# Tutorial 0: What is Calculus

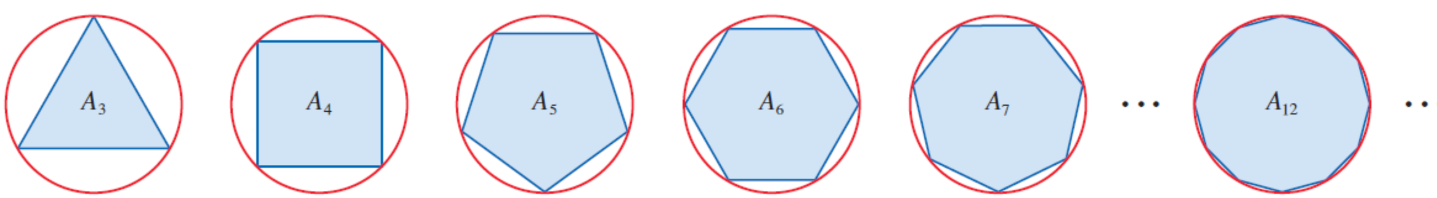
## Contents

# A Preview of Calculus

- It is concerned with change and motion; it deals with quantities that approach other quantities.
- We would like to be able to analyze quantities or processes that are undergoing continuous change.
- For example, if a stone falls 10 feet each second we could easily tell how fast it is falling at any time the stone falls faster and faster, its speed changing at each instant.
- In studying calculus, we will learn how to model (or describe) such instantaneously changing processes and how to find the cumulative effect of these changes.
- Calculus revolves around two key problems involving the graphs of functions
    - The area problem
    - The tangent problem
    - And an unexpected relationship between them.

## The Area Problem

- Finding the area of a circle with inscribed regular polygons.

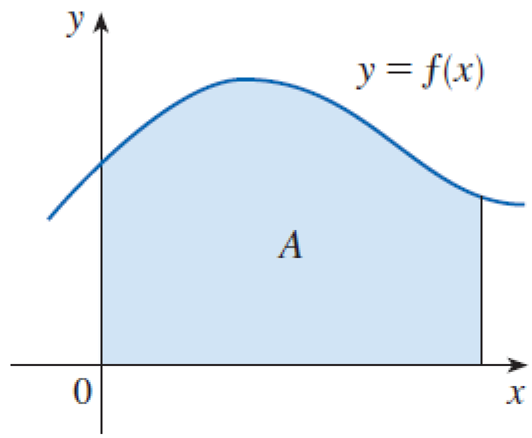



- Let $A_n$ be the area of the inscribed regular polygon of $n$ sides.
- As $n$ increases, it appears that $A_n$ gets closer and closer to the area of the circle.
- We say that the area $A$ of the circle is the limit of the areas of the inscribed polygons, and we write

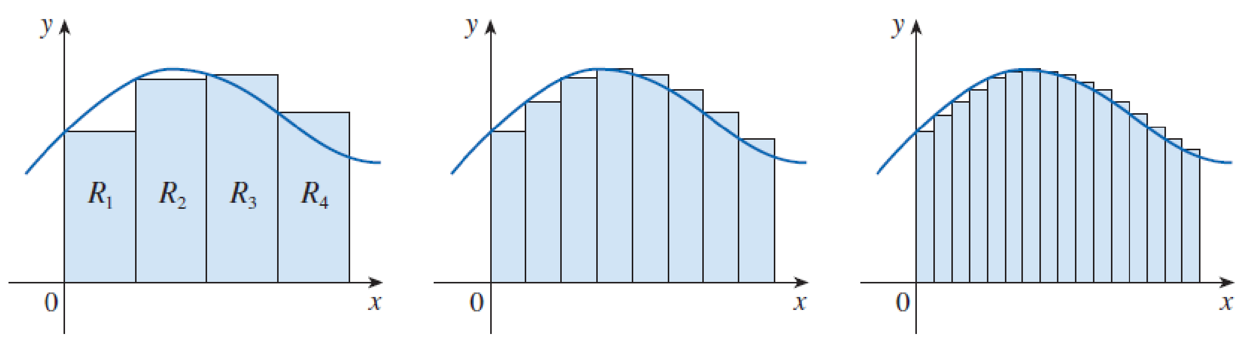
$$A = \lim_{n \to \infty} A_n$$

- Later, by indirect reasoning, it was proofed that the area of the circle: $A = \pi r^2$.

- Finding the area under the curve



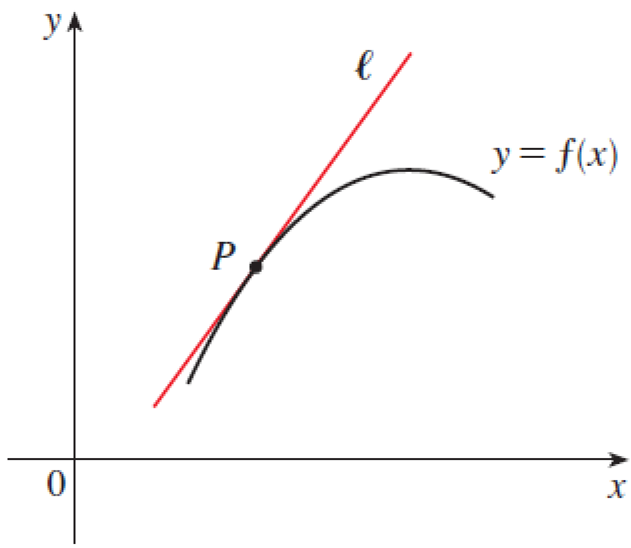

    - We approximate such an area by areas of rectangles.



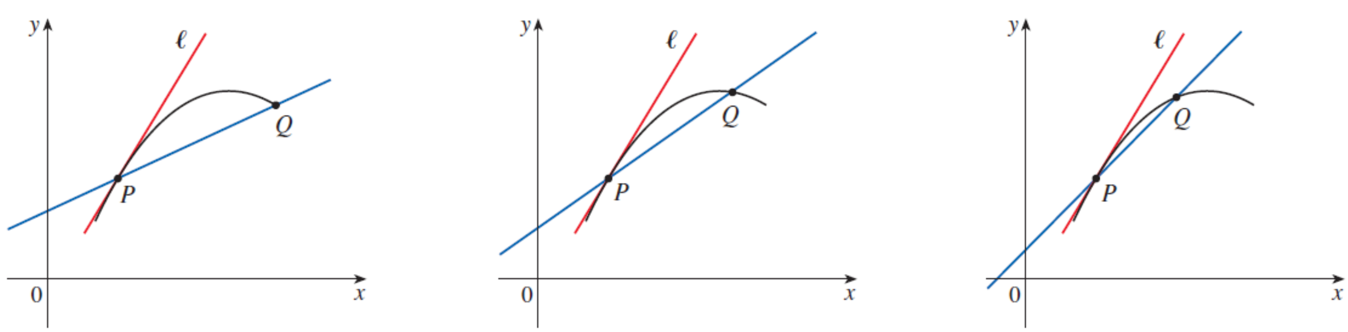

    - If we approximate the area $A$ of the region under the graph of $f$ by using $n$ rectangles $R_1, R_2, \ldots, R_n$, then the approximate area is
$$A_n = R_1 + R_2 + \cdots + R_n$$
    - Now imagine that we increase the number of rectangles (as the width of each one decreases) and calculate $A$ as the limit of these sums of areas of rectangles: $A = \lim_{n\to\infty} A_n$
- The area problem is the central problem in the branch of calculus called **integral calculus**.

## The Tangent Problem

- How to find an equation of the tangent line $L$ to a curve with equation $y = f(x)$ at a given point $P$.




  - To find the function of the line $L$, we need its slope $m$.
  - But, to find the slope $m$, we need two points.
- To get around the problem we need an approximation to $m$ by taking a nearby point $Q$ on the curve and computing the slope $m_{PQ}$ of the secant line $PQ$.
- As $Q$ approaches $P$, the secant line $PQ$ rotates and approaches the tangent line $L$ as its limiting position.

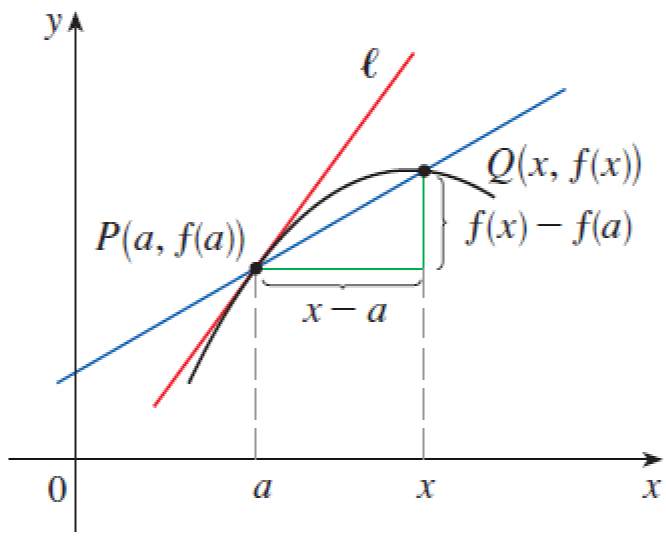



  - This means that the slope $m_{PQ}$ of the secant line becomes closer and closer to the slope $m$ of the tangent line. We write

$$m = \lim_{Q \to P} m_{PQ}$$

- We say that $m$ is the limit of $m_{PQ}$ as $Q$ approaches $P$ along the curve.

- If $P$ is the point $(a, f(a))$ and $Q$ is the point $(x, f(x))$, then




$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$

Because $x$ approaches $a$ as $Q$ approaches $P$, an equivalent expression for the slope of the tangent line is

$$\therefore m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- **The tangent problem has given rise to the branch of calculus called differential calculus.**

# Applications of calculus in computer science

1. Scientific computing: writing software programs and libraries for solving problems/equations involving integrals and differentiations.
   a. Examples: Matlab, Scipy
2. Computer graphics and simulations
   a. Examples: Fourier transformers, wavelet
3. Optimization:
   a. Gradient Descent algorithm
4. Automation:
   similar to robotics, automation can require quantifying a lot of human behavior.