GAs to Solve the TSP

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Computer Science 3201
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Outline

- The Team
- Our Approach
  - Population Size
  - Mating Pool Size
- Fitness Scoring
  - Euclidean Distance in $\mathbb{R}^2$
  - Individual Fitness
- Crossover
  - The Inver-Over Crossover Operator
- Mutation
  - The $\mu + \lambda$ Mutation Operator
- Demonstration

The Team

Omar Mohamed	Project management Programmer
Nabil Miri	Algorithm implementation Debugging
Jacob House	Technical management Code quality control
Hassan El-Khatib	Programmer

- For a route with n cities, we have

$$R := n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1 = n!$$

  possible routes that cover all cities
- As n grows, so does R
- Population size P should also grow with n
- We define $P := 2n$ and choose P (not necessarily distinct) permutations of the set $\{0, 1, \ldots, n-1\}$ as the population

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Our Approach

Mating Pool Size

- Due to the large number of permutations of cities $c_1, c_2, c_3, \ldots c_n$, many of our candidate solutions are likely very low in fitness (*i.e.*, their total distance is very high)
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Fitness Scoring

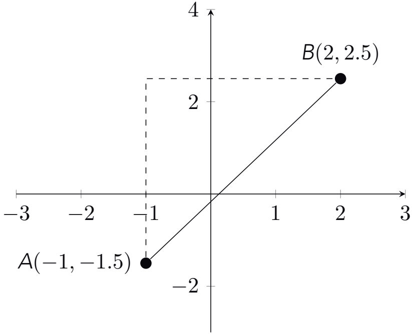
Euclidean Distance in $\mathbb{R}^2$

- Euclidean distance is computed using the formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

- Line $\overrightarrow{AB}$ measures

$$\left\|\overrightarrow{AB}\right\| = \sqrt{(2+1)^2 + (2.5+1.5)^2}$$
$$= \sqrt{9+16}$$
$$= 5$$



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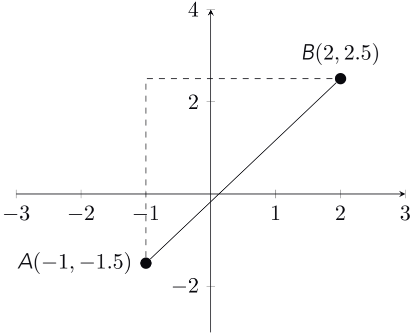

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- Let I_m with $0 \leqslant m < P$ be a candidate solution of the form

$$I_m = \left(C_{c_m(\bar{1})}, C_{c_m(\bar{2})}, C_{c_m(\bar{3})}, \ldots, C_{c_m(\bar{n})}\right),$$

where $c_m \colon \mathbb{Z}_n \to \{0, 1, 2, \ldots, n-1\}$ is a bijection between congruence classes of indices of the n-tuple I_m and the indices of cities.

- For example, if $n = 7$ and c_1 is defined by

$$c_1 : \bar{0} \mapsto 4 \qquad c_1 : \bar{1} \mapsto 6 \qquad c_1 : \bar{2} \mapsto 2 \qquad c_1 : 3 \mapsto 1$$
$$c_1 : \bar{4} \mapsto 3 \qquad c_1 : \bar{5} \mapsto 5 \qquad c_1 : \bar{6} \mapsto 0$$

then I_1 looks like

$$I_1 = (C_4, C_6, C_2, C_1, C_3, C_5, C_0).$$

Fitness Scoring

Individual Fitness

- Let I_m with $0 \leqslant m < P$ be a candidate solution of the form

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- Then define I_m's overall fitness score $F(I_m)$, to be the summation

$$F(I_m) := \sum_{j=0}^{n-1} \left\| \overrightarrow{C_{c_m(\bar{j})} C_{c_m(\overline{j+1})}} \right\|,$$

where $\left\| \overrightarrow{C_{c_m(\bar{j})} C_{c_m(\overline{j+1})}} \right\|$ is the Euclidean distance between city $C_{c_m(\bar{j})}$ and the following city on route m, $C_{c_m(\overline{j+1})}$.

- Hence, the fittest individuals have the *lowest* score.

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The Inver-Over Crossover Operator

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Demonstration