GAs to Solve the TSP

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Computer Science 3201
Fall 2018

The Team

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Outline

- Our Approach
  - Population Size
  - Mating Pool Size
- Fitness Scoring
  - Euclidean Distance in $\mathbb{R}^2$
  - Individual Fitness
- Crossover
  - The Inver-Over Crossover Operator
- Mutation
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- Demonstration
- Any Questions?

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Our Approach

Population Size

- For a route with n cities, we have

$$R := n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1 = n!$$

  possible routes that cover all cities
- As n grows, so does R
- Population size P should also grow with n
- We define $P := 2n$ and choose P (not necessarily distinct) permutations of the set $\{0, 1, \ldots, n-1\}$ as the population

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Our Approach

Mating Pool Size

- Due to the large number of permutations of cities $c_1, c_2, c_3, \ldots c_n$, many of our candidate solutions are likely very low in fitness (*i.e.*, their total distance is very high)
- Define the mating pool size M to be

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Fitness Scoring

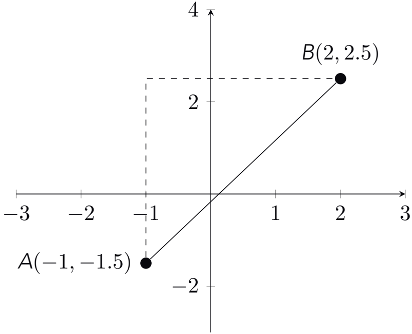
Euclidean Distance in $\mathbb{R}^2$

- Euclidean distance is computed using the formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

- Line $\overrightarrow{AB}$ measures

$$\begin{aligned} \left\|\overrightarrow{AB}\right\| &= \sqrt{(2+1)^2 + (2.5+1.5)^2} \\ &= \sqrt{9+16} \\ &= 5 \end{aligned}$$



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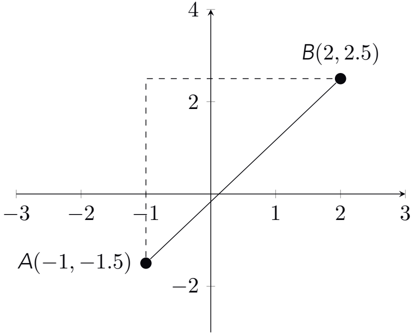

Fitness Scoring
Individual Fitness

- Let I_m with $0 \leqslant m < P$ be a candidate solution of the form

$$I_m = \left(C_{c_m(\bar{1})}, C_{c_m(\bar{2})}, C_{c_m(\bar{3})}, \ldots, C_{c_m(\bar{n})}\right),$$

where $c_m \colon \mathbb{Z}_n \to \{0, 1, 2, \ldots, n-1\}$ is a bijection between congruence classes of indices of the n-tuple I_m and the indices of cities.

- For example, if $n = 7$ and c_1 is defined by

$$c_1 : \bar{0} \mapsto 4 \qquad c_1 : \bar{1} \mapsto 6 \qquad c_1 : \bar{2} \mapsto 2 \qquad c_1 : 3 \mapsto 1$$
$$c_1 : \bar{4} \mapsto 3 \qquad c_1 : \bar{5} \mapsto 5 \qquad c_1 : \bar{6} \mapsto 0$$

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$$I_1 = (C_4, C_6, C_2, C_1, C_3, C_5, C_0)\,.$$

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$$I_1 = (C_4, C_6, C_2, C_1, C_3, C_5, C_0)\,.$$

- Then define I_m's overall fitness score $F(I_m)$, to be the summation

$$F(I_m) := \sum_{j=0}^{n-1} \left\| \overrightarrow{C_{c_m(\bar{j})} C_{c_m(\overline{j+1})}} \right\|,$$

  where $\left\| \overrightarrow{C_{c_m(\bar{j})} C_{c_m(\overline{j+1})}} \right\|$ is the Euclidean distance between city $C_{c_m(\bar{j})}$ and the following city on route m, $C_{c_m(\overline{j+1})}$.
- Hence, the fittest individuals have the *lowest* score.

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For our advanced technique, we have chosen to implement the *inver-over* crossover operator which functions according to the following algorithm.

1. Pick an individual $parent_1$ and copy it to *child*
2. Then pick two loci from $parent_1$ that depend on another individual $parent_2$ from the population
3. Invert everything between these loci in *child*
4. Repeat this process with the resulting offspring and another individual $parent_i$ until a stopping condition is reached

So *inver-over* is a multi-parent crossover operator.

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Mutation

The Scramble Mutation Operator

The scramble mutation operator, given an individual represented as a sequence of integers, typically performs the following.

1. Picks two loci to form a segment
2. Randomly shuffles all information within the selected segments
3. Returns the mutated individual

However, we have slightly modified this operator…

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> *Define a mutation factor $m \in (0, 1)$. Then the distance between the two chosen loci (i.e., the size of the mutation) can be no less than m multiplied by the length of the individual n.*

In other words, we have enforced that the product $m \cdot n$ is the infimum of the possible *severities* of the mutation.

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