Math 1

CH 01: Logic and Proofs

Content

The Foundations: Logic and Proofs
Propositional Logic
Applications of Propositional Logic
Propositional Equivalences
Predicates and Quantifiers
Nested Quantifiers
Rules of Inference
Introduction to Proofs
Proof Methods and Strategy

Propositional Logic

- **Logic rules** are used to distinguish between valid and invalid mathematical arguments.

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
  - Propositions are the basic building blocks of logic.

Propositional Logic

- **EXAMPLE 1:** All the following declarative sentences are propositions.

Proposition	T/F
Washington, D.C., is the capital of the United States of America	True
Qena is the capital of Canada	False
1 + 1 = 2	True
2 + 2 = 3	False

Propositional Logic

- **EXAMPLE 2:** Some sentences that are not propositions.

Proposition	
What time is it?	not declarative sentence
Read this carefully.	not declarative sentence
$x + 1 = 2$.	neither true nor false
$x + y = z$.	neither true nor false

Propositional Logic

Logical Operations

- Negation ($\neg p$)
- Conjunction ($p \land q$)
- Disjunction ($p \lor q$)
- Exclusive ($p \oplus q$)
- Conditional ($p \rightarrow q$)

p	$\neg p$
T	F
F	T

p	q	$p \land q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Propositional Logic

- **<u>EXAMPLE 7:</u>** Let p be the statement *"Maria learns discrete mathematics"* and q the statement *"Maria will find a good job."*
  Express the statement $p \rightarrow q$ as a statement in English.
- **<u>Solution 7:</u>**
  "If Maria learns discrete mathematics, then she will find a good job."

Propositional Logic

- **<u>EXAMPLE 8:</u>**
  What is the value of the variable x after the statement
  $$if\ 2 + 2 = 4\ then\ x := x + 1$$
  if $x = 0$ before this statement is encountered?
- **<u>Solution 8:</u>**
  Because $2 + 2 = 4$ is true, the assignment statement $x := x + 1$ is executed. Hence, x has the value $0 + 1 = 1$ after this statement is encountered.

Propositional Logic

Related conditional statements

$p \rightarrow q$

CONVERSE

CONTRAPOSITIVE

INVERSE

$q \rightarrow p$

$\neg q \rightarrow \neg p$

$\neg p \rightarrow \neg q$

Only the contrapositive always has the same truth value as $p \rightarrow q$.

Propositional Logic

- The **biconditional** statement $p \leftrightarrow q$ is the proposition *"p if and only if q."*
  - The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values and is false otherwise.

- <u>**Example 10:**</u>
  "You can take the flight if and only if you buy a ticket."

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Propositional Logic

- **EXAMPLE 11:**
  Construct the truth table of the compound proposition
  $(p \lor \neg q) \rightarrow (p \land q)$.

TABLE 7 The Truth Table of $(p \lor \neg q) \rightarrow (p \land q)$.

p	q	$\neg q$	$p \lor \neg q$	$p \land q$	$(p \lor \neg q) \rightarrow (p \land q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Propositional Logic

- Precedence of Logical Operators

TABLE 8 Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Exercises

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

Sentence	
$2 + 3 = 5$	
$5 + 7 = 10$	
$x + 2 = 11$.	
Answer this question	

Exercises

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

Sentence	**Answer**
$2 + 3 = 5$	true proposition
$5 + 7 = 10$	false proposition
$x + 2 = 11$.	not a proposition
Answer this question	not a proposition

Exercises

8. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

Sentence	
Smartphone B has the most RAM of these three smartphones.	
Smartphone C has more ROM or a higher resolution camera than Smartphone B.	
Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.	
If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.	
Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.	

Exercises

8. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

Sentence	**Answer**
Smartphone B has the most RAM of these three smartphones.	True
Smartphone C has more ROM or a higher resolution camera than Smartphone B.	True
Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.	False
If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.	False
Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.	False

Exercises

14. Let p, q, and r be the propositions

p :You have the flu; q :You miss the final examination; r :You pass the course.

Express each of these propositions as an English sentence.

Sentence	
$p \rightarrow q$	
$\neg q \leftrightarrow r$	
$q \rightarrow \neg r$	
$p \vee q \vee r$	
$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$	
$(p \wedge q) \vee (\neg q \wedge r)$	

Exercises

14. Let p, q, and r be the propositions

p :You have the flu; q :You miss the final examination; r :You pass the course.

Express each of these propositions as an English sentence.

Sentence	**Answer**
$p \rightarrow q$	If you have the flu, then you miss the final exam.
$\neg q \leftrightarrow r$	You do not miss the final exam if and only if you pass the course.
$q \rightarrow \neg r$	If you miss the final exam, then you do not pass the course.
$p \vee q \vee r$	You have the flu, or miss the final exam, or pass the course.
$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$	if you have the flu, then you do not pass the course or if you miss the final exam then you do not pass the course.
$(p \wedge q) \vee (\neg q \wedge r)$	You have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

Exercises

18. Determine whether these biconditionals are true or false.

Sentence	
1 + 1 = 3 if and only if monkeys can fly	
0 > 1 if and only if 2 > 1.	

Exercises

18. Determine whether these biconditionals are true or false.

Sentence	Answer
1 + 1 = 3 if and only if monkeys can fly	This is $F \leftrightarrow F$, which is true.
0 > 1 if and only if 2 > 1.	This is $F \leftrightarrow T$, which is false.

Exercises

19. Determine whether each of these conditional statements is true or false.

Sentence	
If $1 + 1 = 2$, then $2 + 2 = 5$.	
If $1 + 1 = 3$, then $2 + 2 = 4$.	

Exercises

19. Determine whether each of these conditional statements is true or false.

Sentence	Answer
If $1 + 1 = 2$, then $2 + 2 = 5$.	$T \rightarrow F$, False
If $1 + 1 = 3$, then $2 + 2 = 4$.	$F \rightarrow T$, True

Exercises

30. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Exercises

30. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Converse	Contrapositive	Inverse
If I stay home, then it will snow tonight	If I do not stay at home, then it will not snow tonight.	If it does not snow tonight, then I will not stay home.
Whenever it is a sunny summer day, I go to the beach	Whenever it is not a sunny summer day, I do not go to the beach.	I do not go to the beach. Whenever it is not a sunny day,
If I sleep until noon, then I stayed up late.	If I do not sleep until noon, then I do not stay up late.	If I don't stay up late, then I don't sleep until noon.

Exercises

31. How many rows appear in a truth table for each of these compound propositions?

proposition	
$p \rightarrow \neg p$	
$(p \vee \neg r) \wedge (q \vee \neg s)$	
$q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$	
$(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$	

Exercises

31. How many rows appear in a truth table for each of these compound propositions?

proposition	**Answer**
$p \rightarrow \neg p$	$2^1 = 2$
$(p \lor \neg r) \land (q \lor \neg s)$	$2^4 = 16$
$q \lor p \lor \neg s \lor \neg r \lor \neg t \lor u$	$2^6 = 64$
$(p \land r \land t) \leftrightarrow (q \land t)$	$2^4 = 16$

Exercises

33. Construct a truth table for each of these compound propositions.

$a)\ p \land \neg p$

$b)\ p \lor \neg p$

$c)\ (p \lor \neg q) \rightarrow q$

$d)\ (p \lor q) \rightarrow (p \land q)$

$e)\ (p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$f)\ (p \rightarrow q) \rightarrow (q \rightarrow p)$

Exercises

33. Construct a truth table for each of these compound propositions.

$a)\ p \land \neg p$

$b)\ p \lor \neg p$

p	$\neg p$	$p \land \neg p$	$p \lor \neg p$
T	F	F	T
F	T	F	T

Exercises

33. Construct a truth table for each of these compound propositions.

a) $(p \lor \neg q) \rightarrow q$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\neg\boldsymbol{q}$	$\boldsymbol{p} \lor \neg\boldsymbol{q}$	$(\boldsymbol{p} \lor \neg\boldsymbol{q}) \rightarrow \boldsymbol{q}$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

Exercises

33. Construct a truth table for each of these compound propositions.

$a)\ (p \vee q) \rightarrow (p \wedge q)$

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Exercises

33. Construct a truth table for each of these compound propositions.

a) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Exercises

33. Construct a truth table for each of these compound propositions.

$a)\ (p \rightarrow q) \rightarrow (q \rightarrow p)$

$\boldsymbol{p}$	$\boldsymbol{q}$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Exercises

41. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

Exercises

41. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Exercises

42. Explain, without using a truth table, why

$$(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$$

is true when p, q, and r have the same truth value and it is false otherwise.

Exercises

42. Explain, without using a truth table, why

$$(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$$

is true when $p, q,$ and r have the same truth value and it is false otherwise.

This statement is true if and only if all three clauses, $p \lor \neg q$, $q \lor \neg r$, and $r \lor \neg p$ are true. Suppose p, q , and r are all true. Because each clause has an unnegated variable, each clause is true. Similarly, if p, q , and r are all false, then because each clause has a negated variable, each clause is true.

On the other hand, if one of the variables is true and the other two false, then the clause containing the negation of that variable will be false, making the entire conjunction false; and similarly, if one of the variables is false and the other two true, then the clause containing that variable unnegated will be false, again making the entire conjunction false.

Exercises

46. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

if x + 2 = 3 then x := x + 1	
if (x + 1 = 3) OR (2x + 2 = 3) then x := x + 1	

Exercises

46. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

if x + 2 = 3 then x := x + 1	The condition is true, the statement is executed, so x is incremented and now has the value 2.
if (x + 1 = 3) OR (2x + 2 = 3) then x := x + 1	The condition is false, the statement is not executed, so x is not incremented and now still has the value 1.

Exercises

- In fuzzy logic, a proposition has a truth value that is a number between 0 and 1, inclusive.
- A proposition with a truth value of 0 is false and one with a truth value of 1 is true.
- Truth values that are between 0 and 1 indicate varying degrees of truth.
  - For instance, the truth value 0.8 can be assigned to the statement “Fred is happy,” because Fred is happy most of the time.
  - The truth value 0.4 can be assigned to the statement “John is happy,” because John is happy slightly less than half the time.

Exercises

49. The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. What are the truth values of the statements “Fred is not happy” and “John is not happy?”

Exercises

49. The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. What are the truth values of the statements “Fred is not happy” and “John is not happy?”

For "Fred is not happy," the truth value is 1 - 0.8 = 0.2.

For "John is not happy," the truth value is 1- 0.4 = 0.6.

Exercises

50. The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions. What are the truth values of the statements “Fred and John are happy” and “Neither Fred nor John is happy”?

Exercises

50. The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions. What are the truth values of the statements “Fred and John are happy” and “Neither Fred nor John is happy”?

For "Fred and John are happy," → min(0.8; 0.4) = 0.4

For "Neither Fred nor John is happy" → min(0.2; 0.6) = 0.2

Tasks

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19 (c, d)
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Section 1
46 (c, d, e)
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51 fuzzy logic

Content

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Applications of Propositional Logic

- Translating English Sentences
- System Specifications
- Boolean Searches
- Logic Puzzles
- Logic Circuits

Applications of Propositional Logic

Translating English Sentences

System Specifications

Boolean Searches

Logic Puzzles

Logic Circuits

EXAMPLE 1:

How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Solution:

$a =$ *"You can access the Internet from campus,"*

$b =$ "$You\ are\ a\ computer\ science\ major,$"

$c =$ "$You\ are\ a\ freshman,$"

"$only\ if$" $is\ one\ way\ a\ conditional\ statement$

$$a \rightarrow (b \vee \neg c).$$

Applications of Propositional Logic

- Translating English Sentences
- **System Specifications**
- Boolean Searches
- Logic Puzzles
- Logic Circuits

EXAMPLE 3:
Express the specification using logical connectives.

"The automated reply cannot be sent when the file system is full"

Solution:
$p =$ *"The automated reply can be sent"*
$q =$ *"The file system is full."*

$$q \rightarrow \neg p.$$

Applications of Propositional Logic

- Translating English Sentences
- **System Specifications**
- Boolean Searches
- Logic Puzzles
- Logic Circuits

Check the consistency of the system specification (no conflictions).

We have to prove that **all the specifications be true**.

Applications of Propositional Logic

Example 4: Determine if these system specifications are consistent:

- *"The diagnostic message is stored in the buffer or it is retransmitted."*
- *"The diagnostic message is not stored in the buffer."*
- *"If the diagnostic message is stored in the buffer, then it is retransmitted."*

Solution: let

- p = "The diagnostic message is stored in the buffer"
- q = "The diagnostic message is retransmitted."

Applications of Propositional Logic

The specification can be written as:

- $p \lor q$ = *"The diagnostic message is stored in the buffer or it is retransmitted."*
- $\neg p$ = *"The diagnostic message is not stored in the buffer."*
- $p \to q$ = *"If the diagnostic message is stored in the buffer, then it is retransmitted."*

1. To make $\neg p$ true, p must be **false**.
2. To make $p \lor q$ true, we have p is **false**, then we set q to be **true**.
3. To make $p \to q$ true, we have p is **false** and q is **true**. So, **false** $\to$ **true = true**.

From 1, 2, and 3, we conclude that the specification is **consistent**.

Applications of Propositional Logic

Translating English Sentences

System Specifications

Boolean Searches

Logic Puzzles

Logic Circuits

EXAMPLE 6: Web Page Searching, Information retrieval and Databases

Using Boolean searching to find Web pages about universities in New Mexico, we can look for pages matching
$NEW\ AND\ MEXICO\ AND\ UNIVERSITIES$.

To find pages that deal with universities in New Mexico or Arizona, we can search for pages matching
$(NEW\ AND\ MEXICO\ OR\ ARIZONA)\ AND\ UNIVERSITIES$.

What are Google Search Operators?

Applications of Propositional Logic

Translating English Sentences

System Specifications

Boolean Searches

Logic Puzzles

Logic Circuits

EXAMPLE 7:

An island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B if A says *"B is a knight"* and B says *"The two of us are opposite types?"*

Solution: …TASK

Example 8: TASK … read and comprehend

Applications of Propositional Logic

- Translating English Sentences
- System Specifications
- Boolean Searches
- Logic Puzzles
- **Logic Circuits**

EXAMPLE 9:

Determine the output for the combinatorial circuit in Figure 2.

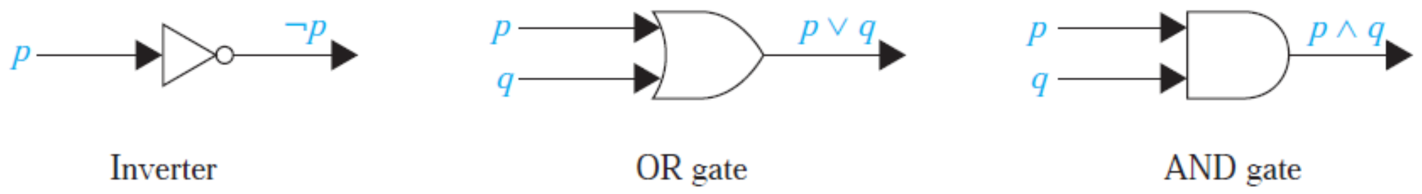


FIGURE 1 Basic logic gates.

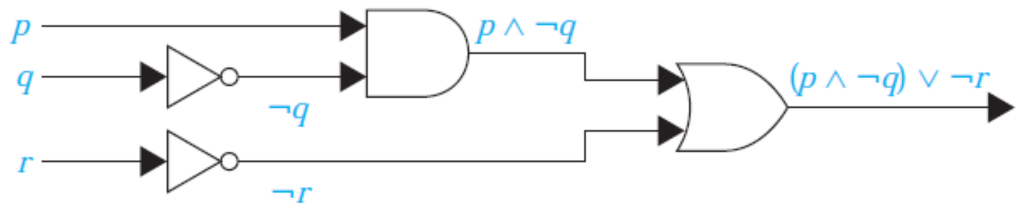


FIGURE 2 A combinatorial circuit.

Exercises

Translate the given statement into propositional logic using the propositions provided.

1. You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of
e: "$You\ can\ edit\ a\ protected\ Wikipedia\ entry$" and
a: "$You\ are\ an\ administrator$."

Exercises

1. You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of
   e: "$You\ can\ edit\ a\ protected\ Wikipedia\ entry$" and
   a: "$You\ are\ an\ administrator.$"

$e \rightarrow a$ (You can edit a protected Wikipedia entry, then you are an admin)

or

$\neg a \rightarrow \neg e$ (if you are not an admin, you cannot edit a Wikipedia entry)

Exercises

3. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of
g: "$You\ can\ graduate,$"
m: "$You\ owe\ money\ to\ the\ university,$"
r: "$You\ have\ completed\ the\ requirements\ of\ your\ major,$" and
b: "$You\ have\ an\ overdue\ library\ book.$"

Exercises

3. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of
g: "*You can graduate,*"
m: "*You owe money to the university,*"
r: "*You have completed the requirements of your major,*" and
b: "*You have an overdue library book.*"

$$\boxed{g \rightarrow (r \land (\neg m) \land (\neg b)).}$$

Exercises

8. Express these system specifications using the propositions

p "*The user enters a valid password,*" q "*Access is granted,*" and

r "*The user has paid the subscription fee*"

and logical connectives (including negations).

Sentence	**Proposition**
"The user has paid the subscription fee, but does not enter a valid password."	
"Access is granted whenever the user has paid the subscription fee and enters a valid password."	
"Access is denied if the user has not paid the subscription fee."	
"If the user has not entered a valid password but has paid the subscription fee, then access is granted."	

Exercises

8. Express these system specifications using the propositions

p *"The user enters a valid password,"* q *"Access is granted,"* and

r *"The user has paid the subscription fee"*

and logical connectives (including negations).

Sentence	Proposition
"The user has paid the subscription fee, but does not enter a valid password."	*"but"* means "and": $r \wedge \neg p$.
"Access is granted whenever the user has paid the subscription fee and enters a valid password."	*"whenever" means "if"*: $(r \wedge p) \rightarrow q$.
"Access is denied if the user has not paid the subscription fee."	*the negation of* q *, so we have* : $\neg r \rightarrow \neg q$.
"If the user has not entered a valid password but has paid the subscription fee, then access is granted."	$(\neg p \wedge r) \rightarrow q$.

Exercises

9. Are these system specifications consistent? “The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.”

Exercises

9. Are these system specifications consistent? “The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.”

The system is in multiuser state	p
it is operating normally	q
the kernel is functioning	r
the system is in interrupt mode	s

Exercises

1. The system is in multiuser state if and only if it is operating normally.
   - $p \leftrightarrow q$ Both p and q be $true$, or both be $false$.
2. If the system is operating normally, the kernel is functioning.
   - $q \rightarrow r$ Anything except $q = true$ and $r = false$.
3. The kernel is not functioning or the system is in interrupt mode.
   - $\neg r \vee s$ $r = false$, or $s = true$.
4. If the system is not in multiuser state, then it is in interrupt mode.
   - $\neg \mathrm{p} \rightarrow s$ Anything except $p = false$ and $s = false$
5. The system is not in interrupt mode.
   - $\neg s$ s must be $false$

The system is in multiuser state	p
it is operating normally	q
the kernel is functioning	r
the system is in interrupt mode	s

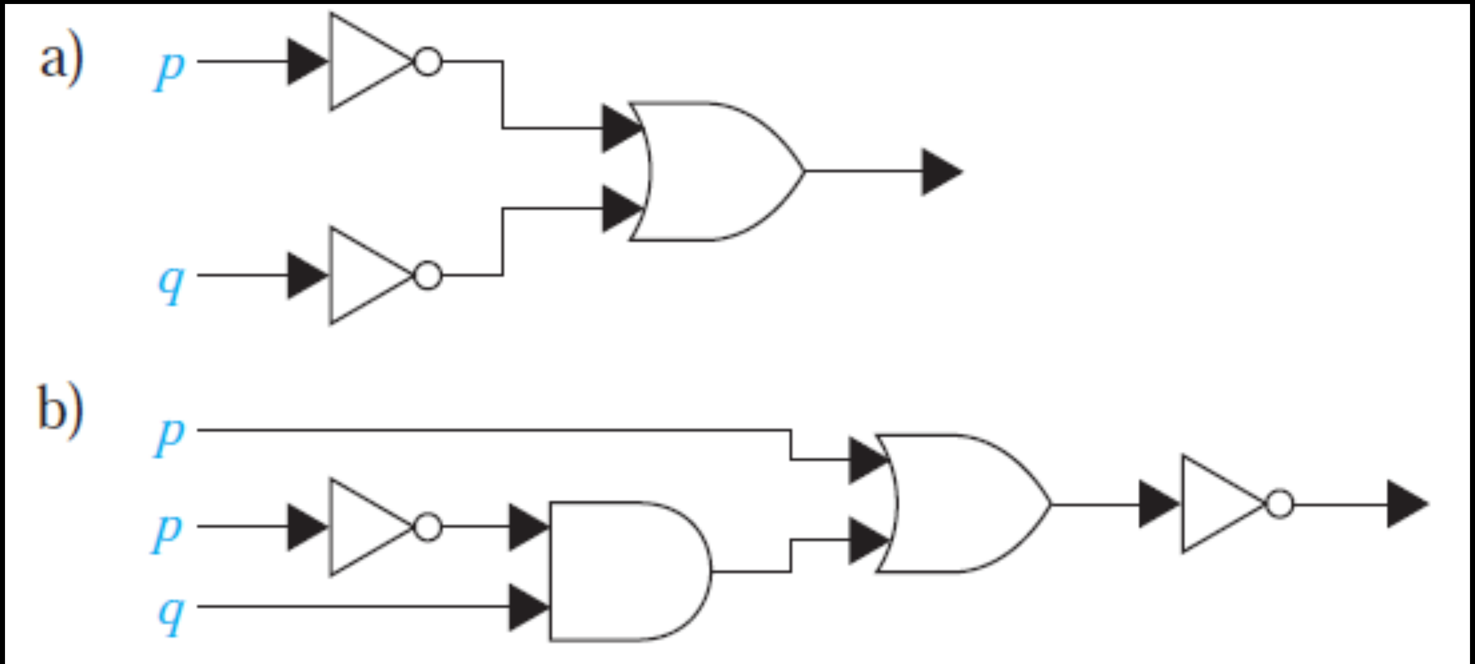
Exercises

- [In step 5 and 4] If we set $s = false$, then we set $p = true$.
- [In step 4 and 3] Since we have $s = false$, then we set $r = false$.
- [In step 3 and 2] Since we have $r = false$, then we set $q = false$.
- [In step 2, 1, and 4] We have $q = false$, but we have $p = true$

So, these specifications are
not consistent

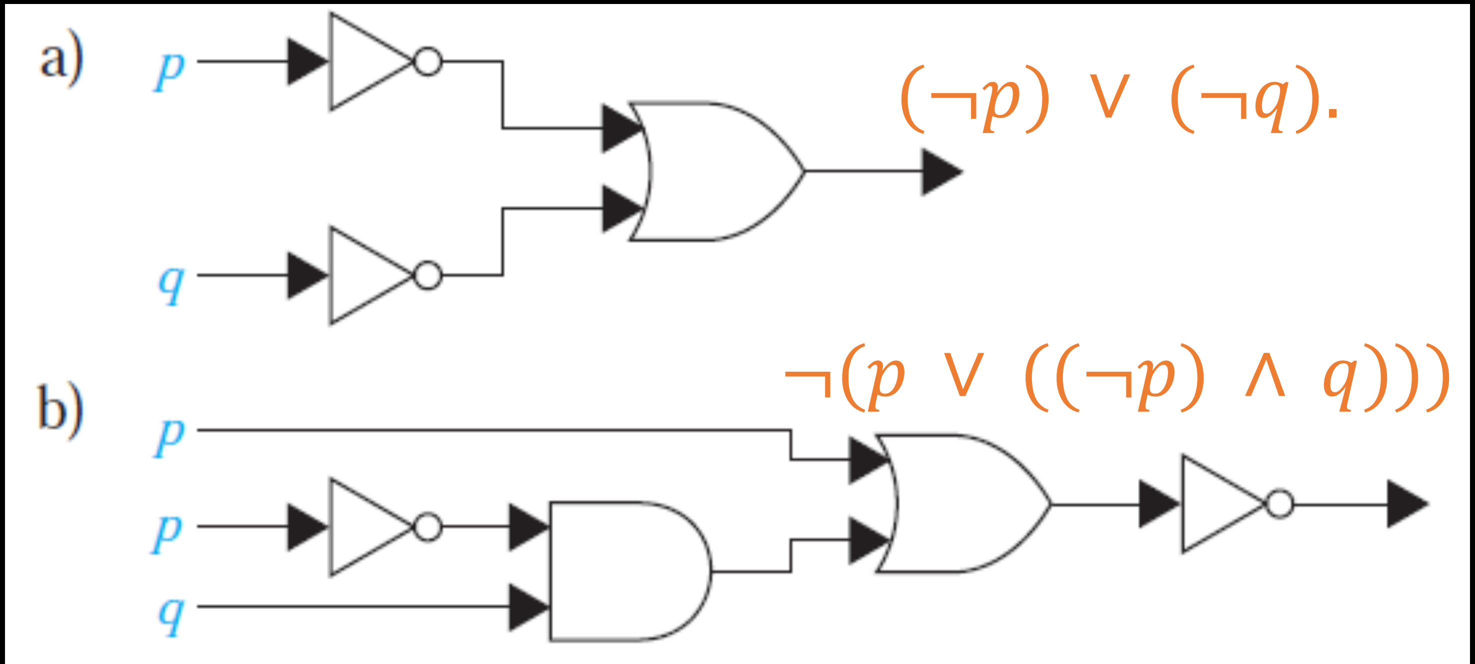
Exercises

44. Find the output of each of these combinatorial circuits.



Exercises

44. Find the output of each of these combinatorial circuits.



Exercises

47. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$$((\neg p \lor \neg r) \land \neg q) \lor (\neg p \land (q \lor r))$$

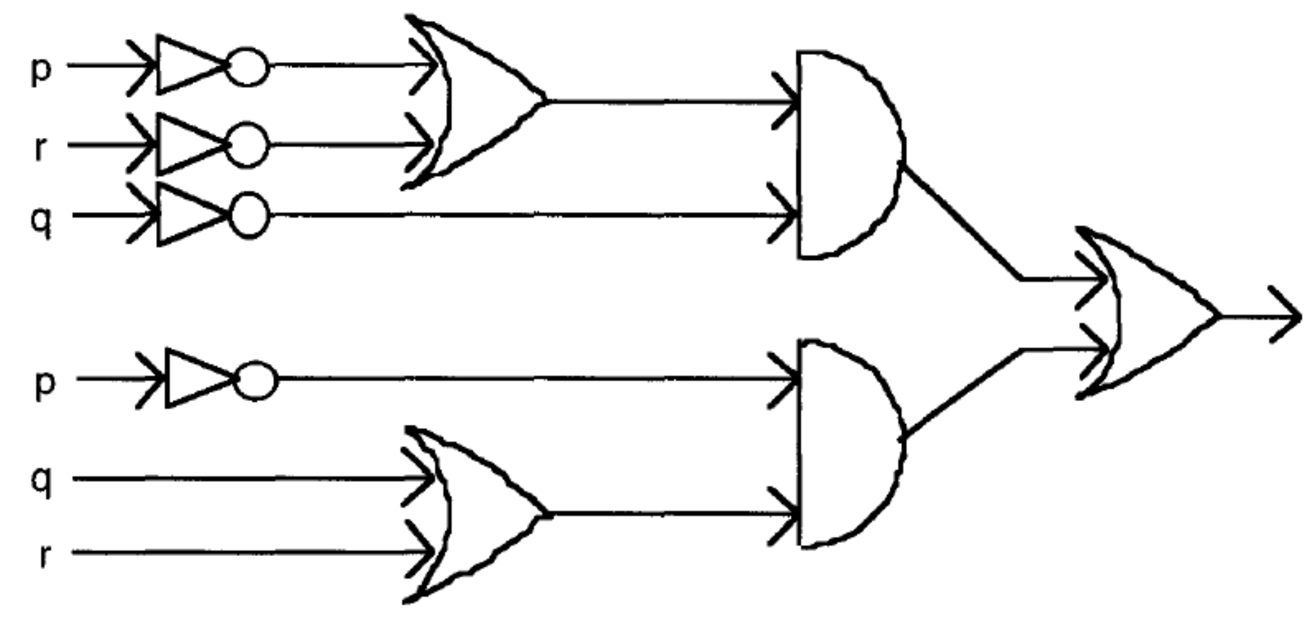
from input bits p, q, and r.

Exercises

47. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$$((\neg p \lor \neg r) \land \neg q) \lor (\neg p \land (q \lor r))$$

from input bits p, q, and r.



TASKS

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