Backstepping-based Tracking Control for a Coaxial Rotor UAV

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***Abstract*—This paper takes a coaxial rotor Unmanned Aerial Vehicle (UAV) as the research object, the problem of controlling the position and orientation of this UAV is solved. The nonlinear mathematical model of the coaxial rotor UAV is novelty present and a hierarchical controller is adopted to solve this problem. The controller includes the attitude controller of inner loop and the position controller of outer loop, which are both adopting backstepping control technique to track the reference position. Numerical simulations present that the coaxial rotor UAV can perfectly track the predefined trajectory, which demonstrates the good performance of the designed control strategy.**

***Index Terms*—Coaxial rotor, Unmanned Aerial Vehicle, Hierarchical controller, Backstepping control**

I. Introduction

Today, Unmanned Aerial Vehicle (UAV) has been widely used in various environments. In some dangerous and obstructed environments, UAV can replace humans to accomplish missions such as disaster relief, reconnaissance attack, environment monitoring, therefore, they bring great convenience and contributions to both civil and military fields. With the development of actuator miniaturization and aircraft design industry over the past few years, the traditional structure of the aircraft, such as traditional fixed wing structure and single rotary wing helicopter structure, may no longer be the optimal designs [1].

As a new type of the UAV, coaxial rotor UAV has attracted extensive attention around the world in the past few years [2]. The coaxial rotor UAV has two rotor wings, the speed of rotors can be adjusted by controlling two BLDC motors placed on the UAV. Looking down on the UAV, it is obvious that its upper and lower rotors rotate in opposite directions. The vertical take-off and landing of the vehicle, as well as the hovering motion in the air are achieved by changing the speed of the two motors. The coaxial rotor UAV's pitch and roll motions are obtained by the motion of a two degrees of freedom turntable, that is actuated by two rotation angles of two steering gears [3]. When the two rotors rotate at different speeds, the reaction torques they produce cannot cancel each other, thus the vehicle obtains a yaw motion. In addition to the advantages of good concealment, low cost and flexible operation, due to coaxial rotor UAV's special structure, it has following advantages compare with the traditional UAV. First of all, the coaxial rotor UAV is small in size and its wings can be folded, which is easy to carry. It can take off and land on the warships or narrow spaces without the help of launching devices. Secondly, compared with single-rotor aircraft, the hovering efficiency of the coaxial rotor UAV is higher. Based on the above advantages, the coaxial rotor UAV is an excellent aircraft configuration scheme.

In the past few years, some scholars had carried out relevant research on the coaxial rotor UAV. In reference [4], Adrien Drouot et al. developed a small-scale coaxial rotorcraft and relevant flight studies were conducted to support the feasibility of the coaxial rotor UAV. Besides, with the development of modern control theory, there are many advanced control algorithms to solve the control problem of small coaxial rotor UAV, such as backstepping control [5], [6], and sliding mode control [3]. Some aerodynamic analysis about the coaxial rotor UAV could be found in [7], [8]. For more information about the coaxial rotor UAV, please refer to [9] and [10].

However, in previous studies, the upper rotor does not have swashplates, thus the direction of lift vector generated by the upper rotor parallel to the axis of the fuselage. The lower rotor has swashplates and generates not only vertical forces but also lateral forces. In this paper, a new type of small coaxial rotor UAV model is proposed as Fig. 1, not only the lower rotor but also the upper rotor has the swashplates. Compared with the coaxial rotor UAV in previous references, the coaxial rotor UAV in this paper has better mobility and flexibility, it can move in all directions more quickly and flexibly.

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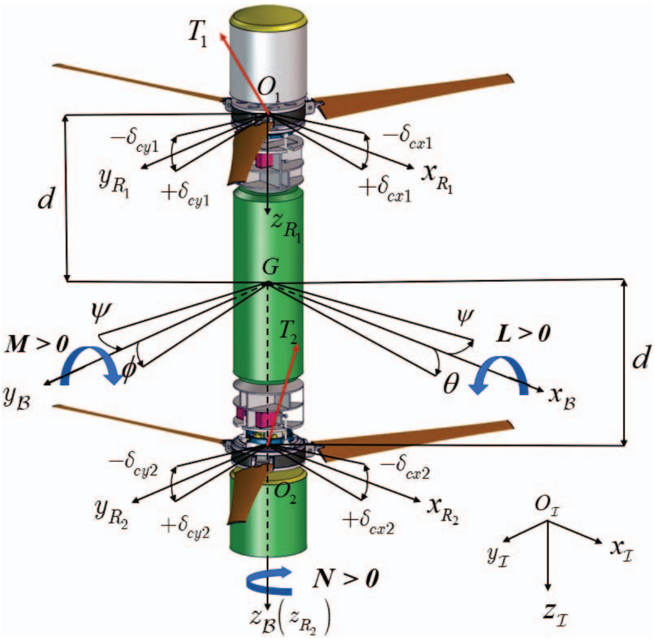

Fig. 1. Coaxial rotor UAV structure with associated coordinate systems

The main work of this paper lies in giving the nonlinear mathematical model of the coaxial rotor UAV, and a hierarchical controller based on backstepping control method is adopted to stabilize the coaxial rotor UAV system and track the desired trajectory.

The structure of this paper is as follows. Section II gives the mathematical model of the coaxial rotor UAV and its derivation process is contained. Then, a hierarchical controller is designed based on backstepping method in section III and numerical simulation results are showed in section IV. Finally, section V draws the conclusion of the whole paper.

II. Mathematical Model

In order to control the coaxial rotor UAV's position and orientation, we first need to know its mathematical model. In this section, some coordinate systems, as well as the dynamic and kinematic model of the coaxial rotor UAV is provided.

A. Assumptions

- The coaxial rotor UAV is a symmetrical rigid body with constant flight mass.
- The the coaxial rotor UAV's center of mass is the origin of the body coordinate.
- The lift of the rotor is proportional to the square of rotation speed.

B. Coordinate systems definition

1) *Inertial coordinate system*: Establish the inertial coordinate system according to the right-hand rule, pointing to North-East-Down directions respectively, it has been noted as $\mathcal{I}\,(O_{\mathcal{I}}, x_{\mathcal{I}}, y_{\mathcal{I}}, z_{\mathcal{I}})$.
2) *Body coordinate system*: The body coordinate system is fixedly connected with the body, it is mainly used to determine the attitude information of UAV during flight. Body coordinate system is noted as $\mathcal{B}\,(G, x_{\mathcal{B}}, y_{\mathcal{B}}, z_{\mathcal{B}})$, where x_B axis pointing to front side and y_B axis pointing to right side.
3) *Upper rotor coordinate system*: The upper rotor coordinate system is a moving coordinate system, which has been noted as $\mathcal{U}\,(O_1, x_{R_1}, y_{R_1}, z_{R_1})$. When there are no angular rotations in the upper coordinate system, its x_{R_1} axis, y_{R_1} axis and z_{R_1} axis are parallel to the x_B axis, y_B axis and z_B axis of the body coordinate system respectively.
4) *Lower rotor coordinate system*: Similar to the upper coordinate system, the lower rotor coordinate system is also a moving coordinate system, it has been noted as $\mathcal{L}\,(O_2, x_{R_2}, y_{R_2}, z_{R_2})$. When there are no angular rotations in the lower coordinate system, its x_{R_2} axis, y_{R_2} axis and z_{R_2} axis are parallel to the x_B axis, y_B axis and z_B axis of the body coordinate system respectively.

C. Angles definition

1) *Euler angles*: Euler angles includes pitch angle ϕ around the x_B axis, roll angle θ around the y_B axis and yaw angle ψ around the z_B axis, they are determined by the transformation relationship between body coordinate system and inertial coordinate system.
2) *Swashplate incidence angles*: Swashplate incidence angles of the upper rotor contain δ_{cx1} around the y_{R_1} axis and δ_{cy1} around the x_{R_1} axis. Swashplate incidence angles of the lower rotor contain δ_{cx2} around the y_{R_2} axis and δ_{cy2} around the x_{R_2} axis. Downward deflection of these angles are defined as positive values and upward deflection are defined as negative values.

D. Transformation between coordinate systems

First of all, the transformation between coordinate systems should be determined to establish the nonlinear model of the coaxial rotor UAV.

$R_\eta : \mathcal{B} \to \mathcal{I}$ is the rotation matrix between body coordinate system and inertial coordinate system, which is given by

$$R_\eta = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\phi + s_\theta c_\psi s_\phi & s_\phi s_\psi + s_\theta c_\psi c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\theta s_\psi s_\phi & -c_\psi s_\phi + s_\theta s_\psi c_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{1}$$

where $s_\kappa = \sin\kappa$, $c_\kappa = \cos\kappa$.

The transformation relationship between the attitude angle change rates ($\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$) and the angular velocity components (p, q, r) in body coordinate system could be expressed as:

$$\dot{\eta} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = C_\eta \Omega \tag{2}$$

where $\Omega = (p, q, r)^\top \in \mathbb{R}^3$.

Here rotate the upper rotor coordinate system continuously to get the body coordinate system by the rotation order: δ_{cy1}-δ_{cx1}, we can get the transformation matrix R_1 between these two coordinates. Similarly, we can get the transformation matrix R_2 from lower rotor coordinate system to the body coordinate system. These two transformation matrices could be defined by

$$R_i = \begin{bmatrix} c_{(\delta_{cxi})} & s_{(\delta_{cxi})}s_{(\delta_{cyi})} & -s_{(\delta_{cxi})}c_{(\delta_{cyi})} \\ 0 & c_{(\delta_{cyi})} & s_{(\delta_{cyi})} \\ s_{(\delta_{cxi})} & -c_{(\delta_{cxi})}s_{(\delta_{cyi})} & c_{(\delta_{cxi})}c_{(\delta_{cyi})} \end{bmatrix} \tag{3}$$

where $s_{(\delta_{cxi})} = \sin\delta_{cxi}$, $c_{(\delta_{cxi})} = \cos\delta_{cxi}$, $i = 1, 2$.

E. Force and moment on UAV

In this section, we obtained the aerodynamic force vector (F) and moment vector (M) by analyzing the motion principle of UAV [3]. The UAV has two rotors of opposite rotation, thus the gyroscopic moments of UAV could be neglect.

1) Forces acting on the coaxial rotor UAV: Thrust is generated by the rotation of rotors. According to the assumption above, the thrust is proportional to the square of rotors rotation speed. The thrust vector (T_1) generated by upper rotor under the coordinate system $\mathcal{U}$ and the thrust vector (T_2) generated by lower rotor under the coordinate system $\mathcal{L}$ can be expressed as

$$T_i = \begin{bmatrix} 0 \\ 0 \\ -c_i\omega_i^2 \end{bmatrix} \quad (i = 1, 2) \tag{4}$$

where $c_1 > 0$ and $c_2 > 0$ are the rotors lift coefficients, ω_1 and ω_2 are the rotation speed of upper rotor and lower rotor, respectively. Both upper rotor and lower rotor have two swashplate incidence angles, thus two rotors generate not only vertical thrusts but also lateral forces. The total thrust vector $T = [T_x, T_y, T_z]^\top$ under coordinate system $\mathcal{B}$ is defined by

$$\begin{aligned} T &= R_1T_1 + R_2T_2 \\ &= \begin{bmatrix} c_1s_{(\delta_{cx1})}c_{(\delta_{cy1})}\omega_1^2 + c_2s_{(\delta_{cx2})}c_{(\delta_{cy2})}\omega_2^2 \\ -c_1s_{(\delta_{cy1})}\omega_1^2 - c_2s_{(\delta_{cy2})}\omega_2^2 \\ -c_1c_{(\delta_{cx1})}c_{(\delta_{cy1})}\omega_1^2 - c_2c_{(\delta_{cx2})}c_{(\delta_{cy2})}\omega_2^2 \end{bmatrix} \end{aligned} \tag{5}$$

Besides, there are also weight G and aerodynamic drag F_f acting on the vehicle. The weight of the vehicle is written in $\mathcal{I}$ as

$$G = \begin{bmatrix} 0 & 0 & mg \end{bmatrix}^\top \tag{6}$$

Assuming that the aerodynamic drags are proportional to the flying speed, we have

$$F_f = \begin{bmatrix} -r_1\dot{x} \\ -r_2\dot{y} \\ -r_3\dot{z} \end{bmatrix} \tag{7}$$

where r_1, r_2, r_3 are drag coefficients, and x, y, z are positions along the three axes in $\mathcal{I}$.

By the rotation matrix R_η, the thrust T could be converted in $\mathcal{I}$. Adding these forces, we can get the sum force in $\mathcal{I}$

$$F = \begin{bmatrix} F_x & F_y & F_z \end{bmatrix}^\top = R_\eta T + G + F_f \tag{8}$$

2) Moments acting on the coaxial rotor UAV: The direction of the thrust vector T_i is not collinear with the symmetry axis of the UAV when the rotor wings have swashplate incidence angles, which will lead to generate pitch and roll moments of the airframe. The moment vector M act on the vehicle can be expressed as

$$M = \begin{bmatrix} M_\phi \\ M_\theta \\ M_\psi \end{bmatrix} = \begin{bmatrix} d(-c_1s_{(\delta_{cy1})}\omega_1^2 + c_2s_{(\delta_{cy2})}\omega_2^2) \\ d(c_1s_{(\delta_{cx1})}c_{(\delta_{cy1})}\omega_1^2 - c_2s_{(\delta_{cx2})}c_{(\delta_{cy2})}\omega_2^2) \\ n_1\omega_1^2 + n_2\omega_2^2 \end{bmatrix} \tag{9}$$

where d is the distance between the center of mass and the rotor hub, both n_1 and n_2 are the yaw moment coefficients, n_1 is positive while n_2 is negative.

F. Complete model of vehicle

The dynamics equation and kinematics equation of the coaxial rotor UAV in inertial coordinate system can be represented as follows:

$$\dot{\xi} = v \tag{10a}$$
$$m\dot{v} = R_\eta T + G + F_f \tag{10b}$$
$$\dot{\eta} = C_\eta \Omega \tag{10c}$$
$$J\dot{\Omega} = -\Omega \times J\Omega + M \tag{10d}$$

where ξ is position vector of direction x, y, z in $\mathcal{I}$, and v is velocity vector, m is the mass of the UAV and $J = diag[j_x, j_y, j_z]^\top$ is inertia matrix. The nonlinear model has six control inputs: rotor rotation speeds ω_1 and ω_2, four swashplate incidence angles δ_{cx1}, δ_{cy1}, δ_{cx2}, δ_{cy2}. Combine (5) with (9), the six control inputs can be written as follows:

$$\begin{cases} \delta_{cx1} = (T_x + M_\theta/d)/2c_1\omega_1^2 \\ \delta_{cx2} = (T_x - M_\theta/d)/2c_2\omega_2^2 \\ \delta_{cy1} = (-T_y + M_\phi/d)/2c_1\omega_1^2 \\ \delta_{cy2} = (-T_y - M_\phi/d)/2c_2\omega_2^2 \\ \omega_1^2 = (n_2T_z + c_2M_\psi)/(c_2n_1 - c_1n_2) \\ \omega_2^2 = (n_1T_z + c_1M_\psi)/(c_1n_2 - c_2n_1) \end{cases} \tag{11}$$

In the stage of model design, the maximum of four swashplate incidence angles is 25 degrees, but these angles usually very small during the flight. In order to simplify the nonlinear mathematical model, we could neglect the lateral forces T_x and T_y compared to the contribution of T_z. Thus dynamics of the coaxial rotor UAV could be rewritten as

$$\begin{bmatrix} \dot{\xi} \\ m\dot{v} \\ \dot{\eta} \\ J\dot{\Omega} \end{bmatrix} = \begin{bmatrix} v \\ R_\eta e_z T_z + G + F_f \\ C_\eta\Omega \\ -\Omega \times J\Omega + M \end{bmatrix} \tag{12}$$

where $e_z = [0, 0, 1]^\top$. Then (11) can be rewritten in the following form:

$$\begin{cases} \delta_{cx1} = (M_\theta/d)/2c_1\omega_1^2 \\ \delta_{cx2} = (-M_\theta/d)/2c_2\omega_2^2 \\ \delta_{cy1} = (M_\phi/d)/2c_1\omega_1^2 \\ \delta_{cy2} = (-M_\phi/d)/2c_2\omega_2^2 \\ \omega_1^2 = (n_2T_z + c_2M_\psi)/(c_2n_1 - c_1n_2) \\ \omega_2^2 = (n_1T_z + c_1M_\psi)/(c_1n_2 - c_2n_1) \end{cases} \tag{13}$$

it is obvious that the six inputs are determined by T_z, M_ϕ, M_θ, M_ψ.

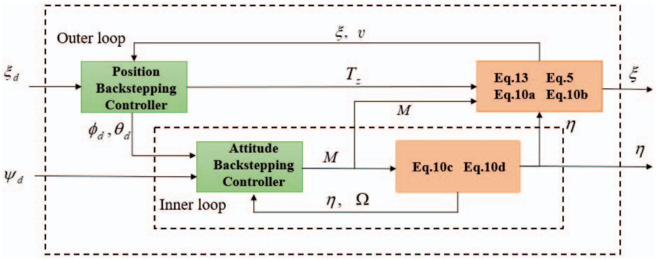

Fig. 2. Structure of hierarchical controller

III. Controller Design

In this section, a hierarchical controller based on backstepping technique [11] will be used to apply on the coaxial rotor UAV. This hierarchical controller is proposed by A.Drouot etal. in [6]. Compared with other control methods, the backstepping method has its own unique advantages, such as the higher-order system can be divided into several lower order subsystems, and the stability of the closed-loop is well ensured by Lyapunov stability theory due to its ingenious design process. Besides, backstepping control technique is adopted because of many real-world applications had been successfully implemented by using backstepping control [12], such as robot manipulator [13], aircraft [14] and satellite [15]. The aim of the control law is design new inputs T_z, M_ϕ, M_θ, M_ψ to ensure the position ξ and attitude angle ψ track the desired values $\xi_d(t)$ and $\psi_d(t)$. A hierarchical control structure is adopted, the outputs ϕ_d and θ_d of the position controller in outer loop are the inputs of the attitude controller in inner loop. The block diagram of the control structure is shown in Figure. 2.

A. Design of position controller

Firstly, in order to ensure the position vector ξ track the desired value ξ_d, we define the error between them, this error is written as

$$e_1 = \xi - \xi_d \tag{14}$$

V_1 is the Lyapunov function corresponding to e_1, which is defined by

$$V_1 = \frac{1}{2} e_1^\top e_1 \tag{15}$$

the derivative of V_1 with respect to time is

$$\dot{V}_1 = e_1^\top (v - \dot{\xi}_d) \tag{16}$$

next we define the desired value of v:

$$v_d = -k_1 e_1 + \dot{\xi}_d \tag{17}$$

thus we could get the error between v and v_d:

$$e_2 = v - v_d = v + k_1 e_1 - \dot{\xi}_d \tag{18}$$

combine (17) with (18), (16) can be rewritten as

$$\dot{V}_1 = -k_1 e_1^\top e_1 + e_1^\top e_2 \tag{19}$$

Then we define the second Lyapunov function:

$$V_2 = V_1 + \frac{1}{2} e_2^\top e_2 \tag{20}$$

with the time derivative is expressed as

$$\begin{aligned}\dot{V}_2 = &-k_1 e_1^\top e_1 + e_1^\top e_2 + e_2^\top (\frac{1}{m} R_\eta e_z T_z \\ &+ g e_z + \frac{1}{m} F_f - \dot{v}_d)\end{aligned} \tag{21}$$

the desired value of the first term in brackets in (21) is given by

$$\frac{1}{m} R_{\eta d} e_z T_{zd} = -e_1 - k_2 e_2 - g e_z - \frac{1}{m} F_f + \dot{v}_d \tag{22}$$

then the third error variable follow as

$$e_3 = \frac{1}{m} R_\eta e_z T_z - \frac{1}{m} R_{\eta d} e_z T_{zd} \tag{23}$$

combine (21), (22) with (23), $\dot{V}_2$ can be rewritten as

$$\dot{V}_2 = -k_1 e_1^\top e_1 - k_2 e_2^\top e_2 + e_2^\top e_3 \tag{24}$$

where the value of last term $e_2^\top e_3$ will be discussed in design of attitude controller. We could define the desired acceleration:

$$a_d = \frac{1}{m} R_{\eta d} e_z T_{zd} + g e_z + \frac{1}{m} F_f \tag{25}$$

where $a_d = [a_{dx}, a_{dy}, a_{dz}]^\top$, $R_{\eta d} = [\phi_d, \theta_d, \psi_d]^\top$ is the desired Euler angles vector. In order to simplify the analysis, we let

$$T_z = T_{zd} \tag{26}$$

according to (25), T_{zd} can be expressed as

$$T_{zd} = -m\sqrt{\tilde{a}_{dx}^2 + \tilde{a}_{dy}^2 + (\tilde{a}_{dz} - g)^2} \tag{27}$$

with

$$\tilde{a}_d = a_d - \frac{1}{m} F_f \tag{28}$$

combine (14), (18), (22) with (26), thus we could get the control law:

$$a_d = -(k_1 k_2 + 1)(\xi - \xi_d) - (k_1 + k_2) v + (k_1 + k_2)\dot{\xi}_d + \ddot{\xi}_d \tag{29}$$

From (25), we can also get the ideal value of roll angle ϕ_d and the ideal value of pitch angle θ_d when $\psi_d \approx 0$:

$$\begin{cases} \phi_d = \arctan\left(\dfrac{-\tilde{a}_{dy}}{\sqrt{\tilde{a}_{dx}^2 + (\tilde{a}_{dz} - g)^2}}\right) \\ \theta_d = \arctan\left(\dfrac{\tilde{a}_{dx}}{\tilde{a}_{dz} - g}\right) \end{cases} \tag{30}$$

B. Design of attitude controller

The error between Euler angles vector and desired Euler angles vector is defined as

$$\epsilon_1 = \eta - \eta_d \tag{31}$$

One purpose of designing attitude controller is ensure η converge to its desired value η_d. Combine (23) with (26), we can assure that $\epsilon_1 \to 0$ lead to $e_3 \to 0$, thus $\dot{V}_2 < 0$, the errors e_1 and e_2 asymptotically convergence to zero.

Similar to the design of position controller, there is another Lyapunov function

$$V_3 = \frac{1}{2}\epsilon_1^\top \epsilon_1 \tag{32}$$

the derivative of V_3 with respect to time is written as

$$\dot{V}_3 = \epsilon_1^\top (C_\eta \Omega - \dot{\eta}_d) \tag{33}$$

the desired value of the first term in brackets in (33) is given by

$$C_\eta \Omega_d = -p_1 \epsilon_1 + \dot{\eta}_d \tag{34}$$

thus the error variable ϵ_2 is defined by:

$$\epsilon_2 = C_\eta (\Omega - \Omega_d) \tag{35}$$

according to (34) and (35), the (33) can be rewritten as

$$\dot{V}_3 = -p_1 \epsilon_1^\top \epsilon_1 + \epsilon_1^\top \epsilon_2 \tag{36}$$

The fourth Lyapunov function is defined as

$$V_4 = V_3 + \frac{1}{2}\epsilon_2^\top \epsilon_2 \tag{37}$$

with the time derivative is expressed as

$$\begin{aligned}\dot{V}_4 =& -p_1 \epsilon_1^\top \epsilon_1 + \epsilon_1^\top \epsilon_2 + \epsilon_2^\top (\dot{C}_\eta (\Omega - \Omega_d) + C_\eta (\dot{\Omega} - \dot{\Omega}_d)) \\ =& -p_1 \epsilon_1^\top \epsilon_1 + \epsilon_1^\top \epsilon_2 + \epsilon_2^\top (\dot{C}_\eta (\Omega - \Omega_d) - C_\eta \dot{\Omega}_d) \\ &+ \epsilon_2^\top C_\eta J^{-1}(-\Omega \times J\Omega + M)\end{aligned} \tag{38}$$

The control law of attitude controller is defined as

$$M = \Omega \times J\Omega + JC_\eta^{-1}(C_\eta \dot{\Omega}_d - \dot{C}_\eta (\Omega - \Omega_d) - \epsilon_1 - p_2 \epsilon_2) \tag{39}$$

according to (39), the (38) can be rewritten as

$$\dot{V}_4 = -p_1 \epsilon_1^\top \epsilon_1 - p_2 \epsilon_2^\top \epsilon_2 \tag{40}$$

It is obvious that $\dot{V}_4 < 0$, the error ϵ_1 and ϵ_2 convergence to a neighborhood of zero in a finite time.

IV. Simulation Results

In this section, simulations are performed on the mathematical model (10a-10d) to illustrate the effectiveness of the proposed control method above. The parameters of coaxial rotor UAV are showed in Table I and initial conditions are chosen to be $\xi_0 = [0, 0, 0.5]^\top m$, $\psi_0 = 0\ rad$. These numerical simulations were implemented in the MATLAB/Simulink software with $0.01\ s$ time step.

The simulation results are showed in Figs. 3-6. Fig. 3 and Fig. 4 show the position tracking results and the attitude tracking results of the coaxial rotor UAV, the actual value quickly tracks the ideal value, which demonstrate the good performance of the designed controller. Fig. 5 displays the trajectories of the rotors rotation speeds, when the position in direction x, y, z have no change over a period of time, the coaxial rotor UAV will hover at the current position, thus the rotors rotation speeds have no change during this period of time. Fig. 6 shows the three-dimensional position trajectory of the vehicle. From these simulation results, the coaxial rotor UAV successfully track the desired trajectory, which demonstrates the validity and good effectiveness of the controller in this work.

TABLE I
PARAMETERS OF COAXIAL ROTOR UAV

Parameter	Value	Unit
m	1.51	kg
g	9.81	m/s^2
d	0.50	m
j_x	1.382×10^{-3}	$kg \cdot m^2$
j_y	1.382×10^{-3}	$kg \cdot m^2$
j_z	2.73×10^{-4}	$kg \cdot m^2$
r_1	6.67×10^{-4}	kg/m
r_2	6.67×10^{-4}	kg/m
r_3	7.54×10^{-4}	kg/m
c_1	-4.6745×10^{-5}	$N/(rad^2 \cdot s^2)$
c_2	-4.8653×10^{-5}	$N/(rad^2 \cdot s^2)$
n_1	2.6355×10^{-6}	$(N \cdot m)/(rad^2 \cdot s^2)$
n_2	-2.4876×10^{-6}	$(N \cdot m)/(rad^2 \cdot s^2)$
k_1	1.2	-
k_2	1.2	-
p_1	4	-
p_2	2	-

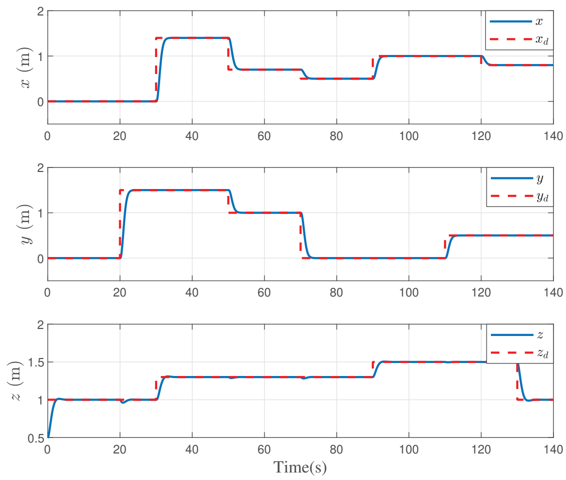

Fig. 3. Position tracking results

V. Conclusion

This work innovatively present a new coaxial rotor UAV, a effective controller is adopted to stabilize this UAV and successfully track the predefined trajectory. First of all, we develop the nonlinear mathematical model of the coaxial rotor UAV according to Newton-Euler equation, where the forces

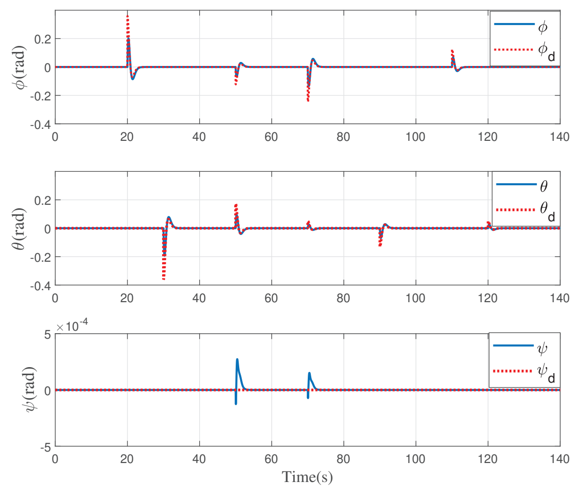

Fig. 4. Attitude tracking results

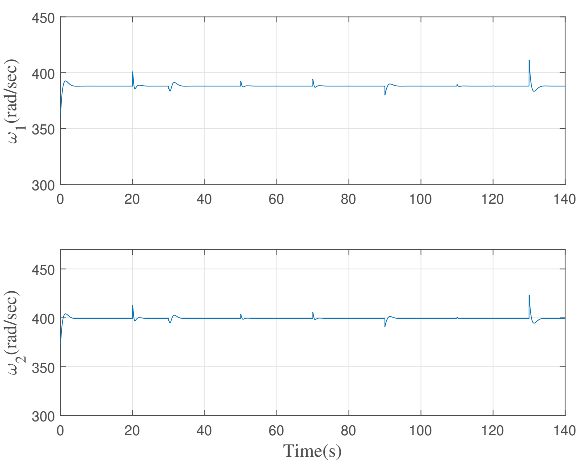

Fig. 5. Rotation speed ω_1 and ω_2

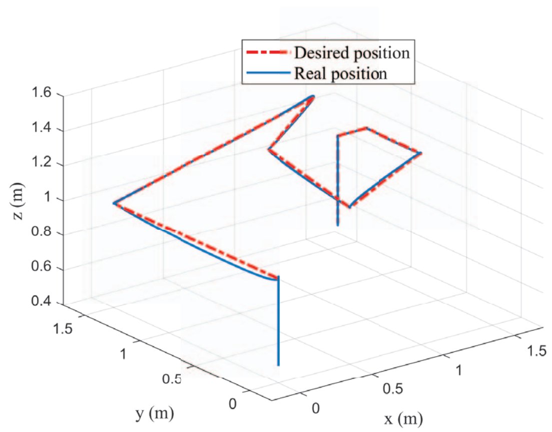

Fig. 6. Path tracking response

and moments are discussed in detail. Then, the backstepping control technique is used to design a hierarchical controller, the attitude control loop controls moments and the position control loop controls the thrust component T_z of UAV. Finally, the numerical simulation results show the controller works effectively and meets the design demand.

Based on the work of this paper, we want to design a physical prototype according to the coaxial rotor UAV model in this paper, and outdoor flight test will be performed in the future.

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