Converted Questions and Options:

Question 1:

If $x = \sec y$, where $y \in \left(\dfrac{\pi}{2}, \pi \right)$, then $\dfrac{dx}{dy} =$ ...

Options:

A. $-x \sqrt{x^{2} - 1}$

B. $x \sqrt{x^{2} - 1}$

C. $-x \sqrt{x^{2} + 1}$

D. $x \sqrt{x^{2} + 1}$

Question 2:

Evaluate $\displaystyle \lim_{x \to 1} \dfrac{e^{x} - e}{x - 1} =$ ...

Options:

A. e

B. $-e$

C. 1

D. -1

Question 3:

If $\displaystyle \lim_{x \to 0} \dfrac{(\ln(x + 1))^{k}}{x} = 4$, then $k =$ ...

Options:

A. 16

B. 4

C. 8

D. 2

Question 4:

If $f(x) = \int \dfrac{1 - (\ln x)^{2}}{x} \, dx$ with $f(1) = 0$, then $f(e) =$ ...

Options:

A. $\dfrac{2}{3}$

B. $-\dfrac{2}{3}$

C. $\dfrac{1}{3}$

D. $-\dfrac{1}{3}$

Question 5:

If $y = \sin(3x)$, then the differential of y is ...

Options:

A. $3 \cos(3x) \, dx$

B. $3 \cos(x) \, dx$

C. $-3 \cos(3x) \, dx$

D. $-3 \cos(x) \, dx$

Question 6:

Evaluate $\displaystyle \int 9^{2x} \ln x \, dx =$ ... $+ \, C$, where C is a constant.

Options:

A. $\dfrac{x^{3}}{2} (\ln x^{3} - 2)$

B. $\dfrac{x^{3}}{2} (\ln x - 2)$

C. $\dfrac{x^{3}}{2} (\ln x^{3} + 2)$

D. $\dfrac{x^{3}}{2} (\ln x + 2)$

Question 7:

If the function f is twice differentiable on the interval $[-1, 1]$ where $f'(x)$ is increasing on $(-1, 0)$ and $f'(x)$ is decreasing on $(0, 1)$, then which statement must be true?

Options:

A. $(0, f(0))$ is an inflection point.

B. $f(0)$ is a local maximum value.

C. The function f is increasing on $(0, 1)$.

D. The function f is decreasing on $(0, 1)$.

Question 8:

If $f\left(\dfrac{1}{2} x \right) = |x|^{3}$, then $f''(-1) =$ ...

Options:

A. 48

B. 14

C. 1

D. -48

Question 9:

Given $y = f(x)$ where $y = 3n^{2} + 7$ and $6n^{2} x + n = 1$, then the normal to the curve at the point with x-coordinate zero is ...

Options:

A. Parallel to the straight line $y = x$

B. Parallel to the X-axis

C. Parallel to the Y-axis

D. Parallel to the straight line $y = -x$

Question 10:

If $y = e^{x} \sec x$, then $\dfrac{dy}{dx} =$ ... at $x = 0$.

Options:

A. 1

B. -1

C. 2

D. -2

Question 11:

The function $f: [-3, -1] \rightarrow \mathbb{R}$ where $f(x) = x + \dfrac{a}{x}$. If the absolute maximum value of f equals -2 and f is increasing on the interval $(-3, -1)$, then $a =$ ...

Options:

A. 1

B. -1

C. 2

D. -2

Question 12:

The slope of the tangent to the curve $y = 5x \log_{5}(x + 1)$ at $x = 0$ equals ...

Options:

A. $\log_{5} e$

B. $\ln 5$

C. 0

D. $5 \log_{5} e$

Question 13:

If $y \times \log_{e}(x^{3}) = 1$ where $x > 1$, then $\dfrac{dy}{dx} =$ ... at $x = 3$.

Options:

A. 1

B. 0

C. $3 \ln 3$

D. $\ln 3$

Question 14:

If $a, b \in \left(0, \dfrac{\pi}{2} \right)$, then $\displaystyle \int_{a}^{b} \tan^{2} x \, dx + \int_{b}^{a} \sec^{2} x \, dx =$ ...

Options:

A. $a - b$

B. $b - a$

C. 1

D. $\tan b - \tan a$

Question 15:

If $y > 0$, then the point on the curve $y^{2} = 8x$ at which $\dfrac{dy}{dx} = \dfrac{dx}{dy}$ is ...

Options:

A. $(2, 4)$

B. $\left(\dfrac{1}{2}, 2 \right)$

C. $(1, 2\sqrt{2})$

D. $(0, 0)$

Question 16:

If $y = a e^{b x}$ and $\dfrac{d^{2} y}{dx^{2}} = y$, then $b^{2} =$ ...

Options:

A. 1

B. 0

C. -1

D. 2

Question 17:

The maximum value of the function $f(x) = \sin x + \cos x$ in the interval $\left(0, \dfrac{\pi}{2} \right)$ is ...

Options:

A. 2

B. $\dfrac{1}{2}$

C. $\sqrt{2}$

D. $1 + \dfrac{\sqrt{3}}{2}$

Question 18:

If $\sin x \cos y = \dfrac{1}{2}$ where x and y are acute angles, then $\dfrac{dy}{dx} =$ ... at $x = \dfrac{\pi}{4}$.

Options:

A. 1

B. -1

C. 0

D. $\dfrac{1}{2}$

Question 19:

If $\dfrac{dz}{d\theta} = \cos^{2} \theta$, $\dfrac{dy}{d\theta} = \sin^{2} \theta$, then $\dfrac{d^{2} y}{dz^{2}} =$ ... at $\theta = \dfrac{\pi}{8}$.

Options:

A. $4\sqrt{2}$

B. 4

C. $2\sqrt{2}$

D. 2

Question 20:

A candle is placed at 3 cm from a wooden block of height L cm, and the wooden block is at a distance 6 cm from a vertical wall. If the length of the candle (x) is decreasing at a rate of 3 cm/hr, then the rate of change of the length of the shadow of the block (y) on the wall is ... cm/hr.

Options:

A. 6

B. -6

C. -3

D. 3

Question 21:

A point $A(x, y)$ is moving along the straight line $y = -2x + 4$ where $0 \leq x \leq 1$. Point B is the projection of A onto the X-axis, and $C(-2, 0)$. The smallest area of triangle ΔABC is ... square units.

Options:

A. 3

B. 4

C. 2

D. 5

Question 22:

If the equation of the tangent to the curve $y = -\ln x$ at the point (a, b) which lies on the curve is $y = m x$, then $a =$ ...

Options:

A. e

B. $\dfrac{3}{2} e$

C. $\dfrac{1}{2} e$

D. $2 e$

Question 23:

The curve $e^{x} + y = 3x - 2$ has a tangent at the point $A(1, -1)$ intersecting the coordinate axes at points B and C respectively. Then A divides $\overline{BC}$ in the ratio ...

Options:

A. $1 : 2$ internally

B. $2 : 1$ internally

C. $1 : 2$ externally

D. $2 : 1$ externally

Question 24:

The figure represents the curves of the functions g and k on the interval $[A, B]$. If f is a function where $f(x) = (g \circ k)(x)$, then the correct statement is ...

Options:

A. The function f is decreasing on the interval (A, B).

B. The function f is increasing on the interval (A, B).

C. The function f is increasing on the interval (A, C) only.

D. The function f is decreasing on the interval (A, C) only.

Question 25:

In the interval $(0, 1)$, the function g is twice differentiable and $g''(x) < 0$. If f is a function where $f(x) = g(x) + g(1 - x)$, then the correct statement is ...

Options:

A. f is decreasing on $\left(\dfrac{1}{2}, 1 \right)$.

B. f is decreasing on $(0, 1)$.

C. f is increasing on $(0, 1)$.

D. f is increasing on $\left(\dfrac{1}{2}, 1 \right)$.
