Differentiation Rules

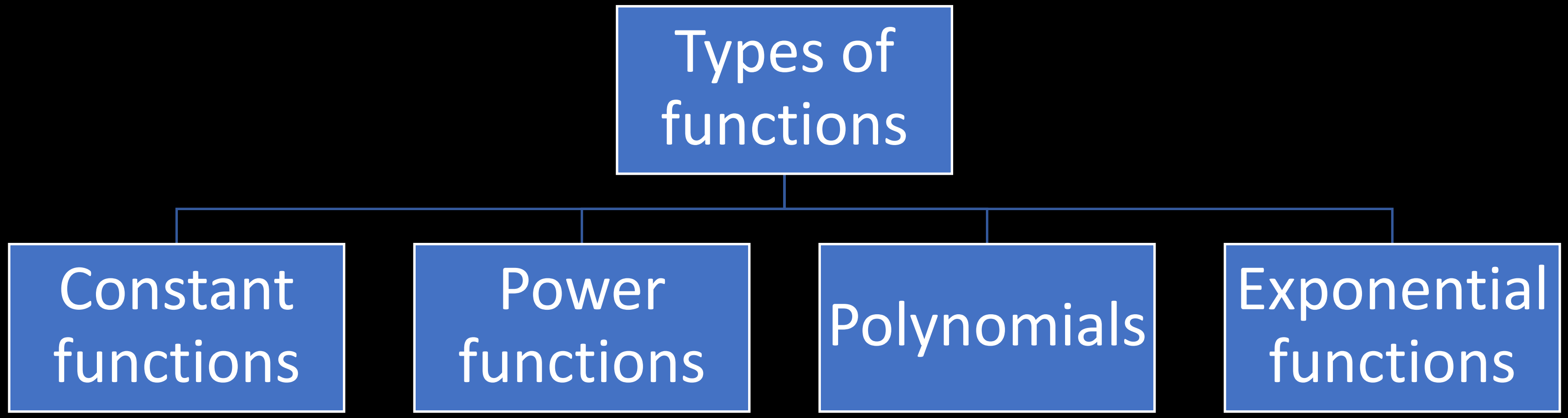
Differentiation Rules

- The definition of a derivative → calculate the derivatives of functions.
- Hard to use the definition, so use differentiation rules.
- Differentiation rules enable us to calculate derivatives of:
    - Polynomials
    - Rational functions
    - Algebraic functions
    - Exponential and logarithmic functions
    - Trigonometric and inverse trigonometric functions

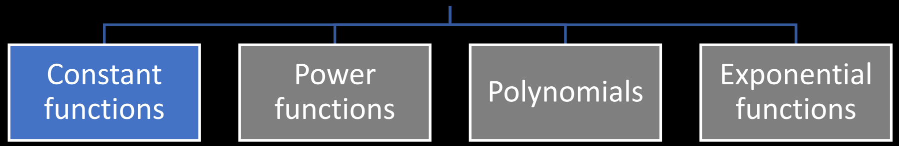
Content
Derivatives of Polynomials and Exponential Functions
The Product and Quotient Rules
Derivatives of Trigonometric Functions

Derivatives of Polynomials and Exponential Functions

- In this section we learn how to differentiate



Derivatives of Polynomials and Exponential Functions



- **Constant fn.**: $f(x) = c$ has a horizontal line $y = c$
- The slope is 0
- So, we must have the derivative $f`(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0$$

- The rule is

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

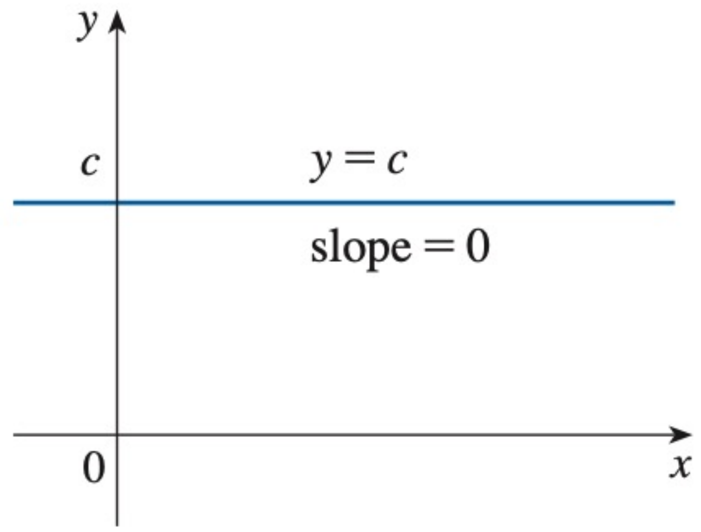


FIGURE 1
The graph of $f(x) = c$ is the line $y = c$, so $f'(x) = 0$.

Derivatives of Polynomials and Exponential Functions

Constant functions
Power functions
Polynomials
Exponential functions

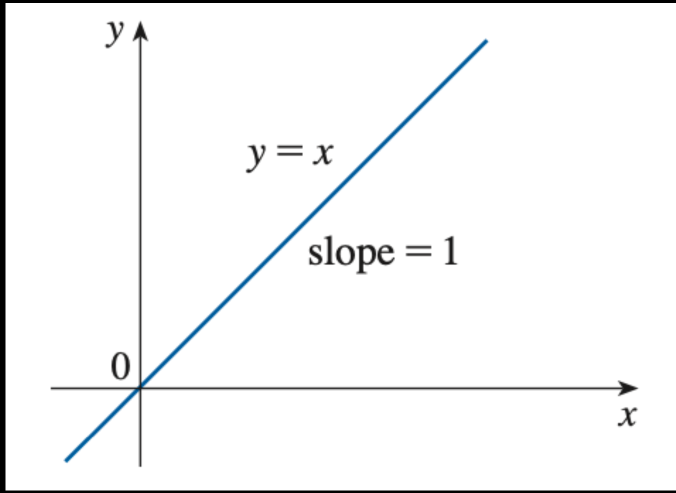
- **Power fn.**: $f(x) = x^n$, n is a real number
- Rule:

The Power Rule (General Version) If n is any real number, then

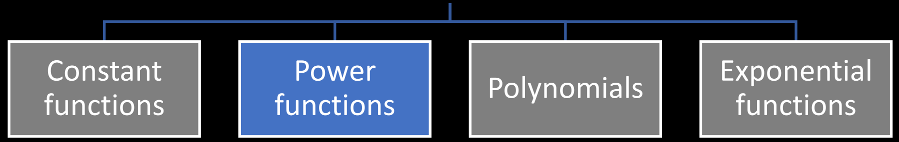
$$\frac{d}{dx}(x^n) = nx^{n-1}$$

- If $n = 1$, then $f(x) = x$ → slope = 1

$$\boxed{\frac{d}{dx}(x) = 1}$$

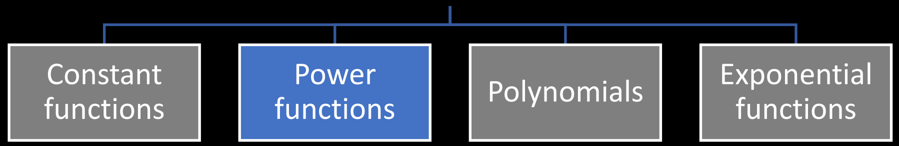


Derivatives of Polynomials and Exponential Functions



- Examples:
- $\frac{d}{dx}(x^2) = 2x$
- $y = t^4, \frac{dy}{dt} = 4t^3$
- $y = x^{1000}, y' = 1000x^{999}$
- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -x^{-2} = -\frac{1}{x^2}$
- $f(x) = \sqrt{x} \rightarrow x^{\frac{1}{2}}, f`(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

Derivatives of Polynomials and Exponential Functions



- Example: differentiate

(a) $f(x) = \frac{1}{x^2}$

$$f(x) = x^{-2}, f`(x) = -2x^{-3}$$

(b) $y = \sqrt[3]{x^2}$

$$y = x^{2/3}, y` = \frac{2}{3}x^{-1/3}$$

```
setenv("PYTHON", "./venv/bin/python");
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clear;

pkg load symbolic;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
syms x;

f = 1/x^2;
df = diff(f, x);
disp(df);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
y = cbrt(x^2);
df = diff(y, x);
disp(df);
```

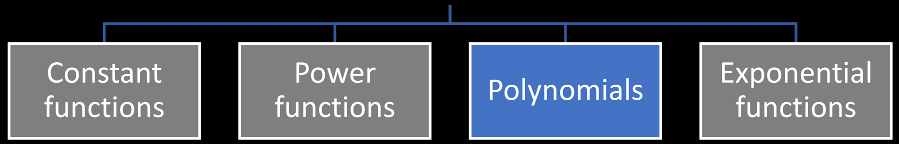
Derivatives of Polynomials and Exponential Functions

Constant functions | Power functions | Polynomials | Exponential functions

- When functions are formed combined by $+$, $-$, $/$, or $*$, their derivatives can be calculated in terms of derivatives of the individual functions.

$\frac{d}{dx}[cf(x)]$, where c is a constant	$c\frac{d}{dx}f(x)$
$\frac{d}{dx}[f(x) \pm g(x)]$	$\frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

Derivatives of Polynomials and Exponential Functions

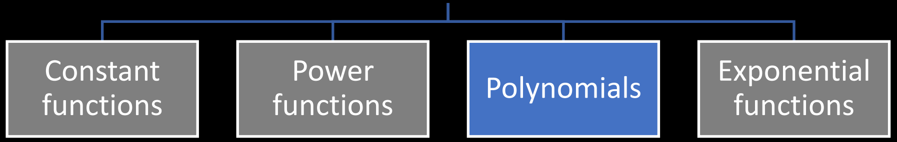


- Example:

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}x^4 = 3\times 4x^3 = 12\,x^3$

(b) $\frac{d}{dx}\,(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5) = \left(\frac{d}{dx}\,x^8 + \frac{d}{dx}12x^5 - \frac{d}{dx}4x^4 + \frac{d}{dx}10x^3 - \frac{d}{dx}6x + \frac{d}{dx}5\right) = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$

Derivatives of Polynomials and Exponential Functions

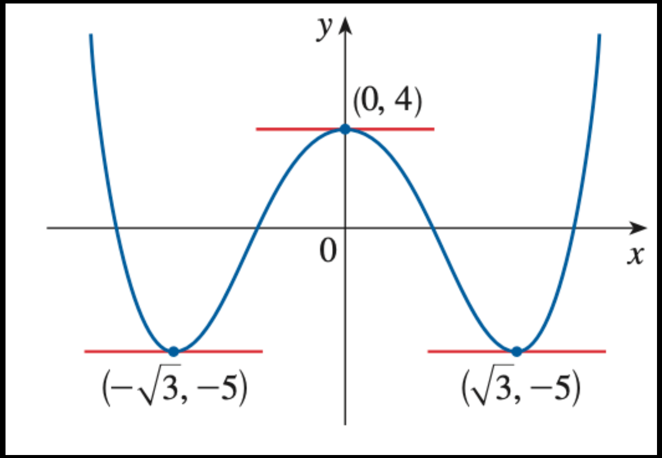


- Example: Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

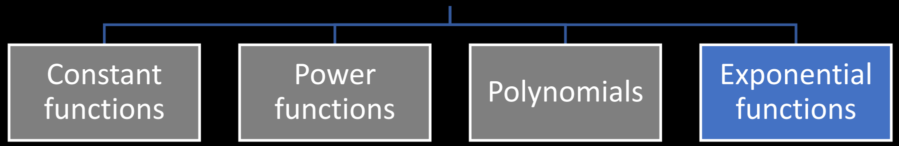
- Horizontal tangent → the derivative = 0

$$\frac{dy}{dx} = \frac{d}{dx}x^4 - \frac{d}{dx}6x^2 + \frac{d}{dx}4 = 4x^3 - 12x = 4x(x^2 - 3)$$

So, the derivative is 0 when $x = 0$ or $x = \pm\sqrt{3}$

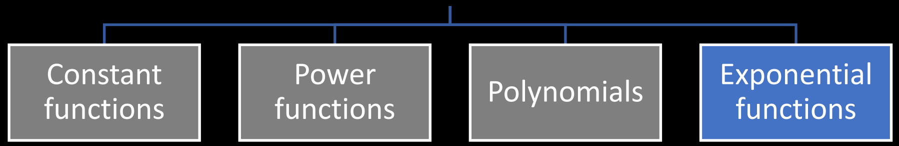


Derivatives of Polynomials and Exponential Functions



$\frac{d}{dx}a^x$	$a^x \ln a$
$\frac{d}{dx}e^x$	e^x

Derivatives of Polynomials and Exponential Functions



- If $f(x) = e^{x} - x$, find $f`$ and $f``$.

$$f`(x) = \frac{d}{dx}(e^{x} - x) = \frac{d}{dx}e^{x} - \frac{d}{dx}x = e^{x} - 1$$

$$f``(x) = \frac{d}{dx}(e^{x} - 1) = e^{x}$$

Exercises

3-34 Differentiate the function

3. $g(x) = 4x + 7$

6. $g(x) = \frac{7}{4}x^2 - 3x + 12$

9. $W(v) = 1.8v^{-3}$

14. $r(t) = \frac{a}{t^2} + \frac{b}{t^4}$

26. $G(t) = \sqrt{5t} + \frac{\sqrt{7}}{t}$

Exercises

3. $g(x) = 4x + 7$ $\boxed{g`(x) = 4}$

6. $g(x) = \frac{7}{4}x^2 - 3x + 12$ $\boxed{g`(x) = \frac{14}{4}x - 3 = \frac{7}{2}x - 3}$

9. $W(v) = 1.8v^{-3}$ $\boxed{W`(v) = -5.4v^{-4}}$

14. $r(t) = \frac{a}{t^2} + \frac{b}{t^4}$ $\boxed{r(t) = at^{-2} + bt^{-4} \rightarrow r`(t) = -2at^{-3} - 4bt^{-5} = \frac{-2a}{t^3} - \frac{4b}{t^5}}$

26. $G(t) = \sqrt{5t} + \frac{\sqrt{7}}{t}$ $\boxed{G(t) = (5t)^{1/2} + \sqrt{7}\,t^{-1} \rightarrow G`(t) = \frac{5}{2}t^{-\frac{1}{2}} - \sqrt{7}\,t^{-2} = \frac{5}{2\sqrt{t}} - \frac{\sqrt{7}}{t^2}}$

Exercises

35–36 Find dy/dx and dy/dt.

35. $y = tx^2 + t^3x$

36. $y = \frac{t}{x^2} + \frac{x}{t}$

Exercises

35–36 Find dy/dx and dy/dt.

35. $y = tx^2 + t^3x$

$\dfrac{dy}{dx} = 2tx + t^3$, $\dfrac{dy}{dt} = x^2 + 3t^2\, x$

36. $y = \dfrac{t}{x^2} + \dfrac{x}{t} \rightarrow y = tx^{-2} + xt^{-1}$

$\dfrac{dy}{dx} = -2tx^{-3} + t^{-1} = -\dfrac{2t}{x^3} + \dfrac{1}{t}$, $\dfrac{dy}{dt} = x^{-2} - xt^{-2} = \dfrac{1}{x^2} - \dfrac{x}{t^2}$

Exercises

37–40 Find an equation of the tangent line to the curve at the given point.

37. $y = 2x^3 - x^2 + 2,\ (1, 3)$

38. $y = 2e^x + x,\ \ (0, 2)$

Exercises

37. $y = 2x^3 - x^2 + 2,\ (1, 3)$

Slope = $y` = 6x\ - 2x$ → (substitute $x$) $y` = 6 - 2 = 4$

$y - y_0 = m(x - x_0) \rightarrow y - 3 = 4(x - 1) \rightarrow y = 4x - 1$

38. $y = 2e^x + x,\ \ (0, 2)$

Slope = $y` = 2e^x + 1$ → (substitute $x$) $y` = 3$

$y - y_0 = m(x - x_0) \rightarrow y - 2 = 3(x - 0) \rightarrow y = 3x + 2$

Exercises

51–52 Find the first and second derivatives of the function. Check to see that your answers are reasonable by comparing the graphs of $f, f`, f``$.

51. $f(x) = 2x - 5x^{3/4}$

52. $f(x) = e^x - x^3$

Exercises

51. $f(x) = 2x - 5x^{3/4}$

$f`(x) = 2 - \frac{15}{4}x^{-\frac{1}{4}}$,, $f``(x) = \frac{15}{16}x^{\frac{-5}{4}}$

52. $f(x) = e^{x} - x^{3}$

$f`(x) = e^{x} - 3x^{2}$,, $f``(x) = e^{x} - 6x$

Exercises

51. $f(x) = 2x - 5x^{3/4}$

```
%% Setting python venv %%
setenv("PYTHON", "./venv/bin/python");
%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clear; clf;
pkg load symbolic
syms x

figure;
%% First Function
f1 = 2*x - 5*x^(3/4);

% Compute the derivatives of f1
f1_prime = diff(f1, x);
f1_2_prime = diff(f1_prime, x);
fplot(f1, [1, 10], 'r', 'LineWidth', 2);
hold on;
fplot(f1_prime, [1, 10], 'g', 'LineWidth', 2);
fplot(f1_2_prime, [1, 10], 'b', 'LineWidth', 2);
hold off;
```

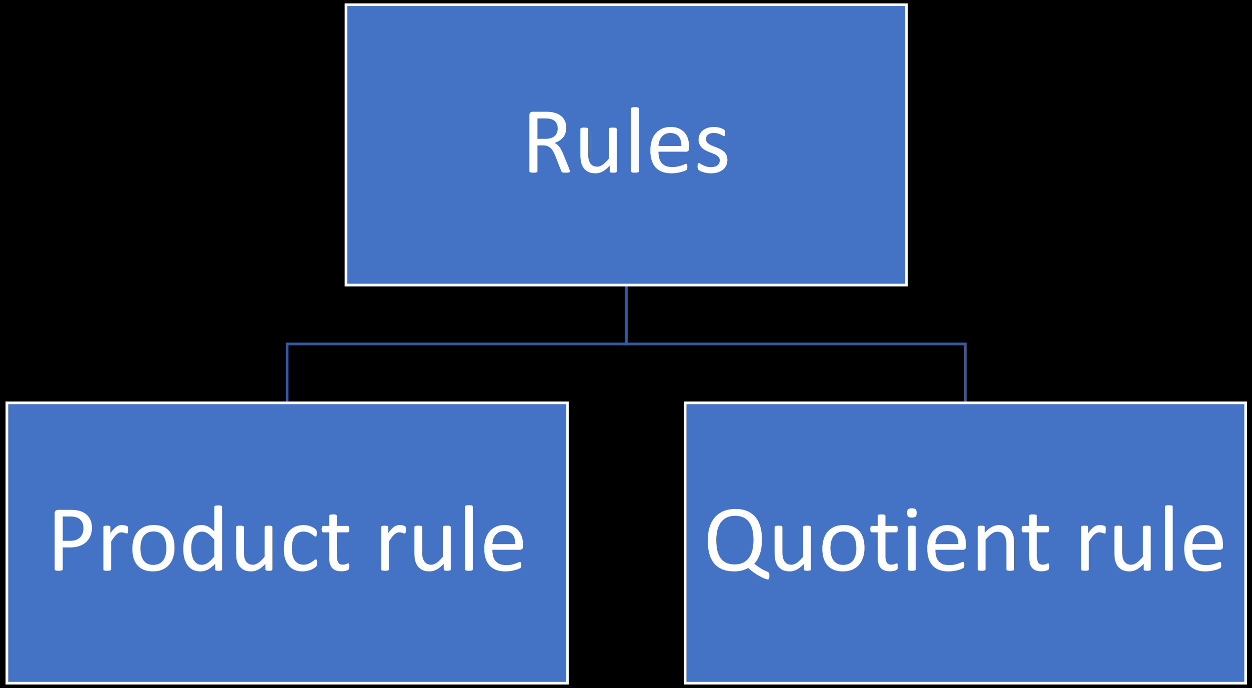
52. $f(x) = e^x - x^3$

```
%%%%%%%%%%%%%%%%%%%%%%%%%
figure;

%% Second Function
f2 = exp(x) - x^3;

% Compute the derivatives of f2
f2_prime = diff(f2, x);
f2_2_prime = diff(f2_prime, x);

fplot(f2, [-5, 6], 'r', 'LineWidth', 2);
hold on;

fplot(f2_prime, [-5, 6], 'g', 'LineWidth', 2);
fplot(f2_2_prime, [-5, 6], 'b', 'LineWidth', 2);

hold off;
```

Content
Derivatives of Polynomials and Exponential Functions
The Product and Quotient Rules
Derivatives of Trigonometric Functions

The Product and Quotient Rules

- Two useful rules for differentiating functions formed from other functions.



The Product and Quotient Rules

- Is this statement correct?
  “The derivative of a product is the product of the derivatives”

The Product and Quotient Rules

- Is this statement correct?
  "The derivative of a product is the product of the derivatives"
- No! assume $f(x) = x$ and $g(x) = x^2$
- We get $f`(x) = 1$ and $g`(x) = 2x$
- The product of derivatives: $f`(x) \cdot g`(x) = 1 \cdot 2x = 2x$
- The derivative of the product: $[f(x) \cdot g(x)]` = [x \cdot x^2]` = [x^3]` = 3x^2$
- $\therefore (fg)` \neq f`g`$

The Product and Quotient Rules

The Product Rule
If f and g are both differentiable, then $$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

- The derivative of a product of two functions is the first function times the derivative of the second function plus the second function times the derivative of the first function.
- This is equivalent to $(fg)` = fg` + f`g$

The Product and Quotient Rules

- **Example**: If $f(x) = xe^x$, find $f`(x)$ and find the nth derivative, $f^{(n)}(x)$

$$f`(x) = \frac{d}{dx}(xe^x)$$
$$= x\frac{d}{dx}e^x + e^x\frac{d}{dx}x$$
$$= xe^x + e^x \cdot 1$$
$$= (x+1)e^x$$

$$f``(x) = \frac{d}{dx}(x+1)e^x$$
$$= (x+1)\frac{d}{dx}e^x + e^x\frac{d}{dx}(x+1)$$
$$= (x+1)e^x + e^x \cdot 1$$
$$= (x+2)e^x$$

$$f```1(x) = \frac{d}{dx}(x+2)e^x$$
$$= (x+2)\frac{d}{dx}e^x + e^x\frac{d}{dx}(x+2)$$
$$= (x+2)e^x + e^x \cdot 1$$
$$= e^x(x+3)$$

$$f````1(x) = \frac{d}{dx}(x+3)e^x$$
$$= (x+3)\frac{d}{dx}e^x + e^x\frac{d}{dx}(x+3)$$
$$= (x+3)e^x + e^x \cdot 1$$
$$= e^x(x+4)$$

The Product and Quotient Rules

- **Example**: If $f(x) = xe^x$, find $f`(x)$ and find the nth derivative, $f^{(n)}(x)$
- $\therefore$ the nth derivative is $e^x(x+n)$

```
f = x*e^x

res1 = diff(f, x);
res2 = diff(res1, x);
res3 = diff(res2, x);
res4 = diff(res3, x);

disp("f`(x): ")
disp(res1)
disp("f``(x): ")
disp(res2)
disp("f```(x): ")
disp(res3)
disp("f`(x): ")
disp(res4)
```

The Product and Quotient Rules

The Quotient Rule
If f and g are both differentiable, then $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left(g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)\right)}{\left(g(x)\right)^2}$$

- The derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.
- This is equivalent to $\left(\frac{f}{g}\right)` = \frac{gf`-fg`}{g^2}$

The Product and Quotient Rules

- **Example**: differentiate $y = \frac{x^2+x-2}{x^3+6}$

$$y` = \frac{\left((x^3+6)\frac{d}{dx}(x^2+x-2)-(x^2+x-2)\frac{d}{dx}(x^3+6)\right)}{(x^3+6)^2}$$

$$= \frac{\left((x^3+6)(2x+1)-(x^2+x-2)(3x^2)\right)}{(x^3+6)^2}$$

$$= \frac{\left((2x^4+x^3+12x+6)-(3x^4+3x^3-6x^2)\right)}{(x^3+6)^2}$$

$$= \frac{-x^4-2x^3-6x^2+12x+6}{(x^3+6)^2}$$

The Product and Quotient Rules

- **Example**: Find an equation of the tangent line to the curve $y = e^x/(1+x^2)$ at the point $(1, ½e)$
  - Equation of the line → slope → differentiate the equation

$$y` = \frac{d}{dx}\left(\frac{e^x}{1+x^2}\right)$$

$$= \frac{\left((1+x^2)\frac{d}{dx}e^x - e^x\frac{d}{dx}(1+x^2)\right)}{(1+x^2)^2}$$

$$= \frac{\left((1+x^2)\cdot e^x - e^x\cdot 2x\right)}{(1+x^2)^2}$$

$$= \frac{e^x\left((1+x^2)-2x\right)}{(1+x^2)^2} = \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$$

Slope $= \frac{e^1(1^2-2\cdot 1+1)}{(1+1^2)^2} = 0$

The slop is a horizontal line.

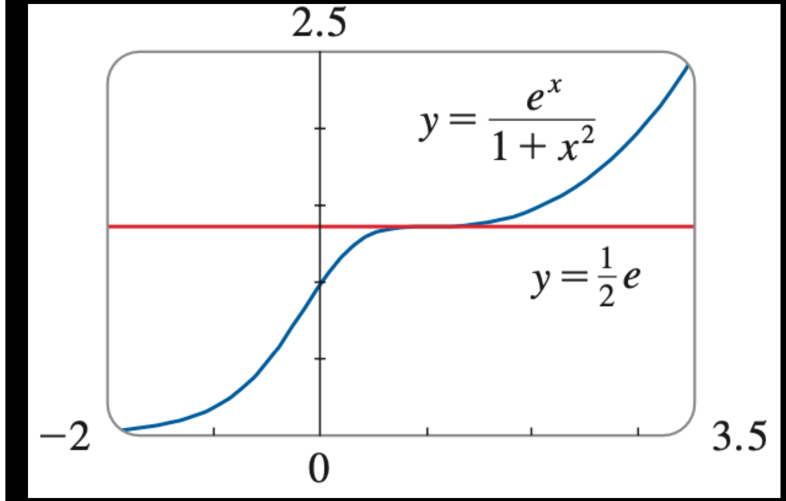
The line equation:

$$(y - y_1) = m\,(x - x_1)$$

$$y = \frac{1}{2}e$$

The Product and Quotient Rules

```
%% Setting python venv %%
setenv("PYTHON", "./venv/bin/python");
%%%%%%%%%%%%%%%%%%%%%%%%%
clc; clear;
pkg load symbolic
syms x

y = e^x / (1+x^2)
x1 = 1;
y1 = 1/2 * e;

% Get the slope equation by differentiating the curve eqn w.r.t x
slope_eq = diff(y, x);

% Get the slope value by substituting for x in the slope equation
slope = subs(slope_eq, x1);
slope

% The line equation will be printed in a different notion
line_eq = slope * (x-x1) + y1;
disp(line_eq)
```



$$\frac{777 * \pi}{1796} = 1/2\, e$$

The Product and Quotient Rules

- Summary the differentiation formulas

Table of Differentiation Formulas

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(e^x) = e^x$$

$$(cf)' = cf' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Exercises

3–30 Differentiate

3. $y = (4x^2 + 3)(2x + 5)$

4. $y = (10x^2 + 7x - 2)(2 - x^2)$

8. $g(x) = (x + 2\sqrt{x})e^x$

10. $y = \frac{e^x}{1-e^x}$

Exercises

3. $y = (4x^2 + 3)(2x + 5)$

$$\begin{aligned} y` &= (8x)(2x+5) + (4x^2+3)(2) \\ &= 16x^2 + 40x + 8x^2 + 6 \\ &= 24x^2 + 40x + 6 \end{aligned}$$

4. $y = (10x^2 + 7x - 2)(2 - x^2)$

$$\begin{aligned} y` &= (20x+7)(2-x^2) + (10x^2+7x\ -2)(-2x) \\ &= (40x - 20x^3 + 14\ - 7x^2) + (-20x^3 - 14x^2 + 4x) \\ &= -40x^3 - 21x^2 + 44x + 14 \end{aligned}$$

Exercises

8. $g(x) = (x + 2\sqrt{x})e^x$

$$g`(x) = \left(1 + x^{-\frac{1}{2}}\right)e^x + (x + 2\sqrt{x})e^x$$
$$= e^x + e^x x^{-\frac{1}{2}} + e^x x + e^x 2\sqrt{x}$$
$$= e^x\left(1 + x^{-\frac{1}{2}} + x + 2\sqrt{x}\right)$$

10. $y = \frac{e^x}{1-e^x}$

$$y` = \frac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2}$$
$$= \frac{e^x\left((1-e^x) + e^x\right)}{(1-e^x)^2}$$
$$= \frac{e^x}{(1-e^x)^2}$$

Exercises

35–36 Find an equation of the tangent line to the given curve at the specified point.

35. $y = \frac{x^2}{1+x}$ at $(1, ½)$

Exercises

35. $y = \frac{x^2}{1+x}$ at (1, ½). Line equation → slope → derivative

Slope = $y` = \frac{2x\,(1+x) - 1(x^2)}{(1+x)^2}$

$= \frac{2x^2+2x - x^2}{(1+x)^2}$

$= \frac{x^2+2x}{(1+x)^2}$

$= \frac{x(x+2)}{(1+x)^2}$

Slope at (1, ½) = $\frac{1(1+2)}{(1+1)^2} = \frac{3}{4}$

Line eqn: $(y - y_1) = m(x - x_1)$

$$\left(y - \frac{1}{2}\right) = \frac{3}{4}(x - 1)$$

$$y = \frac{3}{4}x - \frac{3}{4} + \frac{1}{2} = \frac{3}{4}x - \frac{1}{4}$$

Content
Derivatives of Polynomials and Exponential Functions
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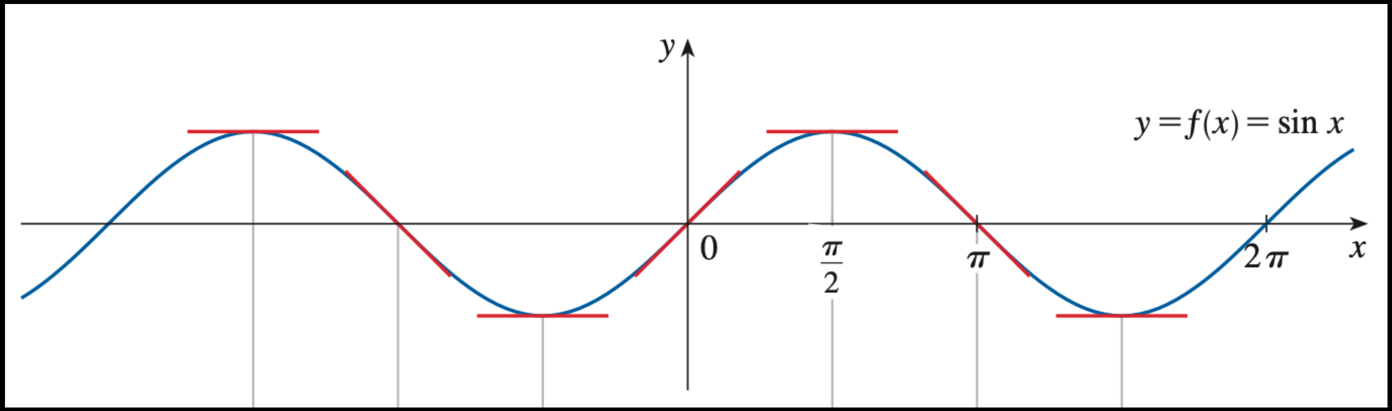
Derivatives of Trigonometric Functions

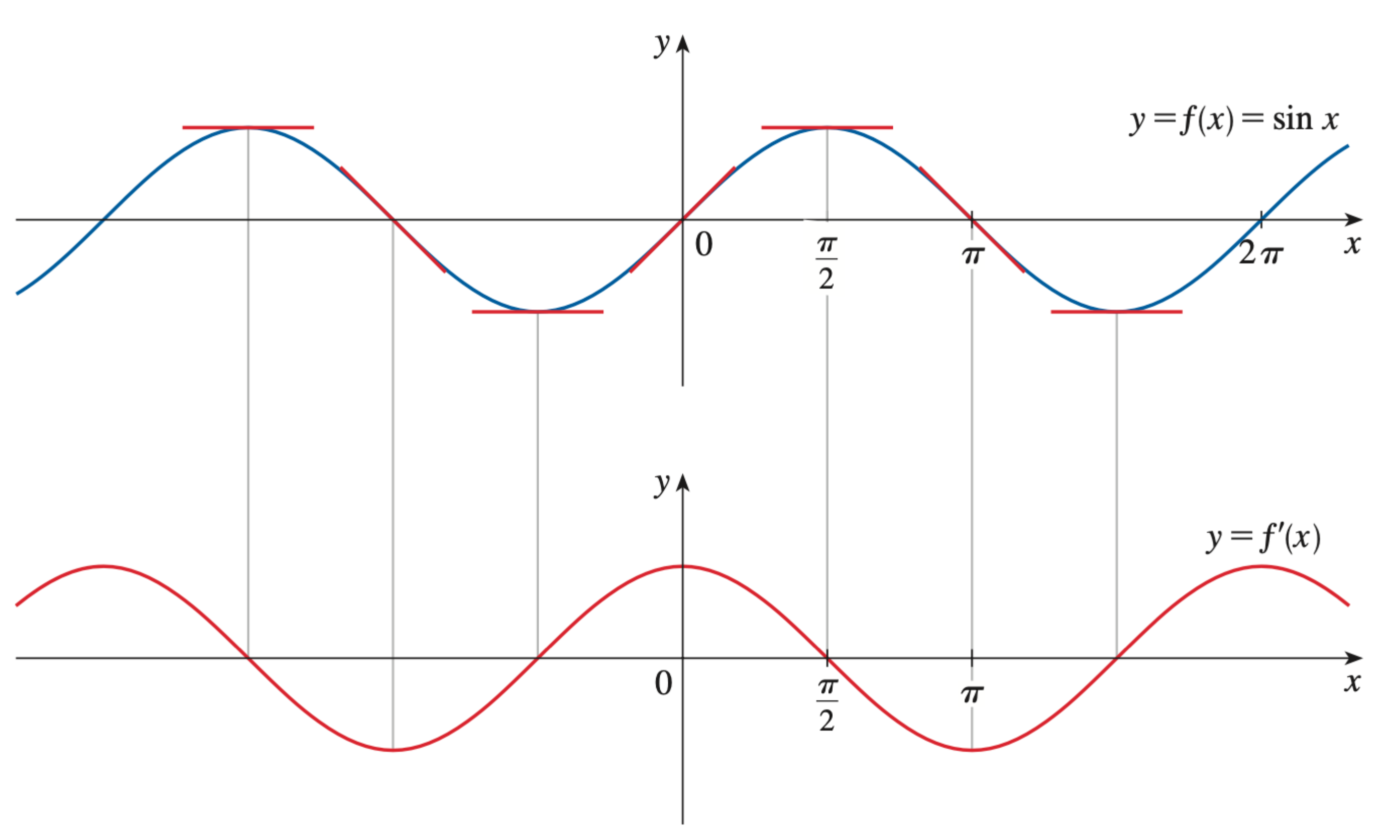
Derivatives of Trigonometric Functions

- The graph of $f = \sin(x)$ vs $f`(x) =$ slope of the tangent line of the curve.



Derivatives of Trigonometric Functions

- The graph of $f = \sin(x)$ vs $f`(x) =$ slope of the tangent line of the curve.
  - It's the same as the $\cos(x)$ → $d/dx \sin(x) = \cos(x)$



Derivatives of Trigonometric Functions

- Example: Differentiate $y = x^2 \sin(x)$

Use the product rule:

$$y` = 2x \sin x + x^2 \cos(x)$$

Derivatives of Trigonometric Functions

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

Derivatives of Trigonometric Functions

- Example: Differentiate $f(x) = \frac{\sec x}{1+\tan x}$. For what values of x does the graph of f have a horizontal tangent?

$$f`(x) = \frac{(1+\tan x)\frac{d}{dx}(\sec x) - \sec x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$= \frac{\left(1+\tan x \cdot (\sec x \tan x) - \sec x \sec^2 x\right)}{(1+\tan x)^2}$$

$$= \frac{\sec x\left(\tan x + \tan^2 x - \sec^2 x\right)}{(1+\tan x)^2}$$

Subsititue $\sec^2 x = \tan^2 x + 1$

$$= \frac{\sec x\left(\tan x + \tan^2 x - \left(\tan^2 x + 1\right)\right)}{(1+\tan x)^2} \rightarrow \frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$$

Derivatives of Trigonometric Functions

- Two special trigonometric limits

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{(\cos \theta - 1)}{\theta} = 0$$

Derivatives of Trigonometric Functions

- Example: Find $\lim\limits_{x\to 0} \frac{\sin 7x}{4x}$

$$\frac{\sin 7x}{4x} = \frac{1}{4}\left(\frac{\sin 7x}{x}\right) = \frac{7}{4}\left(\frac{\sin 7x}{7x}\right)$$ (Note that $\sin 7x \neq 7\sin x$)

$$\therefore \lim_{x\to 0} \frac{\sin 7x}{4x} = \frac{7}{4}\lim_{x\to 0}\left(\frac{\sin 7x}{7x}\right)$$

$$= \frac{7}{4} \times 1 = \frac{7}{4}$$

Derivatives of Trigonometric Functions

- Example: Calculate $\lim\limits_{x\to 0}\frac{\cos\theta-1}{\sin\theta}$

$$\lim_{x\to 0}\frac{\cos\theta-1}{\sin\theta}\quad \text{(divide by } \theta\text{)}$$

$$=\lim_{x\to 0}\frac{\frac{\cos\theta-1}{\theta}}{\frac{\sin\theta}{\theta}}$$

$$=\frac{\lim\limits_{x\to 0}\frac{\cos\theta-1}{\theta}}{\lim\limits_{x\to 0}\frac{\sin\theta}{\theta}}$$

$$\frac{0}{1}=0$$

Exercises

1–22 Differentiate.

1. $f(x) = 3\sin x \; - 2\cos x$

2. $f(x) = \tan x \; - 4\sin x$

5. $h(\theta) = \theta^2 \sin\theta$

Exercises

1. $f(x) = 3\sin x - 2\cos x$

$$f`(x) = 3\cos x + 2\sin x$$

2. $f(x) = \tan x - 4\sin x$

$$f`(x) = \sec^2 x - 4\cos x$$

5. $h(\theta) = \theta^2 \sin\theta$

$$h`(\theta) = 2\theta\sin\theta + \theta^2\cos\theta = \theta(2\sin\theta + \theta\cos\theta)$$

Exercises

23. show that $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Exercises

23. show that $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$$\csc x = \frac{1}{\sin x}$$

$$\frac{d}{dx}\csc x = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \frac{0 \cdot \sin x - \cos x \cdot 1}{(\sin x)^2}$$

$$= -\frac{\cos x}{(\sin x)^2}$$

$$= \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x} \rightarrow -\csc x \cot x$$

Exercises

27-30. Find an equation of the tangent line to the curve at the given point.

30. $y = \frac{1+\sin x}{\cos x}$, $(\pi, -1)$

Exercises

30. $y = \frac{1+\sin x}{\cos x}$, $(\pi, -1)$

Slope = $y` = \frac{d}{dx}\frac{1+\sin x}{\cos x}$

$$= \frac{\cos x \cdot \cos x - (1+\sin x) \cdot - \sin x}{(\cos x)^2}$$

$$= \frac{(\cos^2 x + \sin x + \sin^2 x)}{(\cos x)^2}$$

Substitute for $\cos^2 x + \sin^2 x = 1$

$$= \frac{(1+\sin x)}{(\cos x)^2}$$

Slope = $y`(\pi) = \frac{(1+\sin \pi)}{\cos^2 \pi} = \frac{1+0}{(-1)^2} = 1$

The equation of the tangent line:

$$y - (-1) = 1(x - \pi)$$

$$y = x - \pi - 1$$

TASK

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