = |F(u, v)|^2 = R^2(u, v) + I^2(u, v)$$

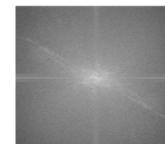
- ❖ **Phase angle** (phase spectrum)

$$\phi(u, v) = \tan^{-1} \left[\frac{I(u, v)}{R(u, v)} \right]$$

`angle(TF)`



Original



Amplitude

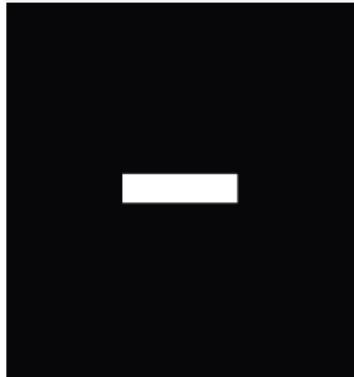


Phase

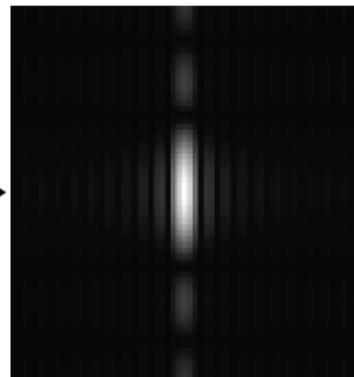
Note: $R(u, v)$ and $I(u, v)$ are real and imaginary parts of $F(u, v)$, respectively.

Fourier Transform Examples

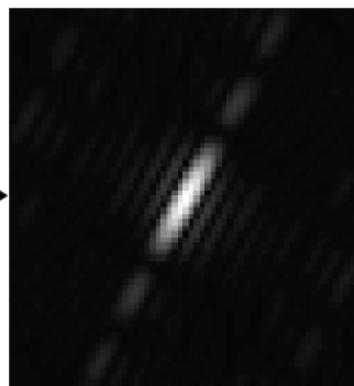
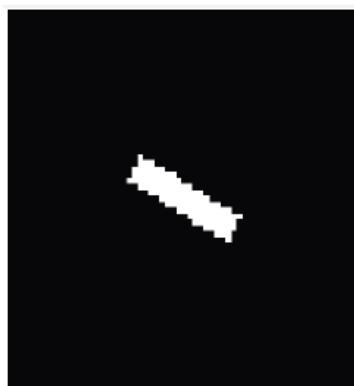
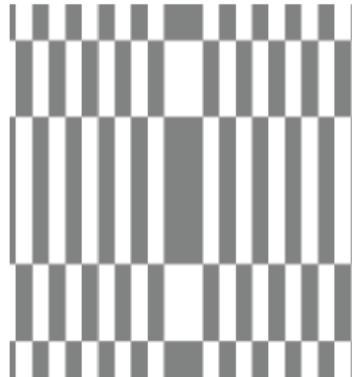
Original
Image



Magnitude DFT



Phase DFT



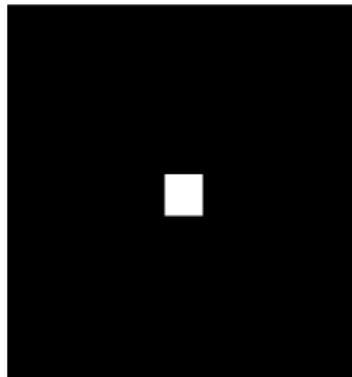
Orientation

A line transforms
to a line oriented
perpendicularly
to the first

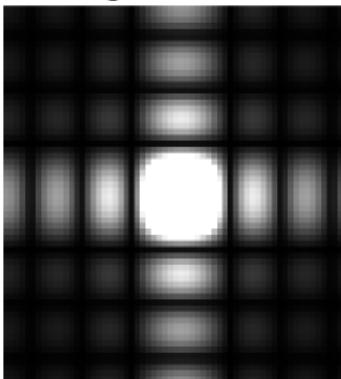


Fourier Transform Examples

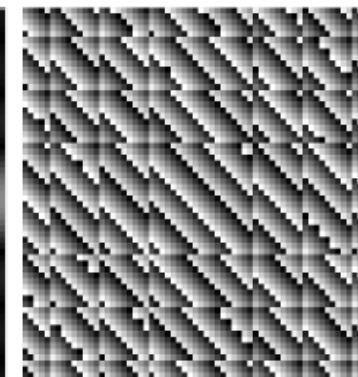
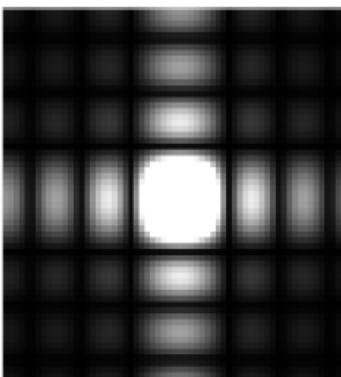
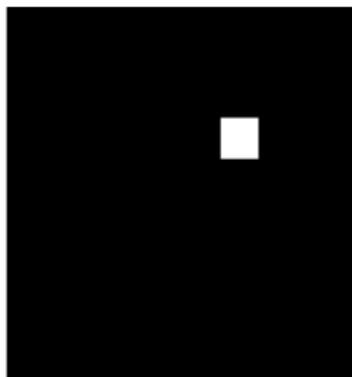
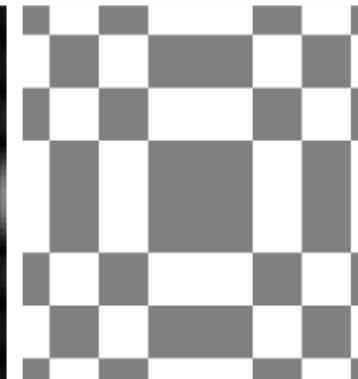
Image



Magnitude DFT

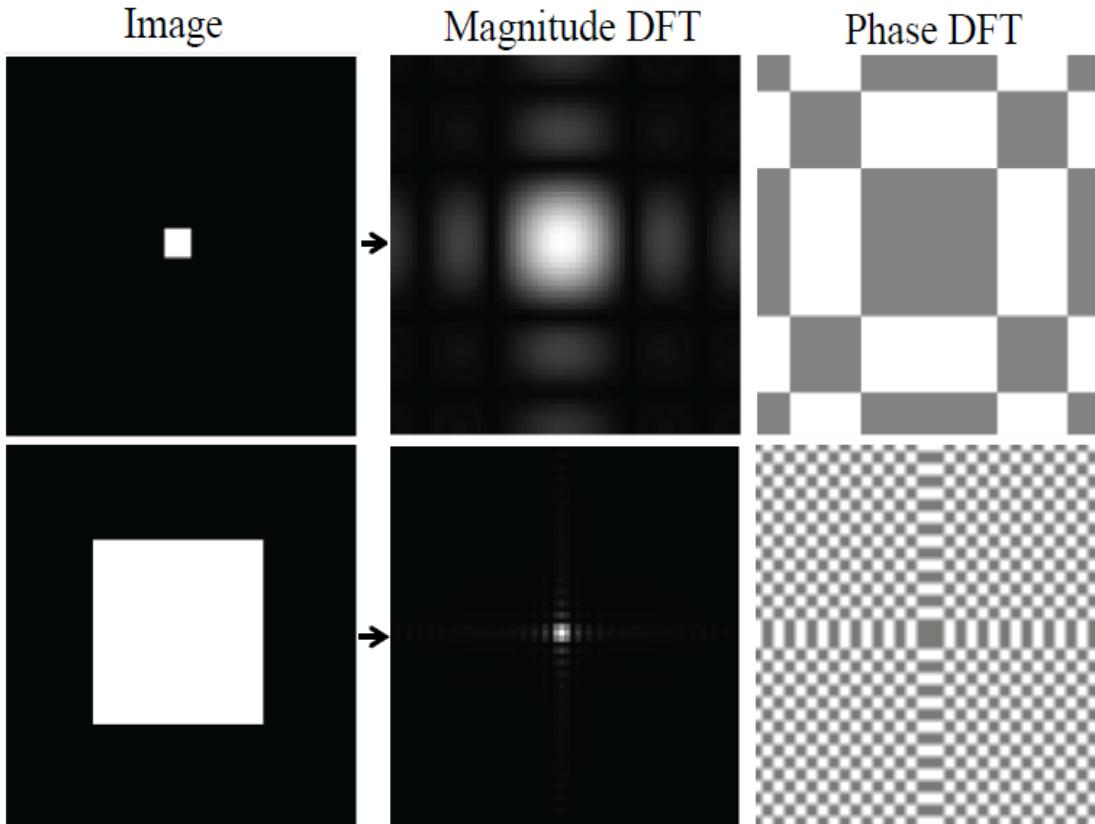


Phase DFT



Translation
Shifts of an image
only produce
changes on the
phase of the DFT

Fourier Transform Examples



Scale
Small image
details produce
content in high
spatial frequencies

⑤ Basic Image Operators

① Thresholding

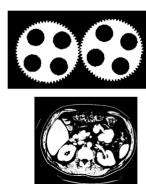
what?

Colored IMg \rightarrow binary IMg

Does not work well with natural scenes.

why?

- document analysis (e.g. read digits)
- Industrial inspection
- Medical imaging



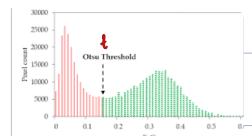
How

histogram

Otsu's Method

- assumes the histogram is bimodal and tries to minimize the variation within the same group of values

Method: find the threshold t that minimizes the weighted sum of within-group variances for the two groups that result from separating the gray tones at value t .



In practice, this operator works very well for true bimodal distributions and not too badly for others.

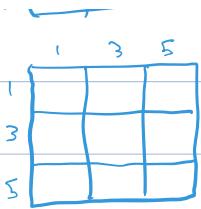
② Resampling (Resize the image)

① reduce the size = sub-Sampling

1	2	3	4	5
1				
2				
3				
4				
5				

delete even rows and columns

Throw away every other row and column to create a $\frac{1}{2}$ size image



② increase the size = up-sampling

① Simplest approach:

repeat each row and column 10 times

② More efficient approach:

Interpolation (estimate a new value)



We can do this by looking at the values nearby. Methods include:

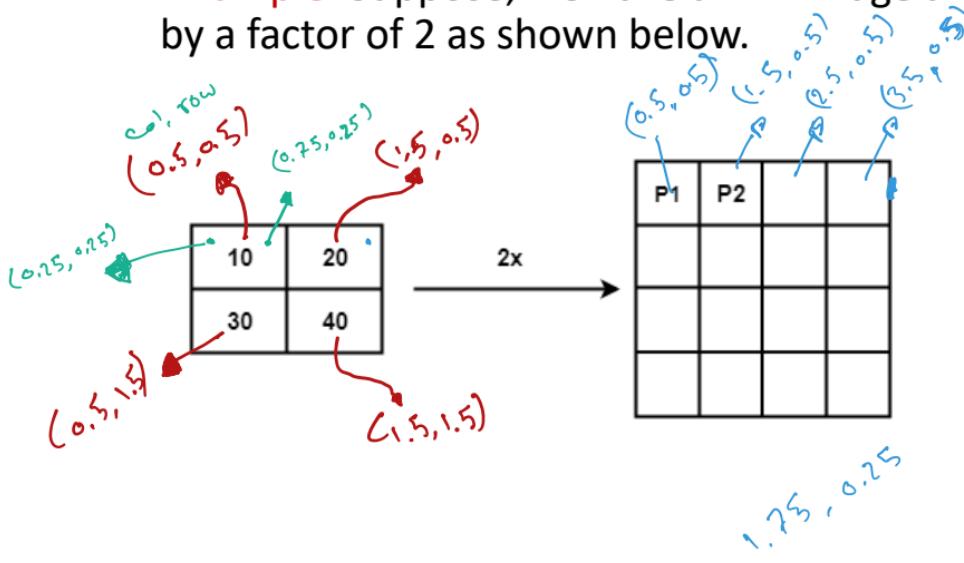
- 1 Nearest neighbor – just take the value of the closest neighbor
- 4 Bilinear – take a combination of the four closest neighbors
- 16 Bicubic – use the closest 16 neighbors (most computationally expensive, but best results)

Review: Nearest Neighbor Interpolation

Estimation value
from one point to
one point

- We assign the unknown pixel to the nearest known pixel.

- Example:** Suppose, we have a 2×2 image and let's say we want to upscale this by a factor of 2 as shown below.



$$P_i = (0.5, 0.5) \times \frac{1}{2} = (0.25, 0.25)$$

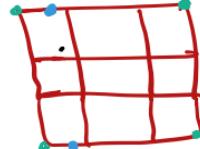
From original
the value = 10

$$P_i = (1.5, 0.5) \times \frac{1}{2} = (0.75, 0.25)$$

From original
the value = 10

Review: Bi-Linear interpolation

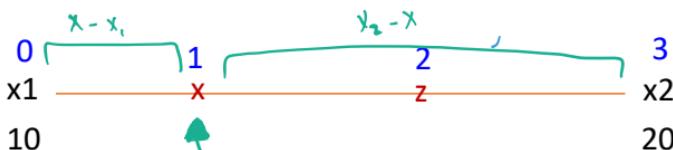
apply twice (vertically, horizontally)



- Let us first review the linear interpolation where more weight is given to the nearest value

- Assume we want to find the pixel value at locations 1 and 2
- Let us call these points: x and z

for vertical
replace x by y



$$f(x) = \frac{x_2 - x}{x_2 - x_1} \times f(x_1) + \frac{x - x_1}{x_2 - x_1} \times f(x_2)$$
$$= \frac{1}{3} \times 20 + \frac{2}{3} \times 10 = 13.3$$

$$f(z) = \frac{x_2 - z}{x_2 - x_1} \times f(x_1) + \frac{z - x_1}{x_2 - x_1} \times f(x_2)$$

W3-P1 - Image filtering

① Image enhancement

- make it more suitable for specific application
 - depends on application
 - methods may different for each image
- Common reasons for enhancement include
 - Improving visual quality,
 - Improving recognition accuracy.

- it can be done in:

① Spatial domain *جُنْدِيَّة*

② Frequency domain *مُوجِيَّة*

③ combinations of method from both domains

* by Spatial domain: (Point or Kernel operators) *بَعْدِيَّة*

* Point operations:

Deals with pixel intensity values individually

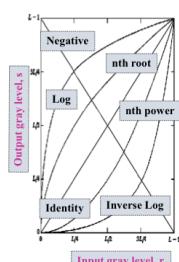
Intensity Transformation function:-

general \rightarrow * Gray-level $S = T(r)$ *original pixel*



$$g(x,y) = f(x,y) + 20$$

• Linear, Logarithmic: $s = c \log(r+1)$, Power-Law: $S = C r^{\alpha}$

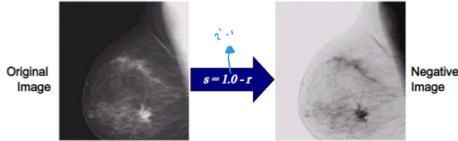


① Image negative

in gray
 $L=256 \leq 2^n$
bits
maximum intensity value
 $s = L - I - r$ original

Image negative is produced by subtracting each pixel from the maximum Intensity value

- Suitable for enhancing white or gray detail embedded in dark regions of an image.



- expand the intensities of dark pixels
- compress the bright (higher-level) intensities

② Log Transformations

dark pixel \rightarrow darker
bright pixel \rightarrow brighter

Constant
original pixel
 $s = c \log(r + 1)$

③ Power-Law (Gamma) Transformation

Constant c is used to correct image's luminance.

$s = cr^\gamma$ or $s = c(r+\epsilon)^\gamma$

$\gamma < 1$ increase Brightness

$\gamma > 1$ increase Darkness

④ Piecewise-Linear Transformation

Commonly types:

① Contrast Stretching

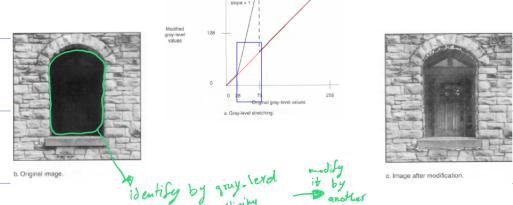
Contrast stretching expands the range of intensity levels in an image.

Contrast stretching is done in three ways:

1. Multiplying each input pixel intensity value with a constant scalar.
Example: $s=2^k r$
2. Using Histogram Equivalent
3. Applying a transform which makes dark portion darker by assigning slope of < 1 and bright portion brighter by assigning slope of > 1 .

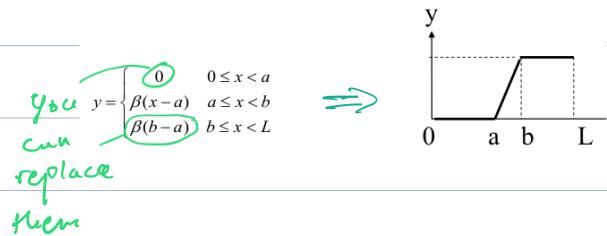
⑦ Array-level Slicing

identify a specific range of gray levels in an ^{image}



③ Clipping (Set two thresholds)

reassign some pixels



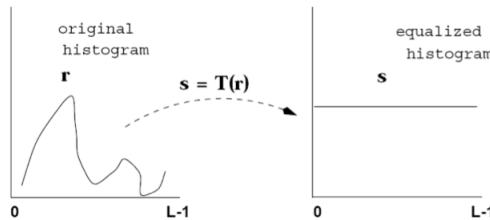
② Histogram Processing

usually if the histogram of an image is uniformly distributed

it will have a high contrast \Rightarrow more details

- The main idea is to redistribute the gray-level values uniformly.

\rightarrow to use all colors



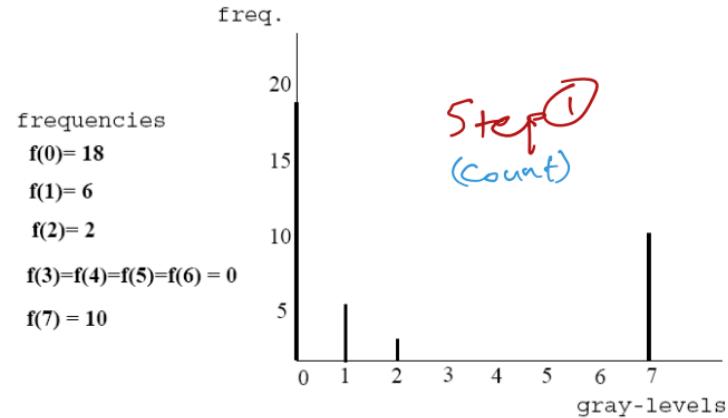
to achieve that



Histogram equalization example

- Histogram equalization algorithm

0	0	1	0	2	0
1	0	7	7	7	0
0	7	0	0	7	0
1	0	0	7	2	0
0	0	7	1	0	1
1	0	7	7	7	0



- Divide frequencies by total number of pixels to represent as probabilities.

$$p_k = n_k / N$$

Step 2
(normalize)

$$P(0) = \frac{f(0)}{36} = \frac{1}{2}$$

$$P(1) = \frac{f(1)}{36} = \frac{1}{6}$$

$$P(2) = \frac{f(2)}{36} = \frac{1}{18}$$

$$P(3) = P(4) = P(5) = P(6) = 0$$

$$P(7) = \frac{f(7)}{36} = \frac{5}{18}$$

never
used

Histogram equalization example

Step ③

$$T(r_k) = \sum_{j=0}^k p_r(r_j)$$

- Compute intensity transformation function

Since there are 36 pixels

$$T(r_0) = \sum_{j=0}^0 p_r(r_k) = p_r(r_0) = \frac{1}{2} = 0.5$$

$$T(r_1) = \sum_{j=0}^1 p_r(r_k) = \frac{1}{2} + \frac{1}{6} = 0.5 + 0.17 = 0.66$$

$$T(r_2) = \sum_{j=0}^2 p_r(r_k) = \frac{1}{2} + \frac{1}{6} + \frac{1}{18} = 0.5 + 0.17 + 0.06 = 0.72$$

$$T(r_3) = T(r_4) = T(r_5) = T(r_6) = 0.72$$

$$T(r_7) = \sum_{j=0}^7 p_r(r_k) = 0.72 + \frac{5}{18} = 1.01$$

$P(0) = \frac{f(0)}{36} = \frac{1}{2}$	$P(1) = \frac{f(1)}{36} = \frac{1}{6}$
$P(2) = \frac{f(2)}{36} = \frac{1}{18}$	$P(3) = P(4) = P(5) = P(6) = 0$
$P(7) = \frac{f(7)}{36} = \frac{5}{18}$	

$T(2) = P(0) + P(1) + P(2)$

$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{18}$$

$$\Rightarrow 0.72$$

- Multiple intensity transformation function by intensity level - 1

8 - 1 = 7

$$S_0 = (L-1) * T(r_0) = 7 * 0.5 = [3.5] = 3$$

Step ⑤

$$S_1 = (L-1) * T(r_1) = 7 * 0.66 = [4.6] = 4$$

$$S_2 = (L-1) * T(r_2) = 7 * 0.72 = [5.04] = 5$$

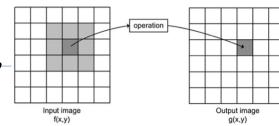
$$S_3 = S_4 = S_5 = S_6 = 7 * 0.72 = [5.04] = 5$$

$$S_7 = (L-1) * T(r_7) = 7 * 1.01 = [7.01] = 7$$

* Kernel / Filter / Mask operators ^{template/window}

- in spatial domain

- it consider the value of neighbours



What Point Operations Can't Do

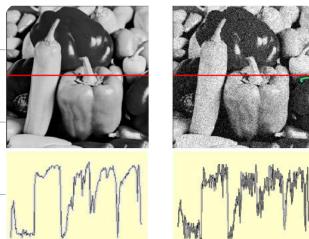
- Combining multiple pixels needed for certain operations:
 - Enhance an image, e.g., denoise, Blurring, Sharpening.
 - Extract information, e.g., texture, edges.
 - Detect patterns, e.g., template matching.

Noise:

• Noise sources

- Light Variations
- Camera Electronics
- Surface Reflectance
- Lens

→ Common types:-



① Salt and pepper (random)

② Gaussian noise

→ How to denoise??

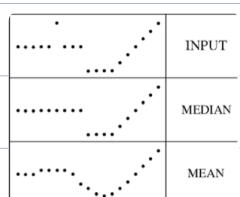
by filters

- The values in a filter sub-image are referred to as **coefficients**, rather than pixels.

- A mask of coefficients is centered on a pixel.
- The mask **coefficients** are **multiplied** by the pixel values in its neighborhood and the products are **summed**.
- The result goes into the corresponding pixel position in the output image.

$$\begin{array}{cccc} 36 & 36 & 36 & 36 \\ 36 & 36 & 45 & 45 \\ 36 & 45 & 45 & 45 \\ 45 & 45 & 54 & 54 \\ 54 & 54 & 54 & 54 \end{array}
 \times
 \begin{array}{c} 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \end{array}
 =
 \begin{array}{cccc} * & * & * & * \\ * & 39 & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{array}$$

Input Image filter Output Image

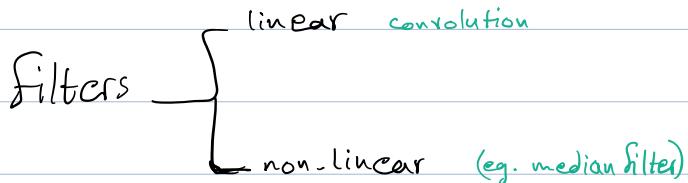


- Some simple neighborhood operations include:

- Min:** Set the pixel value to the **minimum** in the neighborhood
- Max:** Set the pixel value to the **maximum** in the neighborhood
- Average:** Set the pixel value to the **average** of the neighborhoods
- Median:** The median value of a set of numbers is the **midpoint** value in that set (e.g. from the set [1, 7, 15, 18, 24] 15 is the median). Sometimes the median works better than the average

• Image filtering is used to:

- Remove noise
- Sharpen contrast
- Highlight contours
- Detect edges
- Other uses?



W3-P2 - Image filtering

Some properties of linear filters:-

Key properties of linear filters

- 1. **Linearity:** $\text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2)$
- 2. **Shift invariance:** same behavior regardless of pixel location.
 - The value of the output depends on the pattern in the image neighborhood not the position of the neighborhood
 - $\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))$
 - Any linear, shift-invariant operator can be represented as a convolution
- 3. **Commutative:** $a * b = b * a$

Key properties of linear filters

- 4. **Associative:** $a * (b * c) = (a * b) * c$
 - Often apply several filters one after another: $((a * b_1) * b_2) * b_3$
 - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- 5. **Distributes over addition:** $a * (b + c) = (a * b) + (a * c)$
- 6. **Scalars factor out:** $k a * b = a * k b = k (a * b)$
- 7. **Identity:** unit impulse $e = [0, 0, 1, 0, 0]$, $a * e = a$
- 8. **Differentiation:** $\frac{\partial}{\partial x}(f * g) = \frac{\partial f}{\partial x} * g$

Some types of linear filters:-

① Average
for smoothing

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Average filter

increasing Kernel size=more blur

Special type:

Weighted Smoothing Filter

- More effective smoothing filters can be generated by assigning different pixels in the neighborhood different weights in the averaging function

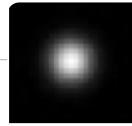
- Pixels closer to the central pixel are more important
- Reduce smoothing effect
- Often referred to as a weighted averaging

1	2	1
2	4	2
1	2	1

② Gaussian

for Smoothing

the largest value in
the center



0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

$$5 \times 5, \sigma = 1$$

σ = Scale / width / spread of the distribution

$\sigma \nearrow \Rightarrow$ Smoother = more blurred
 $\sigma \nwarrow \Rightarrow$ sharper

* Median filter (non-linear)

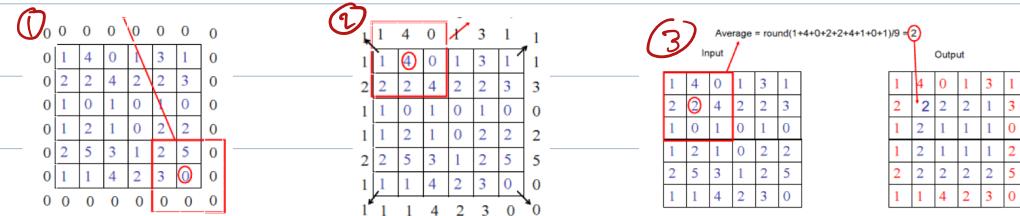
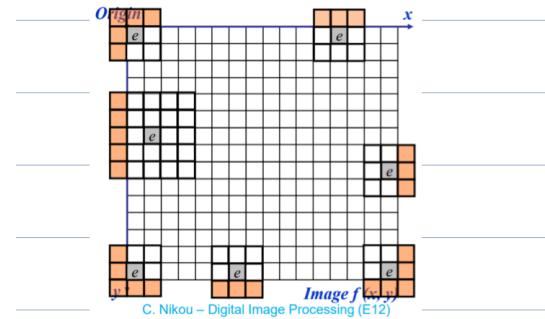
Selecting the median intensity in the Kernel

good for impulses and salt & pepper noise

Boundary issues

- There are a few approaches to deal with missing edge pixels:

- ① Pad the image (padding)
 - Typically with either all white or all black pixels
- ② Replicate border pixels
- ③ Truncate the image *Keep the values of border*
- Allow pixels wrap around the image
 - Can cause some strange image artefacts



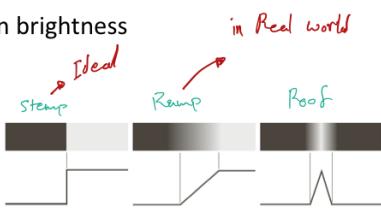
W4-P1-Edge detection

- **Edges** are sharp change in brightness (discontinuities).
- **Goal:** Identify sudden changes in an image

Characteristics of an Edge

- **Edge:** A sharp change in brightness
- Edge models/types:
 - Step
 - Ramp
 - Roof

FIGURE 10.8
From left to right,
models (ideal
representations)
of a step, a ramp,
and a roof edge,
and their corresponding
intensity profiles.



Edge detection

- **Basic idea:** look for a neighborhood with strong signs of change.

- **Problems:**
 - Neighborhood size
 - How to detect change

81	82	26	24
82	33	25	25
81	82	26	24

Differential operators:

- Attempt to approximate the **gradient** at a pixel via **masks**.
- Threshold the gradient to select the **edge pixels**.

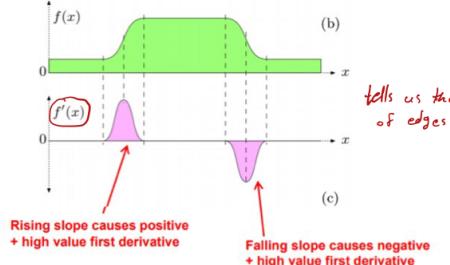
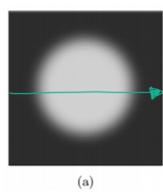
Main Steps in Edge Detection

- 1) **Smoothing:** suppress as much noise as possible, without destroying true edges. *linear filter (Cart sliders)* *take derivative*
- 2) **Enhancement:** apply differentiation to enhance the quality of edges (i.e., sharpening)
- 3) **Thresholding:** determine which edge pixels should be discarded as noise and which should be retained (i.e., threshold edge magnitude).
- 4) **Localization:** determine the exact edge location. *you can use erosion* *to connect them*

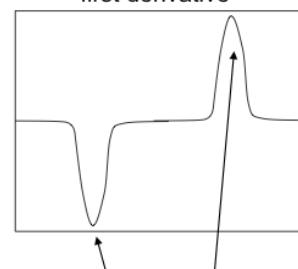
Edge detection using derivatives: (two ways)

①

1st order derivative:



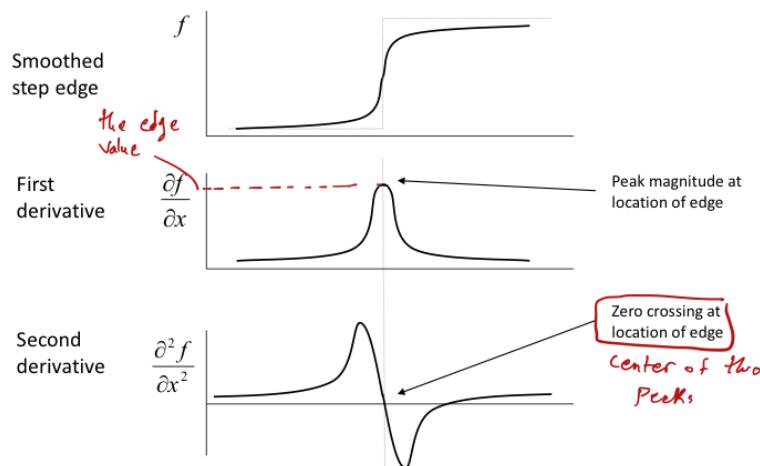
first derivative



edges correspond to extrema of derivative

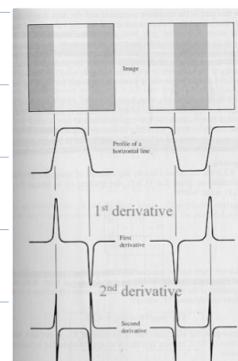
(2)

- 2nd order derivatives enhance fine detail much better



- Often, points that lie on an edge are detected by:

- 1 Detecting the local maxima or minima of the first derivative.
- 2 Detecting the zero-crossings of the second derivative.



Since images are 2D, we use gradient

$$\nabla f = \left[\frac{\partial f}{\partial x}, 0 \right] \quad \begin{matrix} \text{no change on} \\ y \text{ direction} \end{matrix}$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y} \right]$$

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Image gradient

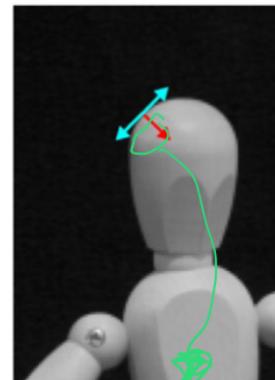
- Gradient is perpendicular to edge contour *always*

- The **gradient direction** is given by:

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- The **edge strength** is given by the gradient **magnitude**

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$



*always to the Right
side*

for 1D

1D discrete derivative filters



- Backward filter:

$$H = [0 \quad 1 \quad -1]$$

next *current* *prev*

$$f(x) - f(x-1) = f'(x)$$

- Forward:

$$H = [-1 \quad 1 \quad 0]$$

$$f(x) - f(x+1) = f'(x)$$

- Central:

$$H = [1 \quad 0 \quad -1]$$

$$f(x+1) - f(x-1) = f'(x)$$

Note: Derivative kernels sum to zero

$$\sum H = 0$$

2D filter

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

in X direction

$$\xrightarrow{\cdot T}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

in Y direction

Effect of noise:

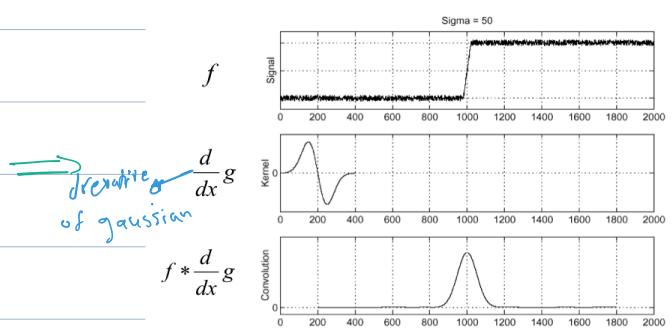
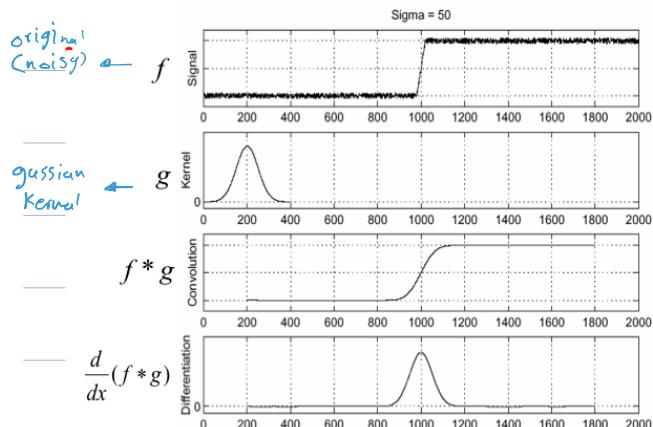
derivative is very sensitive to noise (since it's measure the rate of change)

Solution:

Smoothing first

Small $\sigma \rightarrow$ sharper

Large $\sigma \rightarrow$ smoother (detect large scale edge)



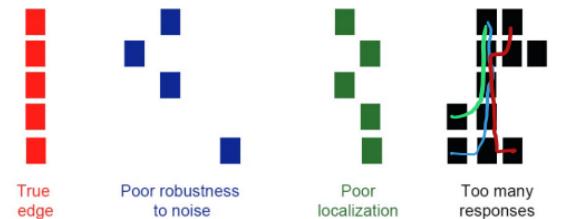
Smoothed derivatives removes noise and finds edges at different "scales"

But it blurs edges

Designing an edge detector

- Criteria for an “optimal” edge detector:

- ① • **Good detection:** the optimal detector must **minimize** the probability of **false positives** (detecting spurious edges caused by noise), as well as that of **false negatives** (missing real edges)
- ② • **Good localization:** the edges detected must be as close as possible to the true edges
- ③ • **Single response:** the detector must return **one point** only for each true edge point; that is, minimize the number of **local maxima** around the true edge



Sobel Operators:

- Smoothing + differentiation

$$\rightarrow \mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} [+1 \quad 0 \quad -1]$$

Gaussian smoothing differentiation

$$\mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} \quad \mathbf{G}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Problems in Sobel:

① poor localization

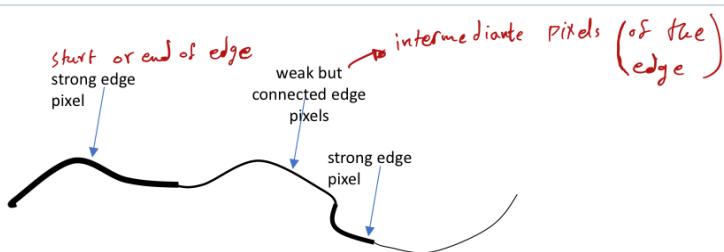
② Too many responses

Canny Edge Operator:

multi-Stage Algo.

- Filter image with x, y derivatives of Gaussian
- Find magnitude and ^{direction} orientation of gradient
- Non-maximum suppression:
 - ^{Edge occurs where gradient reaches Maxima}
 - ^{thinning the edge to get 1 response}
 - Thin multi-pixel wide "ridges" down to single pixel width

Non-maximum suppression can help to suppress all the gradient values (by setting them to 0) except the local maxima
- Thresholding and linking (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them



W4-P2 - Edge detection AND Mathematical Morphology

Hough Transform (parametric function, Accumulator array)

- Main idea:

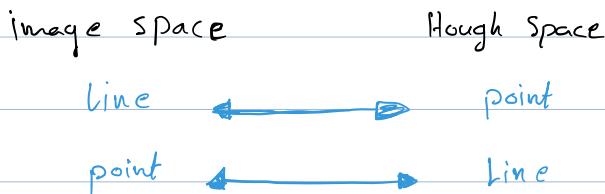
- Record all possible lines on which each edge point lies.
- Look for lines that get many votes.

fitting

- Line detection:** Hough transform

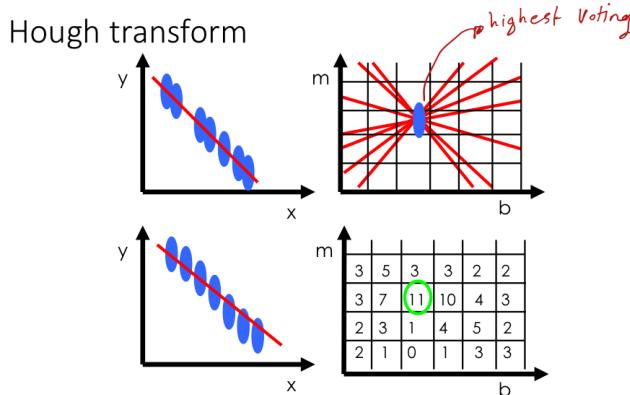
- Simple parametric model, two parameters m, b

$$y = mx + b$$



- Voting strategy**

- Hard to generalize to higher dimensions

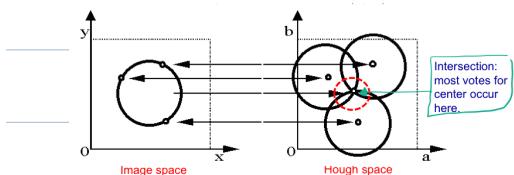


finding circles: (a, b, R)

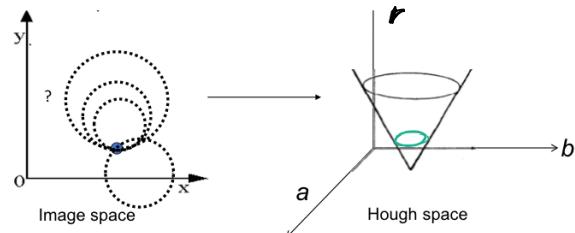
$$\begin{aligned} a &= x + R \cos(\theta) \\ b &= y + R \sin(\theta) \end{aligned}$$

① R is known (a, b) is unknown

- Search can be reduced to 2D



② R is unknown (a, b) is known



Hough transform: pros and cons

- Pros

- All points are processed independently, so can cope with occlusion
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can detect multiple instances of a model in a single pass (Provides multiple good fits)

- Cons

- Complexity of search time increases exponentially with the number of model parameters (Not suitable for more than a few parameters - grid size grows exponentially)
- Some sensitivity to noise.
- Quantization: hard to pick a good grid size

* Summary

- The Hough transform and its variants can be used to find line segments or circles.
- It has also been generalized to find other shapes
- The original Hough method does not work well for line segments, but works very well for circles.

circles are better than lines

Mathematical Morphology

form and structure (Shape of a region)

as long
as it has
an original
point

Morphological operations are a set of operations that process images based on shapes. They apply a structuring element to an input image and generate an output image.

↓
consist of two
operations:
applied on binary images Structuring Elements

OR ① Dilation Expand \oplus
AND ② Erosion Shrink \ominus

a) BOX(3,5)	b) DISK(5)	c) RING(5)

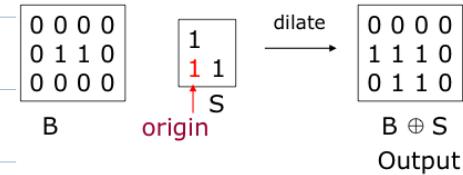
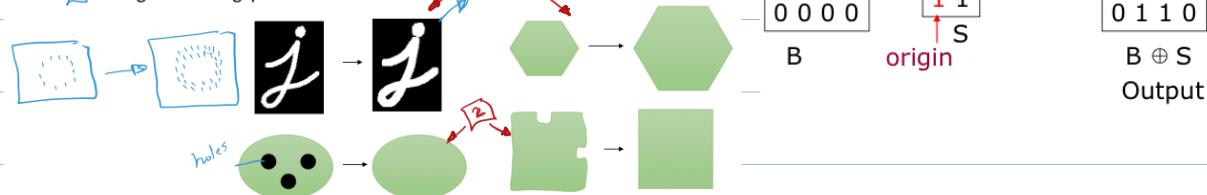
d) L shape	e)	f)

origin = where the value in the image will be changed

Dilation

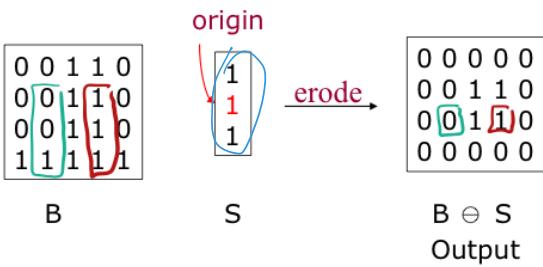
- Dilation expands the connected sets of 1s of a binary image. It can be used for

- Growing features
- Filling holes and gaps



Erosion

inverse of dilation
not exactly



- Erosion shrinks the connected sets of 1s of a binary image. It can be used for:
- Shrinking features
 - Removing bridges, branches and small protrusions



Closing: dilation $\xrightarrow{\text{OR}}$ erosion $\xrightarrow{\text{AND}}$

Opening: erosion $\xrightarrow{\text{AND}}$ dilation $\xrightarrow{\text{OR}}$

with SAME structuring element

1	1	1	1	1	1	1	1
		1	1	1	1		
		1	1	1	1		
		1	1	1	1		
		1	1	1	1		
		1	1	1	1		
		1	1				

a) Binary image B

1	1	1
1	1	1
1	1	1

mask R, filter, Kernel, Structuring element

b) Structuring Element S

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

c) Dilation $B \oplus S$

				1	1		
				1	1		
				1	1		
				1	1		
				1	1		
				1	1		
				1	1		
				1	1		

d) Erosion $B \ominus S$

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

e) Closing $B \bullet S$

				1	1	1	1
				1	1	1	1
				1	1	1	1
				1	1	1	1
				1	1	1	1
				1	1	1	1
				1	1	1	1
				1	1	1	1

f) Opening $B \circ S$

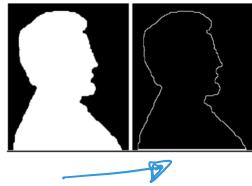
We can Extract boundary using:

Image A

$$A - (A \odot B)$$

\curvearrowright 3×3 square structuring element

Example: Boundary extraction



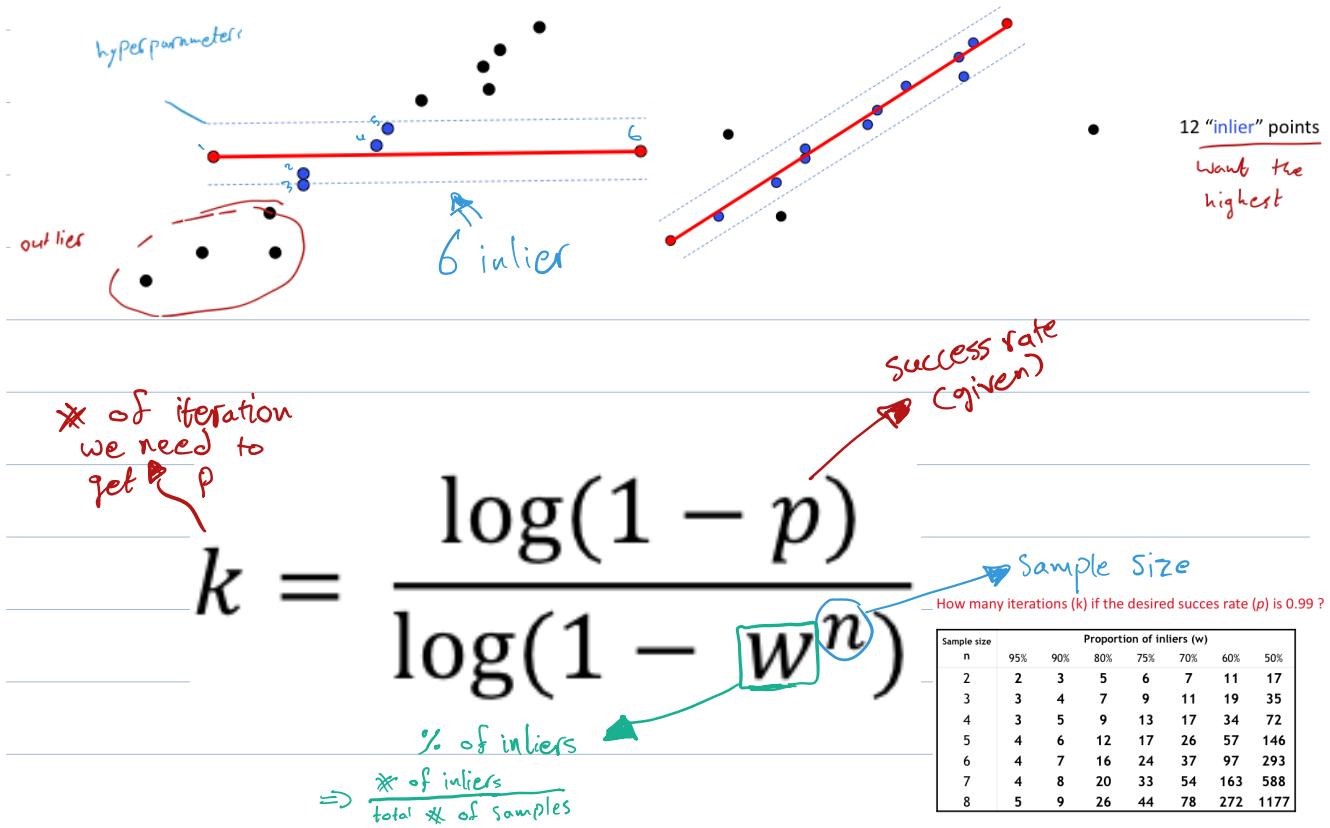
W5 - P1 - Feature detection and matching

fitting (Data association) methods:

- ① Hough Transform
- ② RANSAC

RANdom SAmples Consensus (RANSAC)

- A model fitting method for edge detection
 - RANSAC
- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those.



RANSAC: Pros and Cons

- **Pros:**

- General method suited for a wide range of model fitting problems
- Easy to implement and easy to calculate its failure rate

- **Cons:**

- Only handles a moderate percentage of outliers without cost blowing up
- Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)

Matching Problem:

can be solved by fitting [拟合]

difficulties:-

① different zoom

② different orientation or angle

So we cannot compare pixel by pixel

we use SIFT map (local features, interest points)

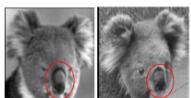
leads
to

Increased robustness to

➤ Occlusions



➤ Intra-category variations

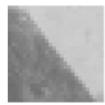


➤ Articulation



Interest points

- 0D structure: single points
 - not useful for matching



- 1D structure: lines
 - edge, can be localized in 1D, subject to the aperture problem*

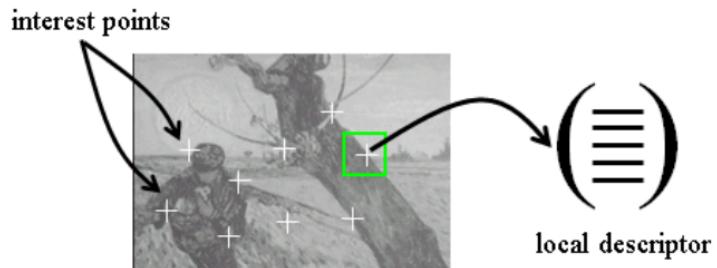
- 2D structure: corners
 - corner or interest point, can be localized in 2D, good for matching
 - Interest Points have 2D structure.



More details: <https://youtu.be/i2ETv2N1z5E?t=526>

We can use matching to generate Panorama images

General Interest Detector/Descriptor Approach



- 1) Extraction of **interest points**
- 2) Computation of **local descriptors**
- 3) Determining **correspondences** *between two images*
- 4) Selection of **similar images**

Common Requirements

- Detect the *same* point *independently* in both images



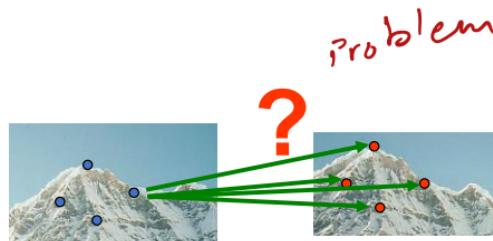
no chance to match!

We need a repeatable detector

Sometimes we
got the problem
in next slide

Common Requirements

- For each point **correctly** recognize the **corresponding** one



We need a reliable and distinctive descriptor

Detectors

to detect interest points

① Harris

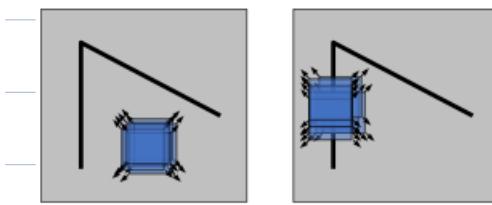
② Laplacian

③ Difference of Gaussians (DoG)

Harris

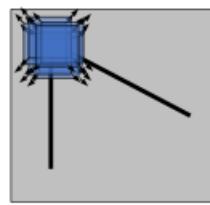
① Small window

② move this window on the image



"flat" region: no change as shift window in all directions
0D

"edge": no change as shift window along the edge direction
just 1D



"corner": significant change as shift window in all directions
2D



$\sum I_x^2$	Large	Corner
$\sum I_y^2$	Large	
$\sum I_x^2$	Small	Edge
$\sum I_y^2$	Large	
$\sum I_x^2$	Small	Nothing
$\sum I_y^2$	Small	

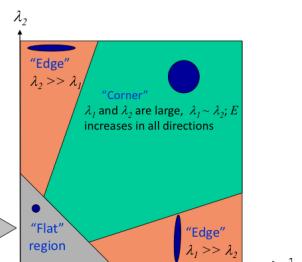
- This measure of change in the shift between two windows can be approximated by:

$$E(u, v) = (u - v) M \begin{pmatrix} u \\ v \end{pmatrix}$$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

both small \Rightarrow flat
one large \Rightarrow edge
both large \Rightarrow corner

λ_1 and λ_2 are small;
 E is almost constant in all directions



We can use another way to avoid computing

Eigenvalues \Rightarrow Response func.

- Measure of cornerness in terms of λ_1 and λ_2

$$\text{Response function} \quad \theta = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

- R depends only on eigenvalues of M

- R is large for a corner

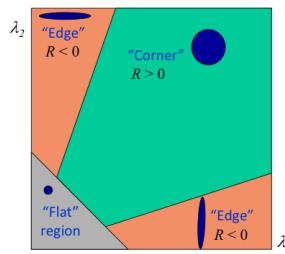
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region

- Fast approximation

- Avoid computing the Eigenvalues

- α : empirical constant

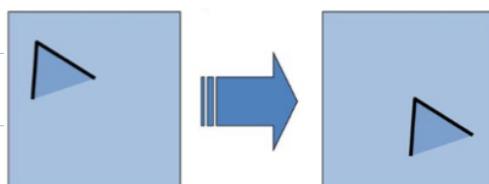
(0.04 to 0.06)



We can Smooth the window with Gaussian to be rotation invariant

Harris Properties:-

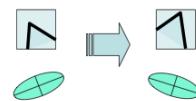
① it is translation invariant



Corner location is equivariant w.r.t. translation

Some information will be detected

② it is rotation invariant

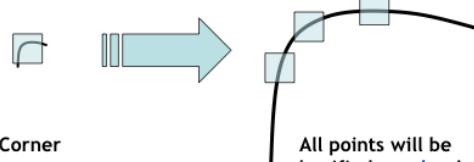


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

③ it is NOT scale invariant

Not same information will be detected



Not invariant to image scale!

Summary: Harris Detector

- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

σ_D : for Gaussian in the derivative calculation

σ_I : for Gaussian in the windowing function

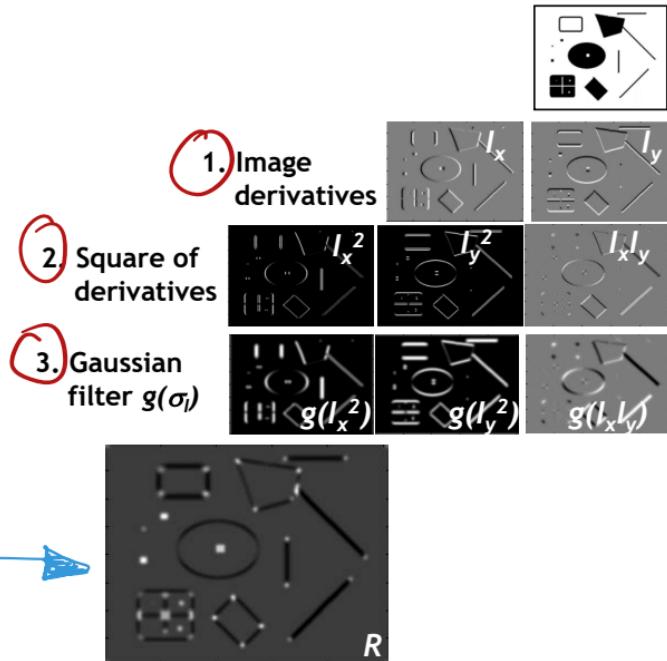
- Cornerness function - two strong eigenvalues

$$R = \det[M(\sigma_I, \sigma_D)] - \alpha[\text{trace}(M(\sigma_I, \sigma_D))]^2$$

not part of Harris

$$= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

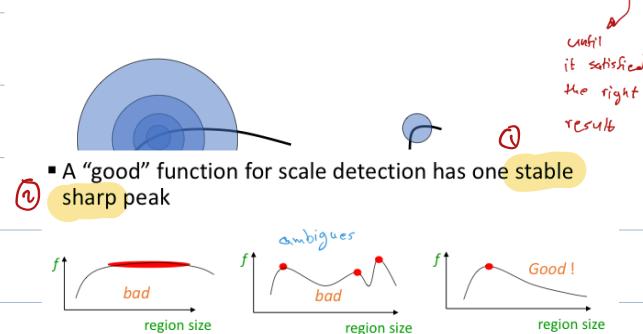
- Perform non-maximum suppression



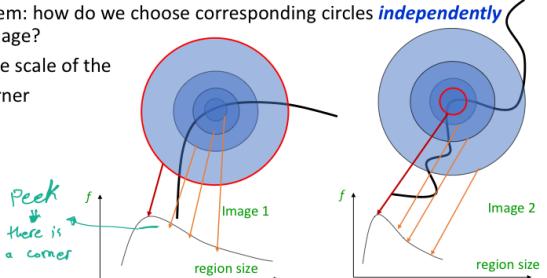
W5 - P2 - Feature detection and matching

We need to solve Scale invariant issue

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



- The problem: how do we choose corresponding circles **independently** in each image?
- Choose the scale of the "best" corner



Important: this scale invariant region size is found in each image **independently**!

- Choices of Kernel: $f = \text{Kernel} * \text{Image}$

2 times derivative

choices of kernels

- (Laplacian) highlight high intensity (like ∇^2)

$$L = \sigma^2 (G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma))$$

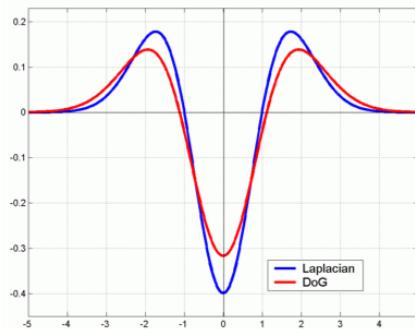
- (Difference of Gaussians)

$$\text{DoG} = G(x, y, k\sigma) - G(x, y, \sigma)$$

where G is the Gaussian function

80

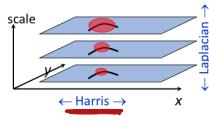
$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



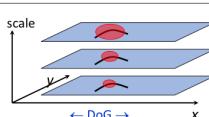
Note: both kernels are invariant to **scale and rotation**

Scale Invariant Detectors

- Harris-Laplacian¹ *combination*
Find local maximum of:
 - Harris corner detector in space (image coordinates)
 - Laplacian in scale



- DoG (from SIFT by Lowe)
Find local maximum of:
 - Difference of Gaussians in space and scale



Scale Invariant Detection: Summary

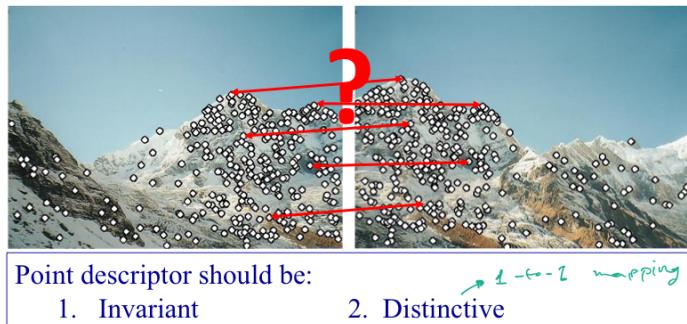
- **Given:** two images of the same scene with a large *scale difference* between them
- **Goal:** find *the same* interest points *independently* in each image
- **Solution:** search for maxima of suitable functions in *scale* and in *space* (over the image)
*the region size
or circle*

Methods:

1. **Harris-Laplacian** [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image
2. **SIFT** [Lowe]: maximize Difference of Gaussians over scale and space

After we extract interest point we need to describe them

- **Descriptor:** Vector that summarizes the content of the keypoint neighborhood.



① Simple descriptors:

interest point

neighborhood around
interest point

Normalized neighborhood
around interest point

Properties:-

- Translation Invariant
- Not scale invariant
- Not rotation invariant
- Somewhat invariant to lighting changes

$$\begin{array}{c} \text{201} \\ - \\ \text{45 56 200} \\ \text{46 201 200} \\ \text{85 101 105} \end{array} \xrightarrow{\text{subtract}} \begin{array}{c} \text{156 145 1} \\ \text{155 0 1} \\ \text{116 100 96} \end{array}$$

② SIFT Algo. :

used for:

① Detect

② Describe

③ Match local features between images

Steps involve in SIFT Algorithm

1. Scale-space Extrema Detection
2. Key point Localization
3. Orientation Assignment
4. Keypoint Descriptor
5. Key point Matching