Project 2

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Kindly note that this is an ***interactive*** script, and it's recommended to be viewed as a live script on MATLAB (.mlx) instead of a normal PDF file.

This ensures the optimum experience intended for the reader.

Part 1

Assume a random variable X has M symbols with the following distribution:

$$f_X\left(x\right) = \frac{1}{6}; \text{for } x = \ 1, 2, 3, 4, 5, \text{ and } 6.$$

Mapping each symbol to a fixed-length code with the minimum number of bits needed to represent *M* symbols:

```
fprintf("Minimum number of bits needed to map each symbol of f(x) using a fixed-length code = %d bits ",ceil(log2(6)));
```

```
Minimum number of bits needed to map each symbol of f(x) using a fixed-length code = 3 bits
```

```
FixedCode_X = ['100' '101' '110' '111' '010' '001'];
dispTable = ["100" "101" "110" "111" "010" "001"];
array2table(dispTable, "VariableNames",{'1' '2' '3' '4' '5' '6'}, "RowNames",{'Codeword'})
```

ans = 1×6 table

	1	2	3	4	5	6
1 Codeword	"100"	"101"	"110"	"111"	"010"	"001"

And of course, since each codeword has the same length, it would make sense that the average length of the codewords is *3.*

Find a Huffman code and calculate its average code length.

```
X_symbols = (1:6); % Symbols vector
X_prob = [1/6 1/6 1/6 1/6 1/6 1/6]; % Symbol probability vector
[codeword_dict_x,average_length_x] = huffmandict(X_symbols,X_prob);
part1HuffmanTable = cell2table(codeword_dict_x);
part1HuffmanTable.Properties.VariableNames = {'Symbol', 'Codeword'};

disp(part1HuffmanTable)
```

```
    Symbol    Codeword
    ______    ________

      1       {[  1 1]}
      2       {[  1 0]}
      3       {[0 0 1]}
      4       {[0 0 0]}
      5       {[0 1 1]}
      6       {[0 1 0]}
```

```
%Generate a binary Huffman code, displaying the average code length and the cell array containing the codeword dictionary.

%codeword_dict_x = cellfun(@num2str,codeword_dict_x(:,2),'UniformOutput',false);
%disp("Codeword of f(x)");
%huffTableX = cell2table(codeword_dict_x);
%disp((huffTableX));

%[dict,avglen] = huffmandict(X_symbols,X_prob)
%dict = cellfun(@num2str,dict(:,2),'UniformOutput',false)
%disp(dict)

H_X = 0;
 for i=1:length(X_prob)
 % Calculating the Entropy
 H_X = H_X + (-1*X_prob(i)*log2(X_prob(i)));
 end

 %fprintf("Entropy of huffman code of f(x)=%.3f bits",H_X);
 fprintf("Source entropy = %.3f bits",H_X);
```

```
Source entropy = 2.585 bits
```

```
 fprintf("Average Length of huffman code = %.3f bits/symbol ",average_length_x)
```

```
Average Length of huffman code = 2.667 bits/symbol
```

Compare the average code lengths in parts 1) and 2) above to each other and to the Entropy of the random variable.

Average Length of Huffman is less than the code of a fixed-length.

The average length of Huffman is between the entropy and entropy + 1

Comment on your observations:

Both codes are optimum codes as they are included in the bounds of the optomal code length (i.e. H <= L < H + 1) but Huffman is better as it has average code length less than the other but due to equiprobable Huffman didn't make that much difference than using normal code.

4) Using the attached .mat file of sample data from the random variable:

• Verify the probability mass function of the random variable.

```
load(strcat(fileparts(matlab.desktop.editor.getActiveFilename),"\ELCN446_Project2_Spring2022.mat"));        %load the attached .mat file
fprintf("The X variable is loaded and it has a length %d ",length(X));
```

```
The X variable is loaded and it has a length 960
```

```
X_uniqueChars = unique(X); % String text has all characters, some are repeatedlenChar=length(uniqueChars);
disp(X_uniqueChars);
```

```
     1     2     3     4     5     6
```

```
%PMF%
f=zeros(1,length(X_uniqueChars));
for i=1:length(X_uniqueChars)
% Count the number of occurence of unique characters
f(i)=length(strfind(X,X_uniqueChars(i)));
end
p=zeros(1,length(X_uniqueChars));
PMF_X = [length(length(X_uniqueChars))]; % declare the Probability Mass Function with size of special chars
for i=1:length(X_uniqueChars)
% Probabilities for each unique character in the file
p(i)=f(i)/length(X);
PMF_X(i) = p(i); %PMF
end
disp("PMF of X")
```

```
PMF of X
```

```
array2table(PMF_X, "RowNames",{'PMF'}, "VariableNames",{'1' '2' '3' '4' '5' '6'})
```

ans = 1×6 table

	1	2	3	4	5	6
1 PMF	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667

Generate the source coded codewords for both the fixed-length code and the Huffman code.

```
%Generate the source coded code words using the fixed length code:
%disp("Using the fixed Code:");
%disp("Encode the Codewords:")
Transmit_X_fixed =0;

for i=1:length(X)
    temp=0;
    for j=1:length(X_uniqueChars)
     if X(i)== X_uniqueChars(j)
         if i==1
             Transmit_X_fixed= FixedCode_X(j+temp*2:j+2+temp*2);
            % disp(Transmit_X_fixed)
         else
             Transmit_X_fixed = append(Transmit_X_fixed,FixedCode_X(j+temp*2:j+2+temp*2));
             % disp(Transmit_X_fixed)
         end
         %disp("test")
     end
     temp = temp + 1;
    end
 end


 disp("The bits that are going to be transmitted using the Fixed code:");
```

```
The bits that are going to be transmitted using the Fixed code:
```

```
disp(Transmit_X_fixed);
```

```
0011001010101000101000100101001001000010010010100011001001011100101101100010101101111101011001111101111001011110110110101010001010010111011011110100111101
```

```
num_of_bits_X_fixed = length(Transmit_X_fixed);
fprintf("The Total number of bits that are going to be transmitted using fixed code = %d", num_of_bits_X_fixed);
```

```
The Total number of bits that are going to be transmitted using fixed code = 2880
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.

disp("Decode the Codewords back to symbols using the fixed code:")
```

```
Decode the Codewords back to symbols using the fixed code:
```

```
Recieve_X_fixed=[];
i=1;
indexX=1;
  while i<=length(Transmit_X_fixed)
      temp = 0;
     for j=1:length(X_uniqueChars)

          if indexX > length(X)
              break;
          end

          if Transmit_X_fixed(i:(i+2)) == FixedCode_X(j+temp*2:j+2+temp*2)

              Recieve_X_fixed(indexX) = X_uniqueChars(j) ;
              i = i+3;
              indexX=indexX+1;


          end
          temp = temp + 1;
     end
  end
 disp(Recieve_X_fixed);
```

```
  Columns 1 through 24

     6     1     2     5     1     5     1     5     5     1     1     1     6     6     6     5     6     1     1     2     3     5     3     3

  Columns 25 through 48

     6     5     3     4     3     2     1     4     3     4     3     5     4     2     2     2     5     1     5     1     2     3     3     4

  Columns 49 through 72

     2     6     4     5     3     1     2     6     2     1     5     4     4     3     2     3     1     6     3     3     4     2     4     6

  Columns 73 through 96

     1     3     5     1     5     4     3     4     3     5     4     3     2     5     3     1     1     3     2     6     5     1     3     5
```

```
if isequal(X,Recieve_X_fixed)
    disp("No losses were found when using the Fixed Code since the recieved bits of X are the same as the transmitted bits");
end
```

```
No losses were found when using the Fixed Code since the recieved bits of X are the same as the transmitted bits
```

```
%Generate the source coded code words using the Huffman code:
disp("Using the Huffman code:");
```

```
Using the Huffman code:
```

```
[dict_sentX,avglen_sentX] = huffmandict(X_uniqueChars,PMF_X);   %X_unique: Symbols
disp("Code Word of X that is Loaded from the .mat file:");
```

```
Code Word of X that is Loaded from the .mat file:
```

```
dict_sentX = cellfun(@num2str,dict_sentX(:,2),'UniformOutput',false);
disp(dict_sentX);
```

```
    {'1  1'   }
    {'1  0'   }
    {'0  0  1'}
    {'0  0  0'}
    {'0  1  1'}
    {'0  1  0'}
```

```
%disp("Test")
%disp((append(regexprep(cell2mat(dict_sentX(1)), '\s', '') , regexprep(cell2mat(dict_sentX(3)), '\s', ''))))
%disp(regexprep(cell2mat(dict_sentX(1)), '\s',''))
%disp(length(regexprep(cell2mat(dict_sentX(1)), '\s','')))
%zxc= regexprep(cell2mat(dict_sentX(1)), '\s','');
%disp(zxc)
```

```
%disp("test")
%disp(transpose(cell2mat(dict_sentX(1))))
%disp(regexprep(cell2mat(dict_sentX(3)), '\s', '') + regexprep(cell2mat(dict_sentX(2)), '\s', ''))

disp("Encode the Codewords:")
```

```
Encode the Codewords:
```

```
Transmit_X =0;
 for i=1:length(X)
    for j=1:length(X_uniqueChars)
     if X(i)== X_uniqueChars(j)
         if i==1
             Transmit_X= regexprep(cell2mat(dict_sentX(j)), '\s', '');
             %disp(Transmit_X)
         else
             Transmit_X = append(Transmit_X,regexprep(cell2mat(dict_sentX(j)), '\s', ''));
         end
         %disp("test")
     end
    end
 end
 disp("The bits that are going to be transmitted using the huffman code:");
```

```
The bits that are going to be transmitted using the huffman code:
```

```
 disp(Transmit_X);
```

```
0101110011110111101101111111101001001001101011111000101100100101001100100000110110000010000010110001010100111101111100010010001001000001100111100101011011
```

```
 num_of_bits_X = length(Transmit_X);
 fprintf("The Total number of bits that are going to be transmitted = %d", num_of_bits_X);
```

```
The Total number of bits that are going to be transmitted = 2560
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.
%sig = huffmandeco(double(Transmit_X),dict_sentX);

disp("Decode the Codewords back to symbols:")
```

```
Decode the Codewords back to symbols:
```

```
Recieve_X=[];
i=1;
indexX=1;
 while i<=length(Transmit_X)
     for j=3:length(dict_sentX)
     for k = 1:2
          if indexX > length(X)
              break;
          end

          if Transmit_X(i:(i+1)) == regexprep(cell2mat(dict_sentX(k)), '\s', '')

              Recieve_X(indexX) = X_uniqueChars(k) ;
              %disp(Recieve_X)  %for the test
              i = i+2;
              indexX=indexX+1;

          elseif  Transmit_X(i:(i+2)) == regexprep(cell2mat(dict_sentX(j)), '\s', '')

              Recieve_X(indexX) = X_uniqueChars(j) ;
              i = i+3;
              indexX=indexX+1;

      end
      end
     end
  end
 disp(Recieve_X);
```

```
  Columns 1 through 24

     6     1     2     5     1     5     1     5     5     1     1     1     6     6     6     5     6     1     1     2     3     5     3     3

  Columns 25 through 48

     6     5     3     4     3     2     1     4     3     4     3     5     4     2     2     2     5     1     5     1     2     3     3     4

  Columns 49 through 72

     2     6     4     5     3     1     2     6     2     1     5     4     4     3     2     3     1     6     3     3     4     2     4     6
```

```
Columns 73 through 96
```

```
if isequal(X,Recieve_X)
    disp("No losses were found when using the Huffman Code since the recieved bits of X are the same as the transmitted bits");
end
```

```
No losses were found when using the Huffman Code since the recieved bits of X are the same as the transmitted bits
```

Compare the total number of bits to be transmitted in each case.

The Total number of bits to be transmitted using the fixed-length code is greater than using the Huffman code since it's 2880 in fixed-length code while it is 2560 in Huffman code

so it is clear that the Huffman code is more efficient than the fixed-length code. (i.e. send less number of bits)

Decode the codewords back to symbols and check for any losses compared to the original data.

No losses were found since the generated vector (The Recieved vector) is equal to the original vector.

Part 2

Map each symbol to a fixed-length code with the minimum number of bits needed to represent *M* symbols:

For F(y):

```
disp("F(y) :");
```

```
F(y) :
```

```
fprintf("Minimum number of bits needed to map each symbol of f(y) using a fixed-length code = %d bits ",ceil(log2(6)));
```

```
Minimum number of bits needed to map each symbol of f(y) using a fixed-length code = 3 bits
```

```
fprintf("Average code length for a fixed-length code = %.2f",ceil(log2(6)));
```

```
Average code length for a fixed-length code = 3.00
```

```
FixedCode_Y = ['100' '101' '110' '111' '010' '001'];
```

So f(Y=1) will have the code: 100

f(Y=2) will have the code: 101

f(Y=3) will have the code: 110

f(Y=4) will have the code: 111

f(Y=5) will have the code: 010

f(Y=6) will have the code: 001

Note: the code may be different every time

2) Find a Huffman code and calculate its average code length.

```
Y_symbols = (1:6); % Symbols vector
Y_prob = [(0.5)^1 (0.5)^2 (0.5)^3 (0.5)^4 (0.5)^5 (0.5)^5]; % Symbol probability vector
[codeword_dict_y,average_length_y] = huffmandict(Y_symbols,Y_prob);
disp("Size of the representation of the code with the minimum number of bits for each Y");
```

```
Size of the representation of the code with the minimum number of bits for each Y
```

```
disp(['   symbol','  ','Size of codeword'])
```

```
   symbol  Size of codeword
```

```
disp(codeword_dict_y);
```

```
    {[1]}    {[       0]}
    {[2]}    {[     1 0]}
    {[3]}    {[   1 1 0]}
    {[4]}    {[  1 1 1 0]}
    {[5]}    {[1 1 1 1 1]}
    {[6]}    {[1 1 1 1 0]}
```

```
%Generate a binary Huffman code, displaying the average code length and the cell array containing the codeword dictionary.

codeword_dict_y = cellfun(@num2str,codeword_dict_y(:,2),'UniformOutput',false);
disp("Codeword of f(y)");
```

```
Codeword of f(y)
```

```
huffTableY = cell2table(codeword_dict_y);
```

```
disp((huffTableY));
```

```
codeword_dict_y
_______________

{'0'           }
{'1  0'        }
{'1  1  0'     }
{'1  1  1  0'  }
{'1  1  1  1  1'}
{'1  1  1  1  0'}
```

```
%[dict,avglen] = huffmandict(X_symbols,X_prob)
%dict = cellfun(@num2str,dict(:,2),'UniformOutput',false)
%disp(dict)

H_Y = 0;
 for i=1:length(Y_prob)
 % Calculating the Entropy
 H_Y = H_Y + (-1*Y_prob(i)*log2(Y_prob(i)));
 end

 fprintf("Source Entropy=%.3f bits",H_Y);
```

```
Source Entropy=1.938 bits
```

```
 fprintf("Average Length of huffman code of f(y)=%.3f ",average_length_y);
```

```
Average Length of huffman code of f(y)=1.938
```

3) Compare the average code lengths in parts 1) and 2) above to each other and to the Entropy of the random variable.

Average Length of Huffman is less than the code of a fixed-length.

The average length of Huffman code is almost equal to the entropy so it's most efficient coding compared to use than the fixed-length code.

Comment on your observations:

The fixed length code isn't an optimal code while Huffman is an optimal code and Huffman is better as it has average code length less than the other.

4) Using the attached .mat file of sample data from the random variable:

• Verify the probability mass function of the random variable.

```
fprintf("The file .mat is already loaded from part I");
```

```
The file .mat is already loaded from part I
```

```
%load("ELCN446_Project2_Spring2022.mat");        %load the attached .mat file    (You don't need to load it again in Part II)
fprintf("The Y variable is loaded and it has a length %d ",length(Y));
```

```
The Y variable is loaded and it has a length 960
```

```
Y_uniqueChars = unique(Y); % String text has all characters, some are repeatedlenChar=length(uniqueChars);
disp(Y_uniqueChars);
```

```
     1     2     3     4     5     6
```

```
%PMF%
f=zeros(1,length(Y_uniqueChars));
for i=1:length(Y_uniqueChars)
% Count the number of occurence of unique characters
f(i)=length(strfind(Y,Y_uniqueChars(i)));
end
p=zeros(1,length(Y_uniqueChars));
PMF_Y = [length(length(Y_uniqueChars))]; % declare the Probability Mass Function with size of special chars
for i=1:length(Y_uniqueChars)
% Probabilities for each unique character in the file
p(i)=f(i)/length(Y);
PMF_Y(i) = p(i); %PMF
end
disp("PMF of Y")
```

```
PMF of Y
```

```
disp(transpose(PMF_Y));
```

```
    0.5000
    0.2500
    0.1250
    0.0625
    0.0312
    0.0312
```

- **Generate the source coded codewords for both the fixed-length code and the Huffman code.**

```
%Generate the source coded code words using the fixed length code:
disp("Using the fixed Code:");
```

```
Using the fixed Code:
```

```
disp("Encode the Codewords:")
```

```
Encode the Codewords:
```

```
Transmit_Y_fixed =0;

for i=1:length(Y)
    temp=0;
    for j=1:length(Y_uniqueChars)
     if Y(i)== Y_uniqueChars(j)
         if i==1
             Transmit_Y_fixed= FixedCode_Y(j+temp*2:j+2+temp*2);

         else
             Transmit_Y_fixed = append(Transmit_Y_fixed,FixedCode_Y(j+temp*2:j+2+temp*2));

         end
         %disp("test")
     end
     temp = temp + 1;
    end
 end


 disp("The bits that are going to be transmitted using the Fixed code:");
```

```
The bits that are going to be transmitted using the Fixed code:
```

```
disp(Transmit_Y_fixed);
```

```
1001001001001001001000101101101001111111111001001001011001011000011011111001001001101101101001011001001011001001001011011001100101001001101010101
```

```
num_of_bits_Y_fixed = length(Transmit_Y_fixed);
fprintf("The Total number of bits that are going to be transmitted using fixed code = %d", num_of_bits_Y_fixed);
```

```
The Total number of bits that are going to be transmitted using fixed code = 2880
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.

disp("Decode the Codewords back to symbols using the fixed code:")
```

```
Decode the Codewords back to symbols using the fixed code:
```

```
Recieve_Y_fixed=[];
i=1;
indexY=1;
  while i<=length(Transmit_Y_fixed)
      temp = 0;
     for j=1:length(Y_uniqueChars)

          if indexY > length(Y)
              break;
          end

          if Transmit_Y_fixed(i:(i+2)) == FixedCode_Y(j+temp*2:j+2+temp*2)

              Recieve_Y_fixed(indexY) = Y_uniqueChars(j) ;
              i = i+3;
              indexY=indexY+1;


          end
          temp = temp + 1;
     end
  end
 disp(Recieve_Y_fixed);
```

```
Columns 1 through 24
```

```
     1     1     1     1     1     1     1     5     3     3     1     4     4     4     1     1     1     2     1     2     1     6     2     4

  Columns 25 through 48

     1     1     1     3     3     3     1     2     1     1     2     1     1     2     1     1     1     2     2     1     3     5     1     1

  Columns 49 through 72

     3     2     5     1     2     2     1     3     1     1     6     1     2     1     2     1     1     1     1     2     3     1     2     3

  Columns 73 through 96
```

```
if isequal(Y,Recieve_Y_fixed)
    disp("No losses were found when using the Fixed Code since the recieved bits of Y are the same as the transmitted bits");
end
```

```
No losses were found when using the Fixed Code since the recieved bits of Y are the same as the transmitted bits
```

```
%Generate the source coded code words using the Huffman code:
disp("Using the Huffman code:");
```

```
Using the Huffman code:
```

```
[dict_sentY,avglen_sentY] = huffmandict(Y_uniqueChars,PMF_Y);   %X_unique: Symbols
disp("Code Word of Y that is Loaded from the .mat file:");
```

```
Code Word of Y that is Loaded from the .mat file:
```

```
dict_sentY = cellfun(@num2str,dict_sentY(:,2),'UniformOutput',false);
disp(dict_sentY);
```

```
    {'0'          }
    {'1  0'       }
    {'1  1  0'    }
    {'1  1  1  0'  }
    {'1  1  1  1  1'}
    {'1  1  1  1  0'}
```

```
%disp("Test")
%disp((append(regexprep(cell2mat(dict_sentX(1)), '\s', '') , regexprep(cell2mat(dict_sentX(3)), '\s', ''))))
%disp(regexprep(cell2mat(dict_sentX(1)), '\s',''))
%disp(length(regexprep(cell2mat(dict_sentX(1)), '\s','')))
%zxc= regexprep(cell2mat(dict_sentX(1)), '\s','');
%disp(zxc)
%disp("test")
%disp(transpose(cell2mat(dict_sentX(1))))
%disp(regexprep(cell2mat(dict_sentX(3)), '\s', '') + regexprep(cell2mat(dict_sentX(2)), '\s', ''))

disp("Encode the Codewords:")
```

```
Encode the Codewords:
```

```
Transmit_Y =0;
 for i=1:length(Y)
    for j=1:length(Y_uniqueChars)
     if Y(i)== Y_uniqueChars(j)
         if i==1
             Transmit_Y= regexprep(cell2mat(dict_sentY(j)), '\s', '');

         else
             Transmit_Y = append(Transmit_Y,regexprep(cell2mat(dict_sentY(j)), '\s', ''));
         end
         %disp("test")
     end
    end
 end
 disp("The bits that are going to be transmitted using the huffman code:");
```

```
The bits that are going to be transmitted using the huffman code:
```

```
 disp(Transmit_Y);
```

```
0000000111111101100111011101110000100100111101011100001101101100100010001000100001010011011111001101011111010100110001111001001000001011001011010001100000110
```

```
 num_of_bits_Y = length(Transmit_Y);
```

```
fprintf("The Total number of bits that are going to be transmitted = %d", num_of_bits_Y);
```

```
The Total number of bits that are going to be transmitted = 1860
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.
%sig = huffmandeco(double(Transmit_X),dict_sentX);

disp("Decode the Codewords back to symbols:")
```

```
Decode the Codewords back to symbols:
```

```
Recieve_Y=[];
i=1;
indexY=1;
 while i<=length(Transmit_Y)
     for j=5:length(dict_sentY)

          if indexY > length(Y)
              break;
          end
         if Transmit_Y(i) == regexprep(cell2mat(dict_sentY(1)), '\s', '')
             Recieve_Y(indexY) = Y_uniqueChars(1) ;
              %disp(Recieve_Y)  %for the test
              i = i+1;
              indexY=indexY+1;
              break;

        elseif Transmit_Y(i:(i+1)) == regexprep(cell2mat(dict_sentY(2)), '\s', '')

              Recieve_Y(indexY) = Y_uniqueChars(2) ;
              %disp(Recieve_Y)  %for the test
              i = i+2;
              indexY=indexY+1;
              break;

        elseif Transmit_Y(i:(i+2)) == regexprep(cell2mat(dict_sentY(3)), '\s', '')


              Recieve_Y(indexY) = Y_uniqueChars(3) ;
              %disp(Recieve_Y)  %for the test
              i = i+3;
              indexY=indexY+1;
              break;

        elseif Transmit_Y(i:(i+3)) == regexprep(cell2mat(dict_sentY(4)), '\s', '')


              Recieve_Y(indexY) = Y_uniqueChars(4) ;
              %disp(Recieve_Y)  %for the test
              i = i+4;
              indexY=indexY+1;
              break;

          elseif  Transmit_Y(i:(i+4)) == regexprep(cell2mat(dict_sentY(j)), '\s', '')

              Recieve_Y(indexY) = Y_uniqueChars(j) ;
             % disp(Recieve_Y)  %for the test
              i = i+5;
              indexY=indexY+1;
              break;

      end
     end
  end
 disp(Recieve_Y);
```

```
  Columns 1 through 24

     1     1     1     1     1     1     1     5     3     3     1     4     4     4     1     1     1     2     1     2     1     6     2     4

  Columns 25 through 48

     1     1     1     3     3     3     1     2     1     1     2     1     1     2     1     1     1     2     2     1     3     5     1     1

  Columns 49 through 72

     3     2     5     1     2     2     1     3     1     1     6     1     2     1     2     1     1     1     1     2     3     1     2     3

  Columns 73 through 96

     2     1     1     3     1     1     1     1     3     2     2     1     2     2     2     6     3     1     3     1     2     1     1     1
```

```
if isequal(Y,Recieve_Y)
    disp("No losses were found when using the Huffman Code since the recieved bits of Y are the same as the transmitted bits");
end
```

```
No losses were found when using the Huffman Code since the recieved bits of Y are the same as the transmitted bits
```

Compare the total number of bits to be transmitted in each case.

The Total number of bits to be transmitted using the naive code is greater than using the Huffman code since it's 2880 in naive code while it is 1860 in Huffman code so it is clear that the Huffman code is more efficient than the naive code. (i.e. send less number of bits)

• Decode the codewords back to symbols and check for any losses compared to the original data.

No losses were found since the generated vector (The Recieved) is equal to the original vector

For F(Z):

1) Map each symbol to a fixed-length code with the minimum number of bits needed to represent *M* symbols.

```
disp("F(z) :");
```

```
F(z) :
```

```
fprintf("Minimum number of bits needed to map each symbol of f(z) using a fixed-length code = %d bits ",ceil(log2(6)));
```

```
Minimum number of bits needed to map each symbol of f(z) using a fixed-length code = 3 bits
```

```
fprintf("Average code length for a fixed-length code = %.2f",ceil(log2(6)));
```

```
Average code length for a fixed-length code = 3.00
```

```
FixedCode_Z = ['100' '101' '110' '111' '010' '001'];
```

So f(Z=1) will have the code: 100

f(Z=2) will have the code: 101

f(Z=3) will have the code: 110

f(Z=4) will have the code: 111

f(Z=5) will have the code: 010

f(Z=6) will have the code: 001

Note: the code may be different every time

2) Find a Huffman code and calculate its average code length.

```
Z_symbols = (1:6); % Symbols vector
Z_prob = [0.05 0.10 0.30 0.25 0.15 0.15]; % Symbol probability vector
[codeword_dict_z,average_length_z] = huffmandict(Z_symbols,Z_prob);
disp("Size of the representation of the code with the minimum number of bits for each Z");
```

```
Size of the representation of the code with the minimum number of bits for each Z
```

```
disp(['   symbol','  ','Size of codeword'])
```

```
   symbol  Size of codeword
```

```
disp(codeword_dict_z);
```

```
    {[1]}    {[1 1 1]}
    {[2]}    {[1 1 0]}
    {[3]}    {[  0 0]}
    {[4]}    {[  1 0]}
    {[5]}    {[0 1 1]}
    {[6]}    {[0 1 0]}
```

```
%Generate a binary Huffman code, displaying the average code length and the cell array containing the codeword dictionary.

codeword_dict_z = cellfun(@num2str,codeword_dict_z(:,2),'UniformOutput',false);
disp("Codeword of f(z)");
```

```
Codeword of f(z)
```

```
huffTableZ = cell2table(codeword_dict_z);
disp((huffTableZ));
```

codeword_dict_z

```
    {'1  1  1'}
    {'1  1  0'}
    {'0  0'   }
    {'1  0'   }
    {'0  1  1'}
    {'0  1  0'}
```

```
%[dict,avglen] = huffmandict(X_symbols,X_prob)
%dict = cellfun(@num2str,dict(:,2),'UniformOutput',false)
%disp(dict)

H_Z = 0;
 for i=1:length(Z_prob)
 % Calculating the Entropy
 H_Z = H_Z + (-1*Z_prob(i)*log2(Z_prob(i)));
 end

 fprintf("Source Entropy = %.3f bits",H_Z);
```

```
Source Entropy = 2.390 bits
```

```
 fprintf("Average Length of huffman code of f(z)=%.3f ",average_length_z);
```

```
Average Length of huffman code of f(z)=2.450
```

3) Compare the average code lengths in parts 1) and 2) above to each other and to the Entropy of the random variable.

Average Length of Huffman is less than the code of a fixed-length so Huffman is more efficient.

The average length for Huffman code is between the entropy and entropy + 1 so it's more optimum.

Comment on your observations:

Both codes are optimum codes as they are included in the bounds of the optomal code length (i.e. H <= L < H + 1) but Huffman is better as it has average code length less than the other but due to equiprobable Huffman didn't make that much difference than using normal code.

4) Using the attached .mat file of sample data from the random variable:

• Verify the probability mass function of the random variable.

```
fprintf("The file .mat is already loaded from part I");
```

```
The file .mat is already loaded from part I
```

```
%load("ELCN446_Project2_Spring2022.mat");        %load the attached .mat file    (You don't need to load it again in Part II)
fprintf("The Z variable is loaded and it has a length %d ",length(Z));
```

```
The Z variable is loaded and it has a length 960
```

```
Z_uniqueChars = unique(Z); % String text has all characters, some are repeatedlenChar=length(uniqueChars);
disp(Z_uniqueChars);
```

```
     1     2     3     4     5     6
```

```
%PMF%
f=zeros(1,length(Z_uniqueChars));
for i=1:length(Z_uniqueChars)
% Count the number of occurence of unique characters
f(i)=length(strfind(Z,Z_uniqueChars(i)));
end
p=zeros(1,length(Z_uniqueChars));
PMF_Z = [length(length(Z_uniqueChars))]; % declare the Probability Mass Function with size of special chars
for i=1:length(Z_uniqueChars)
% Probabilities for each unique character in the file
p(i)=f(i)/length(Z);
PMF_Z(i) = p(i); %PMF
end
disp("PMF of Z")
```

```
PMF of Z
```

```
disp(transpose(PMF_Z));
```

```
    0.0500
    0.1000
    0.3000
    0.2500
    0.1500
    0.1500
```

• Generate the source coded codewords for both the fixed-length code and the Huffman code.

```
%Generate the source coded code words using the fixed length code:
disp("Using the fixed Code:");
```

```
Using the fixed Code:
```

```
disp("Encode the Codewords:")
```

```
Encode the Codewords:
```

```
Transmit_Z_fixed =0;

for i=1:length(Z)
    temp=0;
    for j=1:length(Z_uniqueChars)
     if Z(i)== Z_uniqueChars(j)
         if i==1
             Transmit_Z_fixed= FixedCode_Z(j+temp*2:j+2+temp*2);
            % disp(Transmit_X_fixed)
         else
             Transmit_Z_fixed = append(Transmit_Z_fixed,FixedCode_Z(j+temp*2:j+2+temp*2));
             % disp(Transmit_X_fixed)
         end
         %disp("test")
     end
     temp = temp + 1;
    end
 end


 disp("The bits that are going to be transmitted using the Fixed code:");
```

```
The bits that are going to be transmitted using the Fixed code:
```

```
disp(Transmit_Z_fixed);
```

```
111110111111111111100100111000111110010111100111011000111111001011001010101000111010110100100111111100101001010111000111110101011011010011011011011011011010
```

```
num_of_bits_Z_fixed = length(Transmit_Z_fixed);
fprintf("The Total number of bits that are going to be transmitted using fixed code = %d", num_of_bits_Z_fixed);
```

```
The Total number of bits that are going to be transmitted using fixed code = 2880
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.

disp("Decode the Codewords back to symbols using the fixed code:")
```

```
Decode the Codewords back to symbols using the fixed code:
```

```
Recieve_Z_fixed=[];
i=1;
indexZ=1;
  while i<=length(Transmit_Z_fixed)
      temp = 0;
     for j=1:length(Z_uniqueChars)

          if indexZ > length(Z)
              break;
          end

          if Transmit_Z_fixed(i:(i+2)) == FixedCode_Z(j+temp*2:j+2+temp*2)

              Recieve_Z_fixed(indexZ) = Z_uniqueChars(j) ;
              i = i+3;
              indexZ=indexZ+1;


          end
          temp = temp + 1;
     end
  end
 disp(Recieve_Z_fixed);
```

```
  Columns 1 through 24

     4     3     4     4     4     4     6     6     3     6     4     1     2     4     6     3     3     6     4     3     5     3     5     2

  Columns 25 through 48

     5     6     3     2     2     6     6     4     4     6     5     5     2     3     6     4     2     5     3     3     1     3     3     3

  Columns 49 through 72
```

```
     3     3     3     1     3     4     5     4     5     3     2     6     6     4     4     5     3     3     4     4     5     3     4     4

  Columns 73 through 96

     4     4     6     1     3     1     5     5     3     5     3     4     3     4     3     4     5     6     5     1     2     6     4     3
```

```
if isequal(Z,Recieve_Z_fixed)
    disp("No losses were found when using the Fixed Code since the recieved bits of Z are the same as the transmitted bits");
end
```

```
No losses were found when using the Fixed Code since the recieved bits of Z are the same as the transmitted bits
```

```
%Generate the source coded code words using the Huffman code:
disp("Using the Huffman code:");
```

```
Using the Huffman code:
```

```
[dict_sentZ,avglen_sentZ] = huffmandict(Z_uniqueChars,PMF_Z);   %X_unique: Symbols
disp("Code Word of Z that is Loaded from the .mat file:");
```

```
Code Word of Z that is Loaded from the .mat file:
```

```
dict_sentZ = cellfun(@num2str,dict_sentZ(:,2),'UniformOutput',false);
disp(dict_sentZ);
```

```
    {'1  1  1'}
    {'1  1  0'}
    {'0  0'   }
    {'1  0'   }
    {'0  1  1'}
    {'0  1  0'}
```

```
%disp("Test")
%disp((append(regexprep(cell2mat(dict_sentX(1)), '\s', '') , regexprep(cell2mat(dict_sentX(3)), '\s', ''))))
%disp(regexprep(cell2mat(dict_sentX(1)), '\s',''))
%disp(length(regexprep(cell2mat(dict_sentX(1)), '\s','')))
%zxc= regexprep(cell2mat(dict_sentX(1)), '\s','');
%disp(zxc)
%disp("test")
%disp(transpose(cell2mat(dict_sentX(1))))
%disp(regexprep(cell2mat(dict_sentX(3)), '\s', '') + regexprep(cell2mat(dict_sentX(2)), '\s', ''))

disp("Encode the Codewords:")
```

```
Encode the Codewords:
```

```
Transmit_Z =0;
 for i=1:length(Z)
    for j=1:length(Z_uniqueChars)
     if Z(i)== Z_uniqueChars(j)
         if i==1
             Transmit_Z= regexprep(cell2mat(dict_sentZ(j)), '\s', '');
             %disp(Transmit_X)
         else
             Transmit_Z = append(Transmit_Z,regexprep(cell2mat(dict_sentZ(j)), '\s', ''));
         end
         %disp("test")
     end
    end
 end
 disp("The bits that are going to be transmitted using the huffman code:");
```

```
The bits that are going to be transmitted using the huffman code:
```

```
 disp(Transmit_Z);
```

```
1000101010100100100001010111110100100000010100001100011110011010001101100100101010010011011110000101011001100001110000000000001110010011100110011001001010
```

```
 num_of_bits_Z = length(Transmit_Z);
 fprintf("The Total number of bits that are going to be transmitted = %d", num_of_bits_Z);
```

```
The Total number of bits that are going to be transmitted = 2352
```

```
%Decode the data. Verify that the decoded symbols match the original symbols.
%sig = huffmandeco(double(Transmit_X),dict_sentX);
```

```
disp("Decode the Codewords back to symbols:")
```

```
Decode the Codewords back to symbols:
```

```
Recieve_Z=[];
i=1;
indexZ=1;
 while i<=length(Transmit_Z)
     for j=5:length(dict_sentZ)
     for l=3:4
     for k = 1:2
          if indexZ > length(Z)
              break;
          end

          if Transmit_Z(i:(i+1)) == regexprep(cell2mat(dict_sentZ(l)), '\s', '')

              Recieve_Z(indexZ) = Z_uniqueChars(l) ;
              %disp(Recieve_X)  %for the test
              i = i+2;
              indexZ=indexZ+1;

          elseif  Transmit_Z(i:(i+2)) == regexprep(cell2mat(dict_sentZ(j)), '\s', '')

              Recieve_Z(indexZ) = Z_uniqueChars(j) ;
              i = i+3;
              indexZ=indexZ+1;

          elseif  Transmit_Z(i:(i+2)) == regexprep(cell2mat(dict_sentZ(k)), '\s', '')

              Recieve_Z(indexZ) = Z_uniqueChars(k) ;
              i = i+3;
              indexZ=indexZ+1;

      end
      end
     end
     end
 end
 disp(Recieve_Z);
```

```
  Columns 1 through 24

     4     3     4     4     4     4     6     6     3     6     4     1     2     4     6     3     3     6     4     3     5     3     5     2

  Columns 25 through 48

     5     6     3     2     2     6     6     4     4     6     5     5     2     3     6     4     2     5     3     3     1     3     3     3

  Columns 49 through 72

     3     3     3     1     3     4     5     4     5     3     2     6     6     4     4     5     3     3     4     4     5     3     4     4

  Columns 73 through 96

     4     4     6     1     3     1     5     5     3     5     3     4     3     4     3     4     5     6     5     1     2     6     4     3
```

```
 if isequal(Z,Recieve_Z)
     disp("No losses were found when using the Huffman Code since the recieved bits of Z are the same as the transmitted bits");
 end
```

```
No losses were found when using the Huffman Code since the recieved bits of Z are the same as the transmitted bits
```

Compare the total number of bits to be transmitted in each case.

The Total number of bits to be transmitted using the naive code is greater than using the Huffman code since it's 2880 in naive code while it is 2352 in Huffman code so it is clear that the Huffman code is more efficient than the naive code. (i.e. send less number of bits)

• Decode the codewords back to symbols and check for any losses compared to the original data.

No losses were found since the generated vector (The recieved) is equal to the original vector.

Comment on your observations of Part II compared to Part I

Huffman is always better than the fixed code as it generates an optimum code and efficient code compared to the fixed.

In addition, Huffman generates least number of codewords to the highest probability so decreases the average code length

Part 3

In this part, we are required to compress a text file using *Huffman* codes.

First, We need to acquire the text. You can select a text file by pressing the browse button:

```
textFromFile = false;
[file , path] = uigetfile ('*.txt');

if(~file)
    disp('No file was selected.');
else
    disp(strcat("Selected file: ", file));
    originalText = fileread (strcat(path,file));
    textFromFile = true;
end
```

```
Selected file: lorem.txt
```

Selected text:

```
disp(originalText);
```

```
Lorem ipsum dolor sit amet, consectetur adipiscing elit. Pellentesque congue gravida leo, at euismod ligula fringilla vitae. Proin lobortis eleifend vestib

Proin interdum porttitor nisl, blandit tristique nisl semper ut. Nullam vestibulum vitae justo nec pulvinar. Donec vitae consequat velit. Phasellus pharet

Vestibulum in neque pharetra, auctor tellus pharetra, pharetra nisl. Fusce quis nisi scelerisque, congue augue quis, molestie mi. Pellentesque ligula lacu

Nullam hendrerit porttitor enim, ac viverra odio pharetra sit amet. Fusce fringilla vitae nulla sed fermentum. Quisque varius commodo erat, vel vulputate

Phasellus suscipit sapien dolor, ac ornare lorem tincidunt vitae. Maecenas tincidunt tempus urna, quis tincidunt velit viverra quis. Quisque vel elit lorer
```

Our file should consist of these characters only:

```
charList = char(strcat(char(65:90),char(97:122), char(49:57), ",. "))
```

```
charList = 'ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz123456789,. '
```

We now extract the character data from the file and find its *Probability Mass Function,* after removing all invalid characters:

```
originalText(~ismember(originalText, charList)) = '';
sortedText = sort(originalText);
[labels, charPMF] = PMF(sortedText);
pmfTable = table(transpose(labels), transpose(charPMF), 'VariableNames', {'Character', 'Probability'});
disp(pmfTable)
```

Character	Probability
	0.14225
,	0.014619
.	0.017712
A	0.00056227
C	0.0014057
D	0.001968
F	0.0014057
I	0.0011245
L	0.00028114
M	0.0022491
N	0.0025302
P	0.0036548

Huffman Encoding

We can now use the built in function to generate a Huffman code for our file:

```
% The huffmandict function can't deal with the symbols as chars. We need to
% cast them into doubles.


symbols = double(labels);
dict = huffmandict(symbols, charPMF);
huffTable = cell2table(dict);
huffTable.Properties.VariableNames = {'Character', 'Codeword'};
huffTable.Character = char(huffTable.Character);

disp(huffTable)
```

Character	Codeword
	{[0 1 1]}

```
,       {[         1 0 0 1 1 1]}
.       {[         1 0 0 1 0 0]}
A       {[1 1 1 1 0 1 1 1 0 0 0]}
C       {[    1 1 1 1 0 0 0 1 1]}
D       {[    1 0 0 1 0 1 0 0 1]}
F       {[    1 1 1 1 0 0 0 1 0]}
I       {[  1 1 1 1 0 0 0 0 0 0]}
L       {[1 1 1 1 0 1 1 1 0 0 1]}
M       {[    1 0 0 1 0 1 0 0 0]}
N       {[      1 1 1 1 0 1 1 0]}
P       {[      1 0 0 1 0 1 0 1]}
```

Now, we are ready to encode our text file:

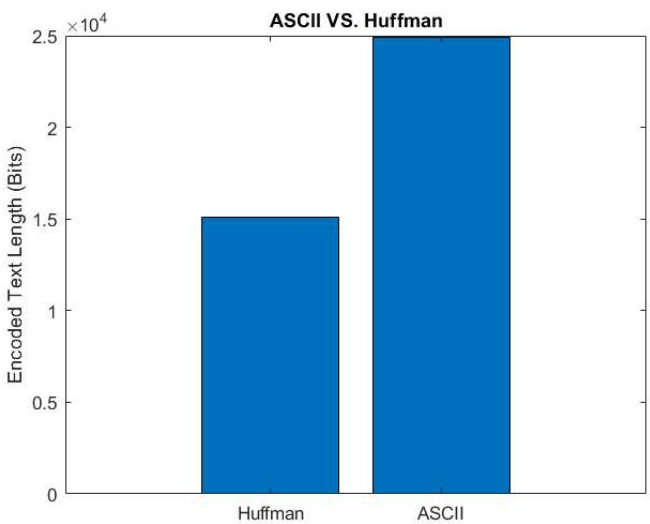
```
encoded = huffmanenco(originalText, dict);
huffman = num2str(encoded);
huffman = strrep(huffman,' ','');
ascii = dec2bin(originalText);
ascii = reshape(ascii',1,[]);

codeTable = cell2table({ascii; huffman});
codeTable.Properties.VariableNames = {'Encoded File'};
codeTable.Properties.RowNames = {'ASCII', 'Huffman'};
disp(codeTable)
```

```
ASCII      {'1001100110111111100101100101110110101000001101001111000011100111110101110110101000001100100110111111011001101111111001001000001110011110
Huffman    {'1111011100100101101111100000001100010000111000010100000011000010001011010001011011011100000010011011010000000110001110011101100100001011110
```

We know find the compression ratio:

```
huffLen = length(huffman);
asciiLen = length(ascii);
lenArr = [huffLen asciiLen];
gra = bar(lenArr);
xticklabels({'Huffman', 'ASCII'});
title('ASCII VS. Huffman');
ylabel('Encoded Text Length (Bits)');
```



We can now calculate the compression ratio by applying the formula:

$$\frac{\text{Length of Huffman code}}{\text{Length of ASCII code}} \times 100\%$$

```
% compRatio = ((asciiLen - huffLen)/asciiLen) * 100;
% fprintf("η = %f %%", compRatio)
compRatio = (huffLen / asciiLen) * 100;
fprintf("η = %f %%", compRatio)
```

```
η = 60.657055 %
```

Decoding

We now decode both the ASCII and the Huffman files to check for errors. We expect none due to the one-to-one mapping in both ASCII and Huffman:

```
decodedHuff = char(huffmandeco(encoded, dict));
```

```
decodedASCII = zeros(1, asciiLen);

for i = 1:7:asciiLen-6
    % We need to decode each 7 bits into a single character

    letterBits = extractBetween(ascii, i, i+6);

    letter = char(bin2dec(letterBits));
    if letter == ' '
        decodedASCII = strcat(decodedASCII," ");
    else
        decodedASCII = strcat(decodedASCII, letter);
    end

end
```

```
Warning: Input should be a string, character array, or cell array of character arrays.
```

```
decodeTable = cell2table({decodedASCII; decodedHuff});
decodeTable.Properties.VariableNames = {'Decoded File'};
decodeTable.Properties.RowNames = {'ASCII', 'Huffman'};
disp(decodeTable);
```

```
    ASCII      "Lorem ipsum dolor sit amet, consectetur adipiscing elit. Pellentesque congue gravida leo, at euismod ligula fringilla vitae. Proin loborti
    Huffman    "Lorem ipsum dolor sit amet, consectetur adipiscing elit. Pellentesque congue gravida leo, at euismod ligula fringilla vitae. Proin loborti
```

```
% And now, we compare the decoded text with the original text:

if(strcmp(decodedHuff, originalText))
    disp("Huffman codes are error free as expected.")
end
```

```
Huffman codes are error free as expected.
```

```
if(strcmp(decodedASCII, originalText))
    disp("ASCII codes are error free as expected.")
end
```

```
ASCII codes are error free as expected.
```

We now output the Huffman decoded file for further inspection:

```
fid = fopen(strcat(path,"huffman.txt"),'w+');
fprintf(fid, decodedHuff);
fclose(fid);
```

Comments

- Huffman coding is a lossless coding method, which by definition means it's error free.
- ASCII codes are made for computers to represent text, not for compression. We have 64 characters in our character list, which means we need only $\log_2(64) = 6\,\text{Bits}$ to represent each charachter, not $7\,\text{Bits}$.

Functions

```
function  [uniqueChars, pmfCharArr] = PMF(text)
            % Returns the probability of each charachter in an array

            textLen = length(text);
            uniqueChars = unique(text);
            uniques = length(uniqueChars);
            pmfCharArr = zeros(1, uniques);

            for idx = 1:uniques
                pmfCharArr(idx) = sum(text == uniqueChars(idx)) / textLen ;
            end
end
```