مدينة زويل للعلوم والتكنولوجيا
Zewail City of Science and Technology

Simulation and Analysis of Bit Error Rate vs. Signal-to-Noise Ratio in M-ary Pulse Amplitude Modulation

CIE 327 - Probability and Stochastic Process

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1.Abstract and Introduction:

This project uses the power of probability theory to tackle this challenge. We'll be simulating a digital channel using Pulse Amplitude Modulation (PAM) to transmit a signal. To make things interesting, we'll add some random noise (called Additive White Gaussian Noise) and see how it affects the signal.

Here's the key part: We'll change the strength of the signal compared to the noise (signal-to-noise ratio) and see how often errors occur in the transmission (Bit Error Rate). By repeating this process for different levels of PAM, we can create a graph that shows how these errors change based on the signal strength and the type of PAM used.

2. Digital Channel Design (Flow):

1. Generating PAM signals and transmission.
2. Adding Gaussian noise (AWGN) to simulate channel noise.
3. Decision-making based on matched filter output.
4. Calculating BER by comparing detected and transmitted symbols.

3. M-ary Pulse Amplitude Modulation (M-PAM):

Pulse amplitude modulation (PAM) is a method used in communication systems to transmit information by changing the amplitude of a train of pulses. It is an analog pulse modulation model, which means that both the message signal and the pulse amplitude fluctuations are continuous.

Basic principle:

PAM encodes a message signal (which can be analog) into a sequence of carrier pulses.
The amplitude of each pulse is directly proportional to the instantaneous amplitude of the message signal at the time of sampling.
Larger amplitudes in the message signal correspond to larger amplitude pulses in the PAM waveform and vice versa.

Demodulation process:

At the receiving end, the demodulator restores the original message signal by measuring the amplitude level of each pulse of the received PAM signal.

<u>Explanation of PAM:</u>

Core Principle:

- Standard PAM employs a limited set of amplitude levels to encode the message signal.
- M-PAM expands on this by utilizing **M distinct amplitude levels** to represent **M-ary symbols**.
- The number of amplitude levels (M) directly influences the amount of information each symbol can carry.

Example: 4-PAM

Imagine we want to transmit digital data using 4-PAM. Here's how it works:

1. **Number of Levels (M):** We have M = 4 distinct amplitude levels, denoted as A0, A1, A2, and A3.
2. **Bits per Symbol:** Since M = 4, each symbol can represent 2 bits (log2(4) = 2).
3. **Symbol Mapping:** We establish a mapping between the 2-bit binary combinations and the corresponding amplitude levels. For instance:
    - 00 -> A0 (Lowest amplitude)
    - 01 -> A1
    - 10 -> A2
    - 11 -> A3 (Highest amplitude)
4. **Modulation:**
    - The incoming binary data stream is divided into groups of 2 bits each.
    - Each 2-bit group is mapped to its corresponding M-ary symbol based on the defined mapping.
    - The amplitude level associated with the chosen symbol is used to modulate the carrier signal, creating variations in its amplitude.

Example Scenario:

Suppose we want to transmit the binary data stream "101100".

- We split the data into 2-bit groups: 10 11 00
- Using the symbol mapping, these groups translate to: A2 A3 A0 (referring to the corresponding amplitude levels)

- The carrier signal is modulated with these amplitude levels (A2, A3, A0) in sequence, creating the transmitted M-PAM signal.

Demodulation:

At the receiver, the M-PAM signal is demodulated to recover the original data:

1. The received signal is analyzed to estimate the transmitted amplitude levels.
2. Based on the pre-defined symbol mapping, each estimated level is converted back to its corresponding 2-bit binary sequence.
3. In our example, if the demodulator accurately estimates the amplitude levels A2, A3, and A0, it correctly recovers the original data stream "101100".

Equation:

$$N = \log_2 M$$

Where, N is the number of bits necessary
M is the number of conditions, levels, or combinations possible with N bits 2
The equation can be rea arranged as:

$$M = N^2$$

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4. **Understanding Additive White Gaussian Noise (AWGN)**

Additive White Gaussian Noise (AWGN) is a crucial concept in communication systems and signal processing. It describes a type of random noise introduced into a signal during transmission or processing. To fully grasp AWGN, let's break down its components: Additive, White, and Gaussian.
Additive: The term "additive" signifies that the noise is added to the original signal. This means that as the signal travels through a communication channel or undergoes processing, the noise combines with it. Think of it as adding an extra layer of sound over a piece of music while it's being played. Mathematically, this can be represented as
$T(t)$=$S(t)$+$W(t)$
where $T(t)$ is the transmitted signal, $S(t)$ is the original signal, and $W(t)$ is the added noise.
White: "White" noise is defined by its constant power spectral density across all frequencies. This means that the noise has equal energy at all frequency components. An analogy would be white light, which contains all colors of the visible spectrum with equal intensity. Similarly, white noise comprises all audible frequencies with uniform strength.

Gaussian: The "Gaussian" part refers to the statistical distribution of the noise values. Gaussian noise follows a Gaussian (or normal) distribution, often visualized as a bell curve. In this distribution, most noise values cluster around a mean value, with the probability of encountering values decreasing as you move further from the mean. Imagine a classroom test where most students score around the average, with fewer students scoring very high or very low.
Understanding AWGN is essential for analyzing and designing communication systems. It helps engineers predict and mitigate the effects of noise on signal integrity, ensuring clearer and more reliable transmissions.

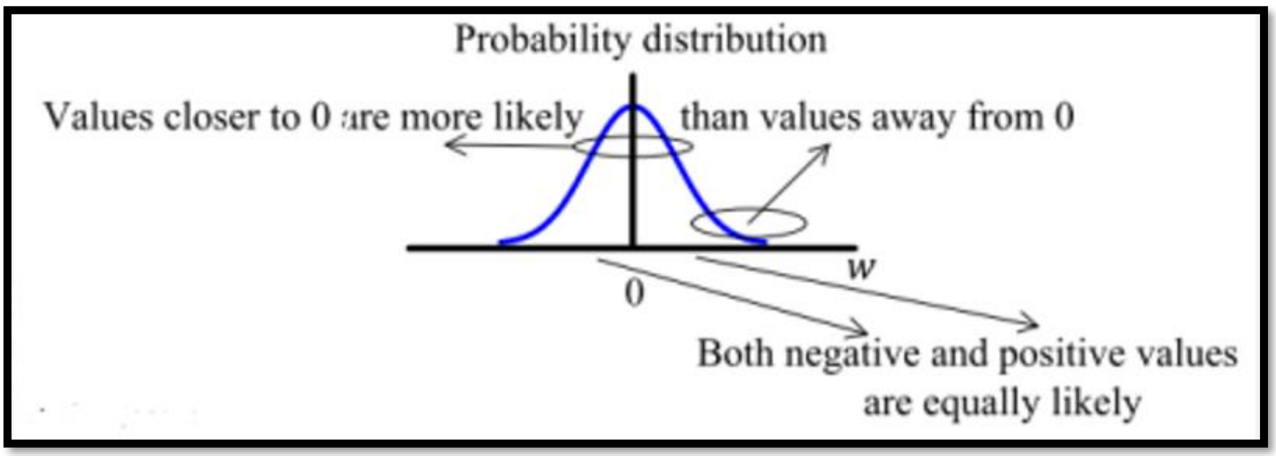

Figure 1: Brief illustration of the Gaussian Distribution

$$N \sim Guassian(\mu = 0, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})}$$

The effect of AWGN: it affects the quality of transmitted signals by introducing random variations in amplitude. This can lead to errors in the received signal, especially when the signal is weak or the SNR is low.

SNR: The Signal-to-Noise Ratio (SNR) is a crucial parameter that quantifies the ratio of the signal power to the noise power. A higher SNR indicates a better-quality signal, while a lower SNR indicates that the noise is stronger relative to the signal.

AWGN Channel: In simulations, the AWGN channel is a common model used to represent the noise encountered during transmission. This channel adds Gaussian noise to the signal

5. Matched Filter:

The matched filter is a filter that maximizes the signal to noise ratio **(SNR)** by reducing the noise's spectral bandwidth to that of the wavelet, and in addition, reduces the noise within the wavelet's bandwidth by the shape of the wavelet's spectrum, it is used in communication systems to improve the detection of transmitted symbols.

$$h(t) = w(T - t)$$

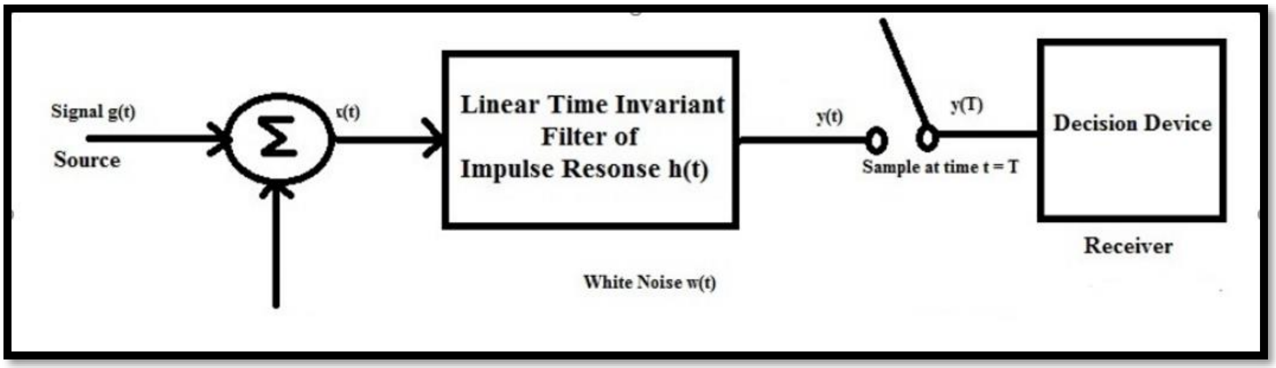

Figure 2: Illustration of the Matched Filter System

The working of a matched filter in signal processing is done by comparing a recognized template or delayed signal with an unidentified signal to notice the existence of the template within the unknown signal. So, this is analogous to convolving the unidentified signal throughout a conjugated time-reversed template version. The impulse response of matched filter is reversed and delayed version of the input signal. So, the impulse response should match the input of the filter.

6. Bit Error Rate (BER):

BER is a critical performance metric used in digital communication systems to quantify the accuracy of transmitted data. It represents the ratio of incorrectly received bits to the total number of transmitted bits, providing insight into the system's reliability and error resilience.

Mathematically, the Bit Error Rate (BER) is calculated as:

$$\text{the Bit Error Rate (BER)} = \frac{Number\ of\ Incorrectly\ Received\ Bits}{Total\ Number\ of\ Transmitted\ Bits}$$

Relation between Bit Error Rate (BER) and Signal-to-Noise Ratio (SNR):

BER is closely related to the Signal-to-Noise Ratio (SNR) in a communication channel. As SNR improves, BER generally decreases, indicating better transmission quality.

7.Simulink:

1. Block Diagram

Random Integer Generator:

- This block acts as the source of data for the simulation.
- It generates a sequence of random integers uniformly distributed between 0 and M-1 (inclusive).
- M represents the number of amplitude levels used in the M-PAM scheme, which also dictates the number of bits per symbol (bit width).

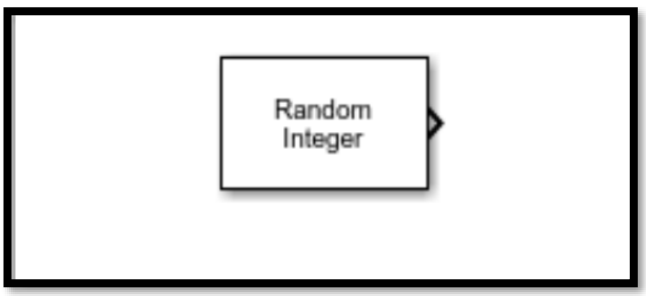

Figure 3: Random Integer Generator Block

M-PAM Modulator:

- This block takes the random integer sequence as input and performs M-PAM modulation.
- It maps each integer value to a specific amplitude level based on a pre-defined mapping scheme.
- The mapping scheme assigns unique amplitude levels to each possible combination of bits represented by the integer (e.g., in 4-PAM, 00 maps to A0, 01 to A1, etc.).

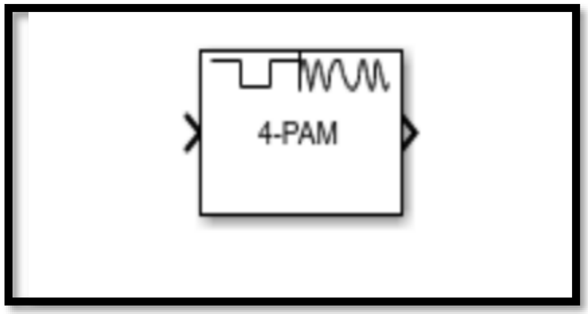

Figure 4: 4-PAM modulator Block

AWGN Channel:

- This block simulates the transmission channel, where the modulated signal encounters Additive White Gaussian Noise (AWGN).
- AWGN is a mathematical model for noise that is:

- Additive: It gets added to the transmitted signal.
- White: It has a constant power spectral density across all frequencies.
- Gaussian: The noise values follow a normal distribution.
- The AWGN channel introduces random variations in the amplitude of the transmitted signal, potentially corrupting the information it carries.

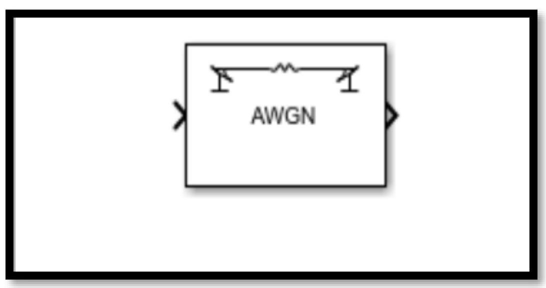

Figure 5: AWGN Channel Block

M-PAM Demodulator:

- This block receives the noisy signal from the AWGN channel and performs demodulation.
- The demodulator aims to recover the original data from the received signal despite the presence of noise.
- It estimates the most likely transmitted amplitude level for each symbol based on the received signal characteristics.

- Based on the pre-defined mapping scheme used in the modulator, the demodulator then maps the estimated amplitude levels back to their corresponding integer values (representing bits)..

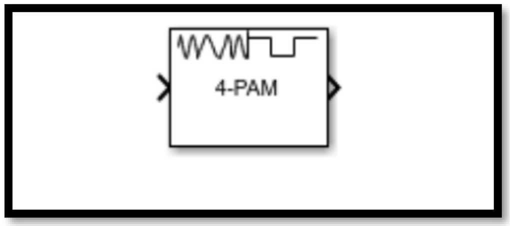

Figure 6: 4-PAM Demodulator Block

Error Rate Calculation:

- This block compares the original data sequence generated in block 1 with the demodulated data sequence from block 4.
- It calculates the Bit Error Rate (BER) by counting the number of bits that were incorrectly received in the demodulated sequence compared to the original data.
- BER is a crucial metric used to assess the performance of the communication system under noisy channel conditions

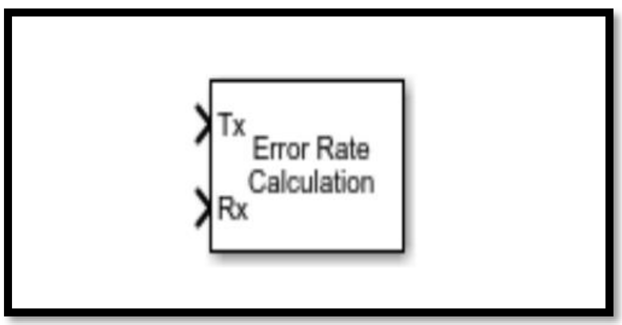

Figure 7: BER Calculation Block

To Workspace:

- This block serves as a data collection point for the simulation.
- It saves the calculated BER value as an array variable, allowing you to store multiple BER values for different simulation runs or parameter settings.
- This data can then be used for further analysis, visualization (e.g., plotting BER vs. signal-to-noise ratio), or performance evaluation of the M-PAM system.

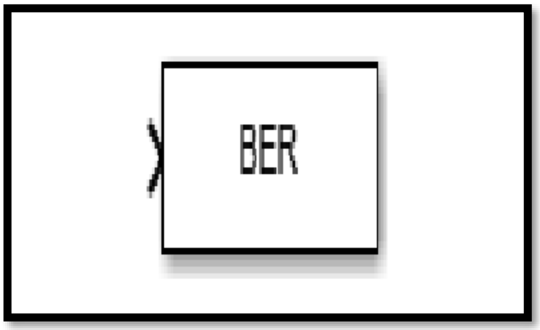

Figure 8: BER Output Block

<u>Here is the Diagram of the Used Simulink Blocks:</u>

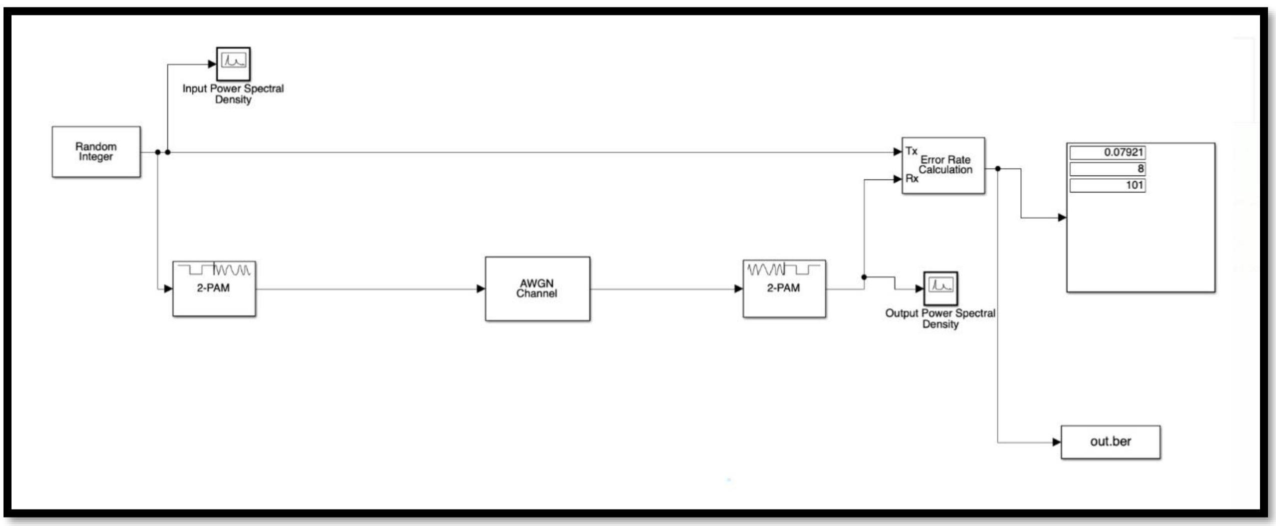

Figure 9: Simulink Project Diagram

7.2 <u>Results:</u>

The next three curves show the relationship between the bit error rate (BER) and the ratio Eb/N0, the signal-to-noise ratio (SNR). The pie charts show the relative observations for the different M levels showing the two correlations. n 1. Increasing the SNR ratio reduces the errors in the compared bits and makes the bit comparison process more successful. As the value of M increases, errors increase because the number of bits per symbol (bit width) increases.), the more amplitude levels used, the larger the medians for the same SNR ratio, and the more difficult the comparison process. For example, at the beginning of the three dot plots for different M levels, the SNR ratio is the same, but the BER is higher for the high modulation method. The graph also shows that at lower order levels the BER decays faster than at higher order levels, with slower BER decay rates as SNR increases.

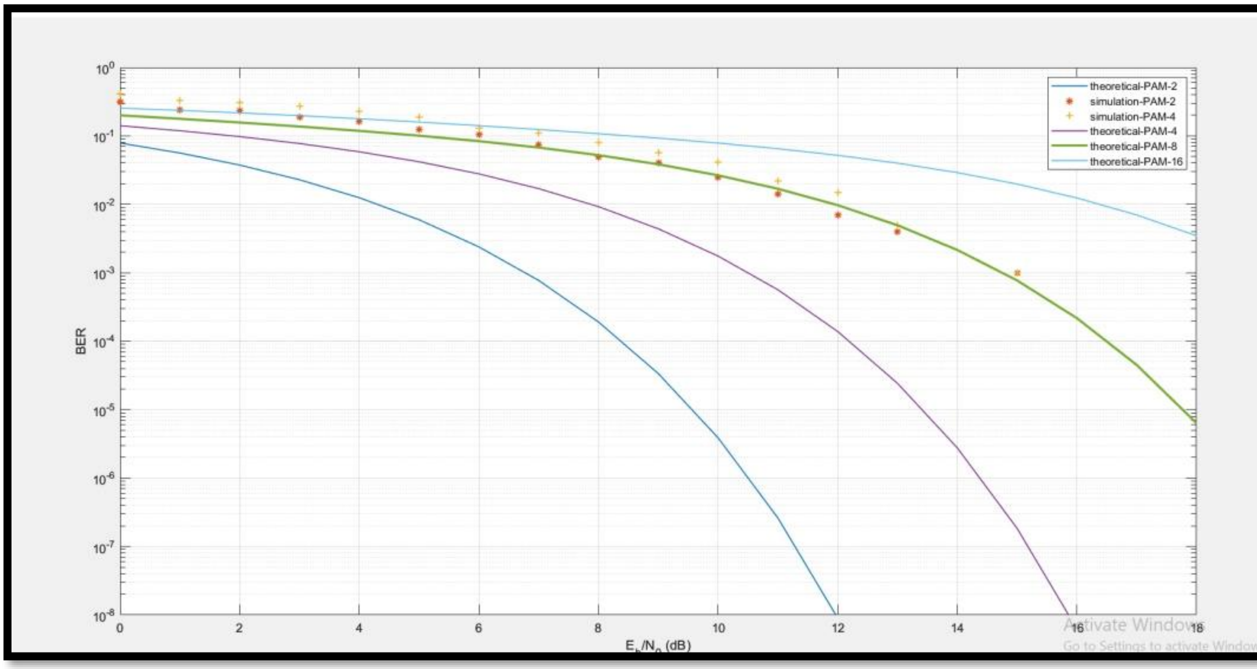

Figure 10: Simulink Simulation Results

8 MATLAB Code:

Below is a screenshot for the whole MATLAB code used in the project:

```matlab
clear all;
% Define the set of M values for PAM modulation
M_vals = [4, 16, 64];
% Define the range of SNR values in dB and the number of bits to transmit
SNR_dB_range = 0:1:20; % SNR values in decibels
num_bits = 1000; % Number of bits to transmit

% Initialize a matrix to store BER values for each M
BER_results = zeros(length(M_vals), length(SNR_dB_range));

% Select an SNR value for signal plotting (e.g., 10 dB)
plot_SNR_dB = 10;
plot_SNR_linear = 10^(plot_SNR_dB / 10);

% Loop over each M value
for idx = 1:length(M_vals)
    M = M_vals(idx);
    % Generate random bits for transmission
    tx_bits = randi([0, M-1], 1, num_bits);
    % Modulate the bits using PAM
    modulated_signal = pammod(tx_bits, M);

    % Loop over each SNR value
    for j = 1:length(SNR_dB_range)
        SNR_linear = 10^(SNR_dB_range(j)/10); % Convert SNR from dB to linear scale
        % Transmit the modulated signal through an AWGN channel
        received_signal = awgn(modulated_signal, SNR_linear, 'measured');
        % Demodulate the received signal
        demodulated_bits = pamdemod(received_signal, M);
        % Calculate the Bit Error Rate (BER)
        bit_errors = sum(demodulated_bits ~= tx_bits);
```

```matlab
        bit_errors = sum(demodulated_bits ~= tx_bits);
        BER_results(idx, j) = bit_errors / num_bits;
    end

    % For the selected SNR value, plot the modulated and demodulated signals
    if plot_SNR_dB == 10
        received_signal_plot = awgn(modulated_signal, plot_SNR_linear, 'measured');
        demodulated_bits_plot = pamdemod(received_signal_plot, M);
    end
end

% Plot SNR vs. BER for different M values
figure;
semilogy(SNR_dB_range, BER_results(1,:), '-o', 'DisplayName', ['M = ' num2str(M_vals(1))]);
hold on;
semilogy(SNR_dB_range, BER_results(2,:), '-s', 'DisplayName', ['M = ' num2str(M_vals(2))]);
semilogy(SNR_dB_range, BER_results(3,:), '-^', 'DisplayName', ['M = ' num2str(M_vals(3))]);
grid on;
xlabel('Signal to Noise Ratio (SNR) in dB');
ylabel('Bit Error Rate (BER)');
title('BER vs SNR using PAM for Different M Values');
legend('Location', 'best');
hold off;

% Plot the pulse signal before transmission and after demodulation for M = 4
figure;
subplot(2,1,1);
stem(modulated_signal(1:50), 'filled');
title('Modulated Signal (First 50 Samples)');
xlabel('Sample Index');
ylabel('Amplitude');
```

```
subplot(2,1,2);
stem(demodulated_bits_plot(1:50), 'filled');
title('Demodulated Signal (First 50 Samples)');
xlabel('Sample Index');
ylabel('Amplitude');

% Optional: Plot the received signal after AWGN
figure;
subplot(2,1,1);
stem(received_signal_plot(1:50), 'filled');
title('Received Signal after AWGN (First 50 Samples)');
xlabel('Sample Index');
ylabel('Amplitude');

subplot(2,1,2);
stem(demodulated_bits_plot(1:50), 'filled');
title('Demodulated Signal (First 50 Samples)');
xlabel('Sample Index');
ylabel('Amplitude');
```

Figure 11: MATLAB Code Implementation

And here is the resulting plot:

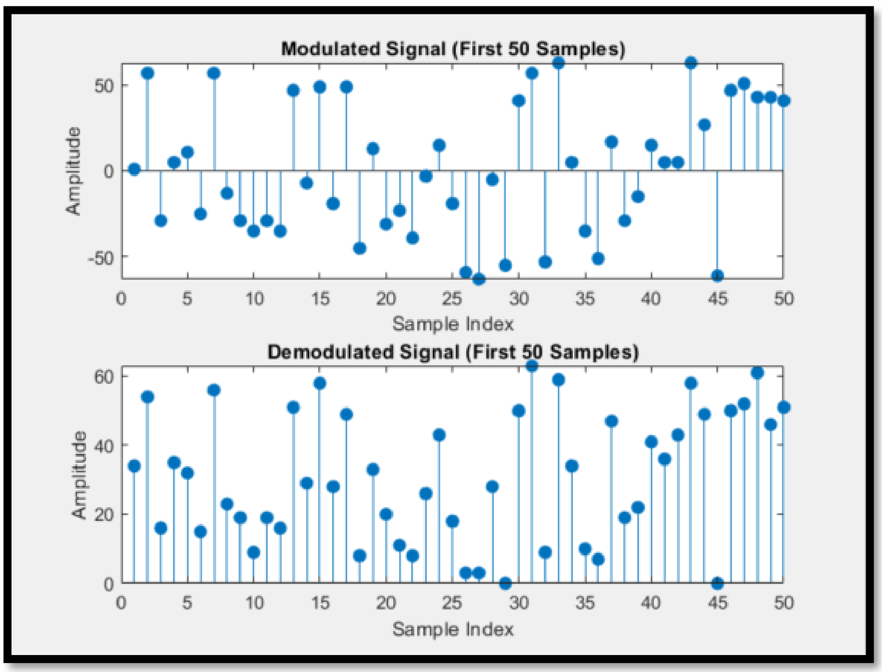

Figure 12: The Original Modulated Signal Pulse and Demodulated Signal Pulse

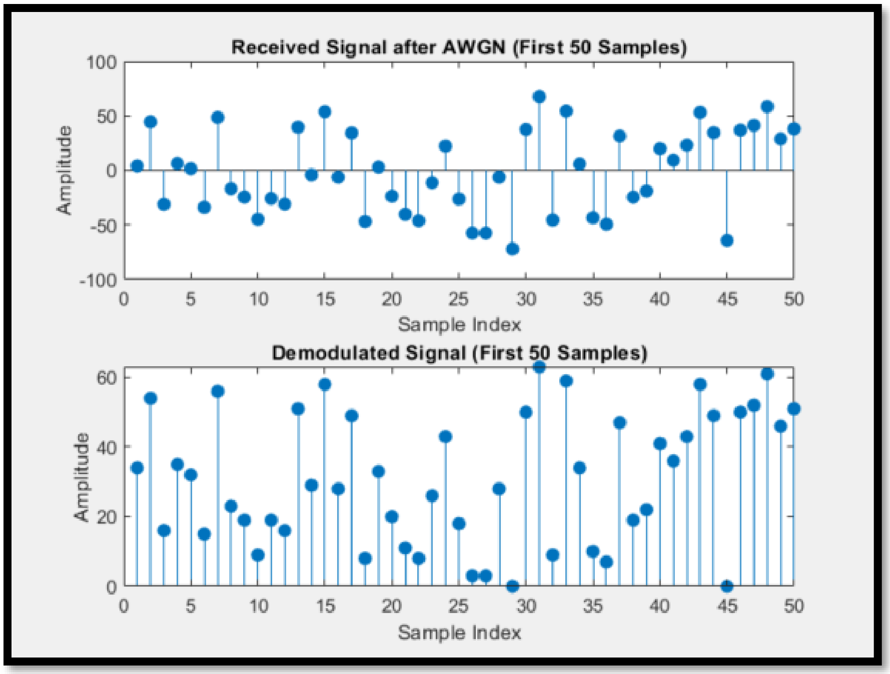

Figure 13: The Received Signal Pulse only after the AWGN channel and the Demodulated Signal Pulse

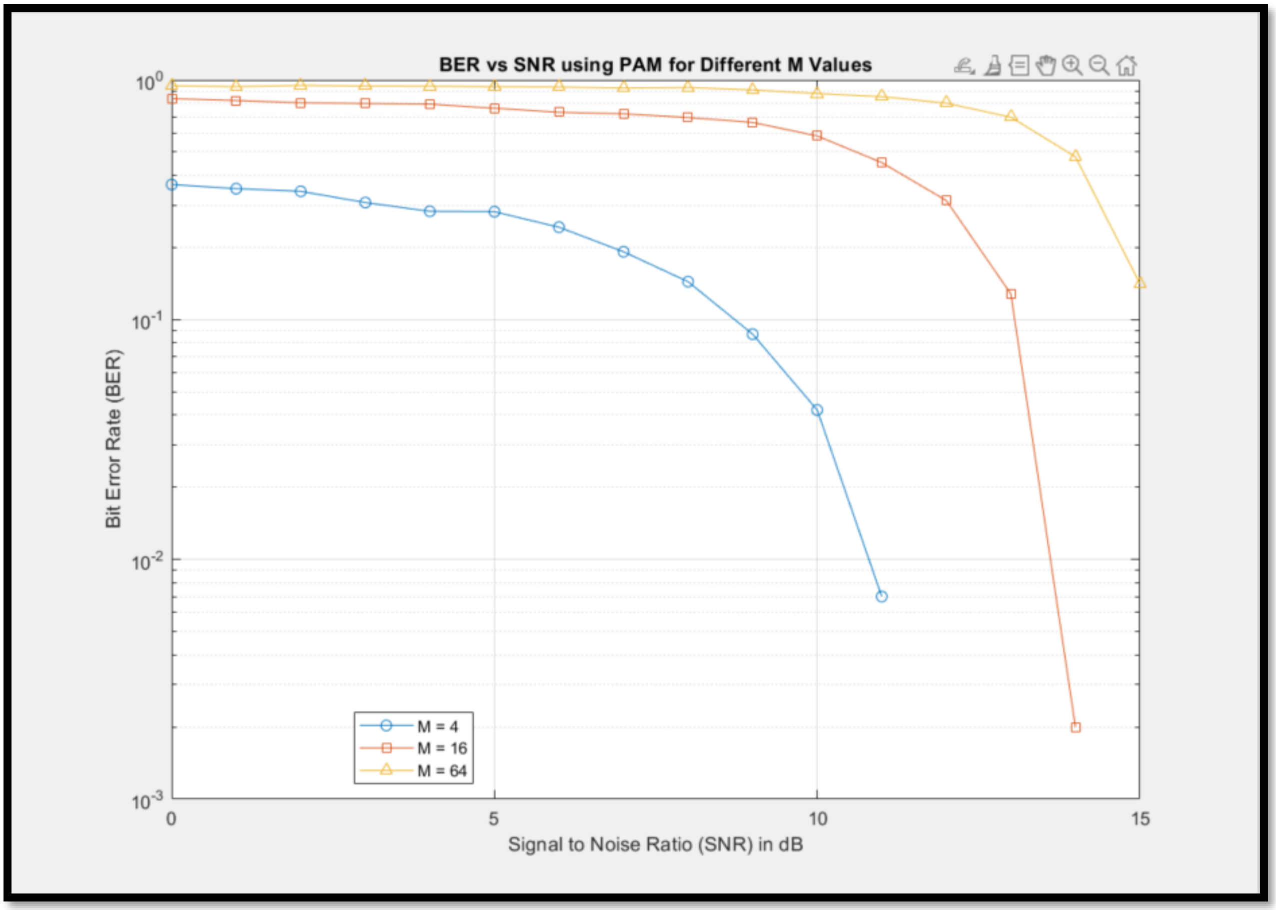

Figure 14: The BER to SNR Graph Project Output

As it showed the same results as the Simulink.

9. Conclusion:

In this experiment, we explored signal modulation using the Pulse amplitude modulation. This process involves segmenting input signals into blocks and normalizing the resulting signal. Notably, using more levels (M levels) and achieving higher Signal-to-Noise Ratio (SNR) ratios can significantly impact communication quality, this can be implemented using a matched filter.

A matched filter is a signal processing technique that optimally detects signals in the presence of noise. It's designed to maximize the Signal-to-Noise Ratio (SNR), improving signal detection accuracy. By aligning the filter's response with the shape of the expected signal, it enhances the ability to extract weak signals from noisy backgrounds, this will greatly enhance the channel output resulting in far less BERs for the same M-level Modulator used.

10. References:

1. What is pulse amplitude modulation? (2023, August 11). Collimator. https://www.collimator.ai/reference-guides/what-is-pulse-amplitude-modulation

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3. Sakib, Meer & Liboiron-Ladouceur, Odile. (2013). A Study of Error Correction Codes for PAM Signals in Data Center Applications. Photonics Technology Letters, IEEE. 25. 2274-2277. 10.1109/LPT.2013.2286105.

4. Bit Error Rate and Signal to Noise Ratio Performance Evaluation of OFDM System with QPSK and QAM M-array Modulation Scheme in Rayleigh, Rician and AWGN Channel Using MATLAB/Simulink

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