Calculus Exam with Multiple-Choice Options

1. If $x = \cos y$ where $y \in \left(0, \dfrac{\pi}{2} \right)$, find $\dfrac{dx}{dy}$.

A. $-\sin y$

B. $\sin y$

C. $-\cos y$

D. $\cos y$

2. Evaluate the limit $\displaystyle \lim_{x \to 0} \dfrac{e^{x} - 1}{x}$.

A. 0

B. 1

C. e

D. -1

3. If $\displaystyle \lim_{x \to 0} \dfrac{\ln(1 + x^{k})}{x} = 5$, find the value of k.

A. 5

B. 1

C. $\dfrac{1}{5}$

D. $\dfrac{1}{2}$

4. Compute the definite integral $\displaystyle \int_{0}^{\ln 2} e^{2x} \, dx$.

A. $\dfrac{1}{2}$

B. $\dfrac{3}{4}$

C. $\dfrac{1}{4}$

D. $\dfrac{3}{2}$

5. If $y = \cos 2x$, find the differential dy.

A. $-2 \sin 2x \, dx$

B. $2 \sin 2x \, dx$

C. $-\sin 2x \, dx$

D. $\sin 2x \, dx$

6. Evaluate the indefinite integral $\displaystyle \int x e^{x} \, dx$.

A. $x e^{x} - e^{x} + C$

B. $x e^{x} + e^{x} + C$

C. $e^{x} - x e^{x} + C$

D. $x^{2} e^{x} + C$

7. Suppose a function f is twice differentiable on the interval $[0, 2]$ and $f''(x) > 0$ for all $x \in (0, 2)$. Determine the concavity of f on $(0, 2)$.

A. Concave up

B. Concave down

C. Linear

D. Neither concave up nor concave down

8. If $f(x) = x^{4}$, calculate $f''(-1)$.

A. 0

B. 12

C. 24

D. 4

9. Find the equation of the normal to the curve $y = x^{3} + 3x^{2} + x$ at the point where $x = 0$.

A. $y = x$

B. $y = -x$

C. $y = 2x$

D. $y = -2x$

10. If $y = \ln(x) \cdot e^{x}$, find $\dfrac{dy}{dx}$ when $x = 1$.

A. 0

B. 1

C. e

D. -1

11. Determine the absolute maximum value of the function $f(x) = -x^{2} + 4x$ on the interval $[0, 3]$.

A. 5

B. 4

C. 7

D. 9

12. Find the slope of the tangent to the curve $y = x \ln x$ at $x = 1$.

A. 0

B. 1

C. -1

D. $\ln e$

13. If $y \cdot \ln x = 1$ where $x > 0$, compute $\dfrac{dy}{dx}$ at $x = e$.

A. 0

B. 1

C. $\dfrac{1}{e}$

D. $-\dfrac{1}{e}$

14. Evaluate $\displaystyle \int_{a}^{b} \sec^{2} x \, dx + \int_{b}^{a} \tan^{2} x \, dx$ where $a, b \in \left(0, \dfrac{\pi}{2} \right)$.

A. 0

B. $a - b$

C. $b - a$

D. $\tan b - \tan a$

15. Find all points on the curve $y^{2} = 4x$ where $\dfrac{dy}{dx} = \dfrac{dx}{dy}$.

A. $(1, 2)$

B. $(2, 4)$

C. $(4, 4)$

D. $(0, 0)$

16. Given $y = a e^{b x}$ and $\dfrac{d^{2} y}{dx^{2}} = y$, determine the value of b^{2}.

A. 1

B. 0

C. -1

D. 2

17. Find the maximum value of the function $f(x) = \sin x + \cos x$ in the interval $\left(0, \dfrac{\pi}{2} \right)$.

A. $\sqrt{2}$

B. 1

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{1}{\sqrt{2}}$

18. If $\sin x + \cos y = 1$ where x and y are acute angles, calculate $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$.

A. 0

B. 1

C. -1

D. $\dfrac{1}{\sqrt{2}}$

19. Given $\dfrac{dz}{d\theta} = \sin^{2} \theta$ and $\dfrac{dy}{d\theta} = \cos^{2} \theta$, find $\dfrac{d^{2} y}{dz^{2}}$ at $\theta = \dfrac{\pi}{4}$.

A. 2

B. $2\sqrt{2}$

C. 4

D. $\sqrt{2}$

20. A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 2 ft/s. How fast is the top of the ladder sliding down the wall when the bottom is 6 feet from the wall?

A. $-\dfrac{12}{5}$ ft/s

B. $-\dfrac{8}{5}$ ft/s

C. $\dfrac{12}{5}$ ft/s

D. $\dfrac{8}{5}$ ft/s

21. A rectangle is inscribed in a circle of radius 5 units. Determine the dimensions of the rectangle with the maximum possible area.

A. Width = 5 units, Height = 5 units

B. Width = $5\sqrt{2}$ units, Height = $5\sqrt{2}$ units

C. Width = 5 units, Height = 5 units

D. Width = 10 units, Height = 0 units

22. Find the equation of the tangent to the curve $y = \ln x$ at the point where $x = e$.

A. $y = x - e$

B. $y = x + e$

C. $y = \dfrac{1}{e} x - 1$

D. $y = \dfrac{1}{e} x + 1$

23. The curve $y = e^{x} + x$ has a tangent at $x = 0$ that intersects the axes at points A and B. Find the ratio in which point A divides the segment OB, where O is the origin.

A. $1:2$ internally

B. $2:1$ internally

C. $1:2$ externally

D. $2:1$ externally

24. Given functions g and k where g is decreasing and k is increasing on the interval $[a, b]$, and $f(x) = g(k(x))$, determine whether f is increasing or decreasing on $[a, b]$.

A. f is increasing on $[a, b]$

B. f is decreasing on $[a, b]$

C. f is constant on $[a, b]$

D. Cannot be determined

25. Let g be twice differentiable on $(0, 1)$ with $g''(x) > 0$ for all $x \in (0, 1)$. Define $f(x) = g(x) + g(1 - x)$. Decide whether f is increasing or decreasing on $\left(0, \dfrac{1}{2} \right)$.

A. f is increasing on $\left(0, \dfrac{1}{2} \right)$

B. f is decreasing on $\left(0, \dfrac{1}{2} \right)$

C. f is constant on $\left(0, \dfrac{1}{2} \right)$

D. Cannot be determined

Note: Choose the correct option for each question without any external assistance.