Assignment no. 2

Exercise 2 (10 points) Consider a one-dimensional classification problem with $X = \mathbb{R}$ and $Y = \{-1, 1\}$. The marginal distribution of labels is given as follows:

$$p(y = -1) = \tfrac{3}{4} \qquad\qquad p(y = +1) = \tfrac{1}{4}$$

The conditional distribution $p(x \mid y)$ is given in the following way:

$$p(x \mid y = -1) = \frac{1}{2\sqrt{\pi}} e^{-\frac{(x+1)^2}{4}} \qquad \text{Gaussian with } \mu = -1 \text{ and } \sigma^2 = 2$$

$$p(x \mid y = +1) = \frac{\sqrt{2}}{\sqrt{\pi}} e^{-2(x-3)^2} \qquad \text{Gaussian with } \mu = 3 \text{ and } \sigma^2 = \tfrac{1}{4}$$

Visualize the marginal distribution $p(x)$ and the conditional distributions $p(y = -1 \mid x)$ and $p(y = +1 \mid x)$. Guess from the visualization of $p(y = -1 \mid x)$ and $p(y = +1 \mid x)$ what the Bayes-optimal classifier is like (**hint:** visualize the two conditional distributions in one plot).

Exercise 3 (20 points) Consider the following one-dimensional regression task: inputs x are uniformly distributed in $[-1, 3]$ and targets y are given as

$$y = f(x) = 0.6x^4 + 2x^3 - 8x^2$$

plus independent normally distributed noise with $\mu = 0$ and $\sigma^2 = 0.09$. What are $\mathrm{E}(y \mid x_0)$ and the unavoidable error $\mathrm{Var}(y \mid x_0)$ in this setting?

Perform polynomial regression to illustrate the bias-variance decomposition. To this end, perform the following steps for each degree $n = 1, \ldots, 7$:

1. Create 200 training sets with $l = 20$ samples each.
2. For each of the training sets, train a polynomial model with degree n and compute the predicted value for $x_0 = 1.8$.
3. Estimate the squared bias and the variance from the 200 predicted values and compute an overall estimate for the expected prediction error for $x_0 = 1.8$.

After having followed these steps, visualize your results appropriately. Discuss how the results illustrate the bias-variance decomposition.

Submission: electronically via Moodle:

`https://moodle.jku.at/jku2015/course/view.php?id=2634`

Please take the submission instructions into account! Deadline: Monday, November 20, 2017, 1:00pm.