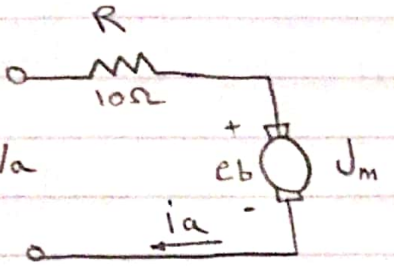
Q1)

Due to the amp. we have

$$V_a = 50\, V_i \quad "0"$$

The motor :



where e_b is the back emf

by writing the KVL eqn. on the motor circuit we get

$$V_a = R i_a + e_b \quad "1"$$

We know that the back emf is proportional to the angular velocity of the motor , this means that

$$V_a = R i_a + K_b \dot{\theta} \quad "2"$$

and from newton's 2nd law we find

$$(J_m + J_L)\, \ddot{\theta} = K_T\, i_a \quad , \text{where } J_L : \text{load inertia} \quad "3"$$

$$\therefore J_m + J_L = 2J$$

from the given equations

$$q_o = 51\, h(t) \quad "4"$$

$$q_1 = 82\, \theta \quad "5"$$

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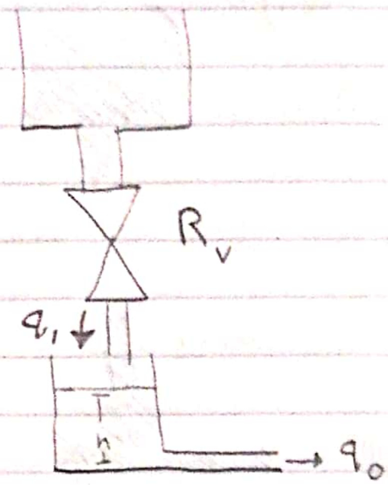
Applying the tank equations yields

$$q_1 - q_o = A\frac{dh}{dt} \quad \text{"6"}$$

$$h = q_o R_v \quad \text{"7"}$$



$$\therefore \quad 82\theta - q_o = A\frac{dh}{dt} \quad \text{"8"}$$

$$A\underline{x} + B\underline{u} = \dot{\underline{x}} \quad \text{"9"}$$

$$C\underline{x} + D\underline{u} = \underline{y} \quad \text{"10"}$$

We choose the states:

$X_1 = \theta$, $X_2 = \dot{\theta}$, $X_3 = h(t)$

the input to our system is V_i, and the output is $h(t)$

Get the state equations:

$$\dot{X}_1 = X_2 \quad \text{"11"}$$

from 4, 5, 6 : $82X_1 - 51X_3 = A\dot{X}_3$

$$\dot{X}_3 = \frac{82}{A}X_1 - \frac{51}{A}X_3 \quad \text{"12"}$$

from 3: $2J\ddot{\theta} = K_T i_a \longrightarrow \dot{X}_2 = \frac{K_T}{2j} i_a$ "13"

Get ia in terms of our chosen states:

from 0, 2: $50u = R i_a + K_b X_2$

$50u - K_b X_2 = R i_a$

$i_a = \frac{-K_b}{R} X_2 + \frac{50}{R} u$ "14"

then, plug eqn 14 into 13

$\dot{X}_2 = \frac{-K_T K_b}{2Rj} X_2 + \frac{50 K_T}{2Rj} u$ "15"

So we have:

$\dot{X}_1 = X_2$ "11"

$\dot{X}_2 = \frac{-K_T K_b}{2Rj} X_2 + \frac{50 K_T}{2Rj} u$ "15"

$\dot{X}_3 = \frac{82}{A} X_1 - \frac{51}{A} X_3$ "12"

Get the output eqn:

$y = X_3$ "16"

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$\therefore$ Our state-space Representation is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -K_T K_b/2RJ & 0 \\ 82/A & 0 & -51/A \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 50K_T/2RJ \\ 0 \end{bmatrix}}_{B} \underline{u}$$

$$y = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$