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Hybrid Fuzzy-PID Control and Modeling of Coaxial Rotor Helicopter

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***Abstract*— In this paper, the hybrid fuzzy-PID controller for a coaxial rotor helicopter is developed as a supplement to the conventional PID controller. The coaxial rotor helicopter uses one motor and two swash plates linked to each other. The helicopter system is modeled by nonlinear dynamics. The PID control system works well with a moderate velocity change. However, it provides unsatisfactory performance with a large velocity change. The fuzzy-PID controller is applied to improve the system behavior. Simulation results are given to show the effectiveness of the fuzzy-PID controller for the nonlinear model.**

Keywords—coaxial rotor; nonlinear modeling; hybrid fuzzy-PID control;

I. INTRODUCTION

Recently, attention and demand about unmanned aerial vehicle (UAV) have been growing among researchers and hobby-drone communities. Multirotor type UAVs, such as quadcopters, are in the spearhead of this growth, based on its structural simplicity, compactness, and safety benefits [1,2]. However, this does not mean that the multirotor totally replaces the helicopter which is a conventional UAV platform. A multirotor has relatively small propellers compare to a helicopter which is the same size. This means figure of merit (FOM) of each propeller, denoting static thrust efficiency, is smaller than that of a helicopter [2]. In addition, it is hard to install gasoline engine on multirotor [3] which allows longer flight time. These points make a helicopter unreplaceable in transportation and exploration missions.

A conventional helicopter uses the main rotor to generate lift, and a smaller tail rotor to generate thrust in the lateral direction for canceling antitorque and controlling z-axis attitude. The tail rotor thrust does not contribute to lift generation, but it dissipates up to 10% of available power to stabilize attitude [4]. A coaxial rotor is proposed as an alternative to the use of the tail rotor. In this type of rotor, two contra-rotating rotors cancel reaction torque of each other. Furthermore, difference of reaction torques is used to control z-axis attitude, which known as yaw angle. These characteristics result in two advantages. First, a helicopter can be manufactured in a compact size. Second, additional rotor increases stability which is caused by its gyroscopic effect.

These characteristics of the coaxial rotor have led research on various topics. Control techniques such as H-infinite control [5], fuzzy logic control [6] have been developed. Also nonlinear modeling and system identification of miniature coaxial rotor helicopters have been also conducted [7-8]. These research adopted nonlinear control methods instead of linear control methods such as PID control. The reason is that the dynamics of the coaxial rotor system is nonlinear and each parameter is highly coupled. Meanwhile, most of previous research focused on coaxial rotors which control angular velocity of two rotors separately. However, majority of real scale coaxial rotor helicopters control collective and cyclic pitch of rotors rather than rotational velocities of rotors.

According to this background, the objective of this paper is as follows. First, we modified a nonlinear model of previous works [9] to describe a coaxial rotor helicopter which uses one motor. Second, we developed PID controller and hybrid fuzzy-PID controller, and compared the performance of each controller with the coaxial rotor helicopter model.

Fuzzy logic control has been chosen to handle nonlinearity of coaxial rotor helicopter model in several works [10-12]. It is because the fuzzy logic controller retains robustness against outer disturbance, model and parameter uncertainty. In addition, human experience about the plant and the controller can be directly reflected in controller design.

The paper is organized in following order. In Section II, a nonlinear model of coaxial rotor helicopter is derived. Section III describes PID and fuzzy controller design which is conducted base on the model. Section IV shows MATLAB Simulink simulation result. In section V, conclusion and future works are provided.

II. NONLINEAR MODELING

In this section, a nonlinear model of coaxial rotor helicopter is derived. Generally, there are two types of coaxial rotor. The method of changing moment of the rotors is main difference. This can be explained by the equation below.[13]

$$Q = c_Q \pi \rho R^5 \Omega^2 = c_Q k_Q \Omega^2 \tag{1}$$

where Q is moment caused by drag force of a rotor, c_Q is drag coefficient, ρ is atmosphere density, R is radius of a rotor, and Ω is angular velocity of a rotor. In (1), c_Q and Ω can change Q. One type changes Ω using motor speed, the other type changes c_Q using collective pitch angle of rotor.

The first type coaxial rotor helicopter changes angular velocities of rotors independently to control z-axis attitude. This type of coaxial rotor helicopter's structure is very simple because collective pitches of two rotors are fixed. Furthermore, cyclic pitch of upper rotor is fixed too. This means only lower rotor is connected to a swash plate. However, when the rotor size gets bigger, its inertia getting bigger simultaneously. This causes time lag of angular velocity control. As a result, this type of coaxial rotor is used in miniature to medium scale helicopters.

The other type changes upper or lower rotor's collective pitch individually to generate torque difference. Each rotor connected to each swash plate, and the two swash plates are linked to each other. This means cyclic pitch angles of both two rotors are controlled by swash plates, unlike the first type. Therefore, controllability of the second type is higher than those of the first type. On the other hand, linked swashplate structure is complex enough to make additional drag to the helicopter. As a rule, this type is applied in real-scale helicopters like Kamov Ka-50 or Sikorsky S-69.

The remainder of section II will introduce the nonlinear model of the second type coaxial rotor helicopter based on the model of [8, 9].

A. Coordinate frame and translational matrices

North-east-down (NED) frame is chosen to describe body fixed frame *B*. If we define ϕ, θ, ψ as Euler angles of a helicopter, the transformation matrix from *B* to inertial frame *I* is defined as below.

$$T_{BI} = \begin{bmatrix} c\psi c\theta & s\psi c\theta & -s\theta \\ c\psi s\theta s\phi - s\psi c\phi & s\psi s\theta s\phi + c\psi c\phi & c\theta s\phi \\ c\psi s\theta c\psi + s\psi s\phi & c\psi s\theta c\phi - c\psi s\phi & c\theta c\phi \end{bmatrix} \quad (2)$$

c and *s* mean cosine and sine, respectively. The Jacobian matrix which transforms angular velocity to Tait-Bryan angles derivatives is R_{BI}.

$$R_{BI} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -\phi & c\theta c\phi \end{bmatrix} \quad (3)$$

B. Rigid body dynamics

The state vector consists of 14 states. 12 states are related to Newton-Euler equation, $\{x, y, z, \phi, \theta, \psi, u, v, w, p, q, r\}$ and 2 states are related to tilting angles of tip path plane (TPP) $\{\alpha, \beta\}$. $\{x, y, z\}$ means three axis position of a coaxial rotor helicopter expressed in *I* frame. $\{u, v, w\}$ and $\{p, q, r\}$ mean three axis velocity and angular velocity in *B* frame.

Using Newton-Euler equation, rigid body dynamics in body frame are equations as below.

$$F_B = \frac{d}{dt} m V_B = m\left(\dot{V}_B + \Omega_B \times V_B\right)$$
$$\therefore \dot{V}_B = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m} F_B - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (4)$$

where F_B, V_B, Ω_B are force, velocity, angular velocity in *B* frame, u, v, w and p, q, r are three axis component of V_B *and* Ω_B. m means body mass.

$$M_B = \frac{d}{dt} J_B \Omega_B = J_B \dot{\Omega}_B + \Omega_B \times J_B \Omega_B$$
$$\therefore \dot{\Omega}_B = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = J_B^{-1}\left(M_B - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times J_B \begin{bmatrix} u \\ v \\ w \end{bmatrix}\right) \quad (5)$$

where M_B means moment in body fixed frame, and J_B means body moment of inertia.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = T_{IB} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = R_{IB} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (7)$$

In order to compute (4) and (5), total force acting on the body F_B and total moment M_B have to be determined.

1) Force components

The body force vector is the sum of several components.

$$F_B = T_{up} + T_{lw} + D_B + G_B \quad (8)$$

T_{up}, T_{lw} mean thrust vectors of upper and lower rotors, where the subscripts '*up*' and '*lw*' mean upper rotor and lower rotor, respectively. The rotor thrust vector T_i is composed of thrust magnitude $T_{i,s}$ and direction vector $n_{T,i}$, where subscribe *i* includes both *up* and *lw*. Thrust magnitude $T_{i,s}$ is defined as below [8].

$$\begin{aligned} T_{i,s} &= c_{T,i} \pi \rho R^4 \Omega^2 = c_{T,i} k_T \Omega^2 \\ &= \tfrac{1}{2} \rho b c \Omega^2 R^3 \int_0^1 x^2 C_L dx \end{aligned} \quad (9)$$

In this equation, ρ means atmosphere density, R means rotor length, Ω means rotor rotational velocity which is constant in this model, *b* and *c* mean number of rotor blades and chord length of the rotor. $c_{T,i}$ stands for thrust coefficient. This coefficient is varying with change of rotor blade pitch, and the second row of equation implies that the thrust is linearly proportional to lift coefficient C_L of rotor airfoil. Assume that the cross section of the rotor is constant along lateral direction and lift coefficient is linearly proportional to collective pitch angle. As a result, we can descript thrust coefficient as

$$c_{T,up} = c_{T,up,0} + c_{T,u_{col}} u_{col} + c_{T,u_{tail}} u_{tail} \quad (10)$$

$$c_{T,lw} = c_{T,lw,0} + c_{T,u_{col}} u_{col} \quad (11)$$

where $c_{T,i,0}$ means initial value of thrust coefficient when input value u_{col} and u_{tail} is zero. u_{col} means collective pitch input, and u_{tail} means tail input. $c_{T,u_{col}}$ and $c_{T,u_{tail}}$ are constants which decide amount of contribution of collective pitch and tail input on thrust coefficient.

In addition, thrust direction vector $n_{T,i}$ is defined as (12).

$$n_{T,i} = \frac{1}{\sqrt{(1 - sin^2(\alpha) sin^2(\beta))}} \begin{bmatrix} -c_\alpha s_\beta \\ s_\alpha c_\beta \\ -c_\alpha \end{bmatrix} \quad (12)$$

α, β mean tilting angle of the tip path plane of two rotors in x axis and y axis of body fixed frame *B*. Because two swash plates are linked, tilting angles α, β of two rotor are equivalent to each other.

D_B of (8) means fuselage drag. Movement of a coaxial rotor helicopter causes aerodynamic drag which act to opposite direction of moving. The drag is proportional to square of velocity.

$$D_B = \begin{bmatrix} -c_{D_x} u^2 sign(u) A_x \\ -c_{D_y} v^2 sign(v) A_y \\ -c_{D_z} w^2 sign(w) A_z \end{bmatrix} \tag{13}$$

where c_{D_x}, c_{D_y}, c_{D_z} and A_x, A_y, A_z mean drag coefficient and area of the helicopter in three axis, defined in B frame.

G_B of (8) means gravity force. Gravity force is always acting on negative direction of z axis in I frame. Thus, gravity force in body fixed frame is

$$G_B = T_{IB}^{-1} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \tag{14}$$

where g means gravitational acceleration.

2) Moment component

The sum of moments acting on body like below

$$\begin{aligned} M_B = Q_{up} + Q_{lw} + M_{flap,up} + M_{flap,up} \\ +Q_{T_{up}} + Q_{T_{lw}} \end{aligned} \tag{15}$$

Q_{up}, Q_{lw} mean rotor drag torgue. Similar to the rotor thrust, the rotor drag vector Q_i is composed of drag magnitude $Q_{i,s}$ and direction vector $n_{Q,i}$.

$$Q_{i,s} = c_{Q,i} \pi \rho R^5 \Omega^2 = c_{Q,i} k_Q \Omega^2 \tag{16}$$

$$c_{Q,up} = c_{Q,up,0} + c_{Q,u_{col}} u_{col} + c_{Q,u_{tail}} u_{tail} \tag{17}$$

$$c_{Q,lw} = c_{Q,lw,0} + c_{Q,u_{col}} u_{col} \tag{18}$$

where $c_{Q,i,0}$ means initial value of drag coefficient when input value u_{col} and u_{tail} is zero. $c_{Q,u_{col}}$ and $c_{Q,u_{tail}}$ are constants which decide amount of contribution of collective pitch input and tail input on drag coefficient.

$$n_{Q,up} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, n_{Q,lw} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{19}$$

The direction of $n_{Q,i}$ is aligned to z axis of body fixed frame, and the sign is determined based on the assumption; upper rotor rotates counter clockwise and lower rotor rotates clockwise.

$M_{flap,up}$, $M_{flap,up}$ mean torque induced by rotor flapping. Unlike conventional helicopter, a coaxial rotor helicopter does not install flapping hinges on the hub of helicopter, because two rotors can collide with each other during flapping motion. This hinge absorbs elastic force of a rotor blade caused by lift acting on the rotor. Consequently, elastic force from rotor flapping has to be considered in the model. As simplified in [5], flapping torque is regarded as reaction torque of torsion spring as below

$$M_{flap_i} = k_{flap} \begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} \tag{20}$$

where k_{flap} denotes spring coefficient of flapping motion.

$Q_{T_{up}}$, $Q_{T_{lw}}$ mean torques induced by thrust. In equation (12), thrust vectors are tilted from z axis of B by tilting angle α and β. This tilting causes induced torque about center of gravity.

$$Q_{T_i} = r_i \times T_i \tag{21}$$

r_i is a vector from center of gravity of the helicopter to center of upper or lower rotor.

C. Swash plate modeling

The longitudinal cyclic input u_{lon} and lateral cyclic input u_{lat} change tip path plane of rotor. The relation of this change is modeled by a simplified first order equations as below. [8]

$$\dot{\alpha} = \frac{1}{c_t}\left(-c_{\alpha_{u_{lon}}} u_{lon} - \alpha\right) \tag{23}$$

$$\dot{\beta} = \frac{1}{c_t}\left(-c_{\beta_{u_{lon}}} u_{lat} - \beta\right) \tag{24}$$

where c_t means time constant. $c_{\alpha_{u_{lon}}}$, $c_{\beta_{u_{lon}}}$ denote amount of contribution of longitudinal and lateral input on tilting angles α, β.

D. Summary of the modeling

The subsection *A.* to *D.* describe the model of the coaxial rotor helicopter. 14 states are used which defined as $\{x, y, z, \phi, \theta, \psi, u, v, w, p, q, r, \alpha, \beta\}$. The derivative of the

TABLE I. PARAMETERS AND THEIR VALUE OF THE MODEL

Parameter	Value	Parameter	Value
m	0.3163kg	$c_{T,u_{col}}$	0.013
R	0.1676m	$c_{T,u_{tail}}$	0.013
ρ	1.26kg/m^3	$c_{Q,u_{col}}$	0.0011
Ω	161.4878	$c_{Q,u_{tail}}$	0.0011
$c_{T,up,0}$	0.02m	$c_{\alpha_{u_{lon}}}$	0.0095
$c_{T,lw,0}$	0.02m	$c_{\beta_{u_{lon}}}$	0.0095
$c_{Q,up,0}$	0.002m	r_{up}	0.1625m
$c_{Q,lw,0}$	0.002m	r_{lw}	0.0997m
c_t	0.5	A_x	0.01526m^2
c_{D_x}	0.136	A_y	0.01526m^2
c_{D_y}	0.136	A_z	0.088247m^2
c_{D_z}	0.236	c_t	0.005

TABLE II. PID GAIN OF THE CONTROL SYSTEM

	PID Gain		
Controller output	P	I	D
u, v command	4.5	0.25	1
θ, ϕ command	6.5	0.1	2
w command	0.04	0.001	0.2
ψ command	1	0.002	0.05

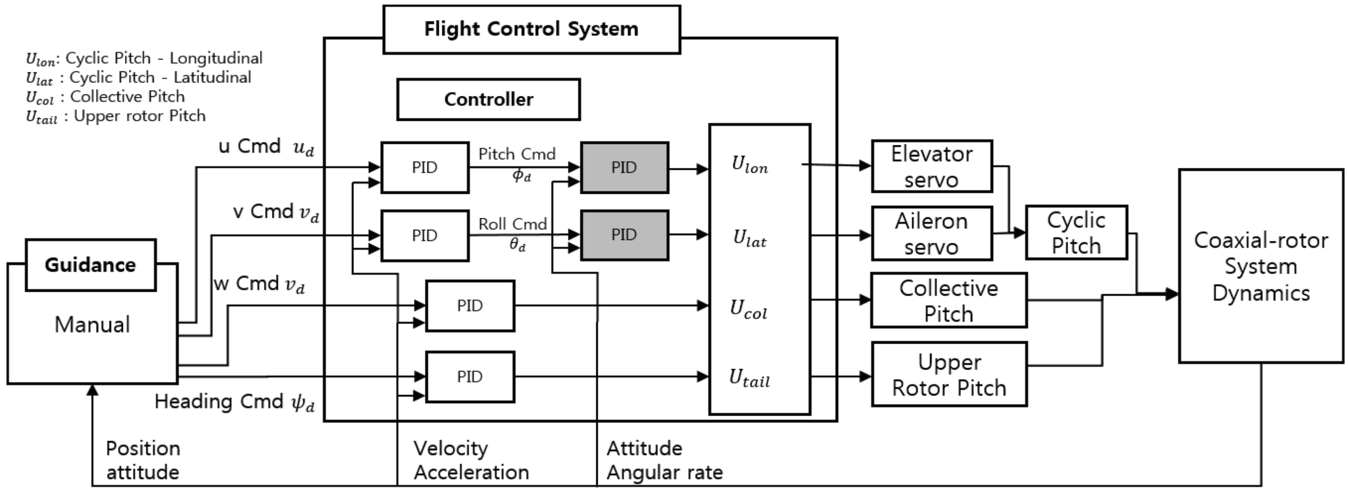

Fig 1. Overall block diagram of control system. Rigid body states feedback to the controller, and the controller generates four inputs.

states can be calculated by the equations (4)~(7) and (23), (24). In order to calculate (4), (5), F_B and M_B have to be calculated by the process described in subsection *B-1) and 2).*

E. Parameter used in the model

Parameters and coefficients used in the model are shown in TABLE I with their values. The values are mainly referred to reference [9] and its simulation, and some values are modified.

III. CONTROL SYSTEM DESIGN

Overall block diagram of the control system is illustrated in Fig. 1. The control system obtains target velocity or target heading angle from guidance algorithm. Guidance algorithm stands outside of the point of the paper, thus target velocity and heading is manually generated. u and v controlled by tilting pitch and roll attitude. Therefore, target u and v converted to target Pitch (θ) and roll (ϕ) angle first and attitude control is followed. In case of w control, such an attitude changing process is not required. On the other hand, heading is directly controlled by heading command. Accordingly, w and heading control use one controller respectively.

As a consequence, the system consists of six controllers to generate four control outputs: u_{lon}, u_{lat}, u_{col} , and u_{tail}.

A. PID control

PID control is a widely used linear control. This control method is preferred in many of real cases because detail of plant is not required. In addition, control parameters which called P, I, D gains are intuitive to tune the control system manually. However, limit of the linear system leads performance degradation of the controller when nonlinearity occurs in model or operating environment.

Each controller is composed of its own P, I, D gain whose values are shown in TABLE II. The gains are determined by trial and error process.

B. Fuzzy logic control

In order to deal with nonlinearity of the model, fuzzy logic control is used. The control system is similar to Fig. 1, but colored two blocks are replaced with fuzzy controllers. This connection of PID controller and fuzzy controller is one type of hybrid fuzzy-control. In this system, applied fuzzy controllers control pitch and roll attitude according to pitch and roll command which generated from PID controllers. This arrangement is applied because controlling attitude is more affected by the nonlinearity of the model than generating velocity command.

The fuzzy controllers are Mamdani type controllers, which uses two inputs. One input is error, and the other input is arctangent value of change of the error. Arctangent function is used to bound change of error value in range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Inputs and output have five membership functions which consist of Negatively Large (NL), Negatively Small (NS), Zero (Z), Positively Small (PS), and Positively Large (PL). The rule table between input and output is shown in TABEL III. For instance, if error and change of error are included in NS and PS respectively, output is selected as Z. The shape of the membership functions are triangular, which is shown in figure 2. Not only shape of function, but the value of functions are same. To tune the control system, however, scaling factor is multiplied to inputs and output. The scaling factors for error and change of error are 0.5 and 0.8, respectably. In case of output, scaling factor 0.9 is used.

TABLE III. RULE TALBE OF THE FUZZY CONTROLLER.

Change of error, atan($\Delta e(t)$)		Error, $e(t)$				
		NL	**NS**	**Z**	**PS**	**PL**
	NL	NL	NL	NL	NS	Z
	NS	NL	NL	NS	Z	PS
	Z	NL	NS	Z	PS	PL
	PS	NS	Z	PS	PL	PL
	PL	Z	PS	PL	PL	PL

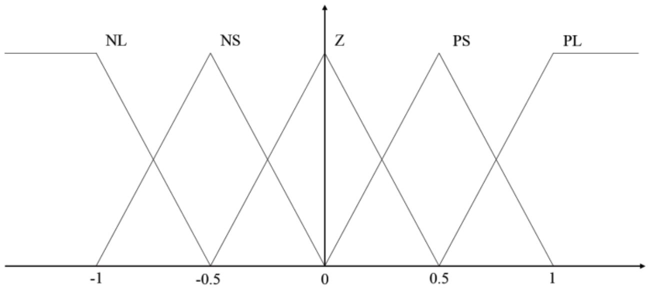

Fig 2. Membership function of the Fuzzy controller

IV. SIMULATION RESULT

The coaxial rotor helicopter model derived in Section II and the control systems described in Section III are simulated by MATLAB Simulink in this section. Manually generated target velocity is entered into the control system. The target velocities are generated as a step function. Narrow lines of velocity graphs of Fig. 3~5 are target velocities. Heading and w maintain zero, which is the initial value. Simulations are performed for 300 seconds, and time step is fixed to 0.01 seconds, which means update rate of 100Hz. In addition, attitude angle limit is configured as -25 degree to 25 degree.

Fig. 3 is velocity and attitude response of the PID control system when moderate target velocity change is entered. As the figure shows, attitude angle tracks its command angle in small error. This leads that velocity converges to target velocity in short time. In order to quantify the performance of the controller, rise time is measured. Rise time is defined as 'the time consumed for response rise from 10% to 90% of its target value'. In Fig. 3, the rise time is measured as 2.5 seconds in average.

However, when target velocity change is increased to a large value, convergence performance get worse. This is shown in Fig. 4. The velocity converges to the target velocity finally, however, it takes longer time than that of Fig. 3. Target velocity change is increased up to $\pm$20m/s. In this case, we have to tune gains of the PID controllers again to stabilize the system. If not, the response gets worse and diverges in the worst case. Especially I gain of the controllers have to be reduced. This cause low performance of convergence. In other words, rise time increased to 7.6

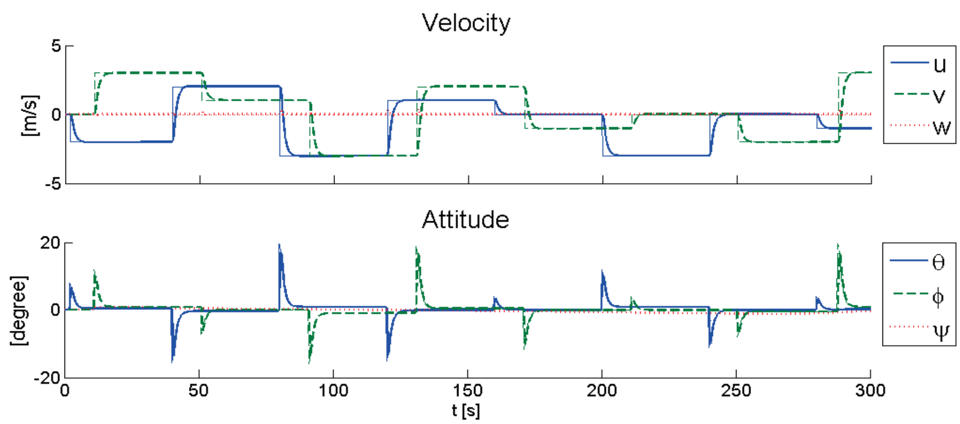

Fig 3. Matlab simulation result of PID control system in moderate target velocity condition.

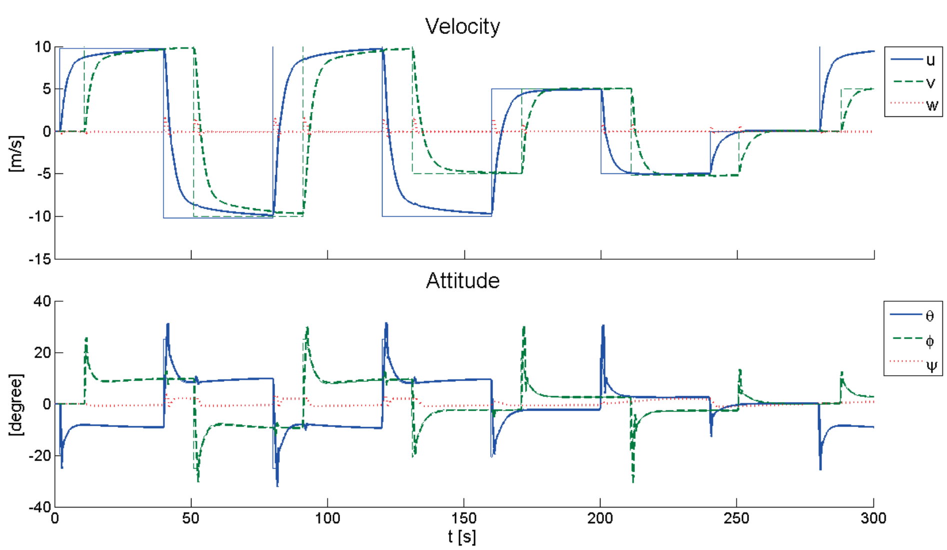

Fig 4. Matlab simulation result of PID control system in larger target velocity condition.

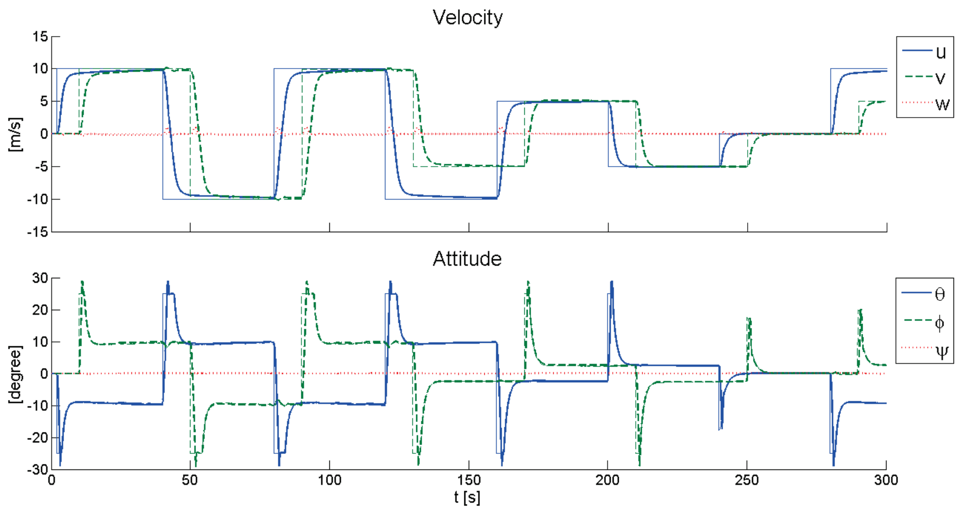

Fig 5. Matlab simulation result of hybrid fuzzy-PID control system in larger target velocity condition.

seconds average

When fuzzy controller applied as described in Section III, velocity and attitude control works normally. The simulation result is shown in figure 5, which can compare with figure 4. The velocity command is same as Fig. 4. The rise time of the response is decreased to 3.9 seconds which means faster response than figure 4. Moreover, the P, I, D gains of controllers which generate pitch and roll command are same as the gains of figure 3. This means conducted hybrid fuzzy-PID controller can deal with larger target velocity change than pure PID controller.

V. CONCLUSION

In this paper, nonlinear modeling of the coaxial rotor helicopter which use one motor is derived. Furthermore, PID control and hybrid fuzzy-PID control is applied on the nonlinear model. The model and the control system is simulated using MATLAB Simulink. The simulation result shows that PID controller can deal with the model in moderate target velocity change. When the change gets intense, however, convergence performance becomes worse. In severe cases, P, I, D gains of the controller have to be tuned again. This performance limit can be compensated by applying fuzzy logic controller as the simulation results show. As a future work, the guidance algorithm or a modified PID controller which can deal with this convergence problem in PID controller will be considered. Meanwhile, onboard computing of proposed hybrid Fuzzy-PID controller will be conducted.

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