Zewail City of Science and Technology

University of Science and Technology

Communications & Information Engineering Program

CIE 206 – Spring 2021

Probability and stochastic process

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Introduction

Having a deep look at the stage where all the world is satisfied with the ease of communication, no need to wait 15 days till the message arrives, no need to travel to see friends, all of that is accomplished by improvement of the digital communication field.

Digital communication represented in sending digital signals in the form of electromagnetic waves was a huge challenge for communication engineers; it was due to the noise added to the signal transmitted.

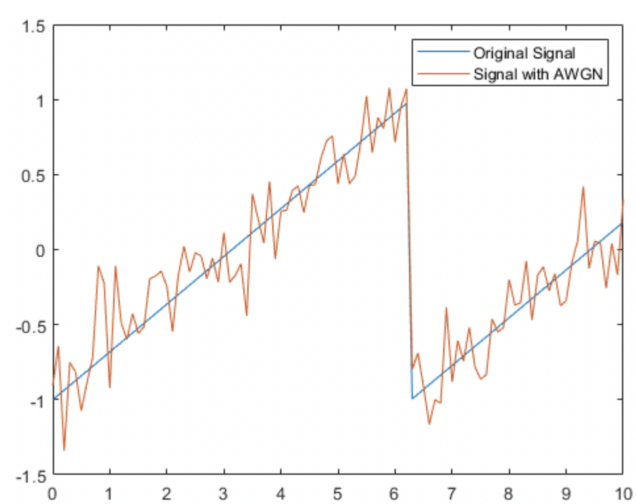
In this project, a simulation for the whole process of sending and receiving binary waves is simulated by SIMULINK, another version is made by code in MATLAB; We present here a very detailed process, including all challenges. The report will have mainly 3 parts; transmission, channel, and receiving.

Overview

We are simulating the process of sending and receiving the signal, afterward; get the Bit error rate(BER) which is the number of error bit per unit time; and also calculate the Signal-to-noise relation(SNR) meaning the ratio between of power of signal and power of noise, and therefore getting the relation between BER and SNR.

For the most accurate results, and for the most appropriate mean, in the transmission part; we are transmitting a large number of bits; about 10,000 bits.

Coming to the receiving part; here the signal is mostly noisy due to many factors like the vibration of atoms in conductors, radiation of objects,..etc. the summation of all these random variables results in Gaussian distribution; according to the “Central limit theorem”. The noise here is called Additive White Gaussian noise (AWGN); each word here means something; Additive is used here as the noise is added to the signal, White here stands for having uniform power for all frequencies like white color, Gaussian stands for having gaussian distribution.



For the received signal it might be a little corrupted like in figure 1, still can be improved in the receiving part by removing the noise. But some noise could be large enough to destroy the signal like in figure 2. This is controlled by SNR.

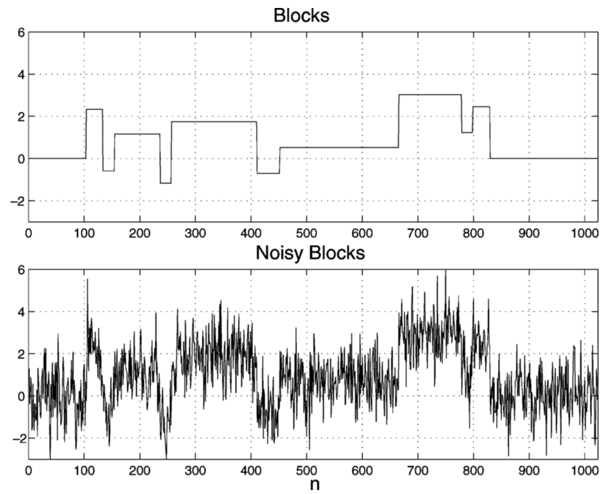
SNR is the signal to noise ratio

$$SNR = \frac{P_{Signal}}{P_{Noise}} \quad (1)$$

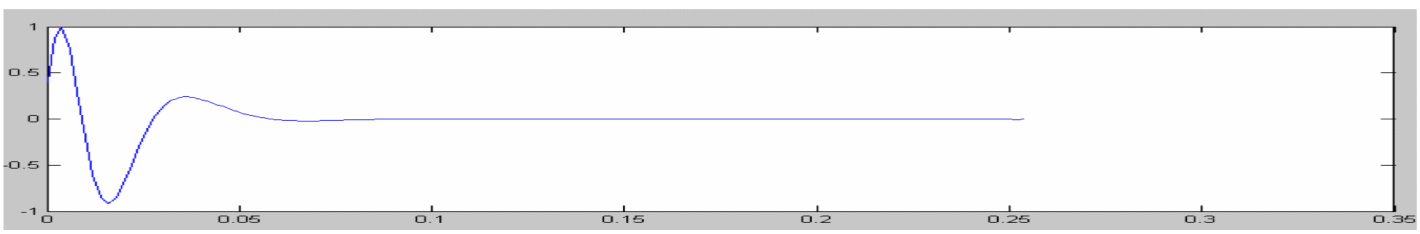
Figure(1)

So considering the large value of SNR meaning the signal will not be corrupted totally, but getting a very small SNR will destroy it, which is used to modulate the noise in simulation.

For more understanding of SNR, consider the signal in figure (3), this the signal transmitted, in figure (4), it shows the received signal in different SNR values



Figure(2)



Figure(3)

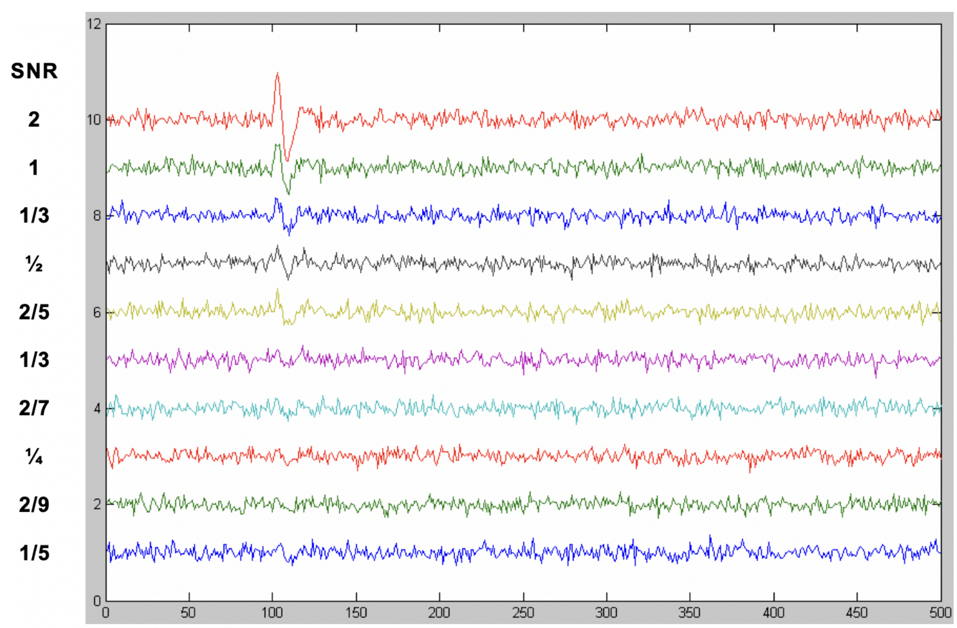
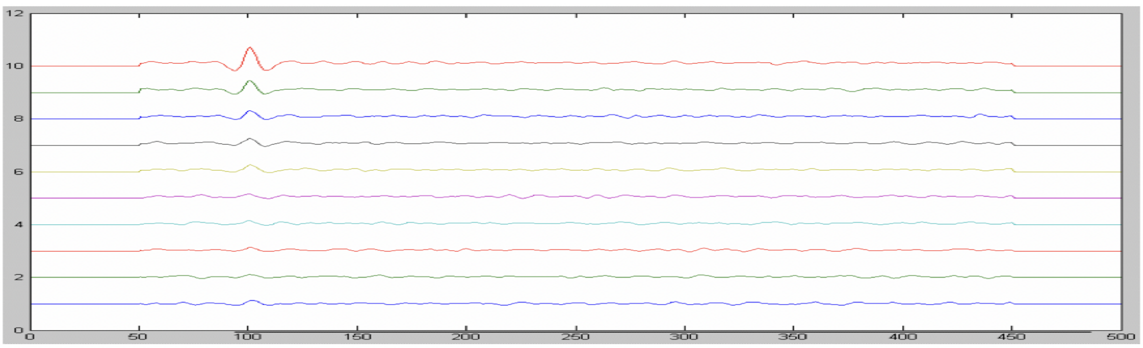


Figure (4)

We see here when the SNR is so small the signal is very noisy. And vice versa.

Now getting back again to the Receiving part, the signal is so noisy; scientists found out that they could improve the quality of input signal by using "Matched filter", mainly it increases the SNR, so get rid of most of the noise.

The matched filter is a linear filter, usually used in maximizing the signal-to-noise ratio. In figure (5), it shows the results of Figure(5) after going through the matched filter.



Figure(5)

The way the matched filter works is quite similar to convolution; it reverses the input signal and convolves it with the original signal(unreversed), by which it maximizes the SNR. by that it could also be used in radar to detect weak signals

The math behind matched filter:

Returning back to the definition of convolution; it could be thought of as the measure of how two signals are alike, so the main purpose here is to get an LTI system, with impulse response that maximizes the output, following the formula (2)

$$y(\tau) = \int_{-\infty}^{\infty} s(\tau - t) * h(t)\, dt \quad (2)$$

Where s*(t)* is the desired finite energy signal, the *h(t)* is the impulse response of the system. So the question here is what *h(t)* will produce a maximum response at time *t0*? But this question is very loosely phrased, as getting *2h(t)* will produce a larger response than *h(t),* so if we subject this formula too $\int_{-\infty}^{\infty} |h(t)|^2 dt = E = \int_{-\infty}^{\infty} |s(t)|^2 dt$.

But Cauchy-Schwarz inequality is proved to be (3)

$$y(t_0) = \int_{-\infty}^{\infty} s(t_0 - t) * h(t)\, dt \le \sqrt{\int_{-\infty}^{\infty} |s(t_0 - t)|^2 dt} \sqrt{\int_{-\infty}^{\infty} |h(t)|^2 dt} = E \quad (3)$$

But that inequality is satisfied only if given that $h(t) = \lambda s(t_0 - t)$, so taking $\lambda = 1$, then, producing maximum response at *t0.*

Producing finally the formula at (4)

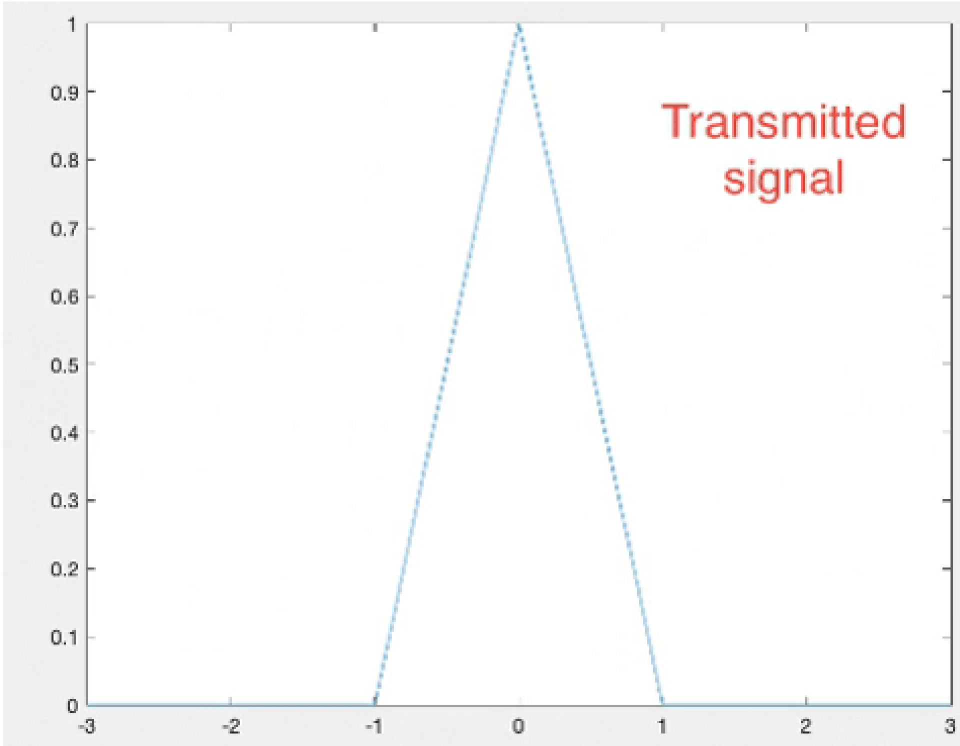
$$y(\tau) = \int_{-\infty}^{\infty} s(\tau - t) * s(- \tau)\, d\tau \text{(4)}$$

But what about SNR? In any digital communication channel, there is noise, considering AWGN noise with two-sided power spectral$\frac{N_0}{2}$, the output noise will be zero mean stationary gaussian with autocorrelation function$\frac{N_0}{2}Rs(t)$, whereas h(t) is the impulse response of the LTI system. Then variance will be calculated via equation (5)

$$\sigma^2 = \frac{N_0}{2}R_s(0) = \frac{N_0}{2}\int_{-\infty}^{\infty} |h(t)|^2 dt \text{ (5)}$$

Knowing that, so what will maximize SNR? It turned out that, so maximizing output, will maximize SNR. Quick GIF for the whole process.

$$SNR = \frac{y(t_0)}{\sigma} = \sqrt{\frac{2E}{N_0}}$$



BER vs SER

Obviously, Bit Error Rate (BER) is the number of bit errors per unit time. For example, if the transmitted signal is 1 0 1 1 and the received signal is 1001 so there is an error in just one bit (BER = number of errors/ total number of bits = 1/4). Meanwhile, the Symbol Error Rate (SER) is mainly the same concept but in terms of symbols, not bits. For example, if the transmitted signal is 00 11 10 and the received signal is 11 11 10 so there is an error (SER = number of errors /total number of symbols).

	BER	SER
M (levels number)	2	>2
Concerning about	bits	symbols

Definition	the number of bit errors per unit time.	the number of symbol errors per unit time.

Transmission Techniques

1) Baseband Transmission

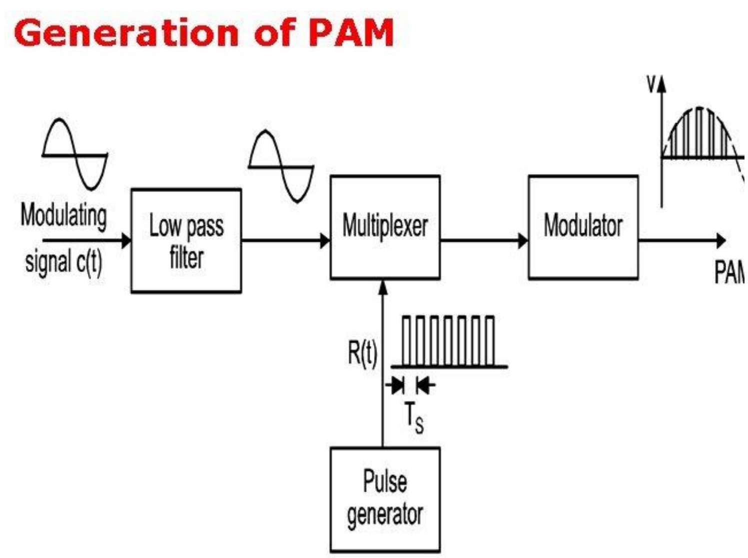
This type of transmission requires no modulation, which means that the signal does not need to be changed from a digital signal to analog signal. This process requires a low pass channel, a channel whose bandwidth starts from zero, yet this process is only used for wired communication since the baseband signal frequency varies from 20Hz-20000 Hz which is a small frequency range. In air, a short distance to be travelled by those baseband signals is enough to get it suppressed .

2) PAM

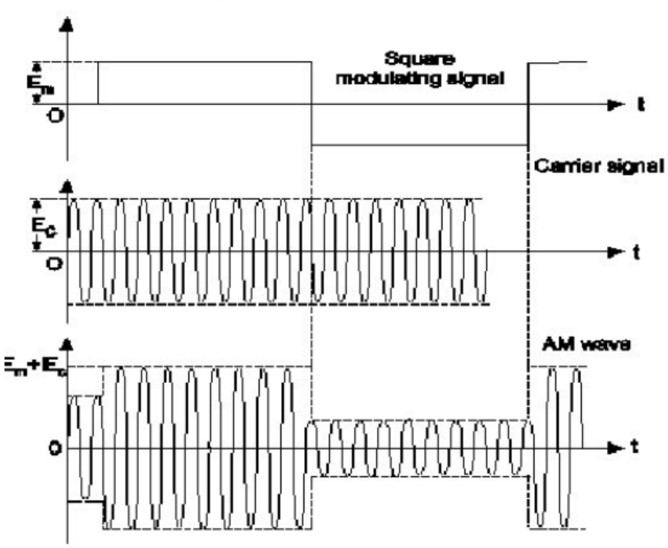
Pulse Amplitude Modulation is a type of modulation where the amplitude of a train of carried pulses is altered according to the message information desired to be transmitted. Modulation is a process where high frequency carriers are used to superimpose low frequency information.

1) How does the PAM get generated?

The modulating signal(message information) is passed through a Low Pass filter, the low pass filter is responsible for allowing signals to pass as long as they are not greater than the cutoff frequency, for the parts of the signal having a greater frequency than the cutoff frequency, they are attenuated, this creates a baseband. Another use for the low pass filter is to prevent the aliasing of samples at the beginning of the process through having a frequency nearly half the sampling frequency, and this takes us to the next part, the sampling. Sampling is used to convert the analog signal which is continuous in time into a discrete time signal. This is done through a multiplexer and a pulse generator, where the pulse generator generates sampling pulses, and the multiplexer multiplies the analog signal with the sampling pulses.Finally, the last step is modulation where the carrier signal is altered according to the signal resulting from the multiplexer.



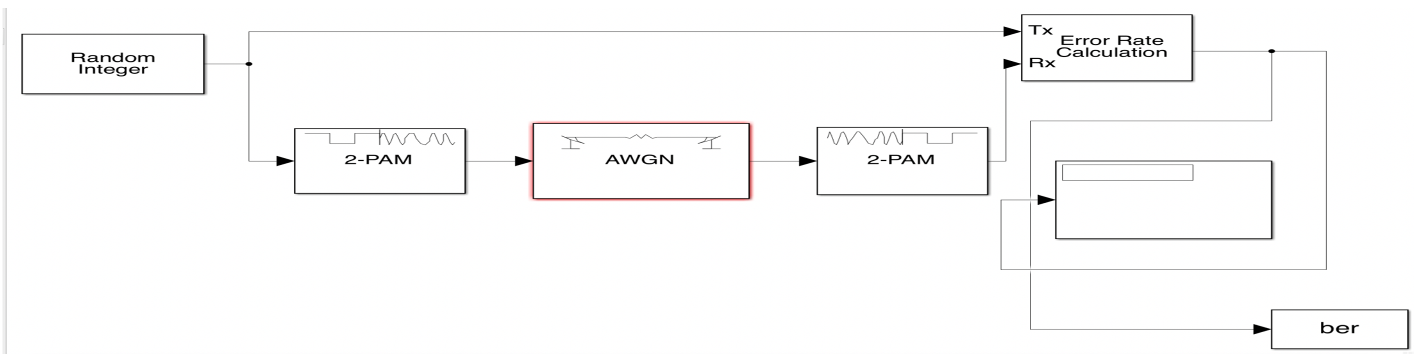
The figure below shows an example of a square wave as the modulating signal, and a sinusoidal wave as the carrier signal.



Simulink

Simulink is a simulation tool, provided with Matlab, we simulated the whole process here using it, A random integer was used to transmit the signal of bits, followed by an M-PAM modulator, getting the transmitted signal into a channel, adding AWGN noise, and received by M-PAM demodulator, and finally the error rate was calculated, the results were extracted and given to bertool, to plot the graph between SNR and BER.

Here is the design, associated with the 2-PAM modulator and demodulator, at figure(6)



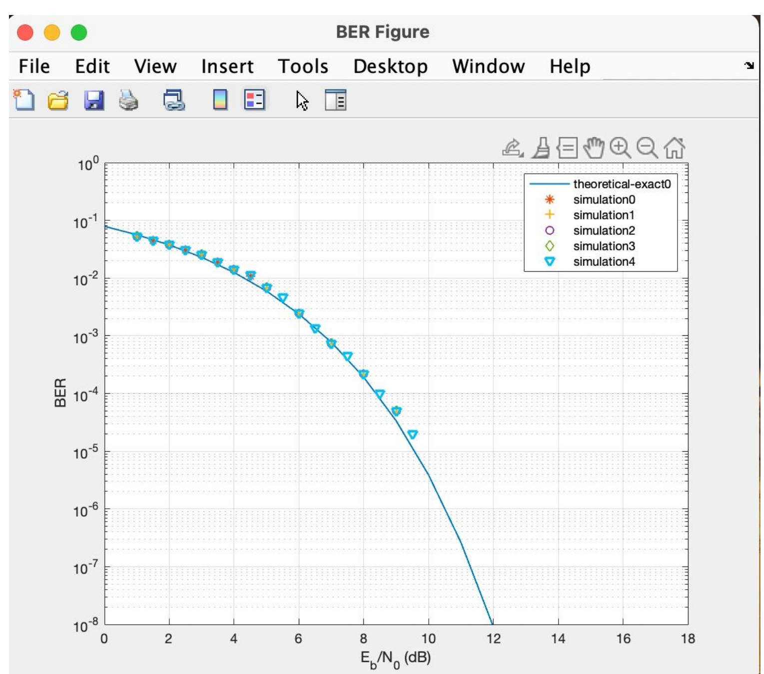
Figure(6)

In figure (7). The SNR vs BER is shown, where the points fit exactly on the theoretical curve.

Taking into consideration that BER has been calculated via equation (6)

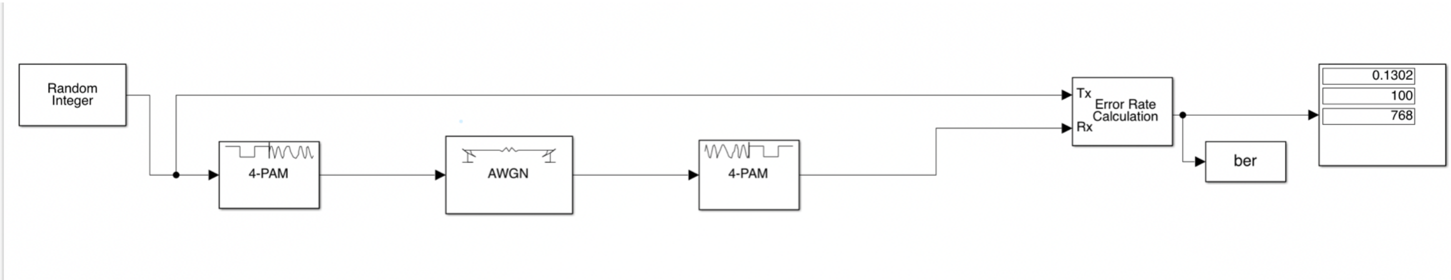
$$P_e = \frac{M-1}{M} erfc(\frac{SNR}{\sqrt{2(M-1)}}) \quad (6)$$

Where m, is the modulator number used by m-pam.

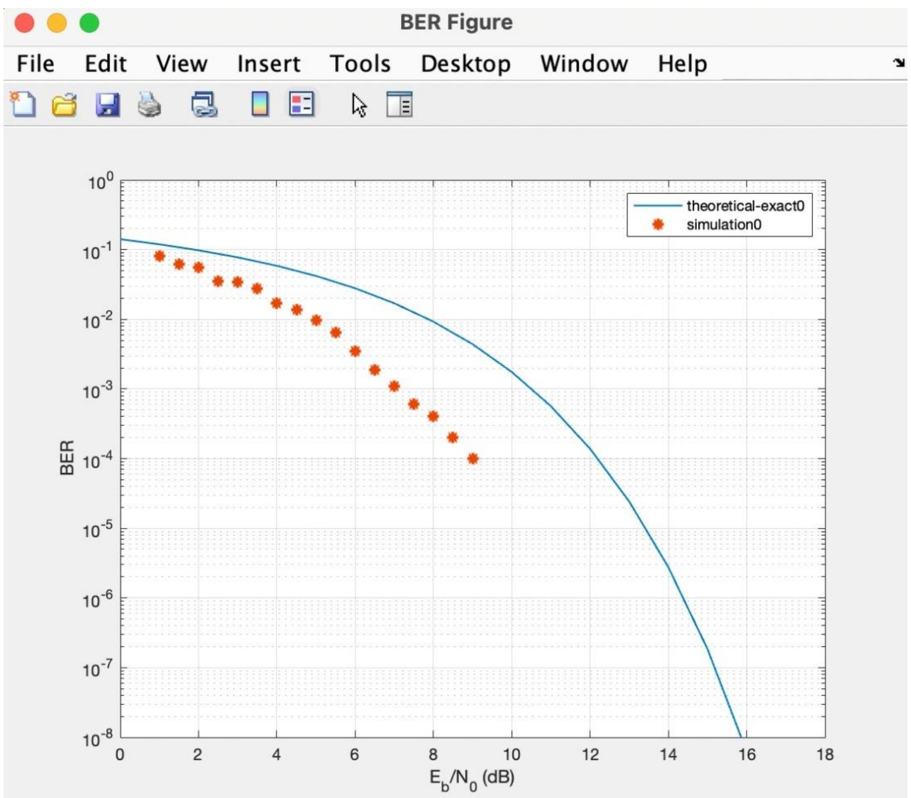


Considering the larger value for m, 4-pam, the design is given at figure(8), and graph for SER vs. SNR is given at figure(9), and for 8-pam, they are provided in figure(10), and figure(11)

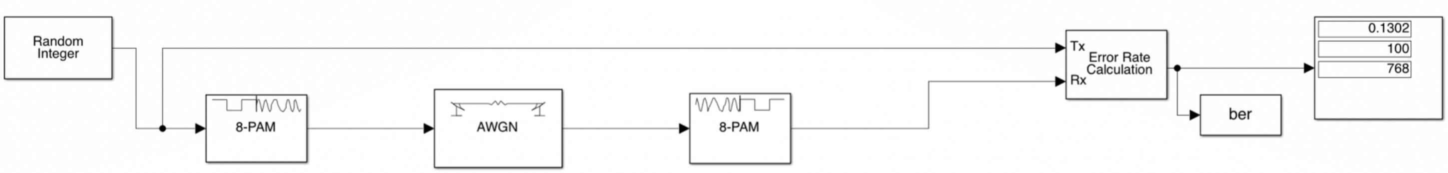
Figure(7)



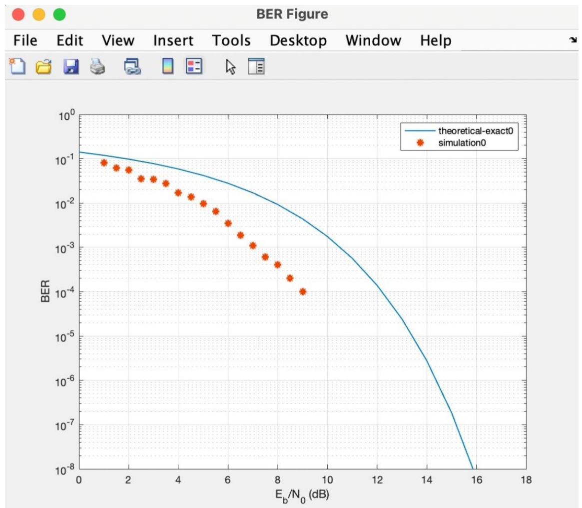
Figure(8)



Figure(9)



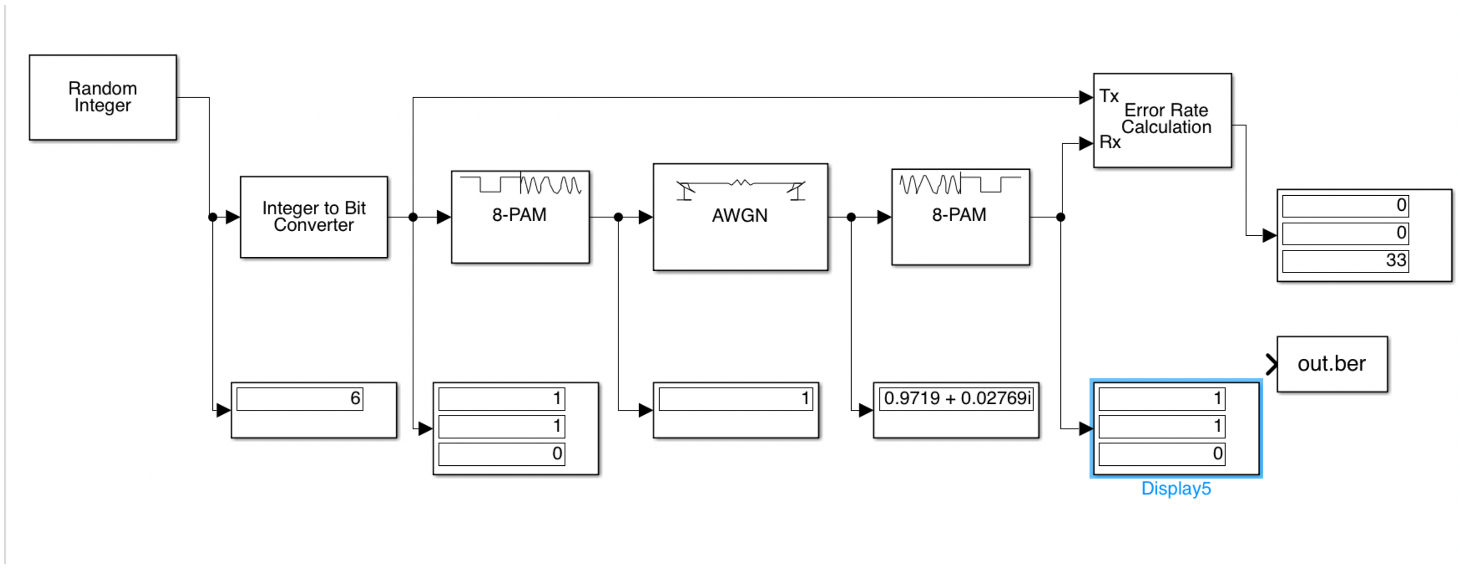
Figure(10)



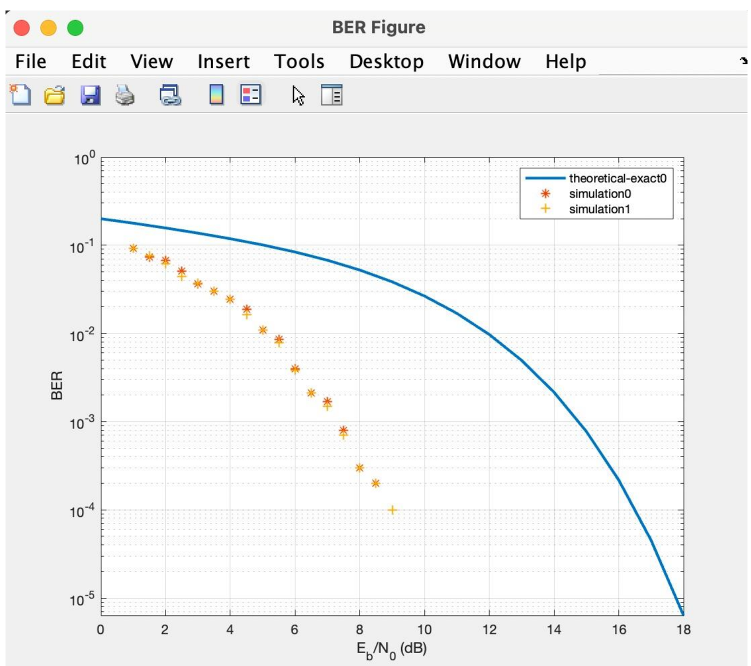
Figure(11)

In 4-pam, and 8-pam, the point didn't fit exactly the theoretical curve, it could be related to simulation-related problems, same as for 8-pam, the design is at figure(10), and curve is at figure (11)

We tried to plot the SER for by figure (12)



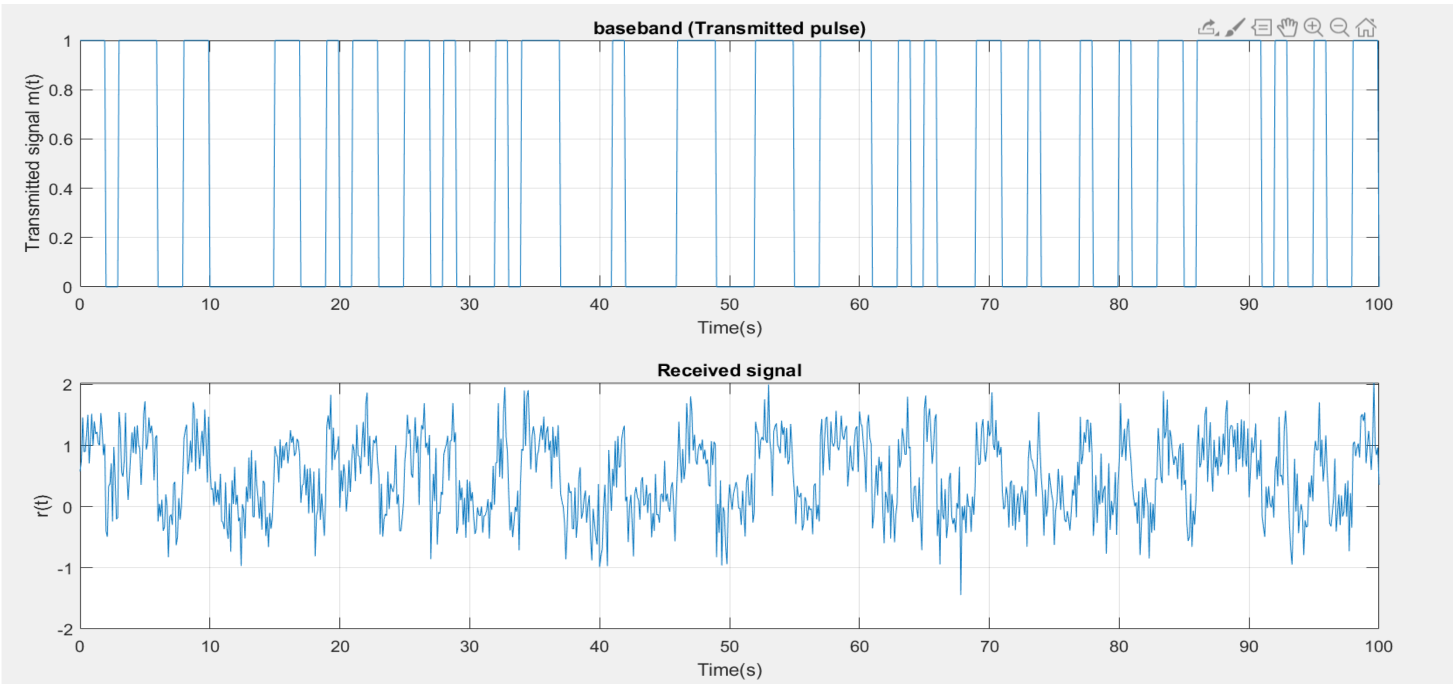
Figure(12)



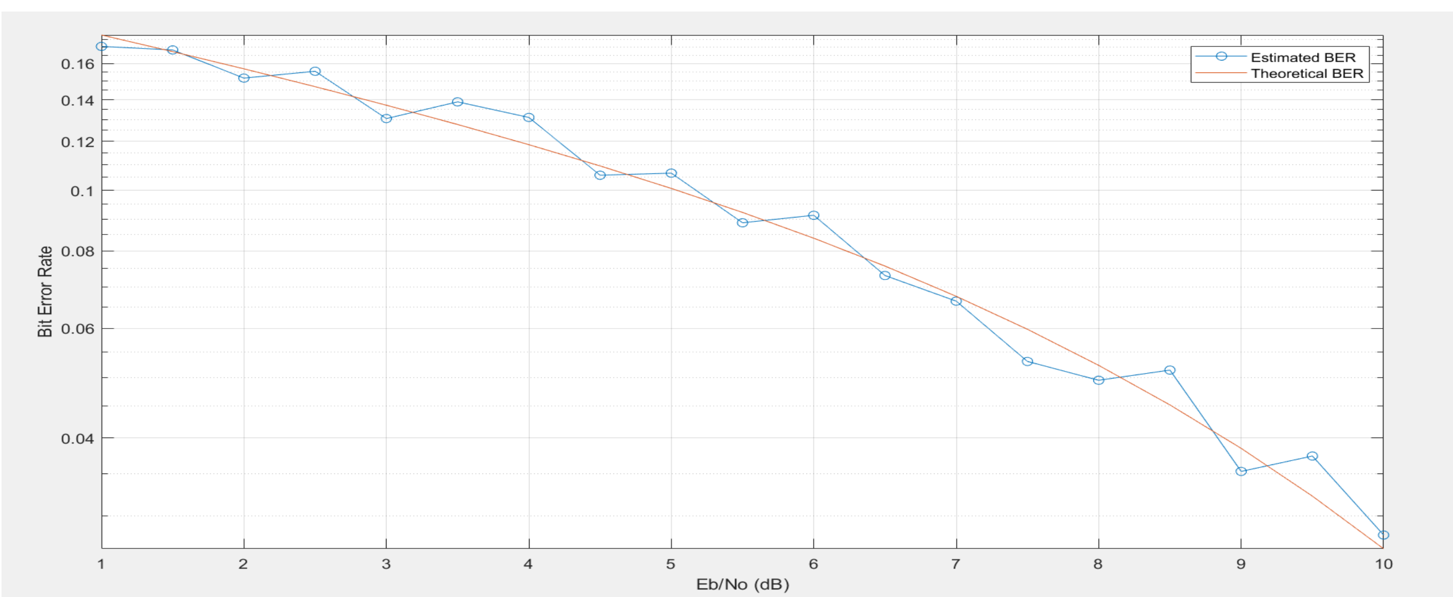
Figure(13)

Code Curves:

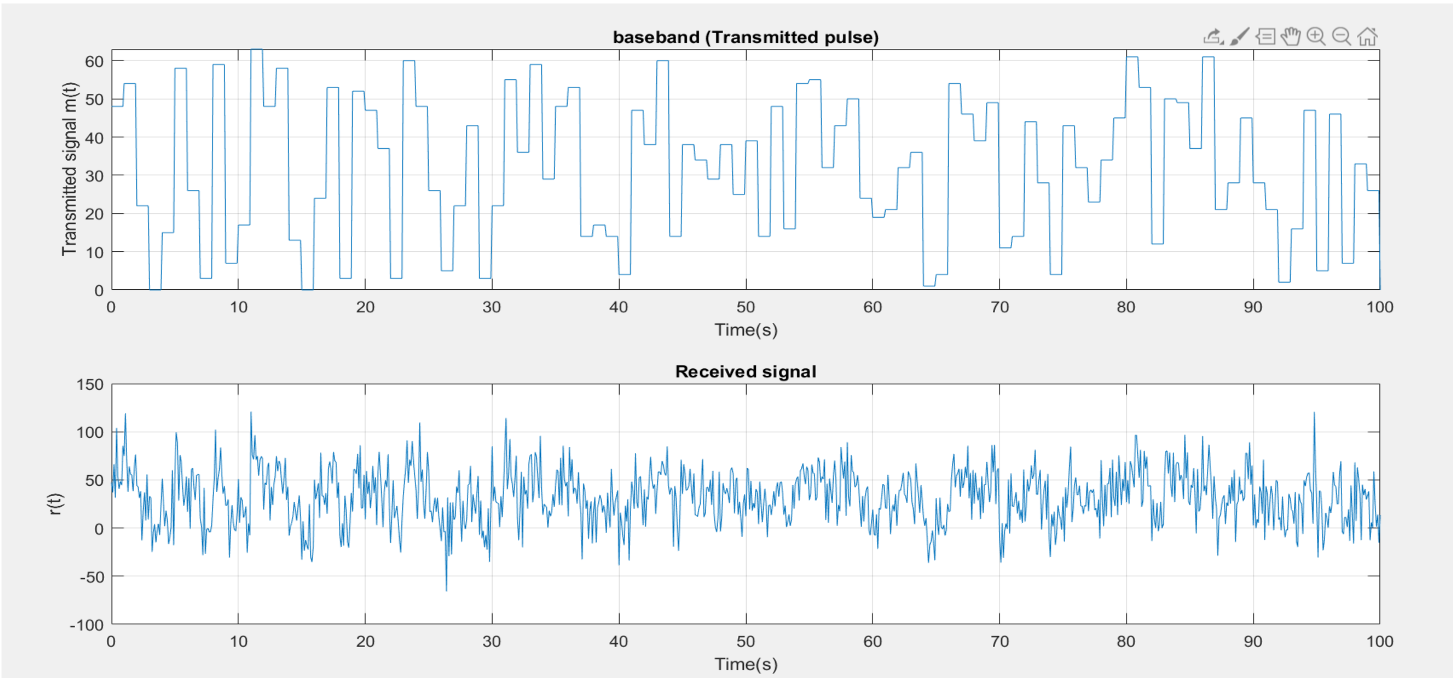
(Transmitted & Received signal) IF M = 2



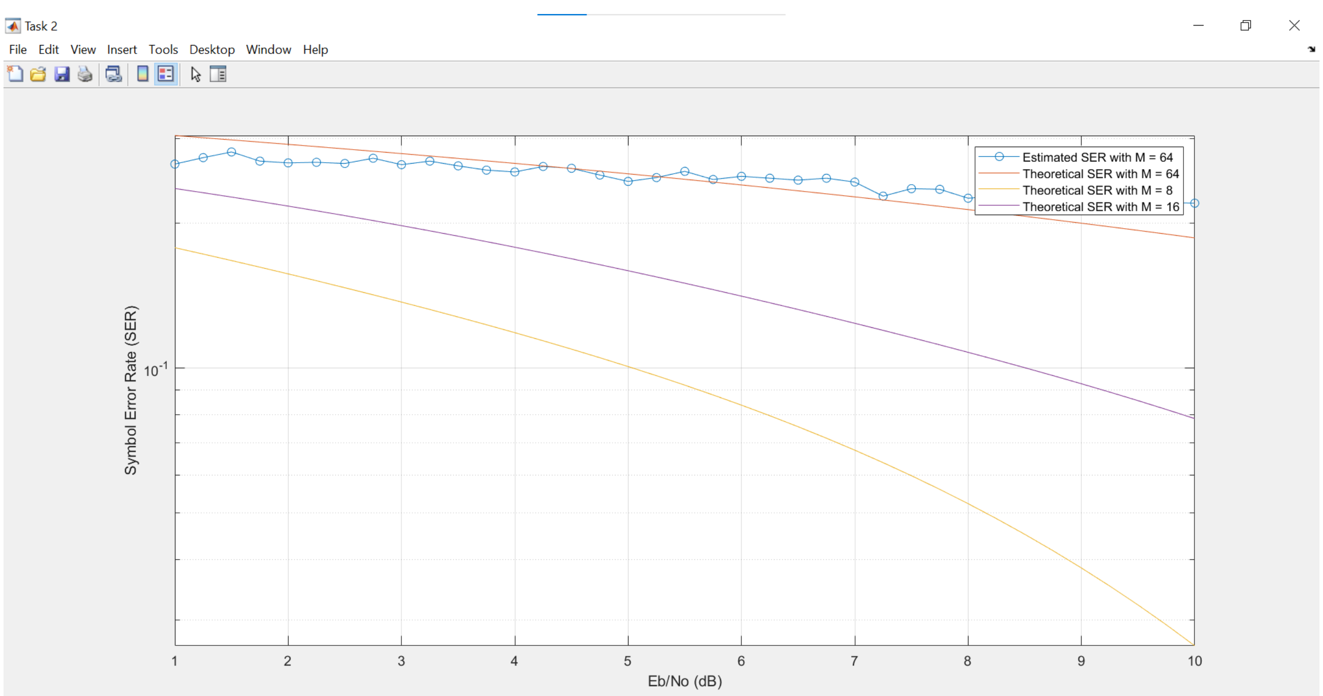
BER vs SNR Curve



(Transmitted & Received signal)IF M = 64



SER vs SNR Curve



Special Notes:

1) Why is SER directly proportional to M(number of levels)?

It is clear that the greater M is, the more errors increase. This makes sense because while increasing the scale of numbers the expected number of errors would increase. For example, if M = 4, the levels would be -3,-1,1,3 . So, if the received signal has 4 after passing through the channel it is most probable that it was 3. If M = 8 the levels would be -5,-3,-1,1,3,5 .So, if the received signal has 4 after passing through the channel it would be 3, or 5. (if the noise is a little high, it would be 3,5, or 1). Consequently, the number of errors increases, and the symbol error rate (SER) would increase.

2) Why are SER and BER inversely proportional to SNR?

Because higher SNR means the ratio of the signal power is greater than the noise power. So, when the signal power is high with respect to the noise power, there would be fewer errors (lower SER, BER). Therefore, the higher SNR is, the lower SER is.

References

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Dilip SarwateDilip Sarwate, Jason RJason R 23.1k22 gold badges6060 silver badges7171 bronze badges, Alexander GrauAlexander Grau 2111 bronze badge, QMCQMC 75433 silver badges1111 bronze badges, & AaronAaron 1. (1961, October 1). *Understanding the matched filter*. Signal Processing Stack Exchange. https://dsp.stackexchange.com/questions/9094/understanding-the-matched-filter.