Calculus Exam with Multiple-Choice Options
1. If \(x = \cos y \) where \(y \in \left(0, \dfrac{\pi}{2} \right) \), find \(\dfrac{dx}{dy} \).
A. \(-\sin y \)
B. \(\sin y \)
C. \(-\cos y \)
D. \(\cos y \)
2. Evaluate the limit \(\displaystyle \lim_{x \to 0} \dfrac{e^{x} - 1}{x} \).
A. \(0 \)
B. \(1 \)
C. \(e \)
D. \(-1 \)
3. If \(\displaystyle \\lim_{x \to 0} \dfrac{\\ln(1 + x^{k})}{x} = 5 \), find the value of \(k \).
A. \(5 \)
B. \(1 \)
C. \(\dfrac{1}{5} \)
D. \(\dfrac{1}{2} \)
4. Compute the definite integral \(\displaystyle \int_{0}^{\ln 2} e^{2x} dx \).

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5. If $(y = \cos 2x)$, find the differential (dy).

6. Evaluate the indefinite integral $\ (\ x e^{x} \ , dx \).$

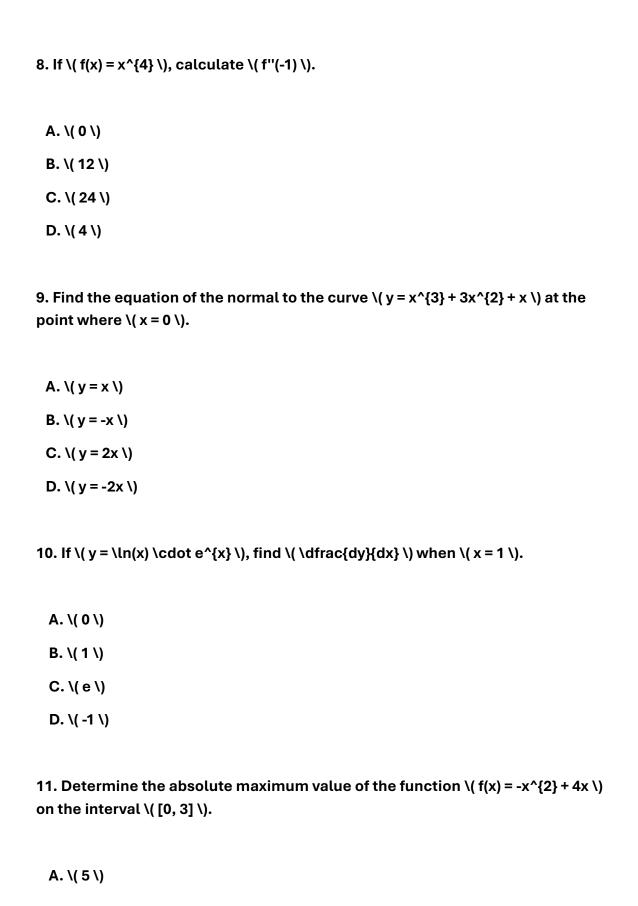
B.
$$(x e^{x} + e^{x} + C)$$

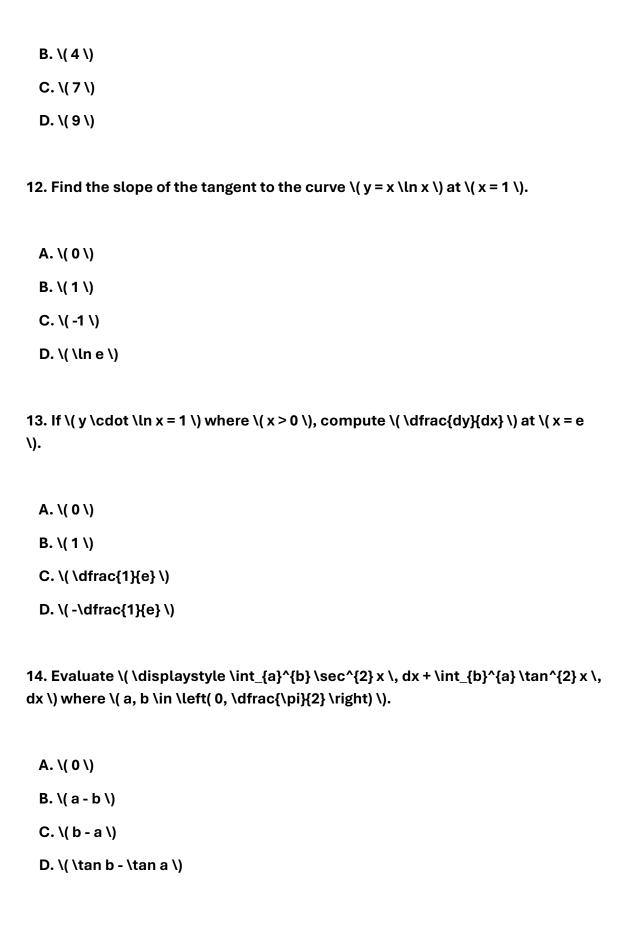
C.
$$(e^{x} - x e^{x} + C)$$

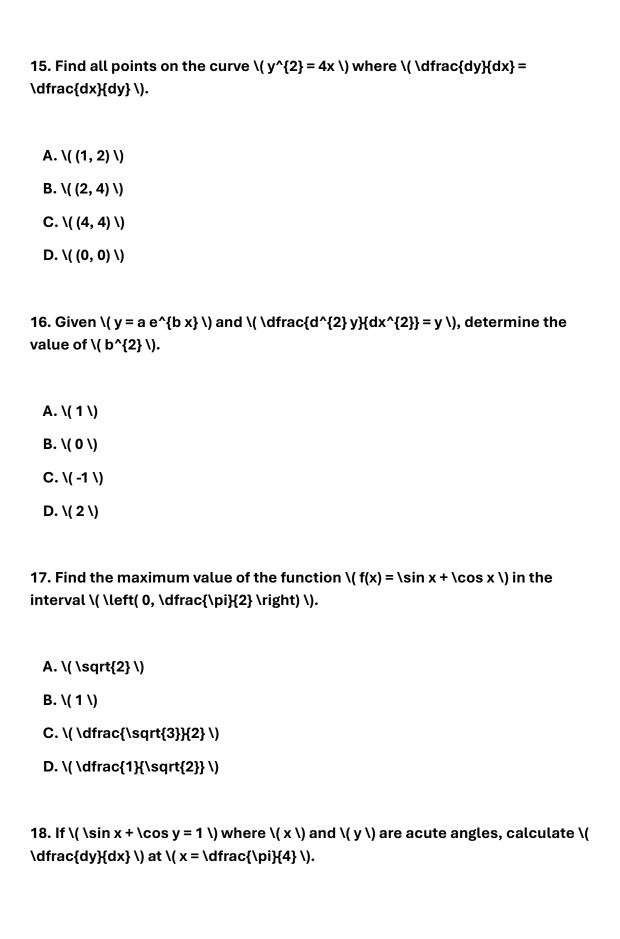
D.
$$(x^{2} e^{x} + C)$$

7. Suppose a function \(f \) is twice differentiable on the interval \([0, 2] \) and \(f''(x) > 0 \) for all \(x \in (0, 2) \). Determine the concavity of \(f \) on \((0, 2) \).

- A. Concave up
- B. Concave down
- C. Linear
- D. Neither concave up nor concave down







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A. \( 0 \)
B. \( 1 \)
C. \( -1 \)
D. \( \dfrac{1}{\sqrt{2}} \)
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19. Given \(\dfrac{dz}{d\theta} = \sin^{2} \theta \) and \(\dfrac{dy}{d\theta} = \cos^{2} \theta \), find \(\dfrac{d^{2} y}{dz^{2}} \) at \(\theta = \dfrac{\pi}{4} \).

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A. \( 2 \)
B. \( 2\sqrt{2} \)
C. \( 4 \)
D. \( \sqrt{2} \)
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20. A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 2 ft/s. How fast is the top of the ladder sliding down the wall when the bottom is 6 feet from the wall?

21. A rectangle is inscribed in a circle of radius 5 units. Determine the dimensions of the rectangle with the maximum possible area.

A. Width =
$$\ (5 \)$$
 units, Height = $\ (5 \)$ units

B. Width = $\ (5 \)$ units, Height = $\ (5 \)$ units

- C. Width = (5) units, Height = (5) units
- D. Width = $\(10\)$ units, Height = $\(0\)$ units

22. Find the equation of the tangent to the curve \($y = \ln x \)$ at the point where \($x = e \)$.

A.
$$(y = x - e)$$

B.
$$(y = x + e)$$

C.
$$(y = \frac{1}{e} x - 1)$$

D.
$$(y = \frac{1}{e} x + 1)$$

23. The curve \($y = e^{x} + x \)$ has a tangent at \($x = 0 \)$ that intersects the axes at points \(A \) and \(B \). Find the ratio in which point \(A \) divides the segment \(OB \), where \(O \) is the origin.

- A. \(1:2 \) internally
- B. \(2:1 \) internally
- C. \(1:2 \) externally
- D. \(2:1 \) externally

24. Given functions \(g \) and \(k \) where \(g \) is decreasing and \(k \) is increasing on the interval \([a, b] \), and \(f(x) = g(k(x)) \), determine whether \(f \) is increasing or decreasing on \([a, b] \).

- A. $\ (f \)$ is increasing on $\ ([a, b] \)$
- B. $\ (f \)$ is decreasing on $\ ([a, b] \)$
- C. $\ (f \)$ is constant on $\ ([a, b] \)$

D. Cannot be determined

25. Let \(g \) be twice differentiable on \((0, 1) \) with \(g''(x) > 0 \) for all \(x \in (0, 1) \). Define \((f(x) = g(x) + g(1 - x) \). Decide whether \((f \) is increasing or decreasing on \(\\ left(0, \\ dfrac{1}{2} \\ right) \).

- A. $\ (f \)$ is increasing on $\ (\left\{ 0, \frac{1}{2} \right\} \)$
- B. $\ (f \)$ is decreasing on $\ (\left\{ 0, \left(1\right) \right\} \)$
- C. $\(f\)$ is constant on $\(\left(\frac{1}{2} \right) \)$
- D. Cannot be determined

^{**}Note:** Choose the correct option for each question without any external assistance.