Differentiation Rules

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The Chain Rule
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The Chain Rule

- Derivative of composite functions → chain rule

The Chain Rule
If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f\left(g(x)\right)$ is differentiable at x and $F`$ is given by the product $$F`(x) = f`\left(g(x)\right) \cdot g`(x)$$ In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The Chain Rule

- Example: Find $F`(x)$ if $F(x) = \sqrt{x^2 + 1}$

Express $F(x)$ as $f(u) = \sqrt{u}$ and $u = g(x) = x^2 + 1$

$$F`(x) = f`\big(g(x)\big) \cdot g`(x)$$

$$g`(x) = 2x$$

$$f`(\sqrt{u}) = \frac{1}{2}\, u^{-\frac{1}{2}} = \frac{1}{2\,\sqrt{u}}$$

$$F`(x) = \frac{1}{2\sqrt{u}} \cdot 2x = \frac{x}{\sqrt{u}} \rightarrow \frac{x}{\sqrt{x^2+1}}$$

The Chain Rule

- Example: Differentiate (a) $y = \sin(x^2)$ and (b) $y = \sin^2 x$.

Express y as $f(u) = \sin(u)$ and $u = x^2$

$u` = 2x$

$f` = \cos(u)$

$y` = \cos(u) \cdot u`$

$\quad = \cos(\mathrm{x}^2) \cdot 2x$

$\quad = 2x \cos x^2$

Note that $\sin^2 x = (\sin x)^2$

Express y as $f(u) = u^2$ and $u = \sin x$

$u` = \cos x$

$f`(u) = 2u$

$y` = f`(u) \cdot u`$

$\quad = 2u \cdot u`$

$\quad = 2 \sin x \cdot \cos x$

The Chain Rule

- Special case of the Chain Rule where the outer function f is a power function.

The Power Rule Combined with the Chain Rule
If n is any real number and $u = g(x)$ is differentiable, then $$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$ Alternatively, $$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g`(x)$$

The Chain Rule

- Example: Differentiate $y = (x^3 - 1)^{100}$

Take $g(x) = (x^3 - 1)$ and $n = 100$

$$\frac{dy}{dx} = \frac{d}{dx}\, g(x)^n = ng(x)^{n-1}\frac{d}{dx}g(x)$$

$$= \frac{d}{dx}(x^3 - 1)^{100}$$

$$= 100(x^3 - 1)^{99}\frac{d}{dx}(x^3 - 1)$$

$$= 100(x^3 - 1)^{99} \cdot 3x^2$$

$$= 300x^2(x^3 - 1)^{99}$$

The Chain Rule

- Example: Differentiate $y = \ e^{\sec 3\theta}$

$$y` = \frac{dy}{d\theta}\ e^{\sec 3\theta}$$

$$= e^{\sec 3\theta}\frac{dy}{d\theta}\ (\sec 3\ \theta)$$

$$= e^{\sec 3\theta}\sec 3\theta \tan 3\theta \frac{dy}{d\theta}(3\theta)$$

$$= 3e^{\sec 3\theta}\sec 3\theta \tan 3\theta$$

The Chain Rule

- Derivatives of General Exponential Functions:

$$\boxed{\frac{d}{dx}(b^x) = b^x \ln b}$$

- *Do not confuse with the power function:*

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

  - *Here x is the base*
  - *In exponential functions, x is the exponent*

The Chain Rule

- Example: Find the derivative of: (a) $g(x) = 2^x$ (b) $h(x) = 5^{x^2}$

$$\boxed{g`(x) = 2^x \ln 2}$$

$$\boxed{\begin{aligned} h`(x) &= 5^{x^2} \ln 5 \cdot \frac{d}{dx} x^2 \\ &= 5^{x^2} \ln 5 \cdot 2x \end{aligned}}$$

Exercises

7–52 Find the derivative of the function.

7. $f(x) = (2x^3 - 5x + 4)^5$

8. $f(x) = (x^5 + 3x^2 - x)^{50}$

9. $f(x) = \sqrt{5x + 1}$

10. $f(x) = \frac{1}{\sqrt[3]{x^2 - 1}}$

Exercises

7. $f(x) = (2x^3 - 5x^2 + 4)^5$

$$\boxed{\begin{aligned} f`(x) &= 5(2x^3 - 5x^2 + 4)^4 \cdot \frac{d}{dx}(2x^3 - 5x^2 + 4) \\ &= 5(2x^3 - 5x^2 + 4)^4 \cdot (6x^2 - 10x) \end{aligned}}$$

8. $f(x) = (x^5 + 3x^2 - x)^{50}$

$$\boxed{\begin{aligned} f`(x) &= 50(x^5 + 3x^2 - x)^{49} \cdot \frac{d}{dx}(x^5 + 3x^2 - x) \\ &= 50(x^5 + 3x^2 - x)^{49} \cdot (5x^4 + 6x - 1) \end{aligned}}$$

Exercises

9. $f(x) = \sqrt{5x+1} = (5x+1)^{1/2}$

$$f`(x) = 1/2\ (5x+1)^{-1/2} \cdot \frac{d}{dx}(5x+1)$$

$$= 1/2\ (5x+1)^{-1/2} \cdot 5 \quad = \quad \frac{5}{2\sqrt{5x+1}}$$

10. $\frac{1}{\sqrt[3]{x^2-1}} = \frac{1}{(x^2-1)^{1/3}} = (x^2-1)^{-1/3}$

$$f`(x) = -1/3\ (x^2-1)^{-4/3} \cdot \frac{d}{dx}(x^2-1)$$

$$= -1/3\ (x^2-1)^{-4/3} \cdot 2x \ = \frac{-2x}{3(x^2-1)^{4/3}}$$

Exercises

7–52 Find the derivative of the function.

42. $y = e^{\sin 2x} + \sin(e^{2x})$

44. $f(t) = e^{1/t}\sqrt{t^2 - 1}$

Exercises

42. $y = e^{\sin 2x} + \sin(e^{2x})$

$$
\begin{aligned}
f`(x) &= \frac{d}{dx}\, e^{\sin 2x} + \frac{d}{dx}\sin(e^{2x}) \\
&= e^{\sin 2x} \cdot \frac{d}{dx}\sin 2x + \cos e^{2x} \cdot \frac{d}{dx} e^{2x} \\
&= e^{\sin 2x} \cdot \cos 2x \cdot \frac{d}{dx} 2x + \cos e^{2x} \cdot e^{2x} \cdot \frac{d}{dx}\, 2x \\
&= e^{\sin 2x} \cdot \cos 2x \cdot 2 + \cos e^{2x} \cdot e^{2x} \cdot 2
\end{aligned}
$$

Exercises

44. $f(t) = e^{1/t}\sqrt{t^2 - 1}$

Product rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g`(x) + f`(x)g(x)$

$$\frac{d}{dx}e^{1/t} = e^{1/t} \cdot \frac{d}{dt}\, 1/t$$

$1/t = t^{-1}$

$$= e^{1/t} \cdot -t^{-2}$$

$$= e^{1/t} \cdot \frac{-1}{t^2}$$

$$\frac{d}{dx}(t^2 - 1)^{1/2} = \frac{d}{dx}(t^2 - 1)^{1/2}$$

$$= 1/2\,(t^2 - 1)^{-\frac{1}{2}} \cdot \frac{d}{dx}\,(t^2 - 1)$$

$$= 1/2\,(t^2 - 1)^{-\frac{1}{2}} \cdot 2t = \frac{2t}{2\sqrt{(t^2-1)}} = \frac{t}{\sqrt{(t^2-1)}}$$

$$f`(t) = e^{1/t} \cdot \frac{t}{\sqrt{(t^2-1)}} + e^{1/t} \cdot \frac{-1}{t^2} \cdot \sqrt{t^2 - 1} = e^{1/t}\left(\frac{t}{\sqrt{(t^2-1)}} - \frac{\sqrt{t^2-1}}{t^2}\right)$$

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Implicit Differentiation
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Implicit Differentiation

- The functions we met so far are described by:

$$y = f(x)$$

Such as $y = \sqrt{x^3 + 1}$ or $y = x \sin x$

- Some functions that are defined by **implicit relation** between x and y:

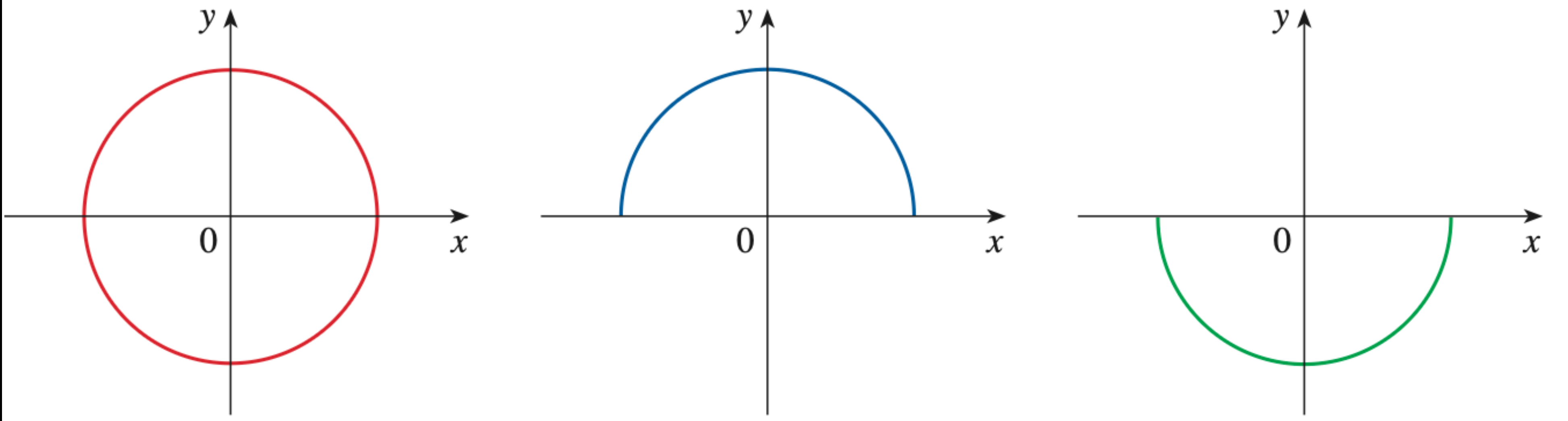
$$x^2 + y^2 = 25 \quad \rightarrow y = \pm\sqrt{25 - x^2}$$

$$x^3 + y^3 = 6xy \rightarrow ?$$

It's not easy to solve for y as a function of x in the 2nd eqn.

Implicit Differentiation

- $x^2 + y^2 = 25 \ \rightarrow y = \pm\sqrt{25 - x^2}$ → circle



(a) $x^2 + y^2 = 25$ (b) $f(x) = \sqrt{25 - x^2}$ (c) $g(x) = -\sqrt{25 - x^2}$

Implicit Differentiation

- $x^3 + y^3 = 6xy$ → **folium of Descartes** curve
- It implicitly defines y as several functions of x

$$x^3 + [f(x)]^3 = 6xy$$

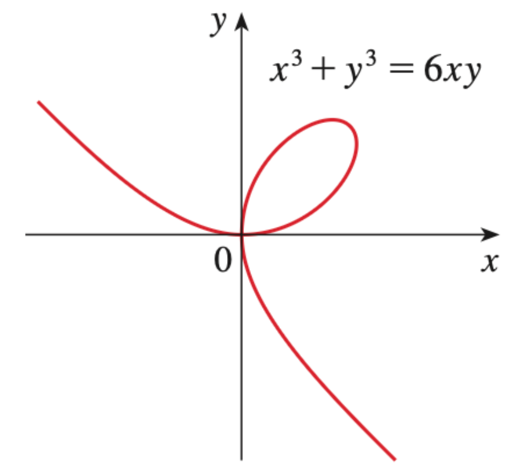


FIGURE 2 The folium of Descartes

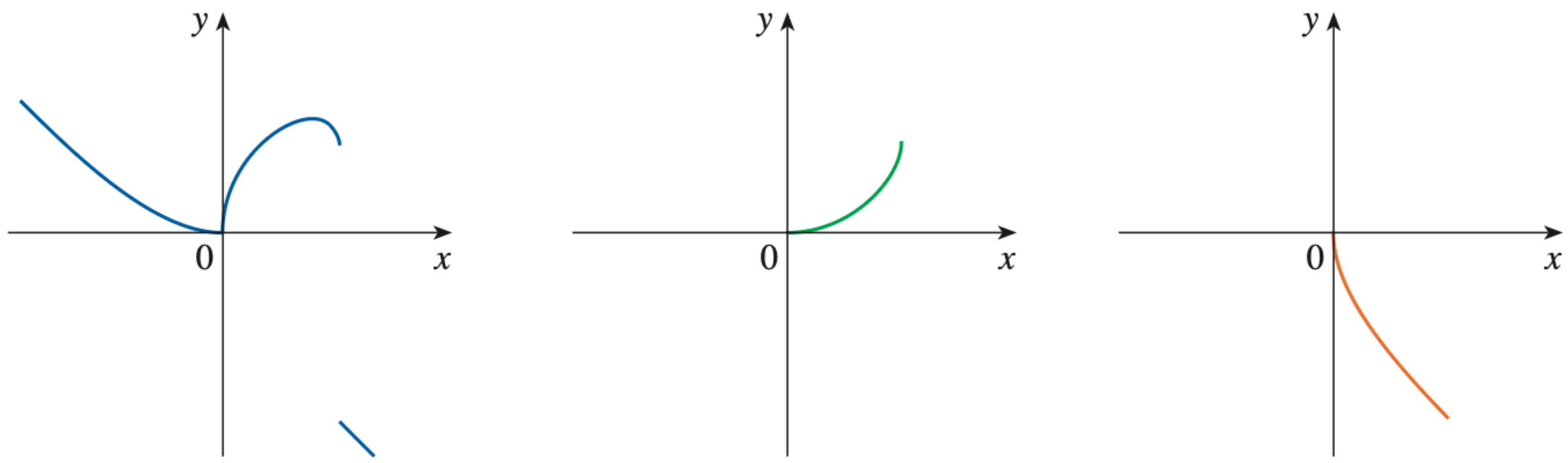


FIGURE 3 Graphs of three functions defined implicitly by the folium of Descartes

Implicit Differentiation

- Do we need to solve $x^3 + y^3 = 6xy$ for y to find $y`$?

Implicit Differentiation

- Do we need to solve $x^3 + y^3 = 6xy$ for y to find y`?
- NO!

- We can use **implicit differentiation**:
  1. Differentiate both sides of the equation w.r.t x
  2. Solve the resulting equation for dy/dx

Implicit Differentiation

- Example: if $x^2 + y^2 = 25$, find dy/dx. Then find an equation of the tangent to the circle at the point $(3, 4)$
- **Solution 1**: using implicit differentiation

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dy}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0 \rightarrow \text{assume } y = f(x)$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\frac{dy}{dx} = 2y\frac{dy}{dx} \leftarrow \text{using chain rule}$$

$$\therefore 2x + 2y\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x}{y}$$

Implicit Differentiation

- Now, we have $\frac{dy}{dx} = -\frac{x}{y}$. We have the point $(3, 4)$
- $\therefore \frac{dy}{dx} = -\frac{3}{4}$

- Tangent line equation:

$$y - 4 = -\frac{3}{4}(x - 3) = 3x + 4y = 25$$

Implicit Differentiation

- **Solution 2**: using differentiation laws

$$y = \ \pm\sqrt{25 - x^2}$$

The point $(3, 4)$ lies on the upper semicircle, so we use $y = \sqrt{25 - x^2}$

$$y` = f`(x) = 1/2(25 - x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(25 - x^2)$$

$$1/2(25 - x^2)^{-\frac{1}{2}} \ \cdot (-2x)$$

$$\frac{-2x}{2(25-x^2)^{\frac{1}{2}}} = \frac{-x}{\sqrt{25-x^2}} \to \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{4}$$

The tangent eqn: $3x + 4y = 25$

Implicit Differentiation

- Example 2: Find $y`$ if $x^3 + y^3 = 6xy$. Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.
- **Solution**

Differentiating both sides with respect to x, regarding y as a function of x

$$3x^2 + 3y^2\, y` = 6y + 6xy`$$

$$x^2 + y^2 y` = 2y + 2xy`$$

$$y^2 y` - 2xy` = 2y - x^2$$

$y` = \frac{2y - x^2}{y^2 - 2x}$ → at $(3,3)$, $y` = \frac{2\cdot 3 - 3^2}{3^2 - 2\cdot 3} = -1$

Tangent eqn:

$$y - 3 = -1(x - 3)$$

$$x + y = 6$$

Implicit Differentiation

- Example 3: Find $y`$ if $\sin(x+y) = y^2 \cos x$

  Differentiate implicitly w.r.t x and remembering that y is a function of x

  $\cos(x+y) \cdot (1 + y`) = 2y\, y` \cos x - y^2 \sin x$

  $\cos(x+y) + \mathrm{y}`\ \cos(x+y) = 2y\, y` \cos x\ - y^2 \sin x$

  $y` \cos(x+y)\ - 2y\, y` \cos x = -\cos(x+y) + y^2 \sin x$

  $y`\,(\cos(x+y)\ - 2y \cos x) = -\cos(x+y) + y^2 \sin x$

  $y` = \frac{-\cos(x+y)+y^2 \sin x}{\cos(x+y)-2y \cos x}$

Implicit Differentiation

- Example 4: Find $y``$ if $x^4 + y^4 = 16$

Differentiate implicitly w.r.t x and remembering that y is a function of x

$4x^3 + 4y^3\, y` = 0$

$y` = \frac{-x^3}{y^3}$

$y`` = \frac{d}{dx}\left(-\frac{x^3}{y^3}\right) = \frac{\left(\left(-3x^2 \cdot y^3\right) - 3y^2 y` \cdot x^3\right)}{(y^3)^2}$

$= \frac{-3x^2y^3 - 3x^3y^2\left(-\frac{x^3}{y^3}\right)}{y^6} = \frac{-3\left(x^2y^3 + x^6\frac{1}{y}\right)}{y^6} = \frac{-3\,(x^2y^4 + x^6)}{y^7} = \frac{-3x^2\left(y^4 + x^4\right)}{y^7}$

$= \frac{-3x^2 \cdot 16}{y^7} = -48\,\frac{x^2}{y^7}$

Exercises

5–22 Find dy/dx by implicit differentiation.

5. $x^2 - 4xy + y^2 = 4$

6. $2x^2 + xy - y^2 = 2$

Exercises

5. $x^2 - 4xy + y^2 = 4$

$2x - 4y + 4xy` + 2y\,y` = 0$

$4xy` + 2y\,y` = 4y - 2x$

$y` = \frac{4y-2x}{4x+2y} = \frac{2y-x}{2x+y}$

6. $2x^2 + xy - y^2 = 2$

$4x + y + xy` - 2y\,y` = 0$

$xy` - 2y\,y` = -4x - y$

$y` = \frac{-4x-y}{x-2y}$

Exercises

5–22 Find dy/dx by implicit differentiation.

9. $\frac{x^2}{x+y} = y^2 + 1$

10. $xe^y = x - y$

Exercises

9. $\frac{x^2}{x+y} = y^2 + 1$

$$\frac{2x\cdot(x+y)-x^2\ (1+y`)}{(x+y)^2} = 2y\, y`$$

$$2x\cdot(x+y) - x^2\ (1+y`) = 2y\, y`\ (x+y)^2$$

$$2x\ (x+y) - x^2 - x^2y` = 2y\, y`\ (x+y)^2$$

$$2x\ (x+y) - x^2 = 2y\, y`\ (x+y)^2 + x^2y`$$

$$2x(x+y) - x^2 = y`(2y(x+y)^2 + x^2)$$

$$2x^2 + 2xy\ - x^2 = y`(2y(x^2+2xy+y^2) + x^2)$$

$$x^2 + 2xy = y`\ (2x^2y + 4xy^2 + 2y^2 + x^2)$$

$$y` = \frac{x(x+2y)}{2x^2y+4xy^2+2y^2+x^2}$$

10. $xe^y = x - y$

$$e^y + xy`e^y = 1\ - y`$$

$$xy`e^y + y` = 1 - e^y$$

$$y`(xe^y + 1) = 1 - e^y$$

$$y` = \frac{1-e^y}{xe^y+1}$$

Exercises

5–22 Find dy/dx by implicit differentiation.

11. $\sin x + \cos y = 2x - 3y$

12. $e^x \sin y = x + y$

Exercises

11. $\sin x + \cos y = 2x - 3y$

$\cos x - y` \sin y = 2 - 3y`$

$3y` - y` \sin y = 2 - \cos x$

$y`(3 - \sin y) = 2 - \cos x$

$y` = \frac{2 - \cos x}{3 - \sin y}$

12. $e^x \sin y = x + y$

$e^x \sin y + e^x y` \cos y = 1 + y`$

$e^x y` \cos y - y` = 1 - e^x \sin y$

$y`(e^x \cos y - 1) = 1 - e^x \sin y$

$y` = \frac{1 - e^x \sin y}{e^x \cos y - 1}$

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Derivatives of Logarithmic and Inverse Trigonometric Functions

- Logarithmic function is the inverse of the exponential function
  - $y = \log_b x \iff y = b^x$

3 Laws of Logarithms If x and y are positive numbers and r is a rational number, then

1. $\ln(xy) = \ln x + \ln y$ **2.** $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ **3.** $\ln(x^r) = r \ln x$

$$\boxed{\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}}$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

$$\boxed{\frac{d}{dx}\ln|x| = \frac{1}{x}}$$

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}}$$

$$\boxed{\frac{d}{dx}(\ln g(x)) = \frac{g`(x)}{g(x)}}$$

Derivatives of Logarithmic and Inverse Trigonometric Functions

- Example: differentiate $y = \ln(x^3 + 1)$
- Use chain rule: assume that $u = x^3 + 1$

$$\boxed{\begin{aligned} \frac{dy}{dx} &= \frac{1}{u} \cdot \frac{du}{dx} \\ &= \frac{1}{x^3+1} \cdot 3x^2 \\ &= \frac{3x^2}{x^3+1} \end{aligned}}$$

Derivatives of Logarithmic and Inverse Trigonometric Functions

- Example: find $\frac{d}{dx}\ln(\sin x)$

Assume $u = \sin x$

$$\frac{d}{dx}\ln(\sin x) = \frac{1}{u} \cdot \frac{d}{dx}(\sin x)$$

$$= \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Derivatives of Logarithmic and Inverse Trigonometric Functions

- The calculation of derivatives of complicated functions can be simplified by taking logarithms.
  - This is called **logarithmic differentiation**.

Steps in Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation $y = f(x)$ and use the Laws of Logarithms to expand the expression.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y' and replace y by $f(x)$.

Derivatives of Logarithmic and Inverse Trigonometric Functions

- Example: differentiate $y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

Take the ln of both sides of the equation:

$$\ln y = \ln\left(x^{\frac{3}{4}}\sqrt{x^2+1}\right) - \ln\left((3x+2)^5\right)$$

$$= \frac{3}{4}\ln x + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$$

$$y` = \frac{1}{y}\frac{dy}{dx} = \frac{3}{4}\cdot\frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+1} - 5\cdot\frac{3}{3x+2}$$

$$y` = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

Derivatives of Logarithmic and Inverse Trigonometric Functions

- The derivatives of the inverse trigonometric functions

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$$

Derivatives of Logarithmic and Inverse Trigonometric Functions

- Example: differentiate $y = \frac{1}{\sin^{-1} x}$

$$\boxed{\begin{aligned} y &= (\sin^{-1} x)^{-1} \\ y` &= -(\sin^{-1} x)^{-2} \cdot \frac{1}{\sqrt{1-x^2}} \\ &= -\frac{1}{(\sin^{-1} x)^2 \cdot \sqrt{1-x^2}} \end{aligned}}$$

Exercises

2–26 Differentiate the function.

2. $g(t) = \ln(3 + t^2)$
3. $f(x) = \ln(x^2 + 3x + 5)$
4. $f(x) = x \ln x - x$
5. $f(x) = \sin(\ln x)$
6. $f(x) = \ln(\sin^2 x)$

Exercises

2. $g(t) = \ln(3 + t^2)$

$$g`(t) = \frac{1}{3+t^2} \cdot 2t \;\rightarrow \frac{2t}{3+t^2}$$

3. $f(x) = \ln(x^2 + 3x + 5)$

$$f`(x) = \frac{1}{x^2+3x+5} \cdot (2x + 3) \rightarrow \frac{2x+3}{x^2+3x+5}$$

4. $f(x) = x \ln x \; - x$

$$f`(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} \; - 1 \;\rightarrow \ln x$$

5. $f(x) = \sin(\ln x)$

$$f`(x) = \cos(\ln x) \cdot \frac{d}{dx} \ln x = \frac{\cos(\ln x)}{x}$$

6. $f(x) = \ln(\sin^2 x)$

$$f(x) = \ln(\sin x)^2 = 2 \ln(\sin x)$$
$$f`(x) = 2 \cdot \frac{1}{\sin x} \cdot \cos x \;\rightarrow \frac{2 \cos x}{\sin x} \;\rightarrow 2 \cot x$$

Exercises

Find an equation of the tangent line to the curve at the given point.

39. $y = \ln(x^2 - 3x + 1)\quad (3, 0)$

Exercises

Find an equation of the tangent line to the curve at the given point.

39. $y = \ln(x^2 - 3x + 1) \quad (3, 0)$

Slope $= y` = \frac{1}{(x^2-3x+1)} \cdot (2x\ - 3)$
$= \frac{2x-3}{(x^2-3x+1)} \rightarrow \frac{2*3-3}{3^2-3*3+1} = 3$
Line eq: $y = slope\ * (x - x_1) + y_1$ $y = 3x\ - 9$

Exercises

45–56 Use logarithmic differentiation to find the derivative of the function.

45. $y = (x^2 + 2)^2 (x^4 + 4)^4$

46. $y = \frac{e^{-x} \cos^2 x}{x^2+x+1}$

Exercises

45. $y = (x^2+2)^2(x^4+4)^4$

$\ln y = \ln\left((x^2+2)^2(x^4+4)^4\right)$
$= \ln\left((x^2+2)^2\right) + \ln\left((x^4+4)^4\right)$
$= 2\ln(x^2+2) + 4\ln(x^4+4)$
$y` = \frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x^2+2} \cdot 2x + 4 \cdot \frac{1}{x^4+4} \cdot 4x^3$
$= \frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}$
$y` = y\left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right) \rightarrow (x^2+2)^2(x^4+4)^4 \cdot \left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right)$

Exercises

46. $y = \frac{e^{-x}\cos^2 x}{x^2+x+1}$

$\ln y = \ln\left(\frac{e^{-x}\cos^2 x}{x^2+x+1}\right)$
$= \ln e^{-x} + \ln\cos^2 x \ - \ln(x^2+x+1)$
$= -x + 2\ln\cos x \ - \ln(x^2+x+1)$
$y` = \frac{1}{y}\cdot\frac{dy}{dx} = -1 + 2\cdot\frac{1}{\cos x}\cdot -\sin x - \frac{(2x+1)}{x^2+x+1}$
$= y\left(-1 - \frac{2\sin x}{\cos x} - \frac{(2x+1)}{x^2+x+1}\right)$
$= \left(\frac{e^{-x}\cos^2 x}{x^2+x+1}\right)\cdot\left(-1 \ - 2\tan x - \frac{(2x+1)}{x^2+x+1}\right)$

TASK

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