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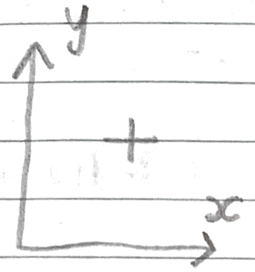
Assignment 3

Question 1: $a_1 = 7$; $a_2 = 6$; $a_3 = 5$; no friction; f?

car	m [kg]	spring	k [N/m]
1	37	1	270
2	46	2	260
3	55	3	250
		4	200



$$F = ma \; ; \; m_1 \frac{d^2 x_1}{dt^2} = -K_1 x_1 + K_2 (x_2 - x_1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = -K_2 (x_2 - x_1) + K_3 (x_3 - x_2)$$

$$m_3 \frac{d^2 x_3}{dt^3} = -K_3 (x_3 - x_2) - K_4 x_3$$

Rearranging,

$$m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 - K_2 x_2 + K_2 x_1 = 0$$

$$m_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 - K_2 x_1 - K_3 x_3 + K_3 x_2 = 0$$

$$m_3 \frac{d^2 x_3}{dt^2} + K_3 x_3 - K_3 x_2 + K_4 x_3 = 0$$

$$x_i'' = -x_i \, w^2 \sin(wt)$$

Rearrange to solve for oscillation amplitudes,

$$\left(-w^2 + \frac{K1}{m1} + \frac{K2}{m1}\right)x_1 \quad \left(\frac{-K2}{m1}\right)x_2 \quad 0 = 0$$

$$\left(\frac{-K2}{m2}\right)x_1 \quad \left(-w^2 + \frac{K2}{m2} + \frac{K3}{m2}\right)x_2 \quad \left(\frac{-K3}{m2}\right)x_3 = 0$$

$$0 \quad \left(\frac{-K3}{m3}\right)x_2 \quad \left(-w^2 + \frac{K3}{m3} + \frac{K4}{m3}\right)x_3 = 0$$

$$\begin{bmatrix} \frac{K1}{m1} + \frac{K2}{m1} - w^2 & \frac{-K2}{m1} & 0 \\ \frac{-K2}{m2} & \frac{K2}{m2} + \frac{K3}{m2} - w^2 & \frac{-K3}{m2} \\ 0 & \frac{-K3}{m3} & \frac{K3}{m3} + \frac{K4}{m3} - w^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

w^2 here is the eigen value, λ; the equation of λ can be found by taking the determinant,

$$-\lambda^3 + \frac{314465\,\lambda^2}{9361} - \frac{2829850\,\lambda}{9361} + \frac{157000}{253} = 0$$

$$\boxed{\lambda = \{20.0732,\ 10.6047,\ 2.9152\}}$$

$$\boxed{\text{eigenvector} = \begin{bmatrix} -0.7523 & -0.6643 & 0.4226 \\ 0.6154 & -0.3516 & 0.6861 \\ -0.2352 & 0.6596 & 0.5922 \end{bmatrix}}$$

Q2:

x	12.9	13.5	13.4	13.6	13.7	14.0	15.0	16.7
y	13.0	12.2	11.8	10.1	8.1	6.2	4.2	2.3
x	18.3	19.0	20.2	21.7	23.3	26.7		
y	1.1	0.8	0.5	0.4	0.3	0.2		

$y = f(x) \mid x =$ mean wind speed [m/s]; y_2? y_3?

$y_2 = a_2 x^2 + a_1 x + a_0$

$$\begin{bmatrix} n & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \end{bmatrix}$$

solving for a_0, a_1, a_2 we get,

$$\boxed{y_2 = 0.1834x^2 - 6.0047x + 66.6208}$$

$\therefore a_0 = 66.6208;\ a_1 = -6.0047;\ a_2 = 0.1834$

$y_3 = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$\begin{bmatrix} n & \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 & \Sigma x_i^5 \\ \Sigma x_i^3 & \Sigma x_i^4 & \Sigma x_i^5 & \Sigma x_i^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \\ \Sigma x_i^3 y_i \end{bmatrix}$$

$$\boxed{y_3 = -0.0206x^3 + 1.3536x^2 - 29.3108x + 209.8834}$$

Q3:

time, t	9	8	7	6	5	4	3	2	1	0
speed, S	4	6	4	15	37	46	42	39	8	0

S [Km/h]
t [S]

Apply Lagrange 1^{st}–5^{th} order to find speed at 1.5 S

1^{st} & 2^{nd} order hand calculations:

the first & second order Lagrange interpolating polynomial are $S_1(t)$ & $S_2(t)$ respectively.

$$S_1(t) = L_1 S(t_1) + L_2 S(t_2) = -2t + 22$$

$$L_1 = \frac{t - t_2}{t_1 - t_2} = \frac{t-8}{9-8} = t-8 ; L_2 = \frac{t-t_1}{t_2-t_1} = -t+9$$

$$S_1(1.5) = -2(1.5) + 22 = \boxed{19 Km/h}$$

$$S_2(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)} S(t_1) + \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)} S(t_2) + \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)} S(t_3) = 2(t-7)(t-8) - 6(t-7)(t-9) + 2(t-8)(t-9) ; S_2(1.5) = \boxed{-78.5 \ km/h}$$

∴ the speed at first order at t=1.5 S is 19 Km/h

∴ the speed at second order at t=1.5 S is −78.5 Km/h

Q4: $y(x) = e^{\frac{10x}{765}}$; y = sales / day ; x = day after launch ;

$x = 360$ days ; $\int_0^{360} y(x)\,dx$?

Analytical solution:

$$\int_0^{360} e^{\frac{2}{153}x}\,dx = \frac{153}{2} e^{\frac{2}{153}x}\Big|_0^{360} = \boxed{\frac{153\left(e^{\frac{80}{17}}-1\right)}{2}} = \boxed{8384.08}$$

the Answers are in matlab file