A Generalized Methodology for Bit-Error-Rate Prediction in Correlation-Based Communication Schemes Using Chaos

Georges Kaddoum, Pascal Chargé, and Daniel Roviras

***Abstract*—The aim of this paper is to present a new and accurate approach to compute the bit-error-rate (BER) performance of coherent and non-coherent chaos-based communication systems. The approach explores the dynamical properties of chaotic sequences and takes into account that the bit energy is varying from one transmitted bit to another. Compared with other widely used approaches in the literature, the proposed methodology gives accurate results even for low spreading factors.**

***Index Terms*—Chaos-based communication systems, bit energy distribution, performance analysis, exact BER expression.**

I. Introduction

CHAOTIC sequences can offer many advantages, such as robustness in multipath environments, resistance to jamming, low probability of interception, and security of transmission. Consequently the exploitation of chaotic signal features in practical communication systems has gained much attention [1], [2], [3], [4]. Many schemes have been proposed and studied, namely chaos shift-keying (CSK) [5], chaos-based DS-CDMA systems [6], [4], and differential chaos shift-keying (DCSK) [2], [3]. For the DCSK system the exact knowledge of the chaotic signal at the receiver side is not required. On the other hand, coherent-detection-based systems, like CSK and chaos-based DS-CDMA, a synchronization unit at the receiver side is used in order to generate a local synchronous chaotic sequence.

Due to the non-periodic nature of chaotic spreading signals, it undoubtedly comes that the transmitted bit energy after spreading by chaotic sequences varies from one bit to another [6] in chaos-based communication schemes. However, many assumptions concerning the bit energy are generally made in order to derive the BER for such systems. In particular, the sum of dependent variables at the output of the correlator is taken as a Gaussian variable [1], the so-called Gaussian assumption. Since chaotic signals are generated with deterministic systems, this assumption suffers from low precision for a small spreading factor. Another approach proposed in [1] integrates the BER expression for a given chaotic map over all possible spreading sequences of given spreading factor produceable by the chaotic map. This latter method is compared to the BER computation under Gaussian assumption in [1] and seems better in matching the exact BER. But, as it is said in [1], it has a high computing charge. A rather drastic approximation is used in [2], [7], in which the transmitted chaotic bit energy is treated as a constant. This approximation yields very imprecise BER performances especially when the considered spreading factor is small. Because previously presented approaches are not valid for small spreading factors or have a high computing charge, another accurate approach was recently developed in [6], [8] to compute the exact BER performance for coherent chaos-based communication systems. The novelty of our proposed work, lies in the generalization of the simple accurate approach [6], [8] to the coherent and non-coherent chaos-based communication systems. Also, in this paper, a comparison with BER prediction results from other widely used approaches is provided. In order to illustrate this purpose. In particular, the chaos-based DS-CDMA and the DCSK systems in mono-user case over an AWGN channel are used as applicative examples. The chaos-based DS-CDMA system is quite similar to the CSK system in [1]. The choices of mono-user and AWGN channel are taken for brevity, but this study can be straightforwardly extended to the multi-user cases and to different types of channels ([6], [9]).

Manuscript received March 26, 2009. The associate editor coordinating the review of this letter and approving it for publication was C. Charalambous.

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Digital Object Identifier 10.1109/LCOMM.2009.090715

II. BER Computing Approach

Since the bit energy changes from one bit to another, the main idea is to associate this bit-energy value to a stationary random process. Then the Probability Density Function (PDF) of the bit energy has to be determined, and an overall BER can be obtained by integrating constant-bit-energy-assumption formula over the bit-energy distribution. This PDF can be evaluated by plotting the histogram of the bit energy for a given chaotic map and for a given number of chaotic chips per bit duration, i.e. spreading factor β.

When a perfect synchronisation is assumed, as in [1], and taking into account the variation of the bit energy, the BER of a chaos-based DS-CDMA system is given by:

$$BER_{coherent} = \int_{0}^{+\infty} \frac{1}{2} erfc\left(\sqrt{\frac{E_{bc}}{N_0}}\right) p\left(E_{bc}\right) dE_{bc} \qquad (1)$$

where $erfc(x)$ is the complementary error function. An expression of the bit energy is $E_{bc} = T_c \sum_{k=1}^{\beta} x_{i\beta+k}^2$, where $x_{i\beta+k}$ is the k^{th} chaotic sample in the i^{th} transmitted bit, and T_c is the chip duration. The spreading factor is then defined as $\beta = Ts/T_c$, with T_s the bit duration. In (1), $N_0/2$ stands for the Gaussian noise variance coming from the channel, and $p(E_{bc})$ is the above mentioned PDF.

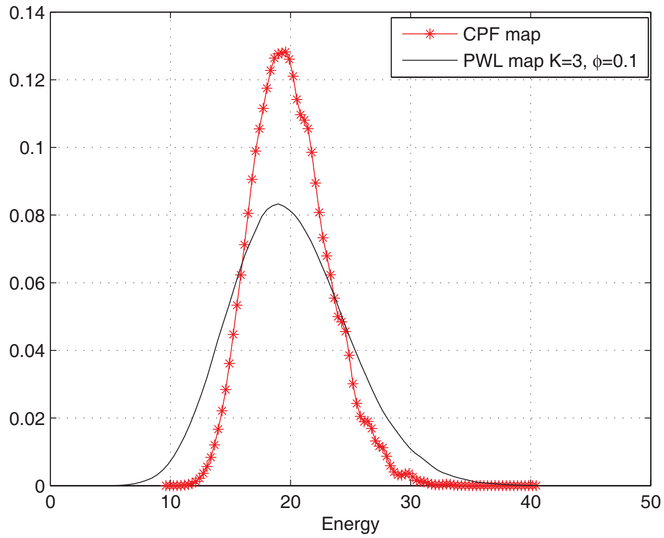

Fig. 1. Simulated distribution of bit energy for a spreading factor of 20 and chaotic sequences generated with CPF and PWL maps.

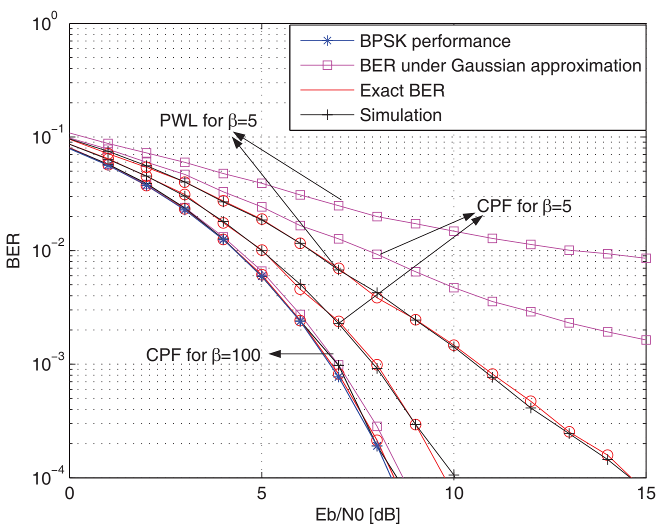

Fig. 2. Exact BER computation, BER computation under Gaussian approximation, Simulations with CPF and PWL maps ($\beta = 5;\ 100$).

Figure 1 shows the histogram of the bit energy. In this study, two different chaotic maps have been chosen in order to illustrate the new approach. The first one is the Chebyshev polynomial function of order 2 (CPF) [1] and the second one is a piecewise linear map (PWL) [10]. Histograms associated to these maps are obtained here by performing ten million chaotic samples. Looking carefully at the PDF in Figure 1, we can predict that the CPF sequence will give better results in terms of BER than the PWL, because the CPF sequence has more centralized distribution of values. Expression (1) can be processed using two different ways. Firstly, when the PDF of the bit energy can not be approximated with a known distribution, as for CPF spreading in Figure 1, an analytical integration of (1) has to be a abandoned. But fortunately, a calculation of (1) can always be performed by running a numerical integration method. This approach can be applied for any type of chaotic sequence with quite simple operations:

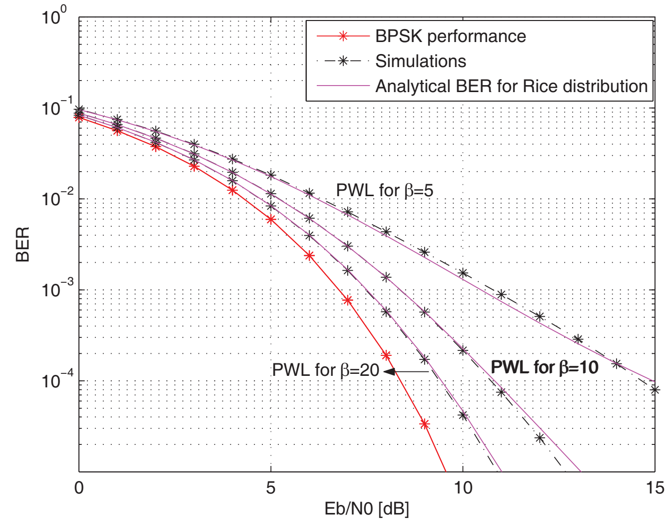

Fig. 3. Analytical and simulated BER for $\beta = 5;\ 10,\ 20$ of PWL map

calculation of the histogram of the bit energy followed by a numerical integration. Figure 2 shows simulation results and the numerical integration output of the BER of equation (1). The BER under the Gaussian approximation given in [1], and the lower bound of chaos based communication systems corresponding to the BPSK case are also provided in this figure. The perfect match between simulation results and the proposed numerical method confirms the accuracy of the approach. In addition, it can be observed that the Gaussian approximation gives poor results for small spreading factor values.

In some cases, the distribution of the bit energy can be associated with a well known distribution. In these cases, an analytical derivation of (1) is available. Concerning the use of the PWL map in a chaos-based DS-CDMA system, the following solution can be proposed:

$$BER_{coherent} = \int_{0}^{+\infty} \frac{1}{2} erfc\left(\sqrt{\frac{Y_2^2}{N_0}}\right) p\left(Y_2\right) dY_2;\ Y_2 = \sqrt{E_{bc}} \tag{2}$$

To compute analytically (2), the PDF of the root square bit-energy is required. This PDF can be approximated with known distributions such that Rayleigh, Nakagami or Rice. Then referring to the intensive work on the analytical expression of BER over Rayleigh, Nakagami or Rice channels, an analytical solution of (2) can be determined, and is provided in [8]. In Figure 3, the BER obtained using the analytical expression with a Rice distribution and the BER given by Monte Carlo simulations for PWL map are compared. The lower bound BER is also plotted for reference. It clearly appears in Figure 3 that there is a perfect match between simulations and the analytical results even for a very small spreading factor.

The DCSK system [7], [2], [3], [11] is probably the most studied non-coherent chaos-based communication system. The autocorrelation and cross-correlation problems were recognized and described in [12], [13]. The bit error rate for DCSK system when the CPF map is used as chaotic generator is given in [5]. As it will be shown, this expression is not valid for low

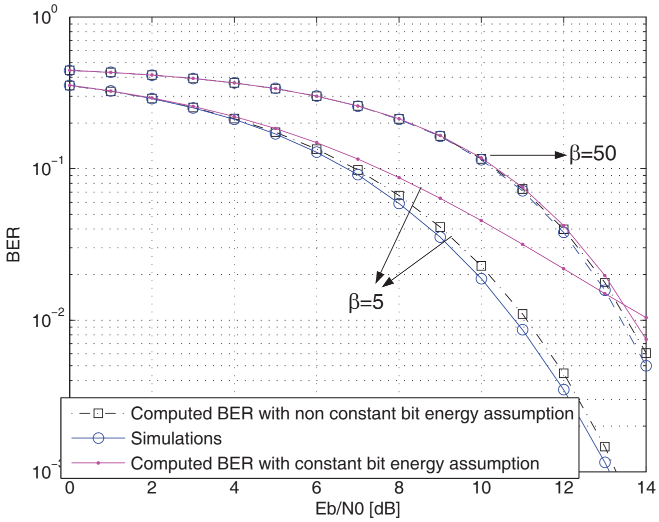

Fig. 4. Computed BER with non constant energy assumption, Simulations, Computed BER expression with constant energy assumption for spreading factors $\beta = 5;\ 50$ of CPF map.

spreading factor values. In order to compute the BER with our approach, the bit error probability has first to be evaluated for a given received energy E_i. Considering the bit energy as a deterministic variable, the decision variable at the output of the correlator is necessarily a random Gaussian variable. It comes that this bit error probability is

$P_e(E_i) = \frac{1}{2}erfc\left(\sqrt{\frac{E_i}{4N_0}\left(1+\frac{\beta N_0}{2E_i}\right)^{-1}}\right)$, where $E_i = 2T_c\sum_{k=1}^{\beta} x_{i\beta+k}^2$ is the given symbol bit energy. Then, the overall BER expression of the DCSK system is given by :

$$BER = \int_0^{+\infty} \frac{1}{2}erfc\left(\sqrt{\frac{E_i}{4N_0}\left(1+\frac{\beta N_0}{2E_i}\right)^{-1}}\right)p\left(E_i\right)dE_i \quad (3)$$

Since an analytical expression seems difficult to obtain, the proposed numerical integration method is used to perform the BER computation. That is, the expression (3) is computed numerically, taking into account the bit-energy variation. Figure 4 presents the BER curves obtained from numerical integration of (3), the Monte Carlo simulations of the DCSK system, and the BER expression given in [5], for spreading factors $\beta = 5,\ 50$. It clearly shows that there is an excellent match between simulations and the proposed numerically integrated BER especially when the spreading factor is low. Moreover, for a low spreading factor it can be observed that the difference between simulations and the BER expression based on the constant bit energy increases when the noise variance decreases. For a large spreading factor the bit-energy variation is small [3], [6]. Nevertheless, for low spreading factor $(\beta = 5)$ our approach gives more accurate results compared to [5].

III. Conclusion

Since chaotic sequences are generated from deterministic systems and have a non-periodic nature, neither the constant bit-energy assumption, or the Gaussian approximation, are expected to provide accurate results. In this paper, we have presented a new simple methodology for computing the BER expression for coherent and non-coherent chaos-based communication systems. The mono-user case has been considered and the AWGN channel assumed. The new approach to derive the BER is based on the bit-energy distribution and gives accurate results with perfect match to simulations. The numerical integration method can be applied to any types of chaos-based communication system and chaotic maps with quite simple operations: calculation of the histogram of the bit energy followed by a numerical integration. For coherent systems, when the PDF of the root square bit energy follows a known distribution, the analytical BER expression can be easily derived. In addition, this approach explores the dynamic properties of chaotic sequences and gives results with a very high accuracy. Moreover due to its low computation charge, this method is implementable for realistic sized systems.

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